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GLOBAL EINSTEIN -AND- VAN CONG RELATIONS, AND NEW OPTICAL-ELECRICAL-THERMOELECTRIC LAWS IN N(P)-TYPE DEGENERATE, COMPENSATED AND VISCOUS InAs(1-x)P(x)-CRYSTALLINE ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW, CONDUCTIVITY MODEL, AND ESPECIALLY ACCURATE FERMI ENERGY EXPRESSION (34)

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ABSTRACT

In degenerate, compensated and viscous $\mathbf{n^+(p^+) - p(n) - X(x) \equiv InAs(1-x)P(x)}$- crystalline alloy, $0 \le x \le 1$, the Global Einstein -and- Van Cong relations, as those given in Equations (24, 31), and various optical-electrical-thermoelectric laws, enhanced by: the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP), as those defined in Eq. (15), our static dielectric constant law given in Equations (1a, 1b), our conductivity model in Eq. (18), and especially our accurate Fermi energy expression in Eq. (11), are now investigated, by basing on the same physical model and the mathematical treatment method, as those used in our recent works.[1-5] Their numerical results are obtained and given in Tables (3-13), suggesting an equivalence between degeneracy-compensation-viscosity concept, given in this X(x)- crystalline alloy. Here, the well-known "local" Einstein relation, being valid only at the lowest degeneracy, lowest viscosity and lowest compensation, is now globalized by our "Global" Einstein -and- Van Cong relations, valid at

any degeneracy, viscosity and compensation, as showed in Equations (24, 31).

KEYWORDS: Conductivity, Mobility, Viscosity coefficient, Diffusion coefficient, Activation energy, Fermi energy.

INTRODUCTION

In the $\mathbf{n^+(p^+) - X(x) \equiv InAs_{1-x}P_x}$-crystalline alloy, $0 \leq x \leq 1$, x being the concentration, the Global Einstein -and- Van Cong relations, as those given in Equations (24, 31), and various optical-electrical-thermoelectric laws, enhanced by : (i) the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP) as those given in Eq. (15), (ii) our static dielectric constant law given in Equations (1a, 1b), (iii) our optical-and-electrical conductivity models obtained in Eq. (18, 20a) and especially (iv) our accurate Fermi energy expression in Eq. (11), with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$[9], affecting strongly all the expressions of optical and electrical coefficients, are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.[1-5] It should be noted here that for x=0, the present obtained numerical results are reduced to those given in the n(p)-type degenerate **InAs –crystal.**[1,6-18]

Then, some important remarks can be summarized as follows.

(1) As observed in Equations (3, 5, 6a, 6b), the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N^{EBT}_{CDn(CDp)}$, being obtained with a precision of the order of $\mathbf{2.76 \times 10^{-7}}$, as given in our recent work.[3] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^*.

(3) Finally, for particular physical conditions, as those given in Eq. (15), one observes that the optical conductivity σ_O has a same form with that of the electrical conductivity, σ_E, as those given in Eq. (20a), but $\sigma_O > \sigma_E$ since $m_r(x) < m_{c(v)}(x)$, $m_{c(v)}$ and m_r, being the unperturbed **reduced** effective electron (hole) mass in conduction (valence) bands and the **reduced** relative carrier mass, respectively. Therefore, by basing on those $\sigma_{O[E]}$-expressions, the Global Einstein -and- Van Cong relations defined in Equations (24, 31) and various optical-electrical-thermoelectric laws are determined. Finally, the numerical results of the corresponding optical-electrical-thermoelectric coefficients are obtained and reported in Tables (3-13), suggesting an equivalence between degeneracy-compensation-viscosity concept, given in this X(x)- crystalline alloy.

Our Static Dielectric Constant Law and Generalized Mott Criterium in the Metal-Insulator Transition (MIT)

First of all, in the degenerate $n^{+}(p^{+}) - \mathbf{X}(\mathbf{x})$- crystalline alloy, at T=0 K[1-5], we denote : the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $\mathbf{r}_{\mathbf{do(ao)}} = r_{\mathbf{As(In)}}$, respectively, the effective averaged numbers of equivalent conduction (valence)-bands by: $g_{c(v)}$, the unperturbed **reduced** effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)$, the unperturbed effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x) \times m_o$, m_o being the free electron mass, the **reduced** relative carrier mass by: $m_r(x) \equiv \frac{m_c(x) \times m_v(x)}{m_c(x)+m_v(x)} < m_{c(v)}(x)$, the unperturbed static dielectric constant by: $\varepsilon_o(x)$, and finally the intrinsic band gap by: $E_{go}(x)$, for a given x, as those given in **Table 1 in Appendix 1**.

Here, the reduced effective carrier mass $m^{*}_{n(p)}(x)$ is equal to $m_{c(v)}(x)$ or $m_r(x)$. Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as: $E_{do(ao)}(x) = \frac{13600 \times [m_{c(v)}(x)]}{[\varepsilon_o(x)]^2}$ meV, and then, the isothermal bulk modulus, by : $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right) \times \left(r_{do(ao)}\right)^3}$.

Our Static Dielectric Constant Law $\left[m^{*}_{n(p)}(x) \equiv m_{c(v)}(x)\right]$

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective dielectric constant $\varepsilon(r_{d(a)}, x)$, are developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volumes : $V = (4\pi/3) \times \left(r_{d(a)}\right)^3$ and $V_{do(ao)} = (4\pi/3) \times \left(r_{do(ao)}\right)^3$, corresponding to the pressures : p, $p_o = 0$, and to the deformation potential energies (or the strain energies) : α, $\alpha_o = 0$. Further, the two important equations, used to determine the α -variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by : $\frac{dp}{dV} = -\frac{B_{do(ao)}(x)}{V}$ and $p = -\frac{d\alpha}{dV}$, which lead to : $\frac{d}{dV}\left(\frac{d\alpha}{dV}\right) = \frac{B_{do(ao)}(x)}{V}$. After integration, one obtains :

$\left[\Delta\alpha(r_{d(a)},x)\right]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) leads to an increase (decrease) in the energy gap $E_{gn(gp)}(r_{d(a)},x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)},x)$ in absolute values, obtained in the effective Bohr model. Those are represented respectively by : $\pm\left[\Delta\alpha(r_{d(a)},x)\right]_{n(p)}$, as :

$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = + \left[\Delta\alpha(r_{d(a)},x)\right]_{n(p)}$,

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = - \left[\Delta\alpha(r_{d(a)},x)\right]_{n(p)}$.

Therefore, the relative dielectric constant $\varepsilon(r_{d(a)},x)$ and the energy band gap $E_{gn(gp)}(r_{d(a)},x)$ are given as follows.

First, for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new $\boldsymbol{\varepsilon(r_{d(a)},x)}$-law**,

$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0,$ (1a)

according to the increase of both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with increasing $r_{d(a)}$ and for a given x. Secondly, for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_o(x)}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being also a **new $\boldsymbol{\varepsilon(r_{d(a)},x)}$-law**,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \qquad (1b)$$

corresponding to the decrease of both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with decreasing $r_{d(a)}$ and for a given x.

It should be noted that, in the following, all the optical and electrical properties strongly depend on this **new $\boldsymbol{\varepsilon}(\mathbf{r}_{\mathbf{d(a)}}, \mathbf{x})$-law.**

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)}, x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}, x) \equiv \frac{\varepsilon(r_{d(a)}, x) \times \hbar^2}{m^*_{n(p)}(x) \times m_o \times q^2} = 0.53 \times 10^{-8}\ \mathrm{cm} \times \frac{\varepsilon(r_{d(a)}, x)}{m^*_{n(p)}(x)}, \qquad (2)$$

where q=e, according to an electron charge equal to : -e.

Generalized Mott Criterium in the MIT $[m^*_{n(p)}(x) \equiv m_{c(v)}(x)]$

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(CDp)}(r_{d(a)}, x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as[3] :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = M_{n(p)},\ \ M_{n(p)} = 0.25, \qquad (3)$$

depending thus on our **new $\boldsymbol{\varepsilon}(\mathbf{r}_{\mathbf{d(a)}}, \mathbf{x})$-law**.

This result can be explained by using the definition of the reduced effective Wigner-Seitz (**WS**) radius $\mathbf{r}_{\mathbf{sn(sp),M}}$, in the Mott's criterium, being characteristic of interactions, by:

$$\mathbf{r}_{\mathbf{sn(sp),M}}(N = N_{CDn(CDp)}(r_{d(a)}, x), r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)}, x)}\right)^{\frac{1}{3}} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)}, x)} = \left(\frac{\mathbf{3}}{\mathbf{4\pi}}\right)^{\frac{\mathbf{1}}{\mathbf{3}}} \times \mathbf{4} \cong 2.4814, \qquad (4)$$

for any $(r_{d(a)}, x)$-values. Then, from Eq. (4), one also has : ≅

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = \mathbf{1/4} = \mathbf{0.25} = M_{n(p)}, \qquad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\boldsymbol{\mathcal{H}}_{\mathbf{n(p)}} = \mathbf{0.47137}$, as those given in our previous work[3], we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction**

(valence)-band tail , $\mathbf{N^{EBT}_{CDn(CDp)}}$, with a precision of the order of $\mathbf{2.76\times 10^{-7}}$, respectively.[3] So,

$$N_{CDn(NDp)}(r_{d(a)}, x) \cong N^{EBT}_{CDn(CDp)}(r_{d(a)}, x). \qquad (6a)$$

It shoud be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen such that those of $N_{CDn(CDp)}$ and $N^{EBT}_{CDn(CDp)}$ are found to be in good agreement with their experimental results.

Therefore, the effective density of electrons (holes) given in parabolic conduction (valence) bands, N^*, can be defined, as that given in compensated materials:

$$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) \cong N - N^{EBT}_{CDn(CDp)}(r_{d(a)}, x) \geq 0. \qquad (6b)$$

One notes here that, with increasing $r_{d(a)}$ and for given x and N, $N_{CDn(NDp)}(r_{d(a)})$ increases, as observed in Ref.[3], and therefore, $N^*(r_{d(a)})$ decreases, according the **increasing compensation**. Numerically, we will show that for given x, N and T, and for such the increasing compensation, the viscosity coefficient, $\mathbb{V}_{O[E]}(r_{d(a)})$, defined in next Eq. (22b), increases, suggesting an **equivalence between the compensation- viscosity concept.**

In summary, as observed in our previous paper[3], for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}(r_{d(a)}, x)$, $N_{CDn(NDp)}(r_{d(a)}, x)$ and $N^{EBT}_{CDn(CDp)}(r_{d(a)}, x)$ increase, affecting strongly all the optical, electrical, and thermoelectric coefficients, as those observed in following Sections.

Physical model

In the degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$ -crystalline alloy, the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^*, is now defined by:

$$\gamma \times r_{sn(sp)}(N^*, r_{d(a)}, x) \equiv \frac{k^{-1}_{Fn(Fp)}}{a_{Bn(Bp)}} < 1 \quad , \quad r_{sn(sp)}(N^*, r_{d(a)}, x) \equiv \left(\frac{3g_{c(v)}}{4\pi N^*}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)}, x)} \quad ,$$

being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{g_{c(v)}}\right)^{\frac{1}{3}}$ is the Fermi wave number.

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to $k_{Fn(kp)}$ is defined by:

$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} <$

1, (7)

being valid at any N^*.

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to:

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \qquad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(snWS)}$ is respectively reduced to:

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N^*)]}{dr_{sn(sp)}}\right), \qquad (9a)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\mathbb{U}_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1 \quad , \quad \mathbb{U}_{n(p)}(N^*, r_{d(a)}, x) \equiv \frac{\sqrt{2\pi \times (\frac{N^*}{g_{c(v)}})}}{\varepsilon(r_{d(a)}, x)} \times$$

$q^2 k_{sn(sp)}^{-1/2}$, (9b)

which gives: $A_{n(p)}(N^*, r_{d(a)}, x) = \frac{E_{Fno(Fpo)}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)}$, $E_{Fno(Fpo)}(N^*, r_{d(a)}, x) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{n(p)}^*(x) \times m_o}$.

Here, one remarks that: (i) the generalized Thomas-Fermi energy $\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)$ can thus be approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{d(a)}, x)}$, C being a constant, and (ii) $\mathbb{U}_{n(p)}(r_{d(a)})$ increases with increasing $r_{d(a)}$ and for given x and N, since $\varepsilon(r_{d(a)})$ decreases, as given in Ref.[3]

Band gap narrowing (BGN)

First, the BGN is found to be given by[1]:

$$\Delta E_{gn(gp)}(N^*, r_{d(a)}, x) \simeq$$

$$a_1 + \frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)} \times N_r^{\frac{1}{3}} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times \left[-E_{CE}(r_{sn(sp)})\right] \times r_{sn(sp)}\right) + a_3 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{5}{4}} \times \sqrt{\frac{m_{v(c)}}{m^*_{n(p)}(x)}} \times N_r^{\frac{1}{4}} + 2a_4 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{1}{2}} \times N_r^{\frac{1}{2}} + 2a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r = \frac{N^*}{9.999 \times 10^{17}\,cm^{-3}}, \quad (10a)$$

Here, for $\Delta E_{gn;N}(N^*, r_d, x)$, one has: $a_1 = 3.8 \times 10^{-5}(eV)$, $a_2 = 6.5 \times 10^{-6}(eV)$, $a_3 = 2.8 \times 10^{-5}(eV)$, $a_4 = 5.597 \times 10^{-5}(eV)$, and $a_5 = 8.1 \times 10^{-6}(eV)$, and for $\Delta E_{gp;N}(N^*, r_a, x)$, one has: $a_1 = 3.15 \times 10^{-5}(eV)$, $a_2 = 5.41 \times 10^{-6}(eV)$, $a_3 = 2.32 \times 10^{-5}(eV)$, $a_4 = 4.12 \times 10^{-5}(eV)$, and $a_5 = 9.8 \times 10^{-7}(eV)$.

Therefore, at T=0 K and $N^* = 0$, and for any x and $r_{d(a)}$, one obtains: $\Delta E_{gn(gp)} = 0$, in good agreement with the metal-insulator transition (MIT).

Secondly, the temperature dependent BGN is proposed by[1]:

$$\Delta E_{gn(gp)}(T, x) = 10^{-6} T^2 \times \left[\frac{7.205 \times x}{T + 94} + \frac{5.405 \times (1 - x)}{T + 204}\right] (eV). \quad (10b)$$

Fermi Energy and Fermi-Dirac Distribution Function

Accurate Fermi Energy Expression

Here, for presentation simplicity, we change the sign of all various parameters, given in the degenerate $p^+ - \mathbf{X(x)}$-crystalline alloy, in order to obtain the same one, as given in the degenerate $n^+ - \mathbf{X(x)}$ - crystalline alloy, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N^*, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N^*, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N^*, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N^*, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N^*, r_{d(a)}, x, T)$, obtained in our previous paper[9], obtained with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)},\ A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

where u is the reduced electron density, $u(N^*, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T,x)}$, $N_{c(v)}(T,x) = 2g_{c(v)} \times \left(\frac{m^*_{n(p)}(x)\times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ (cm^{-3}), $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as **$u \longrightarrow 0$, the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2\times m^*_{n(p)}(x)\times m_o}$ as **$u \longrightarrow \infty$, the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, as $T \to 0$ K, since $u^{-1} \to 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2\times m^*_{n(p)}(x)\times m_o}$, proportional to $(N^*)^{2/3}$, noting that, for a given N^*, $E_{Fno(Fpo)}\left(m^*_{n(p)}(x) = m_r(x)\right) > E_{Fno(Fpo)}\left(m^*_{n(p)}(x) = m_{c(v)}(x)\right)$ since $m_r(x) < m_{c(v)}(x)$ for given x. Further, at T=0 K and $N^* = 0$, being the physical conditions, given for the metal-insulator transition (**MIT**).

In the following, it should be noted that all the optical and electrical-and-thermoelectric properties strongly depend on such an accurate expression of $\xi_{n(p)}(N^*, r_{d(a)}, x, T)$.[9]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

Thus, the average of E^p, calculated using the FDDF-method, as developed in our previous works[1,6] is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \qquad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}.$$

Further, it should be noted that, at T=0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p(E_{Fn(Fp)}) \equiv 1 + E_{Fn(Fp)}^{-p} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E_{Fn(Fp)}^{-\beta} \times I_{\beta}$, where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{(e^{\gamma/2}+e^{-\gamma/2})^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains:

$I_{2n} = 2\int_{0}^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma$.

Now, using the identity $(1 + e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty}(-1)^{s+1}s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_{0}^{\infty} t^{2n}e^{-t}\,dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. From the above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case ($\xi_{n(p)} > 1$) the following ratio:

$G_{p>1}\left(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}} < 1\right) \equiv \frac{\langle E^p \rangle_{FDDF}}{E_{Fn(Fp)}^p} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\ldots(p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n}$,

which can thus be approximated by:

$$G_{p>1}(y < 1) \simeq 1 + p(p-1) \times |B_2| \times y^2,\ B_2 = \frac{1}{6}. \qquad (12)$$

Then, some usual results of $G_{p>1}(y < 1)$ are given in the **Table 2 in Appendix 1**, suggesting that, with increasing T (or decreasing T) and for given (N, r_d, x), since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases).

Optical-and-Electrical Properties

Optico-Electrical Phenomenon (O-EP) and Electro-Optical Phenomenon (E-OP)

In the degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$-crystalline alloy, the following relations hold:

(i) in the **E-OP**, the reduced band gap is defined by:

$$E_{gn2(gp2)} \equiv E_{gn(gp)} - \Delta E_{gn(gp)}(N^*, r_{d(a)}, x) - \Delta E_{gn(gp)}(T, x), \qquad (13)$$

where the intrinsic band gap $E_{gn(gp)}$ is defined in Equations (1a, 1b), $\Delta E_{gn(gp)}(N^*, r_{d(a)}, x)$ and $\Delta E_{gn(gp)}(T, x)$ are respectively determined in Equations (10a, 10b), and

(ii) in the (**O-EP**), the photon energy is defined by: $E \equiv \hbar\omega$, and the optical band gap, as: $E_{gn1(gp1)} \equiv E_{gn2(gp2)} + E_{Fn(Fp)}$.

Therefore, for $E \geq E_{gn1(gp1)}(E_{gn2(gp2)})$, the effective photon energy E^* is found to be given by:

$$E^* \equiv E - E_{gn1(gp1)}(E_{gn2(gp2)}) \geq 0. \qquad (14)$$

From the above Equations, one notes that: $E^* \equiv [E - E_{gn1(gp1)}] = E_{Fn(Fp)}$, given in the O-EP, if $E = [E_{gn1(gp1)} + E_{Fn(Fp)}] \equiv E_{gn(gp)O}$ and $m^*_{n(p)}(x) = m_r(x)$, and $E^* \equiv E - E_{gn2(gp2)} = E_{Fn(Fp)}$, given in the E-OP, if $E = [E_{gn2(gp2)} + E_{Fn(Fp)}] \equiv E_{gn(gp)E}$ and $m^*_{n(p)}(x) = m_{c(v)}(x)$, noting that $E_{Fn(Fp)}(m_r(x)) > E_{Fn(Fp)}(m_{c(v)}(x))$, since $m_r(x) < m_{c(v)}(x)$, for a given x. (15)

Eq. (15) thus shows that, in both O-EP and E-OP, the Fermi energy-level penetrations into conduction (valence)-bands, observed in the $n^+(p^+) -$ type degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$-crystalline alloy, $E_{Fn(Fp)}$, are well defined.

Optical Coefficients

The optical properties for any medium, defined in the O-EP and E-OP, respectively, according to: $[m^*_{n(p)} \equiv m_r(x)[m_{c(v)}(x)]]$, can be described by the complex refraction: $\mathbb{N}_{O[E]} \equiv n_{O[E]} - i\kappa_{O[E]}$, $n_{O[E]}$ and $\kappa_{O[E]}$ being the refraction index and the extinction coefficient, the complex dielectric function: $\mathcal{E}_{O[E]} = \varepsilon_{1\,O[E]} - i\varepsilon_{2\,O[E]}$, where $i^2 = -1$, and $\mathcal{E}_{O[E]} = \mathbb{N}_{O[E]}{}^2$. Further, if denoting respectively the normal-incidence reflectance and the optical absorption by $R_{O[E]}$ and $\propto_{O[E]}$, and the effective joint parabolic conduction (parabolic valence)-band density of states by:

$$JDOS_{n(p)\,O[E]}(E, N^*, r_{d(a)}, x, T) \equiv$$

$$\frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{3/2} \times \sqrt{E_{Fno(Fpo)}(N^*)} \times \left[\frac{E - E_{gn1(gp1)}(E_{gn2(gp2)})}{E - [E_{gn1(gp1)}(E_{gn2(gp2)}) + E_{Fn(Fp)} - E_{Fno(Fpo)}]}\right]^2 ,$$ and

$F_{O[E]}(E) \equiv \frac{\hbar q^2 \times |v(E)|^2}{n_{O[E]}(E) \times cE \times \varepsilon(r_{d(a)}, x) \times \varepsilon_{free\ space}}$, one gets [2]:

$$\propto_{O[E]}(E) = JDOS_{n(p)\,O[E]}(E) \times F_{O[E]}(E) = \frac{E \times \varepsilon_{2\,O[E]}(E)}{\hbar c\, n_{O[E]}(E)} = \frac{2E \times \kappa_{O[E]}(E)}{\hbar c} = \frac{4\pi\sigma_{O[E]}(E)}{c n_{O[E]}(E) \times \varepsilon(r_{d(a)}, x) \times \varepsilon_{free\ space}},$$

$\varepsilon_{1\,O[E]}(E) \equiv {n_{O[E]}}^2 - {\kappa_{O[E]}}^2$, $\varepsilon_{2\,O[E]}(E) \equiv 2\kappa_{O[E]} n_{O[E]}$, and $R_{O[E]}(E) \equiv \frac{[n_{O[E]}-1]^2 + {\kappa_{O[E]}}^2}{[n_{O[E]}+1]^2 + {\kappa_{O[E]}}^2}$. (16a)

Here, one remarks that at the MIT-conditions : T=0K and $N^* = 0$, both $E_{gn1(gp1)}(E_{gn2(gp2)})$=$E_{gn(gp)}$, according to :

$\left[\frac{E-E_{gn(gp)}}{E-E_{gn(gp)}}\right]^2 = \frac{0}{0}$ for E=$E_{gn(gp)}$, and then,

$\left[\frac{E-E_{gn(gp)}}{E-E_{gn(gp)}}\right]^2 = 1$ for $E \gtrsim E_{gn(gp)}$, so that, in such the MIT-conditions,

$JDOS_{n(p)\,O[E]}(E, N^*, r_{d(a)}, x, T) \equiv \frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{\frac{3}{2}} \times \sqrt{E_{Fno(Fpo)}(N^* = 0)} \cong 0,$ for $E \gtrsim E_{gn(gp)}$, which is largely verified since $N_{CDn(NDp)}(r_{d(a)}, x) \cong N^{EBT}_{CDn(CDp)}(r_{d(a)}, x)$ or $E_{gn2(gp2)}(N_{CDn(NDp)}, T = 0K) \cong E_{gn2(gp2)}(N^{EBT}_{CDn(CDp)}, T = 0K) \cong E_{gn(gp)}$, as those given in Equations (6a, 6b). In other words, the critical photon energy can be defined by: E =$E_{gn(gp)}$. Then, Eq. (6a) states that $N_{CDn(CDp)}$, given in parabolic conduction (parabolic valence)-band density of states, is just the density of electrons (holes) localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$, with a precision of the order of $\mathbf{2.76 \times 10^{-7}}$ [3] Therefore, for $E \cong E_{gn(gp)}$, the exponential conduction (valence)-band tail states can be approximated with the same precision as:

$$JDOS^{EBT}_{n(p)O[E]}(E, N^*, r_{d(a)}, x, T) \equiv \frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{\frac{3}{2}} \times \sqrt{E_{Fno(Fpo)}\left(N^* = N_{CDn(NDp)}\right)}. \quad 16b)$$

Here, $\varepsilon_{free\ space} = 8.854187817 \times 10^{-12} \left(\frac{C^2}{N \times m^2}\right)$ is the permittivity of free space, -q (<0) is the charge of the electron, $\left|v_{O[E]}(E)\right|$ denotes the matrix elements of the velocity operator between valence (conduction)-and-conduction (valence) bands, and our approximate expression for the refraction index $n_{O[E]}$ is found to be defined by:

$$n_{O[E]}(E, N^*, r_{d(a)}, x, T) = n_\infty(r_{d(a)}, x) + \sum_{i=1}^{4} \frac{X_i(E_{gn1(gp1)}) \times E + Y_i(E_{gn1(gp1)})}{E^2 - B_i E + C_i}, \quad (17)$$

Going to a constant as $E \to \infty$, since $n(E \to \infty, r_{d(a)}, x) \to n_\infty(r_{d(a)}, x) = \sqrt{\varepsilon(r_{d(a)}, x)} \times \frac{\omega_T}{\omega_L}$, given in the well-known Lyddane-Sachs-Teller relation, in which $\omega_T \simeq 5.1 \times 10^{13}\ s^{-1}$ and $\omega_L \simeq 8.9755 \times 10^{13}\ s^{-1}$ are the transverse (longitudinal) optical phonon frequencies, giving rise to: $n_\infty(r_{d(a)}, x) \simeq \sqrt{\varepsilon(r_{d(a)}, x)} \times 0.568$.

Here, the other parameters are determined by: $X_i(E_{gn1(gp1)}) = \frac{A_i}{Q_i} \times \left[-\frac{B_i^2}{2} + E_{gn1(gp1)}B_i - E_{gn1(gp1)}^2 + C_i\right]$, $Y_i(E_{gn1(gp1)}) = \frac{A_i}{Q_i} \times \left[\frac{B_i \times (E_{gn1(gp1)}^2 + C_i)}{2} - 2E_{gn1(gp1)}C_i\right]$, $Q_i = \frac{\sqrt{4C_i - B_i^2}}{2}$, where, for i=(1, 2, 3, and 4),

$A_i = 4.7314 \times 10^{-4}$, $0.2313655, 0.1117995, 0.0116323$, $B_i = 5.871, 6.154, 9.679$, 13.232, and $C_i = 8.619, 9.784, 23.803, 44.119$.

Now, the optical [electrical] conductivity $\sigma_{O[E]}$ can be defined and expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{n(p)}^*(x) \times m_o}$, k being the wave number, as:

$\sigma_{O[E]}(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\mathbb{U}_{n(p)}}\right)^{\frac{1}{2}}$ $\left(\frac{1}{\Omega \times cm}\right)$, which is thus proportional to ${E_k}^2$,

where $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}\ ohm^{-1}$ and $\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)$ is determined in Eq. (9b).

Then, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E_{Fn(Fp)}^2$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N^*, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N^*, r_{d(a)}, x, T)$ for simplicity. Therefore, for $E_{Fno(Fpo)} > 0$ and T =0K, $G_2(y = 0) = 1$.

Therefore, from above equations (16, 17), if denoting the function $H(N^*, r_{d(a)}, x, T)$ by:

$H(N^*, r_{d(a)}, x, T) =$

$\left[\frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)}, x)] \times \sqrt{A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)}}\right] \times$ $G_2(N^*, r_{d(a)}, x, T)$, which can be approximately expressed in terms of: $E_{Fno(Fpo)}^2(N^*) \times G_2(N^*, r_{d(a)}, x, T) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{1}{4}}} = E_{Fno(Fpo)}^2(N^*) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{1}{4}}} = H(N^*, r_{d(a)}, x, T = 0K)$ for $E_{Fno(Fpo)} > 0$ and T =0K, since as noted in Eq. (9b), $\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)$ is approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{d(a)}, x)}$, C being a constant. Thus, with increasing $r_{d(a)}$ and for given x, T and N, the function $H(r_{d(a)})$ therefore decreases since $\varepsilon(r_{d(a)})$ decreases, as noted in Ref.[3]

Then, our optical [electrical] conductivity models, defined in the O-EP and E-OP, respectively, for a simple representation, can thus be assumed to be as:

$$\sigma_O(E, N^*, r_{d(a)}, x, T) =$$

$$\frac{q^2}{\pi\times\hbar} \times H(N^*, r_{d(a)}, x, T) \times \left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \left(\frac{1}{\Omega\times cm}\right), \text{ and}$$

$$\sigma_E(E, N^*, r_{d(a)}, x, T) =$$

$$\frac{q^2}{\pi\times\hbar} \times H(N^*, r_{d(a)}, x, T) \times \left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \left(\frac{1}{\Omega\times cm}\right). \qquad (18)$$

It should be noted here that:

(i) $\sigma_{O[E]}(E = E_{gn1(gp1)}[E_{gn2(gp2)}]) = 0$, and $\sigma_{O[E]}(E \to \infty) \to$ Constant for given $(N, r_{d(a)}, x, T)$ −physical conditions, and

(ii) as $T \to 0$ K and $N^* = 0$ [or $E_{Fno(Fpo)}(N^*) = 0$], according to: $H(N^*, r_{d(a)}, x, T) = 0$, and for a given E, $[E - E_{gn1(gp1)}] = [E - E_{gn(gp)}]$=Constant, then from Equations (16-18), $n_{O[E]}$ (E)= Constant, $\sigma_{O[E]}(E) = 0$, $\kappa_{O-EP[E-OP]}(E) = 0$, $\varepsilon_{1\,O[E]}(E) = (n_\infty)^2 =$ Constant, $\varepsilon_2(E) = 0$, and $\propto_{O[E]}$ $(E) = 0$.

This result (18) should be new, in comparison with that, obtained from an improved Forouhi-Bloomer parameterization, as given in our previous work.[2]

Using Equations (16-18), one obtains all the analytical results as:

$$\frac{|v(E)|^2}{E} = \frac{8\pi^2\hbar}{(2m_r)^{\frac{3}{2}}\times\sqrt{\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)}} \times \left[\frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)}, x)]\right] \times$$

$$G_2(N^*, r_{d(a)}, x, T), \qquad (19a)$$

$$\kappa_O(E) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)}, x)\times\varepsilon_{free\ space}\times E} \times H(N^*, r_{d(a)}, x, T) \times$$

$$\left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \quad \text{and}$$

$$\kappa_E(E) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)}, x)\times\varepsilon_{free\ space}\times E} \times H(N^*, r_{d(a)}, x, T) \times$$

$$\left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2, (19b)$$

which gives: $\kappa_{O[E]}(E = E_{gn1(gp1)}[E_{gn2(gp2)}]) = 0$, and $\kappa_{O[E]}(E \to \infty) \to 0$, as those given in Ref.[2],

$$\varepsilon_{2\,O}(E) = \frac{4q^2}{\varepsilon(r_{d(a)}, x)\times\varepsilon_{free\ space}\times E} \times H(N^*, r_{d(a)}, x, T) \times \left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \text{ and}$$

$$\varepsilon_{2\,E}(E) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{free\ space}\times E} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2, \quad (19c)$$

which gives: $\varepsilon_{2O-EP[2E-OP]}\left(E = E_{gn1(gp1)}[E_{gn2(gp2)}]\right) = 0$, and $\varepsilon_{2O-EP[2E-OP]}(E \to \infty) \to 0$, as those given in Ref.[2],

$\propto_O (E) =$

$$\frac{4q^2}{\hbar cn(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{free\ space}} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \left(\frac{1}{cm}\right) \text{ and}$$

$\propto_E (E) =$

$$\frac{4q^2}{\hbar cn(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{free\ space}} \times H\left(N^*, r_{d(a)}, x, T\right) \times$$

$$\left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \left(\frac{1}{cm}\right), \quad (19d)$$

which gives: $\propto_{O[E]} \left(E = E_{gn1(gp1)}[E_{gn2(gp2)}]\right) = 0$, and $\propto_{O[E]} (E \to \infty) \to \text{Constant}$.

Furthermore, from Equations (16, 17, 19b), we can also determine $\varepsilon_{1\,O[E]}(E)$ and $R_{O[E]}(E)$. Now, from Equations (18, 19b, 19c, 19d), using Eq. (15) as $E \equiv E_{gn(gp)O[E]}$, one obtains respectively, as:

$\sigma_O\left(N^*, r_{d(a)}, x, T\right) = \frac{q^2}{\pi\times\hbar} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{\Omega\times cm}\right) =$

$\frac{q^2}{\pi\times\hbar} \times H\left(N^*, r_{d(a)}, x, T = 0K\right) \left(\frac{1}{\Omega\times cm}\right)$, as noted above, for $E_{Fn(Fp)} > 0$ and T =0K, which has the same form as discussed in Eq. (15) by:

$\sigma_E\left(N^*, r_{d(a)}, x, T\right) = \frac{q^2}{\pi\times\hbar} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{\Omega\times cm}\right) =$

$\frac{q^2}{\pi\times\hbar} \times H\left(N^*, r_{d(a)}, x, T = 0K\right) \left(\frac{1}{\Omega\times cm}\right)$, (20a)

noting here that for given physical conditions $\left(N^*, r_{d(a)}, x, T\right)$ we obtain: $\sigma_O > \sigma_E$ since $m_r(x) < m_{c(v)}(x)$,

$\kappa_O\left(N^*, r_{d(a)}, x, T\right) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{free\ space}\times(E_{gn1(gp1)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times$

$\left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$ and

$\kappa_E\left(N^*, r_{d(a)}, x, T\right) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{free\ space}\times(E_{gn2(gp2)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times$

$\left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$, (20b)

$$\varepsilon_{2\,\mathrm{O}}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn1(gp1)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$$

and

$$\varepsilon_{2\,\mathrm{E}}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn2(gp2)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2,$$

(20c) $\propto_{\mathrm{O}}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{cm}\right)$ and

$$\propto_{\mathrm{E}}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{cm}\right). \quad (20d)$$

Further, from Equations (16, 17, 20b), we can also determine $\varepsilon_{1\,\mathrm{O[E]}}(E)$ and $R_{\mathrm{O[E]}}(E)$.

Going back to **Eq. (20a)**, one remarks that at T=0K the function $\sigma_{\mathrm{O[E]}}\left(N^*, r_{d(a)}, x, T = 0K\right)$ can thus be approximately expressed in terms of $E^2_{Fno(Fpo)}\left(N^*, r_{d(a)}, x\right) \times \frac{\sqrt{\varepsilon(r_{d(a)},x)}}{(N^*)^{\frac{1}{4}}}$, being proportional to: $\sqrt{\varepsilon(r_{d(a)},\ x)} \times (N^*)^{\frac{13}{12}}$, for $E_{Fno(Fpo)} > 0$ and T =0K.

Then, from Equations (3, 6a, 6b), it should be noted that the metal-insulator transition (MIT) occurs as T=0K and $N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) \cong N - N^{EBT}_{CDn(CDp)}\left(r_{d(a)}, x\right) = 0$, according, for $E \geq E_{gn(gp)}$, to : $E_{Fno(Fpo)}(N^* = 0) \equiv \frac{\hbar^2\times k^2_{Fn(Fp)}(N^*)}{2\times m^*_{n(p)}(x)\times m_o} = 0$, $\kappa^{MIT}_{\mathrm{O[E]}}(E, N^* = 0) = 0$, $\varepsilon^{MIT}_{2\,\mathrm{O[E]}}(E, N^* = 0) = 0$, $\sigma^{MIT}_{\mathrm{O[E]}}(E, N^* = 0) = 0$, and $\propto^{MIT}_{\mathrm{O[E]}}\ (E, N^* = 0) =0$, since $\sigma_{\mathrm{E[O]}}(E, N^* = 0)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^* = 0)^{\frac{4}{3}} = 0$. However, for such the same physical conditions: T=0 K, $N^* = 0$ and $E = E_{gn(gp)}$, we obtain other numerical results such as: $n^{N-MIT}_{\mathrm{O[E]}}(N^* = 0, E) \neq 0$, $\varepsilon^{N-MIT}_{1\,\mathrm{O[E]}}(N^* = 0, E) \neq 0$ and $R^{N-MIT}_{\mathrm{O[E]}}(N^* = 0, E) \neq 0$, according to the non-MIT (N-MIT), as showed in **Tables 3, reported in Appendix 1**. These Tables also state that, at T=0 K and $N^* = 0$, and for $E = E_{gn(gp)}$, there is the **[O-EP]-[E-OP] transition**, characterized by: $n^{N-MIT}_{\mathrm{O}} = n^{N-MIT}_{\mathrm{E}}$, $\varepsilon^{N-MIT}_{1\,\mathrm{O}} = \varepsilon^{N-MIT}_{1\,\mathrm{E}}$ and $R^{N-MIT}_{\mathrm{O}} = R^{N-MIT}_{\mathrm{E}}$, since in this case, $E_{gn1(gp1)} = E_{gn2(gp2)} = E_{gn(gp)}$.

Then, from Equations (16b, 18, 19b, 19c, 19d), for $E = E_{gn(gp)}$, one can determine the exponential conduction (valence)-band tail states, due to those coefficients: $\sigma^{EBT}_{\mathrm{O[E]}}(E =$

$E_{gn(gp)}, N^* = N_{CDn(NDp)})$, $\kappa^{EBT}_{O[E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$, $\varepsilon^{EBT}_{2O[2E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$ and $\propto^{EBT}_{O[E]} (E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$, and then their numerical results are obtained and given in **Table 4, reported in Appendix 1.**

Further, the numerical results of $n_{O[E]}(E, N^*, r_{d(a)}, x, T)$, $\kappa_{O[E]}(E, N^*, r_{d(a)}, x, T)$, $\varepsilon_{2O[2E]}(E, N^*, r_{d(a)}, x, T)$ and $\varepsilon_{1\ O[E]}(E, N^*, r_{d(a)}, x, T)$, are obtained by using Equations (17, 19b, 19c and 16a), and expressed as functions of N for given (E=3.2 eV, $r_{d(a)}, x, T$=20 K)-conditions, and also, as functions of T for given (E=3.2 eV, $r_{d(a)}, x, N = 10^{20}\text{cm}^{-3}$)-conditions, and given in **Tables 5n, 5p, 6n and 6p, reported in Appendix 1**, respectively.

Finally, for T=20K and $N = 10^{20}\text{cm}^{-3}$, and for given x and r_d, the numerical results of $\sigma_{O[E]}$ (E), $\varepsilon_{2O[2E]}$(E) and $\propto_{O[E]}$ (E), are obtained by using Equations (18, 19c, 19d), and given in **Tables 7n and 7p, reported in Appendix 1.**

In the following, we will determine the electrical-and-thermoelectric laws, by basing on our $\sigma_{O[E]}$-models, given in Eq. (20a).

Optical [Electrical] Properties $[m^*_{n(p)} \equiv m_r(x)[m_{c(v)}(x)]]$

Here, if denoting, for majority electrons (holes), the thermal conductivity by: $\sigma_{Th.\ O[E]}(N^*, r_{d(a)}, x, T)$ in $\frac{W}{cm\times K}$, and the Lorenz number L by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W\times ohm}{K^2}\right) = 2.4429637 \times 10^{-8}\ (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\sigma_{Th.O[E]}}{\sigma_{O[E]]}}$, due to the O-EP [E-OP], is proportional to the temperature T(K), as:

$$\frac{\sigma_{Th.O[E]}(N^*, r_{d(a)}, x, T)}{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)} = L \times T. \quad (21)$$

Further, the resistivity is found to be given by: $\rho_{O[E]}(N^*, r_{d(a)}, x, T) \equiv 1/\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$.

In Eq. (20a), one notes that at T= 0 K, $\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^*)^{\frac{4}{3}}$. Thus, from Eq. (21), one has: $\sigma_{O[E]}(N^* = 0, r_{d(a)}, x, T = 0K) = 0$ and also $\sigma_{Th.\ O[E]}(N^* = 0, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the MIT occurs.

New Optical [Electrical] Coefficients

The relaxation time $\tau_{O[E]}$ is related to $\sigma_{O[E]}$ by[1]:

$\tau_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \sigma_{O[E]}(N^*, r_{d(a)}, x, T) \times \frac{m^*_{n(p)}(x) \times m_o}{q^2 \times (N^*/g_{c(v)})}$. Therefore, the mobility $\mu_{O[E]}$ is given by:

$$\mu_{O[E]]}(N^*, r_{d(a)}, x, T) = \frac{q \times \tau_{O[E]}(N^*, r_{d(a)}, x, T)}{m^*_{n(p)}(x) \times m_o} = \frac{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)}{q \times (N^*/g_{c(v)})} \left(\frac{cm^2}{V \times s}\right), \qquad (22a)$$

being expressed in terms of $\frac{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)}{N^*}$. Further, as noted in above Eq. (20a) for $\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$, $\mu_{O[E]]}(N^*, r_{d(a)}, x, T)$ can thus be expressed in terms of:

$$E^2_{Fn(Fp)}(N^*, r_{d(a)}, x, T) \times G_2(N^*, r_{d(a)}, x, T) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{5}{4}}}.$$

Then, from the well-known idea of Stokes, Einstein, Sutherland and Reynolds, we can define our viscosity coefficient, $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, and its reduced one, $R\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, by:

$$\frac{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)}{q} \equiv \frac{1}{6\pi \times \mu_{O[E]]}(N^*, r_{d(a)}, x, T) \times R_{WS}(N^*, x)} \left(\frac{V}{cm} \times \frac{s}{cm^2}\right) \quad , \quad R\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \frac{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)}{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T = 0K)}, \qquad (22b)$$

Where $R_{WS}(N^*, x) \equiv \left(\frac{3g_{c(v)}(x)}{4\pi N^*}\right)^{1/3}$ is the effective Wigner-Seitz radius, decreasing with increasing N^*.

Further, as noted above for $\mu_{O[E]]}(N^*, r_{d(a)}, x, T)$, $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$ can thus be expressed in terms of $\frac{(N^*)^{\frac{19}{12}} \times [N_{CDn(NDp)}(r_{d(a)}, x)]^{\frac{1}{6}}}{E^2_{Fn(Fp)}(N^*, r_{d(a)}, x, T) \times G_2(N^*, r_{d(a)}, x, T)}$, giving raise to some concluding remarks as follows.

−**With increasing $r_{d(a)}$** and for given x, low T and high N($N^* \simeq N$),

−**with decreasing T** and for given x, high N and high $r_{d(a)}$, and

−**with increasing N** and for given x, low T and high $r_{d(a)}$,

$\mathbb{V}_{O[E]}(N)$ **thus increases, suggesting an equivalence between degeneracy-compensation-viscosity concept,** and **being used to explain the numerical results of $\mathbb{V}_{O[E]}$, given in Tables 8n, …, 12p.**

Then, the activation energy, $AE_{O[E]}(N^*, r_{d(a)}, x, T)$, can be defined as[17]:

$$AE_{O[E]}\left(N^*, r_{d(a)}, x, T\right) \equiv k_B T \times \mathrm{Ln}\left(R\mathbb{V}_{O[E]}\left(N^*, r_{d(a)}, x, T\right)\right) \le 0 \text{ eV}, \qquad (22c)$$

according to the reduced activation energy, $RAE_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$, given by:

$$RAE_{O[E]}\left(N^*, r_{d(a)}, x, T\right) \equiv \frac{AE_{O[E]}(N^*, r_{d(a)}, x, T)}{k_B T} \equiv \mathrm{Ln}\left(R\mathbb{V}_{O[E]}\left(N^*, r_{d(a)}, x, T\right)\right) \le 0.$$

Furthermore, the Hall factor is defined by:

$$r_{HO[HE]}(N^*, r_{d(a)}, x, T) \equiv \frac{\langle \tau_{O[E]}{}^2 \rangle_{FDDF}}{\left[\langle \tau_{O[E]} \rangle_{FDDF}\right]^2} = \frac{G_4(y)}{[G_2(y)]^2},\ y \equiv \frac{\pi}{\xi_{n(p)}(N^*, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, r_{d(a)}, x, T)}, \text{ and}$$

the Hall mobility is found to be given by:

$$\mu_{HO[HE]}\left(N^*, r_{d(a)}, x, T\right) \equiv \mu_{O[E]}\left(N^*, r_{d(a)}, x, T\right) \times r_{HO[HE]}\left(N^*, r_{d(a)}, x, T\right) \left(\frac{cm^2}{V \times s}\right). \qquad (23)$$

It should be noted that, as T= 0 K and for a given value of $E_{Fn(Fp)}$, since $G_{p>1}(y=0) = 1$ as given in Eq. (12) and Table 2, one obtains: $r_{HE[HO]}(N^*, r_{d(a)}, x, T) = 1$, and therefore: $\mu_{HO[HE]}(N^*, r_{d(a)}, x, T) \equiv \mu_{O[E]}(N^*, r_{d(a)}, x, T)$.

Global Einstein -and- Van Cong relations

By taking into account Equations (22a, 22b) and for **any $N^*, r_{d(a)}, x$ and T, our global relations** are found to be defined by[1]:

$$\mathbb{R}_{E[O]}\left(N^*, r_{d(a)}, x, T\right) \equiv \frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\mu_{O[E]}(N^*, r_{d(a)}, x, T)} \equiv \frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)} \times q \times (N^*/g_{c(v)}) \equiv D_{O[E]}\left(N^*, r_{d(a)}, x, T\right) \times \frac{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)}{q} \times 6\pi \times R_{WS}(N^*, x) \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du}\right),\ \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \qquad (24)$$

where $D_{E[O]}\left(N^*, r_{d(a)}, x, T\right)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), the mobility $\mu_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$ is determined in Eq. (22a), the conductivity $\sigma_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$ is determined in Eq. (20a), and the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$ is defined in Eq. (22b). By differentiating this function $\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}$, with respect to u, being defined in Eq. (11), one obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (24) can also be rewritten as: $\mathbb{R}_{E[O]}(u) = \frac{k_B \times T}{q} \times u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)}$ where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} +$

$2^{-\frac{3}{2}}e^{-du}(1-du)+\frac{2}{3}Au^{B-1}F(u)\left[\left(1+\frac{3B}{2}\right)+\frac{4}{3}\times\frac{bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}}{1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}}\right]$. One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] = 1$, and therefore: $\mathbb{R}_{E[O]}(u \to 0) = \frac{k_B \times T}{q}$, being a **well-known local ($u \to 0$) relation given by Einstein**, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2u^{2B}$ and $u[V' \times W - V \times W'] = \frac{2}{3}au^{2/3}A^2u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, our relation (24) is reduced to a correct result: $\mathbb{R}_{E[O]}(N^*, r_{d(a)}, x, T = 0K) = \frac{2}{3}E_{Fno(Fpo)}(N^*)/q$.

It should be noted that the well-known "local" Einstein relation, being valid only at the lowest degeneracy (or $u \to 0$), is now globalized by our "Global" Einstein -and- Van Cong relations (24), valid at any degeneracy (or any u).

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (24) can be rewritten as:

$$\mathbb{R}_{E[O](VC)}(N^*, r_{d(a)}, x, T = 0K) \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}\right)}{\left(1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}\right)}\right],$$

Where $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

Then, in **Tables 8n and 8p**, reported in Appendix 1, for given ($r_{d(a)}$, x and T), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed as functions of N, are obtained by using Equations (22b, 22a and 24). Here, one notes that those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18]

In **Tables 9n and 9p**, reported in Appendix 1, for given ($r_{d(a)}$, x and T), the numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, expressed as functions of N, are obtained by using Eq. (22b). Here, one also notes that $\mathbb{V}_{O[E]}$ increases with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18]

Finally, in **Tables 10n and 10p**, reported in Appendix 1, for given x, $r_{d(a)}$ and N, the numerical results of reduced Fermi energy $\xi_{nO[E]}(N^*, r_d, x, T)$ and viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using

Equations (11, 22b), respectively. In particular, from these numerical results of $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, one observes that, for such given (x, r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18]

Thermoelectric Coefficients

Here, as noted above, $E_{Fn(Fp)}(m_r(x)) > E_{Fn(Fp)}\left(m_{c(v)}(x)\right)$ or $\xi_{n(p)}(m_r(x)) > \xi_{n(p)}\left(m_{c(v)}(x)\right)$ for a given T, since $m_r(x) < m_{c(v)}(x)$ for given x, corresponding to: $\sigma_O(m_r(x)) > \sigma_E\left(m_{c(v)}(x)\right)$.

Then, from Eq. (20a), obtained for $\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, $S_{E[O]}$, is found to be given by:

$$S_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \left.\frac{\partial \ln \sigma_{O[E]}}{\partial E}\right]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln \sigma_{O[E]}(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$Y_{Sb\ O[E]}(N^*, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3 \times G_2\left(y = \frac{\pi}{\xi_{n(p)}}\right)}\right],$$

$$S_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2Y_{Sb\ O[E]}(N^*, r_{d(a)}, x, T)}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)} = -2\sqrt{L} \times \frac{\sqrt{ZT_{O[E]Mott}}}{1 + ZT_{O[E]Mott}} \left(\frac{V}{K}\right) < 0, \quad ZT_{O[E]Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2},$$

according to:

$$\frac{\partial S_{O[E]}}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{ZT_{O[E]Mott} \times [1 - ZT_{O[E]Mott}]}{[1 + ZT_{O[E]Mott}]^2}. \qquad (25)$$

Here, one notes that: (i) as $\xi_{n(p)} \to +\infty$ or $\xi_{n(p)} \to +0$, one has a same limiting value of $S_{O[E]}$: $S_{O[E]} \to -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S_{O[E]}}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $\left(S_{O[E]}\right)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S_{O[E]} \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit is found to be defined by:

$$ZT_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \frac{S^2 \times \sigma_{O[E]} \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times ZT_{O[E]Mott}}{[1 + ZT_{O[E]Mott}]^2}. \quad (26)$$

Here, one notes that: (i) $\frac{\partial(ZT_{O[E]})}{\partial \xi_{n(p)}} = 2 \times \frac{S_{O[E]}}{L} \times \frac{\partial S_{O[E]}}{\partial \xi_{n(p)}}$, $S_{E[O]} < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial(ZT_{O[E]})}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT_{O[E]})_{max.} = 1$, $ZT_{O[E]Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT_{O[E]} \simeq 0.715$ and $ZT_{O[E]Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient can be defined by:

$$VC1_{O[E]}(N^*, r_{d(a)}, x, T) \equiv -N^* \times \frac{d\, S_{O[E]}}{dN^*}\left(\frac{V}{K}\right) = N^* \times \frac{\partial\, S_{O[E]}}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (27)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

And the second Van-Cong coefficient as:

$$VC2_{O[E]}(N^*, r_{d(a)}, x, T) \equiv T \times VC1_{O[E]}\ (V) \equiv T \times N^* \times \frac{\partial\, S_{O[E]}}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (28)$$

The Thomson coefficient, Ts , by:

$$Ts_{O[E]}\ (N^*, r_{d(a)}, x, T) \equiv T \times \frac{d\, S_{O[E]}}{dT}\left(\frac{V}{K}\right) = T \times \frac{\partial\, S_{O[E]}}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \quad (29)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

And the Peltier coefficient, $Pt_{E[O]}$, as:

$$Pt_{O[E]}(N^*, r_{d(a)}, x, T) \equiv T \times S_{O[E]}\ (V). \quad (30)$$

In **Tables 11n and 11p** in Appendix 1, the numerical results of $\xi_{nO[E]}$, $\mathbb{V}_{O[E]}$, $AE_{O[E]}$, $\sigma_{Th.O[E]}$, $S_{O[E]}$, $VC1_{O[E]}$, $VC2_{O[E]}$, $Ts_{O[E]}$, $Pt_{O[E]}$ and $ZT_{O[E]}$, for given x, $r_{d(a)}$ and N are obtained, as functions of T, by using Equations (11, 22b, 22c, 21, 25, 27, 28, 29, 30, and 26), respectively.

Then, **in Tables 12n and 12p** in Appendix 1, for given x, r_d and T=0K, the numerical results of Fermi energy $E_{Fn-O[E]}(eV)$, $\mathbb{V}_{O[E]}(\frac{eV}{cm} \times \frac{s}{cm^2})$, $\mu_{O[E]}(\frac{10^4 \times cm^2}{V \times s})$, and $D_{O[E]}\left(\frac{10^4 \times cm^2}{s}\right)$, are obtained, as functions of N, by using Equations (11, 22b, 22a, 24), respectively.

Finally, in the O-EP [E-OP] and for given physical conditions: x, $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), since $VC1_{O[E]}(N, r_{d(a)}, x, T)$ and $Ts_{O[E]}(N, r_{d(a)}, x, T)$ are expressed in terms of $\frac{-d\, S_{O[E]}}{dN^*}$ and $\frac{d\, S_{O[E]}}{dT}$, one has: $[VC1_{O[E]}, Ts_{O[E]}] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$, $[VC1_{O[E]}, Ts_{O[E]}] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[\, VC1_{O[E]}, Ts_{O[E]}] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating that for $\boldsymbol{\xi_{n(p)}} = \sqrt{\frac{\pi^2}{3}} \simeq \mathbf{1.8138}$: $S_{O[E]}$, determined in Eq. (25), thus presents **a same minimum** $S_{O[E]\,min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and $ZT_{O[E]}$, determined in Eq. (26), therefore presents **a same maximum:** $ZT_{O[E]\,max.} = 1$, and $(ZT)_{Mott} = 1$. Furthermore, for $\boldsymbol{\xi_{n(p)}} = \mathbf{1}$, the numerical results of $S_{O[E]}$, $ZT_{O[E]}$, $ZT_{O[E]Mott}$, $VC1_{E[O]}$, and $Ts_{O[E]}$, present the **same results**: $-1.322 \times 10^{-4}\,\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\,\frac{V}{K}$, and $1.657 \times 10^{-4}\,\frac{V}{K}$, respectively, as those observed in[4,5], and those given in **Table 13, reported in Appendix 1.**

It seems that these same obtained results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

Finally, it is interesting to remark that the $VC2_{O[E]}$-coefficient, defined in Eq. (28), is related to **our Global Einstein -and- Van Cong relations (24), by**:

$$\frac{k_B}{q} \times VC2_{O[E]}(N^*, r_{d(a)}, x, T) \equiv -\frac{\partial\, S_{O[E]}}{\partial \xi_{n(p)}} \times \frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\mu_{O[E]}(N^*, r_{d(a)}, x, T)} \left(\frac{V^2}{K}\right), \qquad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}},$$

According, in this work, with the use of our Eq. (25), to:

$$VC2_{O[E]}(N, r_{d(a)}, x, T) \equiv T \times VC1_{O[E]}(N, r_{d(a)}, x, T) \equiv -\frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\mu_{O[E]}(N^*, r_{d(a)}, x, T)} \times 2 \times \frac{ZT_{O[E]Mott} \times [1 - ZT_{O[E]Mott}]}{[1 + ZT_{O[E]Mott}]^2}\ (V), \qquad ZT_{O[E]Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}.$$

Therefore, we obtain in this work:

$$\mathbb{R}_{E[O]}(N^*, r_{d(a)}, x, T) \equiv \frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\mu_{O[E]}(N^*, r_{d(a)}, x, T)} \equiv \frac{D_{O[E]}(N^*, r_{d(a)}, x, T)}{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)} \times q \times (N^*/g_{c(v)}) \equiv D_{O[E]}(N^*, r_{d(a)}, x, T) \times \frac{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)}{q} \times 6\pi \times R_{WS}(N^*, x) \equiv -\frac{VC2_{O[E]}(N, r_{d(a)}, x, T)}{2} \times \frac{[1 + ZT_{O[E]Mott}]^2}{ZT_{O[E]Mott} \times [1 - ZT_{O[E]Mott}]} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*},$$ (31) being the new results.

Concluding Remarks

In the $\mathbf{n^+(p^+) - X(x)}$-crystalline alloy, $0 \le x \le 1$, x being the concentration, the optical, electrical and thermoelectric coefficients, enhanced by : **(i)** the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP), given in Eq. (15), **(ii)** our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b), **(iii)** our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq. (11), being accurate with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$[9], affecting all the expressions of optical, electrical and thermoelectric coefficients, and finally **(iv)** our optical-and-electrical conductivity models, given in Eq. (18, 20a), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.[1-5]

Some important concluding remarks can be given and discussed as follows.

First of all, one notes that from Equations (3, 6a, 6b) the MIT occurs as T=0 K and $N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x\ ) \cong N - N^{EBT}_{CDn(CDp)}(r_{d(a)}, x) = 0$, according, for $E \ge E_{gn(gp)}$, to: $E_{Fno(Fpo)}(N^* = 0) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m^*_{n(p)}(x) \times m_o} = 0$, and $\kappa^{MIT}_{O[E]}(E, N^* = 0) = 0$, $\varepsilon^{MIT}_{2\ O[E]}(E, N^* = 0) = 0$, $\sigma^{MIT}_{O[E]}(E, N^* = 0) = 0$ and $\propto^{MIT}_{O[E]}(E, N^* = 0)$=0, since $\sigma_{E[O]}(E, N^* = 0)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^* = 0)^{\frac{4}{3}} = 0$. However, for such the same physical conditions: T=0 K, $N^* = 0$ and $E = E_{gn(gp)}$, we obtain other numerical results such as: $n^{N-MIT}_{O[E]}(N^* = 0, E) \neq 0$, $\varepsilon^{N-MIT}_{1\ O[E]}(N^* = 0, E) \neq 0$ and $R^{N-MIT}_{O[E]}(N^* = 0, E) \neq 0$, according to the non-MIT (N-MIT), as showed in **Tables 3, reported in Appendix 1**. These Tables also state that, at T=0 K and $N^* = 0$, and for $E = E_{gn(gp)}$, there is the **[O-EP]-[E-OP] transition**, characterized by: $n^{N-MIT}_{O} = n^{N-MIT}_{E}$, $\varepsilon^{N-MIT}_{1\ O} = \varepsilon^{N-MIT}_{1\ E}$ and $R^{N-MIT}_{O} = R^{N-MIT}_{E}$, since in this case, $E_{gn1(gp1)} = E_{gn2(gp2)} = E_{gn(gp)}$. Further, it should be noted here that those results well define the properties of **the non-degenerate X-crystalline alloy** (or **the X- insulator**), given in the Mott MIT.

Then, by using Eq. (16b), and from Equations (18, 19b, 19c, 19d), for $E = E_{gn(gp)}$, one can determine the exponential conduction (valence)-band tail states, due to those coefficients: $\sigma^{EBT}_{O[E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$, $\kappa^{EBT}_{O[E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$, $\varepsilon^{EBT}_{2O[2E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$ and $\propto^{EBT}_{O[E]}(E = E_{gn(gp)}, N^* = N_{CDn(NDp)})$, and then their numerical results are given in **Table 4, reported in Appendix 1.**

Further, the numerical results of $n_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{2O[2E]}(E)$ and $\varepsilon_{1\,O[E]}(E)$, are obtained by using Equations (17, 19b, 19c and 16a), expressed as functions of N for (E=3.2 eV and T=20 K)-conditions, and as functions of T for (E=3.2 eV and $N = 10^{20}\text{cm}^{-3}$)-conditions, as those given in **Tables 5n, 5p, 6n and 6p, reported in Appendix 1**, respectively.

For T=20K and $N = 10^{20}\text{cm}^{-3}$, and for given x and r_d, the numerical results of $\sigma_{O[E]}$ (E), $\varepsilon_{2O[2E]}(E)$ and $\propto_{O[E]}$ (E), are obtained by using Equations (18, 19c, 19d), and given in **Tables 7n and 7p, reported in Appendix 1.**

In **Tables 8n and 8p**, reported in Appendix 1, for given ($r_{d(a)}$, x and T), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed as functions of N, are obtained by using Equations (22b, 22a and 24). Here, one notes that those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18]

In **Tables 9n and 9p**, reported in Appendix 1, for given ($r_{d(a)}$, x and T), the numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, expressed as functions of N, are obtained by using Eq. (22b). Here, one also notes that $\mathbb{V}_{O[E]}$ increases with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18]

In **Tables 10n and 10p**, reported in Appendix 1, for given x, $r_{d(a)}$ and N, the numerical results of reduced Fermi energy $\xi_{nO[E]}(N^*, r_d, x, T)$ and viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, expressed in $\left(\frac{\text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}\right)$, are obtained, as functions of T, by using Equations (11, 22b), respectively. In particular, from these numerical results of $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, one observes that, for such given (x, r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18]

In **Tables 11n and 11p** in Appendix 1, the numerical results of $\xi_{nO[E]}$, $\mathbb{V}_{O[E]}$, $AE_{O[E]}$, $\sigma_{Th.O[E]}$, $S_{O[E]}$, $VC1_{O[E]}$, $VC2_{O[E]}$, $Ts_{O[E]}$, $Pt_{O[E]}$ and $ZT_{O[E]}$, for given x, $r_{d(a)}$ and N are obtained, as functions of T, by using Equations (11, 22b, 22c, 21, 25, 27, 28, 29, 30, and 26), respectively.

In **Tables 12n and 12p** in Appendix 1, for given x, $r_{d(a)}$ and T=0K, the numerical results of Fermi energy $E_{Fn-O[E]}(eV)$, $\mathbb{V}_{O[E]}(\frac{eV}{cm}\times\frac{s}{cm^2})$, $\mu_{O[E]}(\frac{10^4\times cm^2}{V\times s})$, and $D_{O[E]}\left(\frac{10^4\times cm^2}{s}\right)$, are obtained, as functions of N, by using Equations (11, 22b, 22a, 24), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions , (T=0K, N=$\mathbf{N_{CDn}}$ or $N^*=0$), and (ii) those values of $E_{Fn-O[E]}\geq 0$, $\mathbb{V}_{O[E]}\geq 0$, $\mu_{O[E]}\geq 0$, and $D_{O[E]}\geq 0$, obtained for $N\geq$ $\mathbf{N_{CDn}}$, thus define the properties of **the degenerate (or viscous) X-crystalline alloy**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}_{O[E]}$, one observes that, for such given (x, r_d and T=0K), they **increase with increasing N**, in good agreement with those, obtained in complex fluids by Wenhao.[18]

Furthermore, from Equations (20a, 21-30), for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: **(i)** for $\xi_{n(p)}=\sqrt{\frac{\pi^2}{3}}\simeq 1.8138$, while the numerical results of $S_{O[E]}$ present a same minimum $S_{O[E]\,min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of $ZT_{O[E]}$ show a same maximum $ZT_{ET[OT]\,max.}=1$, **(ii)** for $\xi_{n(p)}=1$, the numerical results of $S_{O[E]}$, $ZT_{O[E]}$, $ZT_{O[E]\,Mott}$, $VC1_{O[E]]}$, and $Ts_{O[E]}$, present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and finally **(iii)** for $\xi_{n(p)}=\sqrt{\frac{\pi^2}{3}}\simeq 1.8138$, $ZT_{O[E]\,Mott}=1$, as those given in **Table 13**, reported in Appendix 1.

Finally, it should be noted that the well-known "local" Einstein relation, being valid only at the lowest degeneracy (or viscosity), is now globalized by our "Global" Einstein -and- Van Cong relations, valid at any degeneracy, viscosity and compensation, as showed in Equations (24, 31) and in Tables (3-13), suggesting also an equivalence between the degeneracy-and-viscosity concept and another one between the compensation-and-viscosity concept, given in the present X(x)- crystalline alloy.

REFERENCES

1. Van Cong, H. Global Einstein -and- Van Cong Relations, and New Optical-Electrical-Thermoelectric Laws in n(p)-Type Degenerate (or Viscous) GaTe(1-x)P(x)-Crystalline Alloy, Enhanced by: our Static Dielectric Constant Law, Conductivity Model, and Especially Accurate Fermi Energy Expression (28). *WJERT*, 2026; 12(4): 280-334.

2. Van Cong, H. Optical, Electrical and Thermoelectric Laws in n(p)-Type Degenerate "Compensated" GaAs(1-x)Te(x)-Crystalline Alloy, Enhanced by: Optico-Electrical Phenomenon and Electro-Optical Phenomenon, and our Static Dielectric Constant Law, Accurate Fermi Energy and Conductivity Models (VIII). *WJERT*, 2025; 11(12): 180-220.
3. Van Cong, H. New Critical Density in Metal-Insulator Transition, obtained in Various n(p)-Type Degenerate Crystalline Alloys, being Just That of Carriers Localized in Exponential Band Tails. *WJERT*, 2024; 10(4): 05-23.
4. Van Cong, H. Same maximum figure of merit ZT(=1), due to the effect of impurity size, obtained in the n(p)-type degenerate Ge -crystal $(\xi_{n(p)} \geq \mathbf{1})$, at same reduced Fermi energy $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, same minimum Seebeck coefficient $(S)_{min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, same maxi**mum** $(ZT)_{max.} = 1$, and same $(ZT)_{Mott} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1\right)$. *SCIREA Journal of Physics*. 2023; 8(4): 407-430.
5. Van Cong, H. Same Maximal Figure of Merit ZT(=1), Due to the Effect of Impurity Size, Obtained in the n(p)-Type Degenerate GaAs-Crystal $(\xi_{n(p)} \geq 1)$, at Same Reduced Fermi Energy $\xi_{n(p)}(= 1.8138)$ and Same Minimum Seebeck Coefficient $S \left(= -1.563 \times 10^{-4} \frac{V}{K}\right)$, at which Same $(ZT)_{Mott} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1\right)$. *SCIREA Journal of Physics*, 2023; 8(2): 133-157.
6. Van Cong, H. Effects of donor size and heavy doping on optical, electrical and thermoelectric properties of various degenerate donor-silicon systems at low temperatures. *American Journal of Modern Physics*, 2018; 7(4): 136-165.
7. Kim, H. S. et al. Characterization of Lorenz number with Seebeck coefficient measurement. *APL Materials*, 2015; 3(4): 041506.
8. Hyun, B. D. et al. Electrical-and-Thermoelectric Properties of $90\%Bi_2Te_3 - 5\%Sb_2Te_3 - 5\%Sb_2Se_3$ Single Crystals Doped with SbI_3. *Scripta Materialia*, 1998; 40(1): 49-56.
9. Van Cong, H. and Debiais, G. A simple accurate expression of the reduced Fermi energy for any reduced carrier density. *J. Appl. Phys.*, 1993; 73: 1545-1546.
10. Van Cong, H. et al. Size effect on different impurity levels in semiconductors. *Solid State Communications*, 1984; 49: 697-699.
11. Van Cong, H. Diffusion coefficient in degenerate semiconductors. *Phys. Stat. Sol. (b)*, 1984; 101: K27.

12. Van Cong, H. and Doan Khanh, B. Simple accurate general expression of the Fermi-Dirac integral $F_j(a)$ and for j> -1. *Solid-State Electron.*, 1992; 35(7): 949-951.
13. Van Cong, H. New series representation of Fermi-Dirac integral $F_j(-\infty < a < \infty)$ for arbitrary j> -1, and its effect on $F_j(a \geq 0_+)$ for integer j≥ 0. *Solid-State Electron.*, 1991; 34(5): 489-492.
14. Van Cong, H. and G. Mesnard. Thermoelectric effects of heavily doped semiconductors at low temperatures. *Phys. Stat. Sol. (b)*, 1972; 50(1): 53-58.
15. Van Cong, H. Fermi energy and band-tail parameters in heavily doped semiconductors. *Journal of Physics and Chemistry of Solids*, 1975; 36(11): 1237-1240.
16. Van Cong, H. Quantum efficiency and radiative lifetime in degerate n-type GaAs. *Journal of Physics and Chemistry of Solids*, 1981; 36(11): 95-99.
17. Ewell, R. H and Eyring, H. Theory of the Viscosity of Liquids as a Function of Temperature and Pressure. *Journal of Chemical Physics*, 1937; 5: 726-736.
18. Wenhao, Z. Influence of Temperature and Concentration on Viscosity of Complex Fluids. *Journal of Physics: Conference Series, 1965*, 2021; 1: 012064.

APPENDIX 1

Table 1: In the $X(x) \equiv InAs_{1-x}P_x$-crystalline alloy, the different values of energy-band-structure parameters, for a given x, are given in the following.[3]

In the $\mathbf{X(x)}$**-crystalline alloy**, in which $r_{do(ao)} = r_{As(In)}$ =0.118 nm (0.144 nm), we have [3]: $g_{c(v)}(x) = 1 \times x + 1 \times (1-x) = 1$, $m_{c(v)}(x) = 0.077\ (0.5) \times x + 0.09\ (0.3) \times (1-x)$, $\varepsilon_o(x) = 12.5 \times x + 14.55 \times (1-x)$, $E_{go}(x) = 1.424 \times x + 0.43 \times (1-x)$.

Table 2: Expressions for $G_{p>1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, are used to determine the electrical-and-thermoelectric coefficients, suggesting that, with decreasing T and for given (high N, r_d, x), the function $G_{p>1}(y)$ decreases as: $G_{p>1}(y \to 0) \to 1$.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}\right)$	$(1+y^2)$	$\left(1+\frac{35y^2}{24}\right)$	$(1+2y^2)$	$\left(1+\frac{21y^2}{8}\right)$

Table 3: For T=0K and $N=N_{CDn(CDp)}(r_{d(a)}, x)$, and at $E = E_{gn(gp)}$, the numerical results of $n_{O[E]}^{N-MIT}$, $\varepsilon_{1\ O[E]}^{N-MIT}$ and $R_{O[E]}^{N-MIT}$ are obtained, using Equations (17, 16a), suggesting that they decrease (↘) with increasing (↗) $r_{d(a)}$ and $E_{gn(gp)}$, and further they are found to be the same, for given $r_{d(a)}$ and $E_{gn(gp)}$, since $E_{gn1(gp1)} = E_{gn2(gp2)} = E_{gn(gp)}$.

Donor		**As**	Te	Sb	Sn
r_d (nm) [4]	↗	**0.118**	0.132	0.136	0.140
At **x=0**,					
E_{gn}(meV)	↗	430.00 [430.00]	430.78 [430.78]	431.31 [431.31]	431.99 [431.99]
$n_{O[E]}^{N-MIT}$	↘	4.203 [4.203]	4.135 [4.135]	4.094 [4.094]	4.047 [4.047]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	17.66 [17.66]	17.10 [17.10]	16.76 [16.76]	16.38 [16.38]
$R_{O[E]}^{N-MIT}$	↘	0.379 [0.379]	0.373 [0.373]	0.369 [0.369]	0.364 [0.364]
At **x=0.5**,					
E_{gn}(meV)	↗	927.00 [927.00]	927.83 [927.83]	928.40 [928.40]	929.13 [929.13]
$n_{O[E]}^{N-MIT}$	↘	3.816 [3.816]	3.750 [3.750]	3.711 [3.711]	3.665 [3.665]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	14.56 [14.56]	14.06 [14.06]	13.77 [13.77]	13.44 [13.44]
$R_{O[E]}^{N-MIT}$	↘	0.342 [0.342]	0.335 [0.335]	0.331 [0.331]	0.326 [0.326]
At **x=1**,					
E_{gn}(meV)	↗	1424.0 [1424.0]	1424.9 [1424.9]	1425.5 [1425.5]	1426.3 [1426.3]

$n_{O[E]}^{N-MIT}$	↘	3.425 [3.425]	3.362 [3.362]	3.324 [3.324]	3.281 [3.281]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	11.73 [11.73]	11.30 [11.30]	11.05 [11.05]	10.76 [10.76]
$R_{O[E]}^{N-MIT}$	↘	0.300 [0.300]	0.293 [0.293]	0.289 [0.289]	0.284 [0.284]

Acceptor		**Ga**	Mg	In	Cd
r_a (nm)	↗	**0.126**	0.140	0.144	0.148
At **x=0**,					
E_{gp}(meV)	↗	427.45 [427.45]	429.87 [429.87]	430.00 [430.00]	430.13 [430.13]
$n_{O[E]}^{N-MIT}$	↘	4.283 [4.283]	4.207 [4.207]	4.203 [4.203]	4.199 [4.199]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	18.34 [18.34]	17.70 [17.70]	17.66 [17.66]	17.63 [17.63]
$R_{O[E]}^{N-MIT}$	↘	0.386 [0.386]	0.3793 [0.3793]	0.3790 [0.3790]	0.3786 [0.3786]
At **x=0.5**,					
E_{gp}(meV)	↗	923.07 [923.07]	926.80 [926.80]	927.00 [927.00]	927.21 [927.21]
$n_{O[E]}^{N-MIT}$	↘	3.893 [3.893]	3.819 [3.819]	3.816 [3.816]	3.812 [3.812]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	15.16 [15.16]	14.59 [14.59]	14.56 [14.56]	14.53 [14.53]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	0.350 [0.350]	0.342 [0.342]	0.3418 [0.3418]	0.3415 [0.3415]
At **x=1**,					
E_{gp}(meV)	↗	1418.2 [1418.2]	1423.7 [1423.7]	1424.0 [1424.0]	1424.3 [1424.3]
$n_{O[E]}^{N-MIT}$	↘	3.501 [3.501]	3.429 [3.429]	3.425 [3.425]	3.421 [3.421]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	12.26 [12.26]	11.76 [11.76]	11.73 [11.73]	11.71 [11.71]
$\varepsilon_{1\ O[E]}^{N-MIT}$	↘	0.309 [0.309]	0.301 [0.301]	0.3003 [0.3003]	0.2999 [0.2999]

Table 4: For T=0K, $E \cong E_{gn(gp)}$ and $N^* = N_{CDn(NDp)}$, and from Eq. (16b), the numerical results of $\sigma_{O[E]}^{EBT}$, $\kappa_{O[E]}^{EBT}$, $\varepsilon_{2O[2E]}^{EBT}$ and $\propto_{O[E]}^{EBT}$ are obtained, using Equations (18, 19b, 19c, 19d), suggesting that they increase (↗) with increasing (↗) $r_{d(a)}$.

Donor		As	Te	Sb	Sn
r_d (nm) [4]	↗	0.118	0.132	0.136	0.140
At **x=0**,					
$\sigma_{O[E]}^{EBT}\left(\frac{10^2}{\Omega\times cm}\right)$	↗	1.377 [0.981]	1.467 [1.045]	1.525 [1.087]	1.596 [1.137]
$\kappa_{O[E]}^{EBT}\times 10^3$	↗	2.434 [1.732]	2.801 [1.993]	3.053 [2.172]	3.378 [2.403]
$\varepsilon_{2O[2E]}^{EBT}\times 10^2$	↗	2.057 [1.465]	2.329 [1.659]	2.514 [1.791]	2.751 [1.960]
$\propto_{O[E]}^{EBT}\left(\frac{10^2}{cm}\right)$	↗	1.061 [0.755]	1.223 [0.870]	1.334 [0.949]	1.479 [1.052]
At **x=0.5**,					

$\sigma^{EBT}_{O[E]}\left(\frac{10^2}{\Omega\times cm}\right)$ ↗	1.251 [0.979]	1.332 [1.043]	1.385 [1.085]	1.450 [1.135]
$\kappa^{EBT}_{O[E]}\times 10^3$ ↗	1.214 [0.949]	1.400 [1.094]	1.528 [1.194]	1.693 [1.323]
$\varepsilon^{EBT}_{2O[2E]}\times 10^2$ ↗	0.932 [0.730]	1.056 [0.827]	1.141 [0.893]	1.250 [0.978]
$\propto^{EBT}_{O[E]}\left(\frac{10^2}{cm}\right)$ ↗	1.141 [0.892]	1.316 [1.029]	1.437 [1.124]	1.594 [1.246]
At **x=1**,				
$\sigma^{EBT}_{O[E]}\left(\frac{10^2}{\Omega\times cm}\right)$ ↗	1.176 [0.977]	1.252 [1.041]	1.302 [1.082]	1.363 [1.133]
$\kappa^{EBT}_{O[E]}\times 10^3$ ↗	0.894 [0.742]	1.033 [0.857]	1.128 [0.936]	1.252 [1.038]
$\varepsilon^{EBT}_{2O[2E]}\times 10^2$ ↗	0.617 [0.513]	0.700 [0.581]	0.756 [0.628]	0.828 [0.688]
$\propto^{EBT}_{O[E]}\left(\frac{10^2}{cm}\right)$ ↗	1.291 [1.071]	1.491 [1.237]	1.630 [1.352]	1.809 [1.501]

Acceptor	Ga	Mg	**In**	Cd
r_a (nm) ↗	0.126	0.140	**0.144**	0.148
At **x=0**,				
$\sigma^{EBT}_{O[E]}\left(\frac{10^3}{\Omega\times cm}\right)$ ↗	2.452 [0.305]	2.624 [0.326]	2.633 [0.327]	2.642 [0.328]
$\kappa^{EBT}_{O[E]}\times 10^2$ ↗	4.048 [0.497]	4.698 [0.575]	4.734 [0.580]	4.771 [0.584]
$\varepsilon^{EBT}_{2O[2E]}\times 10^1$ ↗	3.439 [0.427]	3.914 [0.486]	3.940 [0.489]	3.966 [0.493]
$\propto^{EBT}_{O[E]}\left(\frac{10^3}{cm}\right)$ ↗	1.750 [0.215]	2.042 [0.250]	2.058 [0.252]	2.075 [0.254]
At **x=0.5**,				
$\sigma^{EBT}_{O[E]}\left(\frac{10^3}{\Omega\times cm}\right)$ ↗	5.675 [0.437]	6.071 [0.467]	6.092 [0.469]	6.113 [0.470]
$\kappa^{EBT}_{O[E]}\times 10^2$ ↗	5.211 [0.390]	6.081 [0.453]	6.129 [0.457]	6.179 [0.460]
$\varepsilon^{EBT}_{2O[2E]}\times 10^1$ ↗	3.963 [0.305]	4.517 [0.348]	4.548 [0.350]	4.579 [0.352]
$\propto^{EBT}_{O[E]}\left(\frac{10^3}{cm}\right)$ ↗	4.866 [0.364]	5.702 [0.425]	5.748 [0.428]	5.796 [0.432]
At **x=1**,				
$\sigma^{EBT}_{O[E]}\left(\frac{10^3}{\Omega\times cm}\right)$ ↗	11.91 [0.591]	12.74 [0.632]	12.79 [0.634]	12.83 [0.636]
$\kappa^{EBT}_{O[E]}\times 10^2$ ↗	8.829 [0.413]	10.38 [0.481]	10.46 [0.484]	10.55 [0.488]
$\varepsilon^{EBT}_{2O[2E]}\times 10^1$ ↗	5.857 [0.290]	6.678 [0.331]	6.723 [0.333]	6.769 [0.336]
$\propto^{EBT}_{O[E]}\left(\frac{10^3}{cm}\right)$ ↗	12.67 [0.592]	14.95 [0.692]	15.07 [0.698]	15.20 [0.703]

Table 5n. In the X(x)-system, at E=3.2 eV and T=20 K, for given r_d and x, the numerical results of $n_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{1O[E]}(E)$ and $\varepsilon_{2O[E]}(E)$, are obtained, as functions of N, by using Equations (17, 19b, 19c and 16), respectively, noting that, with increasing N, $\eta_{o[E]}$ increases [increases], and $E_{gn1\,O[E]}$ increases [decreases], respectively.

N (10^{18} cm^{-3}) ↗	15	26	60	100
At x=0				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	185.6 [142.7]	267.9 [206.1]	468.1 [360.1]	658.1 [506.2]
$E_{gn1\,O[E]}$ in eV	0.43 [0.11]	0.48 [0.02]	0.62 [-0.18]	0.78 [-0.34]
$n_{O[E]}$	5.16 [5.39]	5.13 [5.45]	5.01 [5.58]	4.89 [5.68]
$\kappa_{O[E]}$	0.06 [0.04]	0.10 [0.06]	0.23 [0.13]	0.37 [0.20]
$\varepsilon_{1O[E]}$	26.65 [29.03]	26.27 [29.70]	25.09 [31.09]	23.73 [32.21]
$\varepsilon_{2O[E]}$	0.62 [0.39]	1.04 [0.65]	2.27 [1.41]	3.67 [2.28]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	185.5 [142.7]	267.9 [206.1]	468.1 [360.0]	658.1 [506.2]
$E_{gn1\,O[E]}$ in eV	0.44 [0.12]	0.49 [0.03]	0.64 [-0.16]	0.81 [-0.32]
$n_{O[E]}$	5.11 [5.34]	5.08 [5.40]	4.96 [5.52]	4.82 [5.62]
$\kappa_{O[E]}$	0.06 [0.03]	0.10 [0.06]	0.22 [0.12]	0.37 [0.20]
$\varepsilon_{1O[E]}$	26.16 [28.52]	25.76 [29.17]	24.54 [30.51]	23.15 [31.59]
$\varepsilon_{2O[E]}$	0.60 [0.38]	1.01 [0.63]	2.20 [1.37]	3.55 [2.21]
At x=0.5				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	185.9 [153.8]	268.5 [222.1]	469.1 [388.1]	659.5 [545.6]
$E_{gn1\,O[E]}$ in eV	0.92 [0.60]	0.97 [0.51]	1.11 [0.31]	1.27 [0.14]
$n_{O[E]}$	4.70 [4.95]	4.66 [5.03]	4.54 [5.17]	4.39 [5.29]
$\kappa_{O[E]}$	0.06 [0.04]	0.10 [0.07]	0.24 [0.15]	0.39 [0.23]
$\varepsilon_{1O[E]}$	22.10 [24.56]	21.71 [25.27]	20.52 [26.74]	19.15 [27.93]
$\varepsilon_{2O[E]}$	0.59 [0.42]	0.98 [0.70]	2.15 [1.53]	3.47 [2.46]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	185.9 [153.8]	368.5 [222.1]	469.1 [388.1]	659.5 [545.6]
$E_{gn1\,O[E]}$ in eV	0.93 [0.61]	0.98 [0.52]	1.13 [0.33]	1.30 [0.17]
$n_{O[E]}$	4.65 [4.91]	4.61 [4.98]	4.48 [5.12]	4.33 [5.23]
$\kappa_{O[E]}$	0.06 [0.04]	0.10 [0.07]	0.23 [0.14]	0.39 [0.23]
$\varepsilon_{1O[E]}$	21.66 [24.10]	21.25 [24.78]	20.02 [26.20]	18.63 [27.34]
$\varepsilon_{2O[E]}$	0.57 [0.41]	0.95 [0.68]	2.08 [1.48]	3.36 [2.38]
At x=1				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	192.5 [166.8]	278.0 [240.9]	485.6 [420.8]	682.8 [591.7]
$E_{gn1\,O[E]}$ in eV	1.42 [1.09]	1.47 [1.00]	1.63 [0.80]	1.80 [0.63]
$n_{O[E]}$	4.18 [4.48]	4.13 [4.56]	3.99 [4.72]	3.82 [4.85]
$\kappa_{O[E]}$	0.07 [0.05]	0.12 [0.08]	0.27 [0.17]	0.45 [0.27]
$\varepsilon_{1O[E]}$	17.49 [20.04]	17.08 [20.76]	15.82 [22.27]	14.38 [23.50]
$\varepsilon_{2O[E]}$	0.59 [0.46]	0.98 [0.76]	2.15 [1.66]	3.46 [2.67]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	192.5 [166.8]	278.0 [240.9]	485.6 [420.8]	682.8 [591.7]
$E_{gn1\,O[E]}$ in eV	1.43 [1.10]	1.49 [1.01]	1.65 [0.81]	1.83 [0.65]
$n_{O[E]}$	4.14 [4.43]	4.08 [4.51]	3.93 [4.67]	3.76 [4.80]

$\kappa_{O[E]}$	0.07 [0.05]	0.12 [0.08]	0.26 [0.17]	0.45 [0.27]
$\varepsilon_{1O[E]}$	17.11 [19.63]	16.68 [20.32]	15.38 [21.78]	13.92 [22.96]
$\varepsilon_{2O[E]}$	0.57 [0.45]	0.95 [0.74]	2.08 [1.61]	3.36 [2.59]
N (10^{18} cm^{-3}) ↗	15	26	60	100

Table 5p. In the X(x)-system, at E=3.2 eV and T=20 K, for given r_a and x, the numerical results of $n_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{1O[E]}(E)$ and $\varepsilon_{2O[E]}(E)$, are obtained, as functions of N, by using Equations (17, 19b, 19c and 16a), respectively.

N (10^{18} cm^{-3}) ↗	15	26	60	100
At x=0				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	179.6 [41.43]	263.0 [60.68]	464.3 [107.1]	654.9 [151.1]
$E_{gp1\ O[E]}$ in eV	0.68 [0.38]	0.81 [0.36]	1.13 [0.33]	1.43 [0.31]
$n_{O[E]}$	5.11 [5.34]	5.01 [5.35]	4.74 [5.37]	4.47 [5.39]
$\kappa_{O[E]}$	0.06 [0.005]	0.11 [0.009]	0.26 [0.018]	0.45 [0.03]
$\varepsilon_{1O[E]}$	26.14 [28.6]	25.08 [28.7]	22.4 [28.9]	19.8 [29.1]
$\varepsilon_{2O[E]}$	0.66 [0.06]	1.13 [0.09]	2.51 [0.19]	4.07 [0.31]
For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	178.1 [41.09]	261.8 [60.39]	463.4 [106.9]	654.2 [150.9]
$E_{gp1\ O[E]}$ in eV	0.69 [0.38]	0.82 [0.37]	1.13 [0.34]	1.44 [1.32]
$n_{O[E]}$	5.03 [5.26]	4.93 [5.27]	4.66 [5.29]	4.39 [5.31]
$\kappa_{O[E]}$	0.06 [0.005]	0.11 [0.008]	0.25 [0.017]	0.44 [0.03]
$\varepsilon_{1O[E]}$	25.33 [27.7]	24.28 [27.8]	21.7 [28.0]	19.0 [28.2]
$\varepsilon_{2O[E]}$	0.62 [0.05]	1.06 [0.09]	2.36 [0.19]	3.83 [0.29]
At x=0.5				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	167.6 [28.91]	253.4 [43.74]	457.7 [79.04]	650.0 [112.2]
$E_{gp1\ O[E]}$ in eV	1.18 [0.89]	1.31 [0.88]	1.65 [0.87]	1.96 [0.85]
$n_{O[E]}$	4.62 [4.86]	4.50 [4.87]	4.19 [4.88]	3.87 [4.90]
$\kappa_{O[E]}$	0.06 [0.003]	0.11 [0.006]	0.27 [0.012]	0.49 [0.02]
$\varepsilon_{1O[E]}$	21.36 [23.6]	20.22 [23.7]	17.5 [23.9]	14.8 [24.0]
$\varepsilon_{2O[E]}$	0.57 [0.03]	1.01 [0.06]	2.31 [0.12]	3.80 [0.18]
For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	163.0 [28.12]	249.7 [43.10]	455.0 [78.57]	647.6 [111.8]
$E_{gp1\ O[E]}$ in eV	1.17 [0.90]	1.31 [0.89]	1.65 [0.87]	1.97 [0.86]
$n_{O[E]}$	4.55 [4.78]	4.42 [4.79]	4.11 [4.80]	3.80 [4.82]
$\kappa_{O[E]}$	0.06 [0.003]	0.10 [0.006]	0.26 [0.012]	0.47 [0.02]
$\varepsilon_{1O[E]}$	20.68 [22.8]	19.56 [22.9]	16.8 [23.1]	14.2 [23.2]
$\varepsilon_{2O[E]}$	0.52 [0.03]	0.93 [0.05]	2.16 [0.11]	3.56 [0.17]
At x=1				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	142.9 [19.02]	238.0 [31.73]	456.0 [60.85]	658.0 [87.80]
$E_{gp1\ O[E]}$ in eV	1.64 [1.40]	1.80 [1.39]	2.16 [1.38]	2.50 [1.37]
$n_{O[E]}$	4.11 [4.33]	3.96 [4.34]	3.59 [4.35]	3.22 [4.36]
$\kappa_{O[E]}$	0.05 [0.002]	0.11 [0.004]	0.30 [0.009]	0.57 [0.01]
$\varepsilon_{1O[E]}$	16.90 [18.8]	15.66 [18.9]	12.8 [18.9]	10.0 [19.0]
$\varepsilon_{2O[E]}$	0.43 [0.02]	0.88 [0.03]	2.19 [0.08]	3.67 [0.12]

For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	129.8 [17.27]	227.9 [30.39]	448.8 [59.88]	652.0 [87.00]
$E_{gp1\ O[E]}$ in eV	1.62 [1.41]	1.79 [1.40]	2.15 [1.39]	2.49 [1.38]
$n_{O[E]}$	4.05 [4.26]	3.90 [4.26]	3.52 [4.27]	3.15 [4.28]
$\kappa_{O[E]}$	0.04 [0.002]	0.10 [0.004]	0.29 [0.009]	0.54 [0.01]
$\varepsilon_{1O[E]}$	16.43 [18.1]	15.17 [18.2]	12.3 [18.3]	9.64 [18.4]
$\varepsilon_{2O[E]}$	0.36 [0.02]	0.78 [0.03]	2.02 [0.07]	3.42 [0.11]
$N\ (10^{18}\ cm^{-3})$ ↗	15	26	60	100

Table 6n. In the X(x)-system, at E=3.2 eV and $N = 10^{20} cm^{-3}$, for given r_d and x, the numerical results of $n_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{1O[E]}(E)$ and $\varepsilon_{2O[E]}(E)$, are obtained, as functions of T, by using Equations (17, 19b, 19c and 16), respectively, noting that $\eta_{o[E]}$ and $E_{gn1\ O[E]}$ both decrease with increasing T, respectively.

T ↗	20 K	50 K	100 K	300 K
At x=0				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	658.1 [506.2]	263.2 [202.5]	131.6 [101.2]	43.85 [33.72]
$E_{gn1\ O[E]}$ in eV	0.7858 [-0.3427]	0.7857 [-0.3428]	0.7855 [-0.3429]	0.7843 [-0.344]
$n_{O[E]}$	4.8862 [5.6789]	4.8863 [5.6790]	4.8865 [5.6791]	4.8874 [5.6795]
$\kappa_{O[E]}$	0.37570 [0.20067]	0.37571 [0.20068]	0.37574 [0.20072]	0.3761 [0.2546]
$\varepsilon_{1O[E]}$	23.734 [32.2106]	23.735 [32.2108]	23.736 [32.2117]	23.746 [23.30]
$\varepsilon_{2O[E]}$	3.6716 [2.2793]	3.6717 [2.2794]	3.6721 [2.2799]	3.6764 [2.461]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	658.1 [506.2]	263.2 [202.5]	131.6 [101.2]	43.85 [33.72]
$E_{gn1\ O[E]}$ in eV	0.8106 [-0.3178]	0.8105 [-0.3179]	0.8104 [-0.3180]	0.8091 [-0.3188]
$n_{O[E]}$	4.8258 [5.6235]	4.8259 [5.6236]	4.8260 [5.6237]	4.8270 [5.6241]
$\kappa_{O[E]}$	0.36839 [0.19640]	0.36840 [0.19641]	0.36843 [0.19645]	0.3688 [0.1969]
$\varepsilon_{1O[E]}$	23.153 [31.5860]	23.153 [31.5863]	23.155 [31.587]	23.164 [31.592]
$\varepsilon_{2O[E]}$	3.5556 [2.2089]	3.5557 [2.2090]	3.5561 [2.2095]	3.5602 [2.2145]
At x=0.5				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	659.5 [545.6]	263.8 [218.2]	131.9 [109.1]	43.95 [36.35]
$E_{gn1\ O[E]}$ in eV	1.2757 [0.1438]	1.2756 [0.1437]	1.2753 [0.1435]	1.2739 [0.1425]
$n_{O[E]}$	4.3945 [5.2904]	4.3946 [5.2905]	4.3948 [5.2906]	.3961 [5.2913]
$\kappa_{O[E]}$	0.39488 [0.23253]	0.39489 [0.23254]	0.39491 [0.23258]	0.3952 [0.2330]
$\varepsilon_{1O[E]}$	19.155 [27.934]	19.156 [27.935]	19.158 [27.936]	19.169 [27.94]
$\varepsilon_{2O[E]}$	3.4706 [2.4604]	3.4707 [2.4605]	3.4711 [2.4610]	3.4748 [2.465]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	659.5 [545.6]	263.8 [218.2]	131.9 [109.1]	43.95 [36.35]
$E_{gn1\ O[E]}$ in eV	1.3004 [0.1687]	1.3003 [0.1686]	1.3001 [0.1684]	1.2987 [0.1674]
$n_{O[E]}$	4.3333 [5.2342]	4.3334 [5.2343]	4.3336 [5.2345]	4.3349 [5.2352]
$\kappa_{O[E]}$	0.38787 [0.22777]	0.38788 [0.22778]	0.38790 [0.2278]	0.3882 [0.2282]
$\varepsilon_{1O[E]}$	18.627 [27.345]	18.628 [27.346]	18.629 [27.348]	18.641 [27.355]
$\varepsilon_{2O[E]}$	3.3616 [2.3845]	3.3617 [2.3846]	3.3620 [2.3850]	3.3656 [2.389]
At x=1				
For $\mathbf{r_d} = \mathbf{r_{Te}}$,				
$\xi_{nO[E]} \gg 1$	682.8 [591.7]	273.1 [236.7]	136.5 [118.3]	45.50 [39.42]

	20 K	50 K	100 K	300 K
$E_{gn1\ O[E]}$ in eV	1.8027 [0.6301]	1.8026 [0.6299]	1.8023 [0.6297]	1.8006 [0.6285]
$n_{O[E]}$	3.8193 [4.8559]	3.8194 [4.8560]	3.8197 [4.8562]	3.8214 [4.857]
$\kappa_{O[E]}$	0.453613 [0.27521]	0.453610 [0.27522]	0.453611 [0.27524]	0.4538 [0.2756]
$\varepsilon_{1O[E]}$	14.681 [23.504]	14.382 [23.505]	14.384 [23.507]	14.397 [23.516]
$\varepsilon_{2O[E]}$	3.4649 [2.6728]	3.4650 [2.6729]	3.4653 [2.6733]	3.4681 [2.677]
For $\mathbf{r_d} = \mathbf{r_{Sb}}$,				
$\xi_{nO[E]} \gg 1$	682.8 [591.7]	273.1 [236.7]	136.5 [118.3]	45.50 [39.42]
$E_{gn1\ O[E]}$ in eV	1.8275 [0.6550]	1.8274 [0.6549]	1.8271 [0.6546]	1.8254 [0.6533]
$n_{O[E]}$	3.7573 [4.7991]	3.7574 [4.7992]	3.7576 [4.7994]	3.7593 [4.800]
$\kappa_{O[E]}$	0.446661 [0.26986]	0.446662 [0.26987]	0.446663 [0.2699]	0.4468 [0.2702]
$\varepsilon_{1O[E]}$	13.917 [22.9589]	13.918 [22.9596]	13.920 [22.961]	13.933 [22.971]
$\varepsilon_{2O[E]}$	3.3565 [2.5903]	3.3566 [2.5904]	3.3568 [2.5907]	3.3595 [2.595]
T ↗	20 K	50 K	100 K	300 K

Table 6p. In the X(x)-system, at E=3.2 eV and $N = 10^{20} cm^{-3}$, for given r_a and x, the numerical results of $n_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{1O[E]}(E)$ and $\varepsilon_{2O[E]}(E)$, are obtained, as functions of T, by using Equations (17, 19b, 19c and 16a), respectively, noting that $\xi_{pO[E]}$ and $E_{gp1\ O[E]}$ decrease with increasing T.

T ↗	20 K	50 K	100 K	300 K
At x=0				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	654.9 [151.1]	262.0 [60.44]	131.0 [30.20]	43.64 [9.993]
$E_{gp1\ O[E]}$ in eV	1.433 [0.312]	1.429 [0.307]	1.416 [0.295]	1.337 [0.216]
$n_{O[E]}$	4.472 [5.393]	4.476 [5.396]	4.488 [5.405]	4.560 [5.460]
$\kappa_{O[E]}$	0.4557 [0.02843]	0.4553 [0.02844]	0.4542 [0.02846]	0.4475 [0.02894]
$\varepsilon_{1O[E]}$	19.79 [29.08]	19.83 [29.12]	19.93 [29.21]	20.59 [29.82]
$\varepsilon_{2O[E]}$	4.0760 [0.3067]	4.0761 [0.3069]	4.0765 [0.3077]	4.0809 [0.3161]
For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	654.2 [150.9]	261.6 [60.37]	130.8 [30.16]	43.59 [9.981]
$E_{gp1\ O[E]}$ in eV	1.440 [0.320]	1.436 [0.311]	1.423 [0.303]	1.344 [0.225]
$n_{O[E]}$	4.387 [5.308]	4.391 [5.312]	4.403 [5.321]	4.475 [5.377]
$\kappa_{O[E]}$	0.4371 [0.02754]	0.4367 [0.02755]	0.4356 [0.02757]	0.4291 [0.02803]
$\varepsilon_{1O[E]}$	19.06 [28.18]	19.09 [28.21]	19.20 [28.31]	19.84 [28.91]
$\varepsilon_{2O[E]}$	3.8359 [0.2924]	3.8360 [0.2926]	3.8363 [0.2934]	3.8405 [0.3015]
At x=0.5				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	650.0 [112.2]	260.0 [44.88]	130.0 [22.41]	43.31 [7.369]
$E_{gp1\ O[E]}$ in eV	1.965 [0.852]	1.957 [0.844]	1.939 [0.826]	1.835 [0.723]
$n_{O[E]}$	3.875 [4.898]	3.882 [4.903]	3.901 [4.919]	4.005 [5.001]
$\kappa_{O[E]}$	0.4900 [0.018591]	0.4891 [0.018593]	0.4867 [0.01862]	0.4745 [0.01922]
$\varepsilon_{1O[E]}$	14.77 [23.99]	14.83 [24.04]	14.98 [24.19]	15.82 [25.01]
$\varepsilon_{2O[E]}$	3.7972 [0.1821]	3.7973 [0.1823]	3.7977 [0.1832]	3.8013 [0.1923]
For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	647.6 [111.8]	259.0 [44.72]	129.5 [22.33]	43.16 [7.342]
$E_{gp1\ O[E]}$ in eV	1.967 [0.859]	1.961 [0.852]	1.942 [0.833]	1.838 [0.730]
$n_{O[E]}$	3.796 [4.816]	3.803 [4.822]	3.822 [4.837]	3.927 [4.920]
$\kappa_{O[E]}$	0.4692 [0.018093]	0.4684 [0.018095]	0.4661 [0.01812]	0.4541 [0.01871]
$\varepsilon_{1O[E]}$	14.19 [23.19]	14.25 [23.25]	14.39 [23.40]	15.21 [24.21]

$\varepsilon_{2O[E]}$	3.5627 [0.1743]	3.5629 [0.1745]	3.5632 [0.1753]	3.5666 [0.1841]
At x=1				
For $\mathbf{r_a} = \mathbf{r_{Ga}}$,				
$\xi_{pO[E]} \gg 1$	658.0 [87.80]	263.2 [35.10]	131.6 [17.51]	43.85 [5.705]
$E_{gp1\ O[E]}$ in eV	2.497 [1.370]	2.487 [1.360]	2.462 [1.335]	2.334 [1.208]
$n_{O[E]}$	3.222 [4.366]	3.233 [4.375]	3.261 [4.397]	3.401 [4.511]
$\kappa_{O[E]}$	0.5696 [0.012862]	0.5676 [0.013863]	0.5629 [0.01390]	0.5400 [0.01464]
$\varepsilon_{1O[E]}$	10.06 [19.06]	10.13 [19.14]	10.31 [19.34]	11.28 [20.35]
$\varepsilon_{2O[E]}$	3.6706 [0.1210]	3.6707 [0.1213]	3.6709 [0.1222]	3.6733 [0.1321]
For $\mathbf{r_a} = \mathbf{r_{In}}$,				
$\xi_{pO[E]} \gg 1$	652.0 [87.00]	260.8 [34.78]	130.4 [17.35]	43.45 [5.651]
$E_{gp1\ O[E]}$ in eV	2.494 [1.378]	2.484 [1.368]	2.460 [1.343]	2.332 [1.216]
$n_{O[E]}$	3.152 [4.286]	3.163 [4.295]	3.191 [4.317]	3.331 [4.432]
$\kappa_{O[E]}$	0.5421 [0.013511]	0.5402 [0.013512]	0.5356 [0.01355]	0.5133 [0.01428]
$\varepsilon_{1O[E]}$	9.643 [18.37]	9.716 [18.45]	9.895 [18.64]	10.83 [19.64]
$\varepsilon_{2O[E]}$	3.4178 [0.1158]	3.4179 [0.1161]	3.4181 [0.1170]	3.4204 [0.1266]
T ↗	20 K	50 K	100 K	300 K

Table 7n. For T=20K and $N = 10^{20}\text{cm}^{-3}$, and for given x and r_d, the numerical results of $\sigma_{O[E]}$ (E), $\varepsilon_{2O[2E]}$(E) and $\propto_{O[E]}$ (E), are obtained by using Equations (18, 19c, 19d), noting that, as given in Eq. (15), $E_{gnE} \equiv E_{gn2} + E_{Fn}$ and $E_{gnO} \equiv E_{gn1} + E_{Fn}$, expressed in eV.

E in eV	$\sigma_{O[E]}\ (\frac{10^5}{\Omega\times cm})$	$\varepsilon_{2O[2E]}$	$\propto_{O[E]}\ (\frac{10^5}{cm})$
At x=0 and $r_d = r_{Te}$,			
$\mathbf{-0.3428} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{0.5297} = E_{gnE}$	**… [1.0661]**	**… [13.76]**	**… [0.695]**
$\mathbf{0.7858} = E_{gn1}$	**0** [1.0663]	**0** [9.279]	**0** [0.659]
1.9200 $=E_{gnO}$	**1.7176** [1.0665]	**6.1177** [3.798]	**1.1702** [0.469]
3.5	1.7181 [1.0665]	3.357 [2.084]	1.132 [1.148]
4	1.7181 [1.0665]	2.937 [1.823]	1.501 [0.982]
4.5	1.7182 [1.0666]	2.611 [1.621]	1.322 [0.786]
5	1.7182 [1.0666]	2.350 [1.459]	3.181 [8.340]
5.5	1.7182 [1.0666]	2.136 [1.326]	-1.945 [-24.54]
6	1.7182 [1.0666]	1.958 [1.216]	11.26 [-2.963]
…			
$\mathbf{10^{22}}$	**1.718426 [1.066656]**	**0 [0]**	**2.8367 [1.7608]**
At x=0 and $r_d = r_{Sb}$,			
$\mathbf{-0.3178} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{0.5547} = E_{gnE}$	**… [0.9939]**	**… [12.739]**	**… [0.6806]**
$\mathbf{0.8106} = E_{gn1}$	**0** [0.9940]	**0** [8.718]	**0** [0.645]
1.9448 $=E_{gnO}$	**1.6000** [0.9942]	**5.8489** [3.634]	**1.1445** [0.456]
3.5	1.6004 [0.9942]	3.251 [2.019]	1.559 [1.105]
4	1.6004 [0.9942]	2.845 [1.767]	1.469 [0.959]

4.5	1.6005 [0.9942]	2.529 [1.571]	1.295 [0.769]
5	1.6005 [0.9943]	2.276 [1.414]	3.108 [8.110]
5.5	1.6005 [0.9943]	2.069 [1.285]	24.37 [-1.900]
6	1.6005 [0.9943]	1.896 [1.178]	11.08 [-2.884]
…			
10^{22}	**1.6007159 [0.9943]**	**0 [0]**	**2.8010 [1.7399]**
At x=0.5 and $r_d = r_{Te}$,			
$\mathbf{0.1438} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.0842} = E_{gnE}$	**… [1.0698]**	**… [7.2592]**	**… [0.7908]**
$\mathbf{1.2757} = E_{gn1}$	**0** [1.0699]	**0** [6.1703]	**0** [0.7344]
2.4124 =E_{gnO}	**1.5093** [1.0701]	**4.6028** [3.2635]	**1.1646** [0.4986]
4	1.5097 [1.0702]	2.777 [1.9684]	1.469 [1.0403]
4.5	1.5098 [1.0702]	2.468 [1.7497]	1.327 [0.8715]
5	1.5098 [1.0702]	2.221 [1.5747]	2.549 [3.7431]
5.5	1.5098 [1.0703]	2.019 [1.4316]	6.578 [-4.1540]
6	1.5098 [1.0703]	1.851 [1.3123]	5.460 [-8.2985]
…			
10^{22}	**1.5100158 [1.070349]**	**0 [0]**	**2.7813 [1.9715]**
At x=0.5 and $r_d = r_{Sb}$,			
$\mathbf{0.1687} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.1091} = E_{gnE}$	**… [0.9973]**	**… [6.8773]**	**… [0.7744]**
$\mathbf{1.3004} = E_{gn1}$	**0** [0.9974]	**0** [5.8662]	**0** [0.7383]
2.4372 =E_{gnO}	**1.4062** [0.9976]	**4.4129** [3.1306]	**1.1397** [0.4854]
4	1.4066 [0.9977]	2.689 [1.9076]	1.440 [1.0172]
4.5	1.4066 [0.9977]	2.391 [1.6957]	1.301 [0.8526]
5	1.4066 [0.9977]	2.512 [1.5261]	2.493 [3.6500]
5.5	1.4067 [0.9977]	1.956 [1.3874]	6.429 [-4.0648]
6	1.4067 [0.9977]	1.793 [1.2718]	5.355 [-8.0601]
…			
10^{22}	**1.4068409 [0.997778]**	**0 [0]**	**2.7468 [1.9481]**
At x=1 and $r_d = r_{Te}$,			
$\mathbf{0.6301} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.6499} = E_{gnE}$	**… [1.0742]**	**… [5.1824]**	**… [0.8985]**
$\mathbf{1.8027} = E_{gn1}$	**0** [1.0742]	**0** [4.7435]	**0** [0.8568]
2.9796 =E_{gnO}	**1.3928** [1.0744]	**3.7210** [2.8705]	**1.4112** [0.7467]
3.5	1.3930 [1.0745]	3.168 [2.4438]	1.627 [1.2188]
4	1.3931 [1.0745]	2.772 [2.1383]	1.567 [1.1364]
4.5	1.3932 [1.0745]	2.464 [1.9008]	1.442 [0.9863]
5	1.3932 [1.0745]	2.281 [1.7107]	2.311 [2.7767]
5.5	1.3932 [1.0746]	2.016 [1.5552]	4.123 [-1.3894]
6	1.3933 [1.0746]	1.848 [1.0374]	3.894 [-26.267]
…			
10^{22}	**1.3934648 [1.074673]**	**0 [0]**	**2.8887 [2.2279]**

At x=1 and $r_d = r_{Sb}$,			
$\mathbf{0.6550} = E_{gn2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.6748} = E_{gnE}$	**… [1.0013]**	**… [4.9478]**	**… [0.8797]**
$\mathbf{1.8274} = E_{gn1}$	**0** [1.0014]	**0** [4.5349]	**0** [0.8382]
3.0043 $=E_{gnO}$	**1.2978** [1.0016]	**3.5748** [2.7590]	**1.3933** [0.7417]
3.5	1.2980 [1.0016]	3.069 [2.3683]	1.598 [1.1920]
4	1.2981 [1.0017]	2.685 [2.0723]	1.539 [1.1120]
4.5	1.2981 [1.0017]	2.387 [1.8421]	1.416 [0.9656]
5	1.2982 [1.0017]	2.149 [1.6579]	2.263 [2.7109]
5.5	1.2982 [1.0017]	1.953 [1.5072]	4.036 [-26.003]
6	1.2982 [1.0017]	1.791 [1.3816]	3.819 [25.663]
…			
$\mathbf{10^{22}}$	**1.2984118 [1.001801]**	**0 [0]**	**2.8532 [2.2014]**

Table 7p. For T=20K and $N = 10^{20}\text{cm}^{-3}$, and for given x and r_a, the numerical results of $\sigma_{O[E]}$ (E), $\varepsilon_{2O[2E]}$(E) and $\propto_{O[E]}$ (E) , are obtained, as functions of E, by using Equations (18, 19c, 19d), noting that, as given in Eq. (15), $E_{gpE} \equiv E_{gp2} + E_{Fp}$ and $E_{gpO} \equiv E_{gp1} + E_{Fp}$, expressed in eV.

E in eV	$\sigma_{O[E]}$ $\left(\frac{10^5}{\Omega\times cm}\right)$	$\varepsilon_{2O[2E]}$	$\propto_{O[E]}$ $\left(\frac{10^5}{cm}\right)$
At x=0 and $r_a = r_{Ga}$,			
$\mathbf{0.3117} = E_{gp2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{0.5722} = E_{gpE}$	**… [0.1640]**	**… [1.7142]**	**… [0.1097]**
$\mathbf{1.4331} = E_{gp1}$	**0** [0.1641]	**0** [0.6847]	**0** [0.0918]
2.5620 $=E_{gpO}$	**2.1804** [0.1641]	**5.0902** [0.3830]	**1.3319** [0.0610]
4.5	2.1810 [0.1641]	2.900 [0.2181]	1.506 [0.1043]
5	2.1811 [0.1641]	2.609 [0.1963]	2.608 [0.3318]
5.5	2.1811 [0.1641]	2.372 [0.1784]	5.211 [-1.1780]
6	2.1811 [0.1641]	2.174 [0.1636]	4.680 [134.02]
…			
$\mathbf{10^{22}}$	**2.1814 [0.1641]**	**0 [0]**	**2.9453 [0.2215]**
At x=0 and $r_a = r_{In}$,			
$\mathbf{0.3203} = E_{gp2}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{0.5805} = E_{gpE}$	**… [0.1457]**	**… [1.6113]**	**… [0.1066]**
$\mathbf{1.4401} = E_{gp1}$	**0** [0.1457]	**0** [0.6498]	**0** [0.0890]
2.5676 $=E_{gpO}$	**1.9115** [0.1457]	**4.7797** [0.3645]	**1.2752** [0.0588]
4.5	1.9120 [0.1458]	2.728 [0.2080]	1.444 [0.1011]
5	1.9121 [0.1458]	2.455 [0.1872]	2.528 [0.3314]
5.5	1.9121 [0.1458]	2.232 [0.1701]	5.190 [-0.9788]
6	1.9121 [0.1458]	2.046 [0.1560]	4.639 [-7.7359]
…			
$\mathbf{10^{22}}$	**1.912367 [0.1458]**	**0 [0]**	**2.8718 [0.2189]**
At x=0.5 and $r_a = r_{Ga}$,			

$\mathbf{0.8517} = \mathrm{E_{gp2}}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.0452} = \mathrm{E_{gpE}}$	**… [0.0905]**	**… [0.5572]**	**… [0.0725]**
$\mathbf{1.9646} = \mathrm{E_{gp1}}$	**0** [0.0906]	**0** [0.2966]	**0** [0.0580]
3.0849 $=\mathrm{E_{gpO}}$	**1.8884** [0.0906]	**3.9387** [0.1889]	**1.5526** [0.0545]
4.5	1.8889 [0.0906]	2.701 [0.1295]	1.532 [0.0650]
5	1.8890 [0.0906]	2.431 [0.1165]	2.269 [0.1477]
5.5	1.8890 [0.0906]	2.210 [0.1059]	3.564 [0.7292]
6	1.8891 [0.0906]	2.026 [0.0971]	3.450 [0.4354]
…			
$\mathbf{10^{22}}$	**1.88931 [0.0906]**	**0 [0]**	**2.8463 [0.1364]**
At x=0.5 and $r_a = r_{In}$,			
$\mathbf{0.8592} = \mathrm{E_{gp2}}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.0520} = \mathrm{E_{gpE}}$	**… [0.0807]**	**… [0.5298]**	**… [0.0708]**
$\mathbf{1.9679} = \mathrm{E_{gp1}}$	**0** [0.0807]	**0** [0.2834]	**0** [0.0565]
3.0842 $=\mathrm{E_{gpO}}$	**1.6505** [0.0807]	**3.6964** [0.1808]	**1.4864** [0.0529]
4.5	1.6509 [0.0807]	2.534 [0.1239]	1.466 [0.0632]
5	1.6510 [0.0807]	2.281 [0.1115]	2.189 [0.1463]
5.5	1.6510 [0.0807]	2.073 [0.1014]	3.491 [0.8288]
6	1.6511 [0.0807]	1.901 [0.0929]	3.375 [0.4615]
…			
$\mathbf{10^{22}}$	**1.65129 [0.0807]**	**0 [0]**	**2.7669 [0.1353]**
At x=1 and $r_a = r_{Ga}$,			
$\mathbf{1.3701} = \mathrm{E_{gp2}}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.5214} = \mathrm{E_{gpE}}$	**… [0.0556]**	**… [0.2544]**	**… [0.0540]**
$\mathbf{2.4969} = \mathrm{E_{gp1}}$	**0** [0.0556]	**0** [0.1551]	**0** [0.0406]
3.6310 $=\mathrm{E_{gpO}}$	**1.6877** [0.0556]	**3.2358** [0.1067]	**1.8574** [0.0541]
4.5	1.6880 [0.0556]	2.612 [0.0861]	1.666 [0.0461]
5	1.6881 [0.0556]	2.350 [0.0775]	2.172 [0.0842]
5.5	1.6882 [0.0556]	2.137 [0.0704]	2.953 [0.1907]
6	1.6882 [0.0556]	1.959 [0.0645]	2.952 [0.1656]…
$\mathbf{10^{22}}$	**1.68847 [0.0556]**	**0 [0]**	**2.8629 [0.0943]**
At x=1 and $r_a = r_{In}$,			
$\mathbf{1.3782} = \mathrm{E_{gp2}}$	**… [0]**	**… [0]**	**… [0]**
$\mathbf{1.5281} = \mathrm{E_{gpE}}$	**… [0.0496]**	**… [0.2424]**	**… [0.0528]**
$\mathbf{2.4945} = \mathrm{E_{gp1}}$	**0** [0.0496]	**0** [0.1486]	**0** [0.0397]
3.6183 $=\mathrm{E_{gpO}}$	**1.4639** [0.0496]	**3.0236** [0.1024]	**1.7695** [0.0528]
4.5	1.4642 [0.0496]	2.432 [0.0824]	1.582 [0.0449]
5	1.4642 [0.0496]	2.189 [0.0741]	2.077 [0.0830]
5.5	1.4643 [0.0496]	1.990 [0.0674]	2.854 [0.1943]
6	1.4643 [0.0496]	1.824 [0.0618]	2.853 [0.1675]
…			
$\mathbf{10^{22}}$	**1.464555 [0.0496]**	**0 [0]**	**2.7620 [0.0935]**

Table 8n: For given x, r_d, and T=(4.2 K and 77 K), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed respectively in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22b, 22a and 24). In particular, for given (x, r_d and N), those of $\mu_{O[E]}(T)$ decrease with decreasing T, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), and therefore, those of the viscosity coefficient $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] Further, for given (x, T and r_d), those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (decreasing T or increasing N), both $\xi_{nO[E]}$ and $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between degeneracy-compensation-viscosity concept**.

Donor	Te	Sb
r_d (nm) ↗	**0.132**	0.136
For x=0 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	22.97 [36.66], 11.56 [7.246], 3.917 [1.888]	24.63 [39.26], 10.78 [6.766], 3.653 [1.763]
7	32.17 [51.71], 10.95 [6.814], 6.529 [3.125]	34.53 [55.45], 10.20 [6.355], 6.083 [2.914]
10	37.01 [59.63], 10.72 [6.656], 8.109 [3.872]	39.73 [63.96], 9.990 [6.205], 7.554 [3.609]
For x=0 and at T=77 K		
N (10^{19} cm^{-3})		
3	20.69 [33.39], 12.83 [7.955], 4.347 [2.072]	22.19 [35.76], 11.97 [7.428], 4.054 [1.935]
7	31.07 [50.13], 11.34 [7.030], 6.760 [3.223]	33.35 [53.75], 10.57 [6.556], 6.299 [3.006]
10	36.21 [58.48], 10.96 [6.787], 8.288 [3.948]	38.87 [62.73], 10.21 [6.327], 7.720 [3.680]
For x=0.5 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	26.08 [36.54], 10.18 [7.270], 3.457 [2.042]	27.95 [39.13], 9.502 [6.789], 3.225 [1.906]
7	36.59 [51.53], 9.629 [6.838], 5.753 [3.380]	39.26 [55.26], 8.975 [6.377], 5.361 [3.152]
10	42.12 [59.42], 9.423 [6.679], 7.141 [4.188]	45.21 [63.74], 8.780 [6.227], 6.653 [3.904]
For x=0.5 and at T=77 K		
N (10^{19} cm^{-3})		
3	23.50 [33.28], 11.30 [7.981], 3.837 [2.241]	25.18 [35.64], 10.55 [7.453], 3.579 [2.092]
7	35.34 [49.96], 9.971 [7.054], 5.956 [3.486]	37.92 [53.57], 9.293 [6.578], 5.551 [3.251]
10	41.21 [58.28], 9.301 [6.810], 7.298 [4.270]	44.23 [62.51], 8.973 [6.349], 6.800 [3.980]
For x=1 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	28.22 [36.39], 9.413 [7.299], 3.308 [2.223]	30.24 [38.97], 8.784 [6.815], 3.087 [2.075]
7	39.64 [51.33], 8.890 [6.865], 5.498 [3.680]	42.52 [55.04], 8.287 [6.403], 5.125 [3.431]
10	45.64 [59.18], 8.696 [6.706], 6.823 [4.560]	48.98 [63.48], 8.103 [6.252], 6.357 [4.250]

For x=1 and at T=77 K		
N (10^{19} cm^{-3})		
3	25.42 [33.15], 10.45 [8.012], 3.671 [2.439]	27.24 [35.51], 9.749 [7.481], 3.425 [2.278]
7	38.28 [49.76], 9.205 [7.082], 5.693 [3.795]	41.07 [53.35], 8.580 [6.605], 5.306 [3.539]
10	44.66 [58.04], 8.887 [6.838], 6.973 [4.649]	47.92 [62.26], 8.281 [6.374], 6.497 [4.333]

Table 8p: For given x, r_a, and T=(4.2 K and 77 K), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed respectively in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22b, 22a and 24). In particular, for given (x, r_a and N), those of $\mu_{O[E]}(T)$ decrease with decreasing T, due to the increasing reduced Fermi energy $\xi_{pO[E]}$ (or with increasing degeneracy), and therefore, those of the viscosity coefficient $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] Further, for given (x, T and r_a), those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{pO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (decreasing T or increasing N), both $\xi_{pO[E]}$ and $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor	Ga	In
r_a (nm) ↗	0.126	0.144
For x=0 and at T=4.2 K		
N (10^{19} cm^{-3})		
7	25.10 [326.2], 13.99 [1.077], 8.284 [0.147]	28.54 [365.5], 12.29 [0.960], 7.267 [0.131]
10	28.88 [383.9], 13.71 [1.031], 10.32 [0.179]	32.86 [431.2], 12.04 [0.918], 9.051 [0.159]
20	37.81 [521.9], 13.21 [0.957], 15.82 [0.264]	43.07 [588.3], 11.59 [0.849], 13.87 [0.234]
For x=0 and at T=77 K		
N (10^{19} cm^{-3})		
7	24.23 [31.17], 14.49 [11.27], 8.581 [1.537]	27.55 [34.81], 12.74 [10.08], 7.528 [1.373]
10	28.25 [56.02], 14.02 [7.067], 10.55 [1.226]	32.15 [62.79], 12.31 [6.302], 9.253 [1.092]
20	37.48 [157.7], 13.33 [3.167], 15.96 [0.875]	42.70 [177.6], 11.69 [2.811], 13.99 [0.776]
For x=0.5 and at T=4.2 K		
N (10^{19} cm^{-3})		
7	28.33 [571.3], 12.31 [0.610], 7.201 [0.062]	32.13 [633.9], 10.82 [0.549], 6.300 [0.055]
10	32.69 [682.1], 12.05 [0.577], 9.000 [0.074]	37.14 [759.7], 10.59 [0.518], 7.881 [0.066]
20	42.96 [948.5], 11.60 [0.525], 13.85 [0.108]	48.90 [1062.4], 10.18 [0.469], 12.14 [0.096]
For x=0.5 and at T=77 K		
N (10^{19} cm^{-3})		

7	27.32 [53.09], 12.76 [6.568], 7.467 [0.662]	30.98 [58.36], 11.23 [5.960], 6.535 [0.597]
10	31.97 [97.75], 12.32 [4.031], 9.204 [0.519]	36.31 [108.2], 10.83 [3.635], 8.060 [0.466]
20	42.59 [284.5], 11.70 [1.751], 13.97 [0.361]	48.47 [317.9], 10.27 [1.566], 12.24 [0.322]
For x=1 and at T=4.2 K		
N (10^{19} cm^{-3})		
7	30.11 [872.4], 11.39 [0.393], 6.679 [0.031]	33.98 [954.9], 10.03 [0.357], 5.801 [0.027]
10	34.98 [1061.9], 11.14 [0.367], 8.421 [0.037]	39.60 [1167.7], 9.795 [0.331], 7.338 [0.033]
20	46.30 [1519.5], 10.70 [0.326], 13.09 [0.053]	52.62 [1688.7], 9.397 [0.293], 11.44 [0.047]
For x=1 and at T=77 K		
N (10^{19} cm^{-3})		
7	28.97 [76.16], 11.84 [4.504], 6.942 [0.351]	32.66 [81.33], 10.43 [4.190], 6.036 [0.322]
10	34.17 [146.1], 11.40 [2.666], 8.621 [0.268]	38.66 [158.5], 10.03 [2.447], 7.515 [0.244]
20	45.89 [448.6], 10.80 [1.104], 13.21 [0.180]	52.14 [495.4], 9.483 [0.998], 11.55 [0.162]

Table 9n: The numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained as functions of N, by using Eq. (22b), suggesting that: (i) for given (x, T and N), they increase with increasing r_d, (ii) for given (x, r_d and N) the numerical results of $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and (iii) for given (x, T and r_d) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3n, with increasing degeneracy (decreasing T or increasing N), both the reduced Fermi energy $\xi_{nO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between degeneracy-compensation-viscosity concept**.

Donor		As	Te	Sb	Sn
r_d (nm) [4]	↗	0.118	0.132	0.136	0.140
For x=0 and at T=4.2 K					
N (10^{19} cm^{-3})					
3	↗	20.51 [32.79]	22.97 [36.66]	24.63 [39.26]	26.73 [42.55]
7	↗	28.68 [46.16]	32.17 [51.71]	34.53 [55.45]	37.52 [60.18]
10	↗	32.98 [53.19]	37.01 [59.63]	39.73 [63.96]	43.19 [69.46]
For x=0 and at T=77 K					
N (10^{19} cm^{-3})					
3	↗	18.48 [29.86]	20.69 [33.39]	22.19 [35.76]	24.08 [38.75]
7	↗	27.70 [44.75]	31.07 [50.13]	33.35 [53.75]	36.23 [58.34]
10	↗	32.27 [52.17]	36.21 [58.48]	38.87 [62.73]	42.25 [68.12]
For x=0.5 and at T=4.2 K					

$N\ (10^{19}\ cm^{-3})$					
3	↗	23.29 [32.67]	26.08 [36.54]	27.95 [39.13]	30.33 [42.40]
7	↗	32.63 [46.00]	36.59 [51.53]	39.26 [55.26]	42.65 [59.98]
10	↗	37.54 [53.01]	42.12 [59.42]	45.21 [63.74]	49.13 [69.22]
For x=0.5 and at T=77 K					
$N\ (10^{19}\ cm^{-3})$					
3	↗	20.99 [29.77]	23.50 [33.28]	25.18 [35.64]	27.32 [38.63]
7	↗	31.52 [44.60]	35.34 [49.96]	37.92 [53.57]	41.19 [58.14]
10	↗	36.73 [51.99]	41.21 [58.28]	44.23 [62.51]	48.07 [67.89]
For x=1 and at T=4.2 K					
$N\ (10^{19}\ cm^{-3})$					
3	↗	25.21 [32.55]	28.22 [36.39]	30.24 [38.97]	32.80 [42.24]
7	↗	35.36 [45.82]	39.64 [51.33]	42.52 [55.04]	46.18 [59.74]
10	↗	40.69 [52.79]	45.64 [59.18]	48.98 [63.48]	53.22 [68.94]
For x=1 and at T=77 K					
$N\ (10^{19}\ cm^{-3})$					
3	↗	22.72 [29.65]	25.42 [33.15]	27.24 [35.50]	29.55 [38.48]
7	↗	34.15 [44.42]	38.28 [49.76]	41.07 [53.35]	44.60 [57.91]
10	↗	39.81 [51.78]	44.66 [58.04]	47.92 [62.26]	52.07 [67.62]

Table 9p: The numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained as functions of N, by using Eq. (22b), suggesting that: (i) for given (x, T and N), they increase with increasing r_a, (ii) for given (x, r_a and N) the numerical results of $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and (iii) for given (x, T and r_a) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3p, with increasing degeneracy (decreasing T or increasing N), both the reduced Fermi energy $\xi_{pO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor		Ga	Mg	**In**	Cd
r_a (nm)	↗	0.126	0.140	**0.144**	0.148
----------For x=0 and at T=4.2 K					
$N\ (10^{19}\ cm^{-3})$					
7	↗	25.10 [326.2]	28.36 [363.5]	28.54 [365.5]	28.72 [367.5]
10	↗	28.88 [383.9]	32.66 [428.8]	32.86 [431.2]	33.07 [433.6]
20	↗	37.81 [521.9]	42.80 [584.9]	43.07 [588.3]	43.35 [591.8]
For x=0 and at T=77 K					

N (10^{19} cm^{-3})					
7	↗	24.23 [31.17]	27.37 [34.63]	27.55 [34.81]	27.72 [35.00]
10	↗	28.25 [56.02]	31.95 [62.45]	32.15 [62.79]	32.35 [63.14]
20	↗	37.48 [157.7]	42.43 [176.60]	42.70 [177.63]	42.97 [178.7]
For x=0.5 and at T=4.2 K					
N (10^{19} cm^{-3})					
7	↗	28.33 [571.3]	31.94 [630.8]	32.13 [633.9]	32.33 [637.1]
10	↗	32.69 [682.1]	36.91 [755.8]	37.14 [759.7]	37.38 [763.7]
20	↗	42.96 [948.5]	48.60 [1056.6]	48.90 [1062.4]	49.22 [1068.3]
For x=0.5 and at T=77 K					
N (10^{19} cm^{-3})					
7	↗	27.32 [53.09]	30.79 [58.10]	30.98 [58.36]	31.17 [58.63]
10	↗	31.97 [97.75]	36.09 [107.7]	36.31 [108.2]	36.54 [108.7]
20	↗	42.59 [284.5]	48.17 [316.2]	48.47 [317.9]	48.78 [319.7]
For x=1 and at T=4.2 K					
N (10^{19} cm^{-3})					
7	↗	30.11 [872.4]	33.78 [950.8]	33.98 [954.9]	34.18 [959.0]
10	↗	34.98 [1061.9]	39.36 [1164.3]	39.60 [1169.7]	39.84 [1175.2]
20	↗	46.30 [1519.5]	52.29 [1680.2]	52.61 [1688.7]	52.95 [1697.5]
For x=1 and at T=77 K					
N (10^{19} cm^{-3})					
7	↗	28.97 [76.16]	32.47 [81.09]	32.66 [81.33]	32.85 [81.57]
10	↗	34.17 [146.1]	38.43 [157.9]	38.66 [158.5]	38.90 [159.1]
20	↗	45.89 [448.6]	51.82 [493.1]	52.14 [495.4]	52.47 [497.8]

Table 10n: For given x, r_d and N, the numerical results of Fermi energy $E_{FnO[E](T=0K)}$(eV) [or reduced Fermi energy $\xi_{nO[E]}(N^*, r_d, x, T)$], and viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using Equations (11, 22b), respectively. In particular, from these numerical results of $\mathbb{V}_{O[E]}(T)$, one observes that, for such given (x, r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and from those of $\mathbb{V}_{O[E]}(r_d)$, one observes that, for given (x, T and N), they increase with increasing r_d, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Donor		Te	Sb	Sn
r_d (nm) [4]	↗	0.132	0.136	0.140
For **x=0** and **N=3 × 10^{19} cm^{-3},**				
N_{CDn} in 10^{16} cm^{-3}	↗	**3.0014**	**3.3725**	**3.8688**

$E_{Fno[E](T=0K)}$(eV)	$\searrow$	**0.50853723 [0.39118253]**	**0.50853722 [0.39118252]**	**0.50853721 [0.39118251]**
$\mathbb{V}_{o[E]\,(0K)}$	$\nearrow$	**22.9707 [36.6604]**	**24.6272 [39.2599]**	**26.72673 [42.5482]**
$\xi_{no[E](T=4.2K)}$	$\searrow$	1403.81 [1079.86]	1403.70 [1079.77]	1403.54 [1079.65]
$\mathbb{V}_{o[E]\,(4.2K)}$	$\nearrow$	22.9707 [36.6603]	24.6271 [39.2598]	26.7267 [42.5481]
$\xi_{no[E](T=77K)}$	$\searrow$	76.5878 [58.9223]	76.5815 [58.9174]	76.5731 [58.9109]
$\mathbb{V}_{o[E]\,(77K)}$	$\nearrow$	20.696 [33.39]	22.188 [35.76]	24.079 [38.75]
$\xi_{no[E](T=100K)}$	$\searrow$	58.9812 [45.3812]	58.9763 [45.3775]	58.9698 [45.3725]
$\mathbb{V}_{o[E]\,(100K)}$	$\nearrow$	17.301 [28.41]	18.548 [30.42]	20.128 [32.97]
$\xi_{no[E](T=150K)}$	$\searrow$	39.3382 [30.277]	39.3350 [30.274]	39.3307 [30.271]
$\mathbb{V}_{o[E]\,(150K)}$	$\nearrow$	8.3585 [14.41]	8.9603 [15.43]	9.7228 [16.73]
$\xi_{no[E](T=200K)}$	$\searrow$	29.5221 [22.732]	29.5196 [22.730]	29.5164 [22.727]
$\mathbb{V}_{o[E]\,(200K)}$	$\nearrow$	3.4412 [6.1076]	3.6889 [6.5398]	4.0026 [7.086]
$\xi_{no[E](T=250K)}$	$\searrow$	23.6366 [18.210]	23.6347 [18.209]	23.6321 [18.207]
$\mathbb{V}_{o[E]\,(250K)}$	$\nearrow$	1.5221 [2.732]	1.6316 [2.925]	1.7703 [3.170]
$\xi_{no[E](T=300K)}$	$\searrow$	19.7166 [15.200]	19.7150 [15.199]	19.7128 [15.197]
$\mathbb{V}_{o[E]\,(300K)}$	$\nearrow$	0.7513 [1.3539]	0.8053 [1.4497]	0.8738 [1.5708]

For **x=0.5** and **N=7 × 10^{19} cm^{-3}**,

N_{CDn} in 10^{16} cm^{-3}	$\nearrow$	**2.9843**	**3.3532**	**3.8466**
$E_{Fno[E](T=0K)}$(eV)	$\searrow$	**0.89462423 [0.68817243]**	**0.89462422 [0.68817242]**	**0.89462421 [0.68817241]**
$\mathbb{V}_{o[E]\,(0K)}$	$\nearrow$	**36.5954 [51.5351]**	**39.2637 [55.2575]**	**42.6503 [59.9768]**
$\xi_{no[E](T=4.2K)}$	$\searrow$	2475.96 [2048.36]	2475.87 [2048.29]	2475.75 [2048.19]
$\mathbb{V}_{o[E]\,(4.2K)}$	$\nearrow$	36.5953 [51.5350]	39.2636 [55.2575]	42.6502 [59.9768]
$\xi_{no[E](T=77K)}$	$\searrow$	135.061 [111.740]	135.057 [111.736]	135.050 [111.731]
$\mathbb{V}_{o[E]\,(77K)}$	$\nearrow$	35.341 [49.954]	37.918 [53.567]	41.189 [58.142]
$\xi_{no[E](T=100K)}$	$\searrow$	104.002 [86.0455]	103.998 [86.0425]	103.993 [86.0385]
$\mathbb{V}_{o[E]\,(100K)}$	$\nearrow$	33.095 [47.122]	35.508 [50.525]	38.571 [54.840]
$\xi_{no[E](T=150K)}$	$\searrow$	69.345 [57.3757]	69.342 [57.3736]	69.339 [57.3709]
$\mathbb{V}_{o[E]\,(150K)}$	$\nearrow$	23.401 [34.422]	25.107 [36.908]	27.271 [40.059]
$\xi_{no[E](T=200K)}$	$\searrow$	52.019 [43.0443]	52.017 [43.0428]	52.014 [43.0408]
$\mathbb{V}_{o[E]\,(200K)}$	$\nearrow$	12.937 [19.750]	13.879 [21.176]	15.075 [22.983]
$\xi_{no[E](T=250K)}$	$\searrow$	41.6258 [34.4484]	41.6243 [34.4472]	41.6224 [34.4456]
$\mathbb{V}_{o[E]\,(250K)}$	$\nearrow$	6.6106 [10.333]	7.0922 [11.079]	7.7033 [12.024]
$\xi_{no[E](T=300K)}$	$\searrow$	34.6991 [28.7202]	34.6978 [28.7192]	34.6962 [28.7179]
$\mathbb{V}_{o[E]\,(300K)}$	$\nearrow$	3.4828 [5.5099]	3.7365 [5.9076]	4.0584 [6.4116]

For **x=1** and **N=10^{20} cm^{-3}**,

N_{CDn} in 10^{16} cm^{-3}	$\nearrow$	**2.9644**	**3.3308**	**3.8210**
$E_{Fno[E](T=0K)}$(eV)	$\searrow$	**1.1347723 [0.87290153]**	**1.1347722 [0.87290152]**	**1.1347721 [0.87290151]**
$\mathbb{V}_{o[E]\,(0K)}$	$\nearrow$	**45.64281 [59.1825]**	**48.98180 [63.48433]**	**53.2215 [68.94242]**

$\xi_{no[E](T=4.2K)}$	↘	3251.72 [2817.79]	3251.65 [2817.72]	3251.54 [2817.62]
$\mathbb{V}_{O[E]\ (4.2K)}$	↗	45.64280 [59.1824]	48.98178 [63.48430]	53.2214 [68.94239]
$\xi_{no[E](T=77K)}$	↘	177.374 [153.705]	177.369 [153.702]	177.364 [153.697]
$\mathbb{V}_{O[E]\ (77K)}$	↗	44.658 [58.045]	47.925 [62.264]	52.074 [67.617]
$\xi_{no[E](T=100K)}$	↘	136.581 [118.357]	136.578 [118.354]	136.574 [118.351]
$\mathbb{V}_{O[E]\ (100K)}$	↗	42.829 [55.929]	45.962 [59.995]	49.940 [65.152]
$\xi_{no[E](T=150K)}$	↘	91.0619 [78.914]	91.0596 [78.912]	91.0567 [78.909]
$\mathbb{V}_{O[E]\ (150K)}$	↗	33.802 [45.220]	36.275 [48.507]	39.414 [52.676]
$\xi_{no[E](T=200K)}$	↘	68.304 [59.194]	68.303 [59.193]	68.300 [59.191]
$\mathbb{V}_{O[E]\ (200K)}$	↗	21.368 [29.601]	22.931 [31.752]	24.915 [34.481]
$\xi_{no[E](T=250K)}$	↘	54.6516 [47.365]	54.6502 [47.364]	54.6485 [47.362]
$\mathbb{V}_{O[E]\ (250K)}$	↗	11.958 [17.030]	12.832 [18.267]	13.942 [19.837]
$\xi_{no[E](T=300K)}$	↘	45.551 [39.480]	45.550 [39.479]	45.549 [39.478]
$\mathbb{V}_{O[E]\ (300K)}$	↗	6.6121 [9.5715]	7.0955 [10.267]	7.7092 [11.149]

Table 10p: For given x, r_a and N, the numerical results of the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDp}(r_a, x)$, Fermi energy $E_{Fpo[E](T=0K)}$(eV) [or reduced Fermi energy $\xi_{pO[E]}(N^*, r_a, x, T)$], and viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using Equations (3, 11, 22b), respectively. In particular, from these numerical results of $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, one observes that, for such given (x, r_a and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and from those of $\mathbb{V}_{O[E]}(r_a)$, one observes that, for given (x, T and N), they increase with increasing r_a, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor		**Ga**	Mg	In
r_a (nm)	↗	**0.126**	0.140	0.144
For **x=0** and **N=6×10^{19} cm^{-3}**,				
N_{CDp} in 10^{17} cm^{-3}	↗	**7.4367**	**9.1053**	**9.1996**
$E_{FpO[E](T=0K)}$(eV)	↘	**0.8006 [0.1847]**	**0.7991 [0.1844]**	**0.7990 [0.1843]**
$\mathbb{V}_{O[E]\ (0K)}$	↗	**23.6142 [303.70]**	**26.6736 [338.04]**	**26.8392 [339.88]**
$\xi_{pO[E](T=4.2K)}$	↘	2211.4 [510.3]	2207.3 [509.4]	2207.0 [509.3]
$\mathbb{V}_{O[E]\ (4.2K)}$	↗	23.6142 [303.6]	26.6736 [337.97]	26.8391 [339.81]
$\xi_{pO[E](T=77K)}$	↘	120.634 [27.8808]	120.408 [27.8286]	120.395 [27.8256]
$\mathbb{V}_{O[E]\ (77K)}$	↗	22.61 [23.983]	25.54 [26.602]	25.70 [26.741]
$\xi_{pO[E](T=100K)}$	↘	92.8938 [21.4918]	92.7194 [21.4517]	92.7095 [21.449]
$\mathbb{V}_{O[E]\ (100K)}$	↗	20.86 [8.9438]	23.55 [9.9188]	23.70 [9.9706]
$\xi_{pO[E](T=150K)}$	↘	61.940 [14.377]	61.824 [14.350]	61.817 [14.348]
$\mathbb{V}_{O[E]\ (150K)}$	↗	13.86 [1.8151]	15.63 [2.0128]	15.73 [2.0233]

$\xi_{po[E](T=200K)}$	↘	46.4668 [10.834]	46.3797 [10.814]	46.3747 [10.813]
$\mathbb{V}_{O[E]\,(200K)}$	↗	7.193 [0.5740]	8.103 [0.6365]	8.152 [0.6398]
$\xi_{po[E](T=250K)}$	↘	37.185 [8.7216]	37.116 [8.7058]	37.112 [8.7049]
$\mathbb{V}_{O[E]\,(250K)}$	↗	3.5440 [0.2332]	3.9904 [0.2585]	4.0144 [0.2599]
$\xi_{po[E](T=300K)}$	↘	31.000 [7.3236]	30.942 [7.3105]	30.939 [7.3098]
$\mathbb{V}_{O[E]\,(300K)}$	↗	1.8345 [0.1110]	2.0650 [0.1231]	2.0774 [0.1237]

For **x=0.5** and **N=7** × $\mathbf{10^{19}}$ $\boldsymbol{cm^{-3}}$,

N_{CDp} in 10^{18} cm^{-3}	↗	**2.1947**	**2.6871**	**2.7150**
$E_{FpO[E](T=0K)}$(eV)	↘	**0.8777 [0.1516]**	**0.8735 [0.15085]**	**0.8732 [0.15081]**
$\mathbb{V}_{O[E]\,(0K)}$	↗	**28.3299 [571.39]**	**31.9399 [630.88]**	**32.1348 [634.03]**
$\xi_{po[E](T=4.2K)}$	↘	2424.6 [418.7]	2412.87 [416.7]	2412.20 [416.6]
$\mathbb{V}_{O[E]\,(4.2K)}$	↗	28.3299 [571.30]	31.9399 [630.77]	32.1348 [633.92]
$\xi_{po[E](T=77K)}$	↘	132.261 [22.8941]	131.620 [22.7836]	131.584 [22.7774]
$\mathbb{V}_{O[E]\,(77K)}$	↗	27.32 [53.093]	30.79 [58.103]	30.98 [58.364]
$\xi_{po[E](T=100K)}$	↘	101.846 [17.6573]	101.353 [17.5724]	101.325 [17.5676]
$\mathbb{V}_{O[E]\,(100K)}$	↗	25.52 [19.970]	28.74 [21.842]	28.91 [21.939]
$\xi_{po[E](T=150K)}$	↘	67.907 [11.831]	67.578 [11.775]	67.560 [11.772]
$\mathbb{V}_{O[E]\,(150K)}$	↗	17.84 [4.0582]	20.04 [4.4368]	20.16 [4.4565]
$\xi_{po[E](T=200K)}$	↘	50.9413 [8.9375]	50.6946 [8.8956]	50.6806 [8.8933]
$\mathbb{V}_{O[E]\,(200K)}$	↗	9.745 [1.2783]	10.91 [1.3973]	10.98 [1.4035]
$\xi_{po[E](T=250K)}$	↘	40.764 [7.2164]	40.567 [7.1832]	40.555 [7.1814]
$\mathbb{V}_{O[E]\,(250K)}$	↗	4.9439 [0.5162]	5.5293 [0.5642]	5.5605 [0.5667]
$\xi_{po[E](T=300K)}$	↘	33.981 [6.0791]	33.817 [6.0516]	33.807 [6.0500]
$\mathbb{V}_{O[E]\,(300K)}$	↗	2.5955 [0.2442]	2.9004 [0.2669]	2.9166 [0.2680]

For **x=1** and **N=**$\mathbf{10^{20}}$ $\boldsymbol{cm^{-3}}$,

N_{CDp} in 10^{18} cm^{-3}	↗	**5.4298**	**6.6481**	**6.7170**
$E_{FpO[E](T=0K)}$(eV)	↘	**1.1344 [0.1514]**	**1.1246 [0.1501]**	**1.1241 [0.1500]**
$\mathbb{V}_{O[E]\,(0K)}$	↗	**34.980 [1062.02]**	**39.3621 [1164.47]**	**39.5978 [1169.85]**
$\xi_{po[E](T=4.2K)}$	↘	3133.5 [418.2]	3106.6 [414.57]	3105.0 [414.37]
$\mathbb{V}_{O[E]\,(4.2K)}$	↗	34.9800 [1061.90]	39.3621 [1164.34]	39.5978 [1169.72]
$\xi_{po[E](T=77K)}$	↘	170.928 [22.8635]	169.457 [22.6677]	169.373 [22.6566]
$\mathbb{V}_{O[E]\,(77K)}$	↗	34.17 [146.15]	38.43 [157.87]	38.66 [158.46]
$\xi_{po[E](T=100K)}$	↘	131.618 [17.6338]	130.485 [17.4833]	130.421 [17.4748]
$\mathbb{V}_{O[E]\,(100K)}$	↗	32.67 [56.724]	36.72 [61.179]	36.94 [61.404]
$\xi_{po[E](T=150K)}$	↘	87.753 [11.816]	86.9983 [11.716]	86.9555 [11.710]
$\mathbb{V}_{O[E]\,(150K)}$	↗	25.40 [11.706]	28.44 [12.614]	28.61 [12.659]
$\xi_{po[E](T=200K)}$	↘	65.8232 [8.9259]	65.2570 [8.8517]	65.2249 [8.8475]
$\mathbb{V}_{O[E]\,(200K)}$	↗	15.73 [3.6973]	17.53 [3.9825]	17.63 [3.9968]
$\xi_{po[E](T=250K)}$	↘	52.67 [7.2072]	52.214 [7.1484]	52.188 [7.1451]

$\mathbb{V}_{O[E]\,(250K)}$	↗	8.6721 [1.4942]	9.6320 [1.6089]	9.6825 [1.6147]
$\xi_{po[E](T=300K)}$	↘	43.898 [6.0715]	43.520 [6.0227]	43.499 [6.0200]
$\mathbb{V}_{O[E]\,(300K)}$	↗	4.7545 [0.7071]	5.2706 [0.7611]	5.2977 [0.7638]

Table 11n: For given x, r_d and N, the numerical results of $\xi_{nO[E]}$, $\mathbb{V}_{O[E]}$, $AE_{O[E]}$, $\sigma_{Th.O[E]}$, $S_{O[E]}$, $VC1_{O[E]}$, $VC2_{O[E]}$, $Ts_{O[E]}$, $Pt_{O[E]}$ and $ZT_{O[E]}$ are obtained, as functions of T=(4.2K, 77K), by using Equations (11, 22b, 22c, 21, 25, 27, 28, 29, 30, and 26), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(T)$, one observes that, for such given (x, r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and from those of $\mathbb{V}_{O[E]}(r_d)$, one observes that, for given (x, T and N), they increase with increasing r_d, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Donor		Te	Sb	Sn
r_d (nm) [4]	↗	0.132	0.136	0.140
For **x=0** and **N=3 × 10^{19} cm^{-3}**,				
N_{CDn} in 10^{16} cm^{-3}	↗	**3.0014**	**3.3725**	**3.8688**
$\xi_{nO[E](T=4.2K)}$	↘	**1403.81 [1079.86]**	**1403.70 [1079.77]**	**1403.54 [1079.65]**
$\xi_{nO[E](T=77K)}$	↘	76.5878 [58.9223]	76.5815 [58.9174]	76.5731 [58.9109]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**22.9707 [36.660]**	**24.6271 [39.260]**	**26.727 [42.548]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	20.696 [33.391]	22.188 [35.759]	24.079 [38.753]
$-AE_{O[E](4.2K)}$ (meV× 10^{-6})	↗	0.6017 [0.9237]	0.6018 [0.9238]	0.6019 [0.9241]
$-AE_{O[E](77K)}$ (meV)	↗	0.6919 [0.6197]	0.6920 [0.6198]	0.6922 [0.6199]
$\sigma_{Th.O[E]\,(4.2K)}\left(\frac{10^{-2}\times W}{cm\times K}\right)$	↘	0.5697 [0.3570]	0.5313 [0.3333]	0.4895 [0.3074]
$\sigma_{Th.O[E]\,(77K)}\left(\frac{10^{-1}\times W}{cm\times K}\right)$	↘	1.1593 [0.7185]	1.0812 [0.6709]	0.9961 [0.6189]
$-S_{O[E]\,(4.2K)}\left(\frac{10^{-7}\times V}{K}\right)$	↗	4.0389 [5.2506]	4.0392 [5.2510]	4.0397 [5.2516]
$-S_{O[E]\,(77K)}\left(\frac{10^{-6}\times V}{K}\right)$	↗	7.3990 [9.6136]	7.3996 [9.6144]	7.4004 [9.6155]
$-VC1_{O[E]\,(4.2K)}\left(\frac{10^{-7}\times V}{K}\right)$	↗	2.6926 [3.5004]	2.6928 [3.5006]	2.6931 [3.5010]
$-VC1_{O[E]\,(77K)}\left(\frac{10^{-6}\times V}{K}\right)$	↗	4.9250 [6.3924]	4.9255 [6.3929]	4.9260 [6.3936]
$-VC2_{O[E]\,(4.2K)}\left(\frac{10^{-7}\times V}{K}\right)$	↗	11.309 [14.701]	11.310 [14.702]	11.311 [14.704]
$-VC2_{O[E]\,(77K)}\left(\frac{10^{-4}\times V}{K}\right)$	↗	3.7923 [4.9221]	3.7926 [4.9225]	3.7930 [4.9231]
$-Ts_{O[E]\,(4.2K)}\left(\frac{10^{-7}\times V}{K}\right)$	↗	4.0389 [5.2505]	4.0392 [5.2510]	4.0397 [5.2516]
$-Ts_{O[E]\,(77K)}\left(\frac{10^{-6}\times V}{K}\right)$	↗	7.3876 [9.5886]	7.3882 [9.5893]	7.3890 [9.5904]
$-Pt_{O[E]\,(4.2K)}(10^{-7}\times V)$	↗	16.963 [22.052]	16.965 [22.054]	16.967 [22.057]
$-Pt_{O[E]\,(77K)}(10^{-4}\times V)$	↗	5.6972 [7.4025]	5.6977 [7.4031]	5.6983 [7.4039]

$ZT_{O[E]\,(4.2K)}\ (10^{-6})$	↗	6.6775 [11.285]	6.6786 [11.286]	6.6801 [11.289]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	22.409 [37.832]	22.413 [37.838]	22.418 [37.846]

For **x=0.5** and **N=7** $\times$ $\mathbf{10^{19}\ cm^{-3}}$,

$\mathbf{N_{CDn}}$ **in** $\mathbf{10^{16}\ cm^{-3}}$	↗	**2.9843**	**3.3532**	**3.8466**
$\xi_{nO[E](T=4.2K)}$	↘	**2475.96 [2048.36]**	**2475.87 [2048.29]**	**2475.75 [2048.19]**
$\xi_{nO[E](T=77K)}$	↘	135.061 [111.740]	135.056 [111.736]	135.050 [111.731]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**36.595 [51.535]**	**39.263 [55.257]**	**42.650 [59.977]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	35.341 [49.958]	37.918 [53.567]	41.189 [58.142]
$-AE_{O[E](4.2K)}\ (meV\times 10^{-6})$	↗	0.1935 [0.2637]	0.1936 [0.2638]	0.1937 [0.2639]
$-AE_{O[E](77K)}\ (meV)$	↗	0.2312 [0.2061]	0.2313 [0.2062]	0.2314 [0.2063]
$\sigma_{Th.O[E]\,(4.2K)}\ (\frac{10^{-2}\times W}{cm\times K})$	↘	1.1076 [0.7865]	1.0323 [0.7335]	0.9502 [0.6757]
$\sigma_{Th.O[E]\,(77K)}\ (\frac{10^{-1}\times W}{cm\times K})$	↘	2.1027 [1.4875]	1.9597 [1.3872]	1.8039 [1.2779]
$-S_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.2900 [2.7680]	2.2901 [2.7681]	2.2902 [2.7682]
$-S_{O[E]\,(77K)}\ (\frac{10^{-6}\times V}{K})$	↗	4.1973 [5.0729]	4.1974 [5.0730]	4.1976 [5.0733]
$-VC1_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.5266 [1.8453]	1.5267 [1.8454]	1.5268 [1.8455]
$-VC1_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	2.7968 [3.3794]	2.7969 [3.3796]	2.7970 [3.3797]
$-VC2_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	6.4119 [7.7504]	6.4121 [7.7507]	6.4125 [7.7511]
$-VC2_{O[E]\,(77K)}\ \left(\frac{10^{-4}\times V}{K}\right)$	↗	2.1535 [2.6022]	2.1536 [2.6023]	2.1537 [2.6024]
$-Ts_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.2900 [2.7680]	2.2901 [2.7681]	2.2902 [2.7682]
$-Ts_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	4.1952 [5.0692]	4.1953 [5.0693]	4.1955 [5.0696]
$-Pt_{O[E]\,(4.2K)}(10^{-7}\times V)$	↗	9.6179 [11.625]	9.6183 [11.626]	9.6188 [11.627]
$-Pt_{O[E]\,(77K)}(10^{-4}\times V)$	↗	3.2319 [3.9061]	3.2320 [3.9062]	3.2322 [3.9065]
$ZT_{O[E]\,(4.2K)}\ (10^{-6})$	↗	2.1466 [3.1363]	2.1467 [3.1365]	2.1469 [3.1369]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	7.2114 [10.534]	7.2119 [10.535]	7.2126 [10.536]

For **x=1** and **N=**$\mathbf{10^{20}}$ $\boldsymbol{cm^{-3}}$,

$\mathbf{N_{CDn}}$ **in** $\mathbf{10^{16}\ cm^{-3}}$	↗	**2.9644**	**3.3308**	**3.8210**
$\xi_{nO[E](T=4.2K)}$	↘	**3251.72 [2817.79]**	**3251.65 [2817.72]**	**3251.54 [2817.62]**
$\xi_{nO[E](T=77K)}$	↘	177.374 [153.705]	177.369 [153.701]	177.363 [153.697]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**45.643 [59.182]**	**48.981 [63.484]**	**53.221 [68.942]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	44.658 [58.045]	47.925 [62.264]	52.074 [67.617]
$-AE_{O[E](4.2K)}\ (meV\times 10^{-6})$	↗	0.1142 [0.14415]	0.1143 [0.14416]	0.1144 [0.14417]
$-AE_{O[E](77K)}\ (meV)$	↗	0.1445 [0.12873]	0.1446 [0.12874]	0.1447 [0.12875]
$\sigma_{Th.O[E]\,(4.2K)}\ (\frac{10^{-2}\times W}{cm\times K})$	↘	1.4291 [1.1021]	1.3316 [1.0274]	1.2255 [0.9460]

$\sigma_{Th.O[E]\,(77K)}\,(\frac{10^{-1}\times W}{cm\times K})$	↘	2.6777 [2.0602]	2.4951 [1.9205]	2.2962 [1.7683]
$-S_{O[E]\,(4.2K)}\,(\frac{10^{-7}\times V}{K})$	↗	1.7436 [2.0121]	1.7437 [2.0122]	1.7438 [2.0123]
$-S_{O[E]\,(77K)}\,(\frac{10^{-6}\times V}{K})$	↗	3.1962 [3.6883]	3.1963 [3.6884]	3.1964 [3.6885]
$-VC1_{O[E]\,(4.2K)}\,(\frac{10^{-7}\times V}{K})$	↗	1.1614 [1.3414]	1.1624 [1.3415]	1.1625 [1.3416]
$-VC1_{O[E]\,(77K)}\,\left(\frac{10^{-6}\times V}{K}\right)$	↗	2.1302 [2.4579]	2.1303 [2.4580]	2.1304 [2.4581]
$-VC2_{O[E]\,(4.2K)}\,(\frac{10^{-7}\times V}{K})$	↗	4.8822 [5.6341]	4.8823 [5.6342]	4.8825 [5.6344]
$-VC2_{O[E]\,(77K)}\,\left(\frac{10^{-4}\times V}{K}\right)$	↗	1.6402 [1.8925]	1.6403 [1.8926]	1.6404 [1.8927]
$-Ts_{O[E]\,(4.2K)}\,(\frac{10^{-7}\times V}{K})$	↗	1.7436 [2.0121]	1.7437 [2.0122]	1.7438 [2.0123]
$-Ts_{O[E]\,(77K)}\,\left(\frac{10^{-6}\times V}{K}\right)$	↗	3.1953 [3.6869]	3.1954 [3.6870]	3.1955 [3.6871]
$-Pt_{O[E]\,(4.2K)}(10^{-7}\times V)$	↗	7.3234 [8.4512]	7.3236 [8.4514]	7.3238 [8.4517]
$-Pt_{O[E]\,(77K)}(10^{-4}\times V)$	↗	2.4611 [2.8400]	2.4612 [2.8401]	2.4613 [2.8402]
$ZT_{O[E]\,(4.2K)}\,(10^{-6})$	↗	1.2445 [1.6573]	1.2446 [1.6574]	1.2447 [1.6576]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	4.1818 [5.5685]	4.1820 [5.5687]	4.1823 [5.5691]

Table 11p: For given x, r_a and N, the numerical results of $\xi_{pO[E]}$, $\mathbb{V}_{O[E]}$, $AE_{O[E]}$, $\sigma_{Th.O[E]}$, $S_{O[E]}$, $VC1_{O[E]}$, $VC2_{O[E]}$, $Ts_{O[E]}$, $Pt_{O[E]}$ and $ZT_{O[E]}$ are obtained, as functions of T=(4.2K, 77K), by using Equations (11, 22b, 22c, 21, 25, 27, 28, 29, 30, and 26), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(T)$, one observes that, for such given (x, r_a and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and from those of $\mathbb{V}_{O[E]}(r_a)$, one observes that, for given (x, T and N), they increase with increasing r_a, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor		**Ga**	Mg	In
r_a (nm)	↗	**0.126**	0.140	0.144
For **x=0** and **N=**$\mathbf{6\times10^{19}}\ \boldsymbol{cm^{-3}}$,				
$\mathbf{N_{CDp}}$ **in** $\mathbf{10^{17}\ cm^{-3}}$	↗	**7.4367**	**9.1053**	**9.1996**
$\xi_{pO[E](T=4.2K)}$	↘	**2211.44 [510.335]**	**2207.29 [509.376]**	**2207.05 [509.322]**
$\xi_{pO[E](T=77K)}$	↘	120.6 [27.881]	120.4 [27.828]	120.3 [27.826]
$\mathbb{V}_{O[E]\,(4.2K)}\,\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**23.61 [303.64]**	**26.673 [337.97]**	**26.839 [339.81]**
$\mathbb{V}_{O[E]\,(77K)}\,\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	22.61 [23.98]	25.54 [26.60]	25.70 [26.74]
$-AE_{O[E](4.2K)}\,(eV\times10^{-9})$	↗	0.2424 [69.41]	0.2433 [69.67]	0.2434 [69.69]
$-AE_{O[E](77K)}\,(eV\times10^{-3})$	↗	0.2875 [16.84]	0.2885 [16.868]	0.2886 [16.869]
$\sigma_{Th.O[E]\,(4.2K)}\,(\frac{10^{-2}\times W}{cm\times K})$	↘	1.3753 [0.1070]	1.2130 [0.0957]	1.2053 [0.0952]

$\sigma_{Th.O[E]\,(77K)}\ (\frac{10^{-1}\times W}{cm\times K})$	↘	2.6331 [2.4827]	2.3227 [2.2298]	2.3079 [2.2178]
$-S_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.5639 [11.110]	2.5687 [11.131]	2.5690 [11.132]
$-S_{O[E]\,(77K)}\ (\frac{10^{-6}\times V}{K})$	↗	4.6990 [20.250]	4.7078 [20.288]	4.7084 [20.290]
$-VC1_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.7092 [7.4064]	1.7125 [7.4204]	1.7127 [7.4212]
$-VC1_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	3.1307 [13.344]	3.1366 [13.368]	3.1369 [13.369]
$-VC2_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	7.1789 [31.107]	7.1924 [31.165]	7.1932 [31.169]
$-VC2_{O[E]\,(77K)}\ \left(\frac{10^{-4}\times V}{K}\right)$	↗	2.4106 [10.274]	2.4152 [10.293]	2.4154 [10.294]
$-Ts_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.5639 [11.110]	2.5687 [11.130]	2.5690 [11.132]
$-Ts_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	4.6961 [20.015]	4.7049 [20.052]	4.7054 [20.054]
$-Pt_{O[E]\,(4.2K)}(10^{-6}\times V)$	↗	1.0768 [4.6662]	1.0788 [4.6750]	1.0790 [4.6755]
$-Pt_{O[E]\,(77K)}(10^{-4}\times V)$	↗	3.6182 [15.593]	3.6250 [15.622]	3.6254 [15.624]
$ZT_{O[E]\,(4.2K)}\ (10^{-6})$	↗	2.6908 [50.526]	2.7009 [50.716]	2.7015 [50.727]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	9.0386 [167.86]	9.0726 [168.49]	9.0745 [168.52]

For **x=0.5** and **N=7** $\times$ **10^{19} cm^{-3}**,

N_{CDp} in 10^{18} cm^{-3}	↗	**2.1947**	**2.6871**	**2.7150**
$\xi_{pO[E](T=4.2K)}$	↘	**2424.6 [418.733]**	**2412.9 [416.703]**	**2412.2 [416.588]**
$\xi_{pO[E](T=77K)}$	↘	132.26 [22.894]	131.62 [22.783]	131.58 [22.777]
$\mathbb{V}_{O[E]\,(4.2K)}\ \left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**28.330 [571.29]**	**31.940 [630.77]**	**32.135 [633.92]**
$\mathbb{V}_{O[E]\,(77K)}\ \left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	27.319 [53.093]	30.790 [58.103]	30.977 [58.364]
$-AE_{O[E](4.2K)}\ (eV\times 10^{-9})$	↗	0.2019 [60.22]	0.2039 [60.81]	0.2040 [60.84]
$-AE_{O[E](77K)}\ (eV\times 10^{-3})$	↗	0.2410 [15.76]	0.2433 [15.82]	0.2435 [15.83]
$\sigma_{Th.O[E]\,(4.2K)}\ (\frac{10^{-2}\times W}{cm\times K})$	↘	1.3721 [0.0680]	1.2052 [0.0610]	1.1973 [0.0607]
$\sigma_{Th.O[E]\,(77K)}\ (\frac{10^{-1}\times W}{cm\times K})$	↘	2.6085 [1.3422]	2.2921 [1.2146]	2.2770 [1.2085]
$-S_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.3385 [13.540]	2.3498 [13.606]	2.3505 [13.610]
$-S_{O[E]\,(77K)}\ (\frac{10^{-6}\times V}{K})$	↗	4.2861 [24.611]	4.3069 [24.729]	4.3082 [24.736]
$-VC1_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.5590 [9.0265]	1.5665 [9.0704]	1.5670 [9.0729]
$-VC1_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	2.8559 [16.125]	2.8698 [16.200]	2.8706 [16.204]
$-VC2_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	6.5477 [37.911]	6.5796 [38.096]	6.5814 [38.106]
$-VC2_{O[E]\,(77K)}\ \left(\frac{10^{-4}\times V}{K}\right)$	↗	2.1990 [12.417]	2.2097 [12.474]	2.2104 [12.477]
$-Ts_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	2.3384 [13.539]	2.3498 [13.605]	2.3505 [13.609]
$-Ts_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	4.2839 [24.188]	4.3047 [24.300]	4.3059 [24.307]

$-Pt_{O[E]\,(4.2K)}(10^{-6} \times V)$	↗	0.9821 [5.6870]	0.9869 [5.7147]	0.9872 [5.7163]
$-Pt_{O[E]\,(77K)}(10^{-4} \times V)$	↗	3.3003 [18.951]	3.3163 [19.041]	3.3173 [19.047]
$ZT_{O[E]\,(4.2K)}\ (10^{-6})$	↗	2.2384 [75.049]	2.2603 [75.782]	2.2616 [75.824]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	7.5198 [247.94]	7.5932 [250.32]	7.5974 [250.46]

For **x=1** and **N=10^{20}** cm^{-3},

N_{CDp} in 10^{18} cm^{-3}	↗	**5.4298**	**6.6481**	**6.7170**
$\xi_{pO[E](T=4.2K)}$	↘	**3133.5 [418.171]**	**3106.6 [414.572]**	**3105.0 [414.368]**
$\xi_{pO[E](T=77K)}$	↘	170.93 [22.863]	169.46 [22.667]	169.37 [22.656]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$	↗	**34.980 [1061.9]**	**39.362 [1164.3]**	**39.598 [1169.7]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$	↗	34.169 [146.15]	38.434 [157.87]	38.663 [158.46]
$-AE_{O[E](4.2K)}\ (eV\times 10^{-9})$	↗	0.1230 [40.48]	0.1252 [41.19]	0.1253 [41.23]
$-AE_{O[E](77K)}\ (eV\times 10^{-3})$	↗	0.1556 [13.16]	0.1583 [13.259]	0.1585 [13.265]
$\sigma_{Th\,.O[E]\,(4.2K)}\ (\frac{10^{-2}\times W}{cm\times K})$	↘	1.7316 [0.0570]	1.5125 [0.0511]	1.5020 [0.0508]
$\sigma_{Th\,.O[E]\,(77K)}\ (\frac{10^{-1}\times W}{cm\times K})$	↘	3.2500 [0.7598]	2.8399 [0.6913]	2.8202 [0.6881]
$-S_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.8094 [13.558]	1.8251 [13.676]	1.8260 [13.683]
$-S_{O[E]\,(77K)}\ (\frac{10^{-6}\times V}{K})$	↗	3.3167 [24.644]	3.3455 [24.854]	3.3472 [24.866]
$-VC1_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.2063 [9.0386]	1.2167 [9.1170]	1.2173 [9.1215]
$-VC1_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	2.2105 [16.146]	2.2296 [16.279]	2.2308 [16.287]
$-VC2_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	5.0664 [37.962]	5.1104 [38.291]	5.1129 [38.310]
$-VC2_{O[E]\,(77K)}\ \left(\frac{10^{-4}\times V}{K}\right)$	↗	1.7021 [12.432]	1.7168 [12.535]	1.7177 [12.541]
$-Ts_{O[E]\,(4.2K)}\ (\frac{10^{-7}\times V}{K})$	↗	1.8094 [13.558]	1.8251 [13.675]	1.8260 [13.682]
$-Ts_{O[E]\,(77K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↗	3.3157 [24.219]	3.3445 [24.418]	3.3461 [24.430]
$-Pt_{O[E]\,(4.2K)}(10^{-6} \times V)$	↗	0.7599 [5.6946]	0.7665 [5.7440]	0.7669 [5.7469]
$-Pt_{O[E]\,(77K)}(10^{-4} \times V)$	↗	2.5539 [18.976]	2.5761 [19.137]	2.5773 [19.147]
$ZT_{O[E]\,(4.2K)}\ (10^{-6})$	↗	1.3402 [75.251]	1.3635 [76.563]	1.3649 [76.639]
$ZT_{O[E]\,(77K)}(10^{-4})$	↗	4.5031 [248.60]	4.5816 [252.86]	4.5861 [253.10]

Table 12n: For given x, r_d and T=0K, the numerical results of Fermi energy $E_{Fn-O[E]}(eV)$, $\mathbb{V}_{O[E]}(\frac{eV}{cm}\times\frac{s}{cm^2})$, $\sigma_{O[E]}(\frac{10^4}{\Omega\times cm})$, $\mu_{O[E]}(\frac{10^4\times cm^2}{V\times s})$, and $D_{O[E]}\left(\frac{10^4\times cm^2}{s}\right)$, are obtained, as functions of **N=[$\cong$ N_{CDn}, N1= $5\times10^{18}\ cm^{-3}$, N2 $=5\times10^{19}$, N3 $=10^{20}\ cm^{-3}$**], by using Equations (11, 22b, 20a, 22a, 24), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions, (T=0K, N=N_{CDn} or $N^*=0$), and (ii) those values of $E_{Fn-O[E]}\gtrsim0$, $\mathbb{V}_{O[E]}\gtrsim0$, $\sigma_{O[E]}\gtrsim0$, $\mu_{O[E]}\gtrsim0$, and $D_{O[E]}\gtrsim0$, obtained for $N\gtrsim N_{CDn}$, thus define the properties of **the degenerate (or viscous) X-crystalline alloy**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}_{O[E]}$, one observes that, for such given (x, r_d and T=0K), they **increase with increasing N (or increasing $E_{Fn-O[E]}$)**, in good agreement with those, obtained in complex fluids by Wenhao[18], suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

$N\gtrsim N_{CDn}$		$\cong N_{CDn}$	N1	N2	N3
For x=0, $r_d=r_{Te}$ and N_{CDn}=$3.0014\times10^{16}\ cm^{-3}$,					
$E_{Fn-O[E]}$	↗	≅0 [0]	0.15339 [0.11800]	0.71457 [0.54967]	1.13454 [0.87273]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	10.93 [17.06]	28.17 [45.16]	37.01 [59.63]
$\sigma_{O[E]}$	↗	≅0 [0]	1.063 [0.681]	8.953 [5.585]	17.18 [10.66]
$\mu_{O[E]}$	↘	≅0 [0]	1.336 [0.855]	1.118 [0.697]	1.072 [0.666]
$D_{O[E]}$	↗	≅0 [0]	0.136 [0.067]	0.532 [0.255]	0.811 [0.387]
For x=0, $r_d=r_{Sn}$ and N_{CDn}=$3.8688\times10^{16}\ cm^{-3}$,					
$E_{Fn-O[E]}$	↗	≅0 [0]	0.15322 [0.11786]	0.71449 [0.54961]	1.13448 [0.87268]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	12.63 [19.62]	32.82 [52.50]	43.19 [69.46]
$\sigma_{O[E]}$	↗	≅0 [0]	0.918 [0.591]	7.682 [4.802]	14.72 [9.151]
$\mu_{O[E]}$	↘	≅0 [0]	1.155 [0.743]	0.960 [0.600]	0.919 [0.571]
$D_{O[E]}$	↗	≅0 [0]	0.118 [0.058]	0.457 [0.220]	0.695 [0.332]
For x=0.5, $r_d=r_{Te}$ and N_{CDn}=$2.9843\times10^{16}\ cm^{-3}$,					
$E_{Fn-O[E]}$	↗	≅0 [0]	0.15373 [0.12718]	0.71614 [0.59246]	1.13703 [0.94066]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	12.35 [17.01]	32.02 [45.00]	42.12 [59.42]
$\sigma_{O[E]}$	↗	≅0 [0]	0.941 [0.683]	7.876 [5.604]	15.09 [10.70]
$\mu_{O[E]}$	↘	≅0 [0]	1.182 [0.858]	0.984 [0.700]	0.942 [0.668]
$D_{O[E]}$	↗	≅0 [0]	0.121 [0.073]	0.469 [0.276]	0.714 [0.419]
For x=0.5, $r_d=r_{Sn}$ and N_{CDn}=$3.8466\times10^{16}\ cm^{-3}$,					
$E_{Fn-O[E]}$	↗	≅0 [0]	0.15355 [0.12704]	0.71606 [0.59239]	1.13696 [0.94061]

$\mathbb{V}_{O[E]}$	↗	≅0 [0]	14.24 [19.56]	37.28 [52.32]	49.13 [69.22]
$\sigma_{O[E]}$	↗	≅0 [0]	0.814 [0.593]	6.762 [4.819]	12.94 [9.183]
$\mu_{O[E]}$	↘	≅0 [0]	1.024 [0.746]	0.845 [0.602]	0.808 [0.573]
$D_{O[E]}$	↗	≅0 [0]	0.105 [0.063]	0.403 [0.238]	0.612 [0.359]

For x=1, $\mathbf{r_d = r_{Te}}$ and $\mathbf{N_{CDn}=2.9644 \times 10^{16}\ cm^{-3}}$,

$E_{Fn-O[E]}$	↗	≅0 [0]	0.15916 [0.13792]	0.74142 [0.64248]	1.17716 [1.02007]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	13.31 [16.94]	34.67 [44.82]	45.64 [59.18]
$\sigma_{O[E]}$	↗	≅0 [0]	0.873 [0.686]	7.274 [5.626]	13.93 [10.74]
$\mu_{O[E]}$	↘	≅0 [0]	1.096 [0.861]	0.908 [0.703]	0.869 [0.671]
$D_{O[E]}$	↗	≅0 [0]	0.116 [0.079]	0.449 [0.301]	0.682 [0.456]

For x=1, $\mathbf{r_d = r_{Sn}}$ and $\mathbf{N_{CDn}=3.8210 \times 10^{16}\ cm^{-3}}$,

$E_{Fn-O[E]}$	↗	≅0 [0]	0.15898 [0.13777]	0.74133 [0.64240]	1.17710 [1.02001]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	15.34 [19.49]	40.36 [52.11]	53.22 [68.94]
$\sigma_{O[E]}$	↗	≅0 [0]	0.755 [0.595]	6.248 [4.838]	11.94 [9.220]
$\mu_{O[E]}$	↘	≅0 [0]	0.950 [0.748]	0.780 [0.604]	0.745 [0.576]
$D_{O[E]}$	↗	≅0 [0]	0.101 [0.069]	0.386 [0.259]	0.585 [0.391]

Table 12p: For given x, r_a and T=0K, the numerical results of Fermi energy $E_{Fp-O[E]}$(eV), $\mathbb{V}_{O[E]}(\frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma_{O[E]}(\frac{10^4}{\Omega \times cm})$, $\mu_{O[E]}(\frac{10^4 \times cm^2}{V \times s})$, and $D_{O[E]}\left(\frac{10^4 \times cm^2}{s}\right)$, are obtained, as functions of **N=[** $\cong$ $\mathbf{N_{CDp}}$**,** $\mathbf{N1=7 \times 10^{19}\ cm^{-3}}$**,** $\mathbf{N2 = 10^{20}}$**,** $\mathbf{N3 = 2 \times 10^{20}\ cm^{-3}}$], by using Equations (11, 22b, 20a, 22a, 24), respectively. It should be noted that (i) they are cancelled at the MIT-conditions , (T=0K, N=$\mathbf{N_{CDp}}$ or $N^* = 0$), and (ii) those values of $E_{Fp-O[E]} \gtrsim 0$, $\mathbb{V}_{O[E]} \gtrsim 0$, $\mu_{O[E]} \gtrsim 0$, and $D_{O[E]} \gtrsim 0$, obtained for $N \gtrsim \mathbf{N_{CDp}}$, thus define the properties of **the degenerate (or viscous) X-crystalline alloy**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}_{O[E]}$, one observes that, for such given (x, r_a and T=0K), they **increase with increasing N (or increasing $\mathbf{E_{Fp-O[E]}}$**), in good agreement with those, obtained in complex fluids by Wenhao[18], suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

$N \gtrsim \mathbf{N_{CDp}}$		$\cong \mathbf{N_{CDp}}$	N1	N2	N3
For x=0, $\mathbf{r_a = r_{Ga}}$ and $\mathbf{N_{CDp}=7.4367 \times 10^{17}\ cm^{-3}}$,					
$E_{Fp-O[E]}$	↗	≅0 [0]	0.8883 [0.20499]	1.1291 [0.26057]	1.7969 [0.41466]
$\mathbb{V}_{O[E]}$	↗	≅0 [0]	25.10 [326.2]	28.88 [384.0]	37.81 [521.9]
$\sigma_{O[E]}$	↗ [...]	≅0 [0]	15.52 [1.194]	21.80 [1.640]	42.17 [3.055]
$\mu_{O[E]}$	↘	≅0 [0]	1.399 [0.108]	1.371 [0103]	1.321 [0.096]

$D_{O[E]}$	↗	$\cong$0 [0]	0.828 [0.014]	1.032 [0.018]	1.582 [0.026]
For x=0, $r_a = r_{In}$ and N_{CDp}=9.1996×10^{17} cm^{-3},					
$E_{Fp-O[E]}$	↗	$\cong$0 [0]	0.8868 [0.20464]	1.1278 [0.26026]	1.7958 [0.41442]
$\mathbb{V}_{O[E]}$	↗	$\cong$0 [0]	28.54 [365.5]	32.86 [431.2]	43.07 [588.3]
$\sigma_{O[E]}$	↗ [...]	$\cong$0 [0]	13.61 [1.062]	19.11 [1.457]	36.97 [2.707]
$\mu_{O[E]}$	↘	$\cong$0 [0]	1.229 [0.096]	1.204 [0.092]	1.159 [0.085]
$D_{O[E]}$	↗	$\cong$0 [0]	0.727 [0.013]	0.905 [0.016]	1.387 [0.023]
For x=0.5, $r_a = r_{Ga}$ and N_{CDp}=2.1947×10^{18} cm^{-3},					
$E_{Fp-O[E]}$	↗	$\cong$0 [0]	0.8777 [0.1516]	1.1205 [0.1935]	1.7920 [0.3095]
$\mathbb{V}_{O[E]}$	↗	$\cong$0 [0]	28.33 [571.4]	32.69 [682.2]	42.96 [948.5]
$\sigma_{O[E]}$	↗	$\cong$0 [0]	13.37 [0.663]	18.88 [0.905]	36.75 [1.665]
$\mu_{O[E]}$	↘	$\cong$0 [0]	1.231 [0.061]	1.205 [0.058]	1.159 [0.052]
$D_{O[E]}$	↗	$\cong$0 [0]	0.720 [0.006]	0.900 [0.007]	1.385 [0.011]
For x=0.5, $r_a = r_{In}$ and N_{CDp}=2.7150×10^{18} cm^{-3},					
$E_{Fp-O[E]}$	↗	$\cong$0 [0]	0.8732 [0.15081]	1.1166 [0.19283]	1.7889 [0.30894]
$\mathbb{V}_{O[E]}$	↗	$\cong$0 [0]	32.13 [634.0]	37.14 [759.8]	48.90 [1062.4]
$\sigma_{O[E]}$	↗	$\cong$0 [0]	11.67 [0.591]	16.50 [0.807]	32.18 [1.481]
$\mu_{O[E]}$	↘	$\cong$0 [0]	1.082 [0.055]	1.059 [0.052]	1.018 [0.047]
$D_{O[E]}$	↗	$\cong$0 [0]	0.630 [0.005]	0.788 [0.007]	1.214 [0.009]
For x=1, $r_a = r_{Ga}$ and N_{CDp}=5.4298×10^{18} cm^{-3},					
$E_{Fp-O[E]}$	↗	$\cong$0 [0]	0.8796 [0.11738]	1.1344 [0.15138]	1.8350 [0.24488]
$\mathbb{V}_{O[E]}$	↗	$\cong$0 [0]	30.11 [872.5]	34.98 [1062.0]	46.30 [1519.6]
$\sigma_{O[E]}$	↗	$\cong$0 [0]	11.79 [0.407]	16.88 [0.556]	33.36 [1.017]
$\mu_{O[E]}$	↘	$\cong$0 [0]	1.139 [0.039]	1.114 [0.037]	1.070 [0.033]
$D_{O[E]}$	↗	$\cong$0 [0]	0.668 [0.003]	0.842 [0.004]	1.309 [0.005]
For x=1, $r_a = r_{In}$ and N_{CDp}=6.717×10^{18} cm^{-3},					
$E_{Fp-O[E]}$	↗	$\cong$0 [0]	0.8678 [0.11581]	1.1241 [0.15000]	1.8269 [0.24380]
$\mathbb{V}_{O[E]}$	↗	$\cong$0 [0]	33.98 [955.0]	39.60 [1169.8]	52.62 [1688.6]
$\sigma_{O[E]}$	↗	$\cong$0 [0]	10.17 [0.362]	14.64 [0.495]	29.10 [0.907]
$\mu_{O[E]}$	↘	$\cong$0 [0]	1.003 [0.036]	0.979 [0.033]	0.939 [0.029]
$D_{O[E]}$	↗	$\cong$0 [0]	0.580 [0.002]	0.734 [0.003]	1.144 [0.005]

Table 13: Here, in the O-EP [E-OP] and for given physical conditions: x, $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that (i) for $\xi_{n(p)O[E]} \simeq 1.8138$, while the numerical results of $S_{O[E]}$ present a same minimum $S_{O[E]\,min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of $ZT_{O[E]}$ show a same maximum $ZT_{O[E]\,max.} = 1$, (ii) for $\xi_p = 1$, those of $S_{O[E]}$, $ZT_{O[E]}$, $ZT_{O[E]Mott}$, $VC1_{E[O]}$, and $Ts_{O[E]}$ present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_{n(p)O[E]} \simeq 1.8138$, $(ZT)_{O[E]\,Mott} = 1$.

$\xi_{n(p)O[E]}$ ↘	1.880 [1.880]		**1.8138 [1.8138]**		1.750 [1.750]		**1 [1]**		0.998 [0.998]
$S_{O[E]}\left(10^{-4}\frac{V}{K}\right)$	−1.562 [−1.562]	↘	**−1.563** [**−1.563**]	↗	−1.562 [−1.562]	↗	**−1.322** [**−1.322**]	↗	−1.320 [−1.320]
$ZT_{O[E]}$	0.999 [0.999]	↗	**1** [1]	↘	0.999 [0.999]	↘	**0.715** [**0.715**]	↘	0.713 [0.713]
$(ZT)_{O[E]\,Mott}$ ↗	0.931 [0.931]		**1** [1]		1.074 [1.074]		**3.290 [3.290]**		3.306 [3.306]
$VC1_{E[O]}\left(10^{-4}\frac{V}{K}\right)$	−0.061[−0.061]	↗	0 [0]	↗	0.063 [0.063]	↗	**1.105 [1.105]**	↗	1.109 [1.109]
$Ts_{O[E]}\left(10^{-4}\frac{V}{K}\right)$	−0.092 [−0.092]	↗	0 [0]	↗	0.094 [0.094]	↗	**1.657 [1.657]**	↗	1.663 [1.663]