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GLOBAL VAN-CONG EFFECTIVE KINETIC, COLOMBIAN POTENTIAL, AND TOTAL ENERGIES, OBTAINED IN ELECTRON-PHOTON INTERACTIONS, GIVEN IN THE IONIZED HYDROGEN ATOM, AND ENHANCED BY EMPIRICAL GLOBAL EFFECTIVE KINETIC ELECTRON MASS-AND-CHARGE, EXPRESSED AS FUNCTIONS OF ANY ELECTRON SPEED (42)

Prof. Dr. Huynh Van Cong*

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS), EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

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***Corresponding Author**

Prof. Dr. Huynh Van Cong

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS), EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

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ABSTRACT

In this work, we present the new expressions of global effective kinetic, Colombian potential and total energies, obtained in electron-photon interactions, given in the ionized hydrogen atom, and enhanced by a new global effective kinetic electron mass-and-charge, expressed as functions of any electron speed v. Some concluding remarks are given as follows. At the low speed v (<< c), c being the light speed, given in the non-relativistic case, they are found to be correct, since the numerical results of our global effective total energy, obtained in Eq. (14), for any orbital electron quantum number n and $p > 0$, are found to be approximately equal to the corresponding ones of the ground state Schrodinger energy, determined in Eq. (7), with a high precision, as shown in Table 1. Finally, at v=c, given in the relativistic case, the kinetic energy is found to be equal to: $m \times c^2 \times \left[\frac{2+p}{4}\right]$, for $p > 0$, and, in particular, for p=2, to: $m \times c^2$, in good accordance with that proposed by Einstein in 1905[2], as shown in Tables 2-7.

KEYWORDS: Orbital energy, global effective kinetic-potential-total energies, global effective kinetic electron mass-and-charge, relativity, non-relativity.

INTRODUCTION

First of all, at the electron speed v ($\ll$ c), $c = 2.9979 \times 10^8 (\frac{m}{s})$, being the light speed, it is interesting to present some important results of electron-photon interactions in ionized hydrogen atom, developed by Newton, Bohr, and Schrodinger[1], noting that various values of important parameters are given by : the rest electron mass m (=9.109×10^{-31} Kg), the rest electron charge -q (= -1.602×10^{-19} C), the permittivity of free space in vacuum $\varepsilon_o[= 8.854 \times 10^{-12} \left(\frac{C^2}{N\times m^2}\right)]$, the Coulomb constant $k_C \equiv \frac{1}{4\pi\varepsilon_o}[= 8.9875517923 \times 10^9 \text{ (Kg} \times m^3 \times s^{-4} \times A^{-2})]$, and finally the Planck's constant h [$= 6.625 \times 10^{-34}$ (J $\times$ s)].

Here, some important physical properties are found to be given as follows.
First, the Bohr radius, given for an orbital electron quantum number n (=1, 2, 3, … .), is found to be defined by :

$$r_B(n) \equiv n^2 \times \frac{h^2\times\varepsilon_o}{\pi\times m\times q^2} = n^2 \times 5.2913402 \times 10^{-11} \text{ (m)}. \quad (1)$$

From the 3rd Newton'law, the centrifugal force, $\frac{m\times v^2}{r}$, must constantly counter the centripetal electrical attractive force, $-k_C \times \frac{q^2}{r^2}$, as : $\frac{m\times v^2}{r} - k_C \times \frac{q^2}{r^2} = 0$, giving rise to :

$$m \times v^2 = k_C \times \frac{q^2}{r}. \quad (2)$$

Then, from Equations (1, 2), one can determine the Newton speed,

$$v_N(n) = \sqrt{\frac{k_C\times q^2}{r_B(n)\times m}} = \frac{1}{n} \times 2.18758543 \times 10^6 \left(\frac{m}{s}\right). \quad (3)$$

Therefore, the electron kinetic energy KE(n) can be determined by :

$$KE(n) = \frac{m\times v_N(n)^2}{2} = \frac{1}{n^2} \times 21.79569637 \times 10^{-19} \text{ (J)}, \quad (4)$$

the potential energy PE(n), as :

$$PE(n) = -\frac{k_C\times q^2}{r_B(n)} = -\frac{1}{n^2} \times 43.59139275 \times 10^{-19} \text{ (J)}, \quad (5)$$

and finally, from Equations (4, 5), the total energy TES(n), by :

$$\mathrm{TES(n)} = \mathrm{KE(n)} + \mathrm{PE(n)} = -\frac{1}{n^2} \times 21.79569637 \times 10^{-19}\ (\mathrm{J}). \quad (6)$$

noting that this TES(n)-result is given by Sauerheber[1]

It should be noted that this TES(n) can be compared with the ground state Schrodinger energy SE(n), for any n, as :

$$\mathrm{SE(n)} = -\frac{1}{n^2} \times \frac{m \times q^2}{8 \times \varepsilon_o{}^2 \times h^2} = -\frac{1}{n^2} \times 21.79615871 \times 10^{-19}\ (\mathrm{J}). \quad (7)$$

GLOBAL KINETIC, COULOMB POTENTIAL, AND TOTAL ENERGIES FOR ANY ELECTRON SPEED

First of all, from the expression of the kinetic energy of the electron, in motion with a speed v, being equal to the light speed c, $\frac{2m \times c^2}{2}$, proposed by Einstein in 1905 [2-5], we now propose a **gobal effective kinetic electron mass**, valid at any v, by[5] :

$$m^*(v; p > 0) = m \times (1 + p \times \frac{v^2}{c^2}) \quad (8)$$

and a **gobal effective kinetic electron charge**, valid at any v, as :

$$-q^*(v; p > 0) = -q \times (1 + p \times \frac{v^2}{c^2}), \quad (9)$$

which, in absolute values, increase with increasing v. Further, it should be noted that at lowest v: $m^*(v; p) \simeq m$ and $q^*(v; p) \simeq q$, given in the non-relativistic case, and then, at v=c, given in the relativistic case: $m^*(v = c; p) = m \times (1 + p)$ and $q^*(v = c; p) = q \times (1 + p)$.

Then, from Eq. (8), this electron is in motion, due to the **global effective Newton force**:

$$F^*_{GEN}(v; p) \equiv m^*(v; p) \times \frac{dv}{dt} = m \times \left(1 + p \times \frac{v^2}{c^2}\right) \times \frac{dv}{dt}, \quad (10)$$

where $\frac{dv}{dt}$ is its acceleration. Further, it is attracted by the photon of the charge (+q), due to the **global effective Coulombian force**, defined as:

$$F^*_{GEC}(v; p) \equiv -\frac{k_C \times q \times q^*(v; p)}{r^2} = -\frac{k_C \times q^2 \times (1 + p \times \frac{v^2}{c^2})}{r^2}. \quad (11)$$

From Eq. (10), noting that $\frac{dv}{dt} \times dr = vdv$, our **global effective kinetic energy** is determined by:

$$K^*(v;p) \equiv \int F^*_{GEN}(v;p) \times dr = \int m \times \left(1 + p \times \frac{v^2}{c^2}\right) \times vdv = m \times v^2 \times \left[\frac{2+p\times\frac{v^2}{c^2}}{4}\right]. \quad (12)$$

Eq. (12) is reduced to the correct result at the lowest v ($\ll$ c) as: $\frac{m\times v^2}{2}$, given in the non-relativistic case, and at v=c, to: $K_o(p) \equiv m \times c^2 \times \left[\frac{2+p}{4}\right]$, giving rise to a following assumption, by putting: x=$\frac{2+p}{4}$.

If x is known, then p is determined by: p=4x−2.

For example, **(i)** if x=3/4, then p=1, according to : $K_o(p = 1) = (\frac{3}{4}) \times mc^2$, **(ii)** if x=1, then p=2, according to : $\mathbf{K_o(p = 2) = mc^2}$, in good accordance with that proposed by Einstein in the relativistic case[2], (iii) if x=5/4, then p=3, according to : $K_o(p = 3) = (\frac{5}{4}) \times mc^2$, (iv) if x=3/2, then p=4, according to : $K_o(p = 4) = (\frac{3}{2}) \times mc^2$, and so on. This assumption is used to determine $K_o(p)$, given in next Tables 2-7 in Appendix 1.

Further, from Eq. (11), our global effective Coulombian Potential energy, $U^*(v, r; p)$, is defined by:

$$U^*(v, r; p) \equiv -\int_\infty^r F^*_{GEC}(v;p) \times dx \equiv -\int_\infty^r -\frac{k_C\times q^2\times(1+p\times\frac{v^2}{c^2})}{x^2} dx = -\frac{k_C\times q^2\times(1+p\times\frac{v^2}{c^2})}{r},$$

being reduced to the correct result at the lowest v ($\ll$ c) as: $= -\frac{k_C\times q^2}{r}$, given in the non-relativistic case, and at v=c, to: $-\frac{k_C\times q^2\times(1+p)}{r}$, given in the relativistic case. Here, we choose: $r = r_B(n)$, determined in Eq. (1), and $v_N(n) \le v \le c$, $v_N(n)$ being determined in Eq. (3).

That gives :

$$U^*(v; p, n) = -\frac{k_C\times q^2\times(1+p\times\frac{v^2}{c^2})}{r_B(n)}, \quad (13)$$

according at v=c to: $U_o(p, n) = -\frac{k_C\times q^2\times(1+p)}{r_B(n)}$, given in the relativistic case.

Finally, from Equations (12, 13), the global effective total energy, $E^*(v; p, n)$, is determined by :

$$E^*(v; p, n) = K^*(v; p) + U^*(v; p, n) = m \times v^2 \times \left[\frac{2+p\times\frac{v^2}{c^2}}{4}\right] - \frac{k_C\times q^2\times(1+p\times\frac{v^2}{c^2})}{r_B(n)}, \quad (14)$$

according at v=c to: $E_o(p, n) = m \times c^2 \times \left[\frac{2+p}{4}\right] - \frac{k_C\times q^2\times(1+p)}{r_B(n)}$, given in the relativistic case.

NUMERICAL RESULTS AND CONCLUDING REMARKS

The numerical results will be given and discussed in two following cases.

First, at v=$v_N(n)$, and for n=1, 2, 3, …, 35 and p=1-4, the relative deviations between $TES(n)$, determined in Eq. (6), and $SE(n)$, given in Eq. (7), is calculated by :

$$RD_S(n) = \left|1 - \frac{TES(n)}{SE(n)}\right|, \quad (15)$$

which can be compared with our corresponding results, given by Equations (14, 7), as :

$$RD_{VC}(v = v_N(n); p, n) = \left|1 - \frac{E^*(v;p,n)}{SE(n)}\right|. \quad (16)$$

Then, the numerical results of $RD_S(n)$ and $RD_{VC}(v = v_N(n); p, n)$ are given in Table 1 in Appendix 1, showing a good accordance between those results.

Secondly, at any v and in $v_N(n) \leq v \leq c$, we can evaluate the relative deviations between $K^*(v; p)$ and $E^*(v; p, n)$, given Equations (12, 14), by:

$$RD(v; p, n) \equiv 1 - \frac{E^*(v;p,n)}{K^*(v;p)}. \quad (17)$$

Then, in Tables 2-7 in Appendix 1, the numerical results of $RD(v; p, n)$, $K^*(v; p)/K_o(p)$, $U^*(v; p, n)/U_o(p)$ and $E^*(v; p, n)/E_o(p)$ are obtained for p=1-4 and n=1, 2, 3, 4, 5, 35, respectively, using Equations (7, 4, 5, 6), noting that, for a given p, $RE(v; p, n)$ decreases with increasing v, and at v=c, one obtains $K^*(v; p) \cong E^*(v; p, n)$, with highest precision.

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APPENDIX 1

Table 1. The numerical results of $RD_S(n)$ and $RD_{VC}(v = v_N(n); p, n)$ are given in Equations (15, 16), showing a good accordance between those results.

n	1	2	3	4	5	10	15	20	25	30	35
For p=1											
$RD_{VC}(\times 10^{-5})$	5.86	0.12	1.23	1.62	1.80	2.04	2.09	2.10	2.11	2.11	2.11
$RD_S(\times 10^{-5})$	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12
For p=2											
$RD_{VC}(\times 10^{-5})$	13.8	1.87	0.34	1.12	1.48	1.96	2.05	2.08	2.09	2.10	2.11
$RD_S(\times 10^{-5})$	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12
For p=3											
$RD_{VC}(\times 10^{-5})$	21.8	3.87	0.54	0.62	1.16	1.88	2.01	2.06	2.08	2.09	2.10
$RD_S(\times 10^{-5})$	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12
For p=4											
$RD_{VC}(\times 10^{-5})$	29.8	5.86	1.43	0.12	0.84	1.8	1.98	2.04	2.07	2.08	2.09
$RD_S(\times 10^{-5})$	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12	2.12

Table 2. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=1**, the numerical results of $RD(v;p)$, $K^*(v;p)/K_o(p)$, $U^*(v;p,n)/U_o(p)$ and $E^*(v;p,n)/E_o(p)$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $v_N(1) = 2.18758543 \times 10^6 (\frac{m}{s}) \leq v \leq c$. Further, $\frac{v_N(1)}{c} = 0.00729706$ and $r_B(1) = 5.2913402 \times 10^{-11}$m.

X(v)=v/c	$RD(v;p)$	$K^*(v;p)/K_o(p)$	$U^*(v;p,n)/U_o(p,n)$	$E^*(v;p,n)/E_o(p,n)$
Here, for **p=1**, one notes that at v=c, $K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}$ J, $U_o(p,n) = -8.7182785 \times 10^{-18}$ J and $E_o(p,n) = 6.13909818 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.4199× 10^{-4}.				
0.00729706	**2.00005324**	**3.54989954× 10^{-5}**	**0.50002662**	**-3.55059269× 10^{-5}**
0.09	0.01320046	0.00542187	0.50405	0.00535106
0.30	0.00123422	0.0627	0.545	0.06263151
0.40	0.00071489	0.1152	0.58	0.11513399
0.60	0.00034094	0.2832	0.68	0.28314365
0.80	0.00020674	0.5632	0.82	0.56316353
0.90	0.00016937	0.7587	0.905	0.75867922
0.95	0.00015469	0.87316875	0.95125	0.87315766
1	1.4199× 10^{-4}	**1**	**1**	**1**
Here, for **p=2**, one notes that at v=c, $K_o(p) = mc^2 = 8.18662668 \times 10^{-14}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $U_o(p,n) = -1.30774178 \times 10^{-17}$ J and $E_o(p,n) = 8.18531894 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.5974× 10^{-4}.				
0.00729706	**2.00010648**	**2.66249554× 10^{-5}**	**0.33336883**	**-2.66320447× 10^{-5}**
0.09	0.01325306	0.00408281	0.33873333	0.00402934
0.30	0.00128097	0.04905	0.39333333	0.04899499
0.40	0.00075739	0.0928	0.44	0.09274453
0.60	0.00037412	0.2448	0.57333333	0.24474751
0.80	0.00023133	0.5248	0.76	0.52476242
0.90	0.00019031	0.73305	0.87333333	0.73302759
0.95	0.00017397	0.85850313	0.935	0.85849090
1	1.5974× 10^{-4}	**1**	**1**	**1**
Here, for **p=3**, one notes that at v=c, $K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}$ J, $U_o(p,n) = -1.743655709 \times 10^{-17}$ J and $E_o(p,n) = 1.02315397 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.7039× 10^{-4}.				
0.00729706	**2.00015972**	**2.13005314× 10^{-5}**	**0.25003994**	**-2.13075641× 10^{-5}**
0.09	0.01330525	0.00327937	0.256075	0.00323628
0.30	0.00132401	0.04086	0.3175	0.04081286
0.40	0.00079441	0.07936	0.37	0.07931047
0.60	0.00039955	0.22176	0.52	0.22170917

0.80	0.0002479	0.50176	0.73	0.50172110
0.90	0.00020359	0.71766	0.8575	0.71763617
0.95	0.00018587	0.84970375	0.926875	0.84969060
1	1.7039×10^{-4}	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p, n) = -2.17956964 \times 10^{-17}$ J and $E_o(p, n) = 1.22777604 \times 10^{-13}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of 1.77490251×10^{-4}.

0.00729706	**2.00021296**	**1.77509153×10^{-5}**	**0.2000426**	**$-1.77578474\times 10^{-5}$**
0.09	0.01335702	0.00274374	0.20648	0.00270757
0.30	0.00136377	0.0354	0.272	0.035358
0.40	0.00082694	0.0704	0.328	0.07035427
0.60	0.00041965	0.2064	0.488	0.20635001
0.80	0.00025981	0.4864	0.712	0.48635995
0.90	0.00021277	0.7074	0.848	0.70737504
0.95	0.00019393	0.8438375	0.922	0.84382362
1	1.77490251×10^{-4}	**1**	**1**	**1**

Table 3. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=2**, the numerical results of $RD(v; p)$, $K^*(v; p)/K_o(p)$, $U^*(v; p, n)/U_o(p)$ and $E^*(v; p, n)/E_o(p)$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $v_N(2) = 1.09379272 \times 10^6 (\frac{m}{s}) \leq v \leq c$. Further, $\frac{v_N(2)}{c} = 0.00364853$ and $r_B(2) = 2.11653608 \times 10^{-10}$m.

X(v)=v/c	$RD(v; p)$	$K^*(v; p)/K_o(p)$	$U^*(v; p, n)/U_o(p, n)$	$E^*(v; p, n)/E_o(p, n)$
Here, for **p=1**, one notes that at v=c, $K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}$ J, $U_o(p, n) = -2.1795696 \times 10^{-18}$ J and $E_o(p, n) = 6.13975205 \times 10^{-14}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of 3.54980502×10^{-5}.				
0.00364853	**2.00001296**	**8.87457318×10^{-6}**	**0.50000666**	**$-8.87500326\times 10^{-6}$**
0.09	0.00330011	0.00542187	0.50405	0.00540417
0.30	0.00030856	0.0627	0.545	0.06268288
0.40	0.00017872	0.1152	0.58	0.1151835
0.60	8.52354313×10^{-5}	0.2832	0.68	0.28318591
0.80	5.1683951×10^{-5}	0.5632	0.82	0.56319088
0.90	4.23431336×10^{-5}	0.7587	0.905	0.75869481
0.95	3.86723875×10^{-5}	0.87316875	0.95125	0.87316598
1	3.54980502×10^{-5}	**1**	**1**	**1**

Here, for **p=2**, one notes that at v=c, $K_o(p) = mc^2 = 8.18662668 \times 10^{-14}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $U_o(p, n) = -3.26935445 \times 10^{-18}$ J and $E_o(p, n) = 8.18629974 \times 10^{-14}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of 3.99353065×10^{-5}.

0.00364853	**2.00002662**	**6.65597304$\times 10^{-6}$**	**0.33334221**	**-6.65641602$\times 10^{-6}$**
0.09	0.00331327	0.00408281	0.33873333	0.00406944
0.30	0.00032024	0.04905	0.39333333	0.04903625
0.40	0.00018935	0.0928	0.44	0.09278613
0.60	9.35304019$\times 10^{-5}$	0.2448	0.57333333	0.24478688
0.80	5.7833142$\times 10^{-5}$	0.5248	0.76	0.52479061
0.90	4.75777018$\times 10^{-5}$	0.73305	0.87333333	0.73304440
0.95	4.34937398$\times 10^{-5}$	0.85850313	0.935	0.85850007
1	3.99353065$\times 10^{-5}$	**1**	**1**	**1**

Here, for **p=3**, one notes that at v=c, $K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}$ J, $U_o(p,n) = -4.35913927 \times 10^{-18}$ J and $E_o(p,n) = 1.02328474 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 4.25976603$\times 10^{-5}$.

0.00364853	**2.00003993**	**5.32481387$\times 10^{-6}$**	**0.25000998**	**-5.32525332$\times 10^{-6}$**
0.09	0.00332631	0.00327937	0.256075	0.00326860
0.30	0.000331	0.04086	0.3175	0.04084822
0.40	0.0001986	0.07936	0.37	0.07934762
0.60	9.98862885$\times 10^{-5}$	0.22176	0.52	0.22174730
0.80	6.19744340$\times 10^{-5}$	0.50176	0.73	0.50175028
0.90	5.08980488$\times 10^{-5}$	0.71766	0.8575	0.71765404
0.95	4.64664377$\times 10^{-5}$	0.84970375	0.926875	0.84970046
1	4.25976603$\times 10^{-5}$	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p,n) = -5.4892409 \times 10^{-18}$ J and $E_o(p,n) = 1.22793951 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 4.43725628$\times 10^{-5}$.

0.00364853	**2.00005324**	**4.43737443$\times 10^{-6}$**	**0.20001065**	**-4.43780758$\times 10^{-6}$**
0.09	0.00333925	0.00274374	0.20648	0.00273470
0.30	0.00034094	0.0354	0.272	0.03538950
0.40	0.00020674	0.0704	0.328	0.07038857
0.60	0.00010491	0.2064	0.488	0.20638750
0.80	6.4953258$\times 10^{-5}$	0.4864	0.712	0.48638999
0.90	5.31918762$\times 10^{-5}$	0.7074	0.848	0.70739376
0.95	4.84826793$\times 10^{-5}$	0.8438375	0.922	0.84383403
1	4.43725628$\times 10^{-5}$	**1**	**1**	**1**

Table 4. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=3**, the numerical results of $RD(v;p)$, $K^*(v;p)/K_o(p)$, $U^*(v;p,n)/U_o(p)$ and $E^*(v;p,n)/E_o(p)$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $v_N(3) = 7.29195143 \times 10^5 (\frac{m}{s}) \leq v \leq c$. Further, $\frac{v_N(3)}{c} = 0.00243235$ and $r_B(3) = 4.76220618 \times 10^{-10}$m.

X(v)=v/c	$RD(v;p)$	$K^*(v;p)/K_o(p)$	$U^*(v;p,n)/U_o(p,n)$	$E^*(v;p,n)/E_o(p,n)$
Here, for **p=1**, one notes that at v=c, $K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}$ J, $U_o(p,n) = -9.68697616 \times 10^{-19}$ J and $E_o(p,n) = 6.13987314 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of $1.57769112 \times 10^{-5}$.				
0.00243235	**2.00000591**	**3.94423948**$\times 10^{-6}$	**0.50000296**	**-3.94432502**$\times 10^{-6}$
0.09	0.00146672	0.00542187	0.50405	0.00541400
0.30	0.00013714	0.0627	0.545	0.06269239
0.40	7.94323654×10^{-5}	0.1152	0.58	0.11519267
0.60	3.78824139×10^{-5}	0.2832	0.68	0.28319374
0.80	2.29706449×10^{-5}	0.5632	0.82	0.56319595
0.90	1.88191705×10^{-5}	0.7587	0.905	0.75869769
0.95	1.71877275×10^{-5}	0.87316875	0.95125	0.87316752
1	1.57769112×10^{-5}	**1**	**1**	**1**
Here, for **p=2**, one notes that at v=c, $K_o(p) = mc^2 = 8.18662668 \times 10^{-14}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $U_o(p,n) = -1.45304642 \times 10^{-18}$ J and $E_o(p,n) = 8.18648137 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.77490251×10^{-5}.				
0.00243235	**2.00001182**	**2.95818836**$\times 10^{-6}$	**0.33333728**	**-2.95827585**$\times 10^{-6}$
0.09	0.00147256	0.00408281	0.33873333	0.00407687
0.30	0.00014233	0.04905	0.39333333	0.04904389
0.40	8.41548604×10^{-5}	0.0928	0.44	0.09279384
0.60	4.1569065×10^{-5}	0.2448	0.57333333	0.24479417
0.80	2.57036187×10^{-5}	0.5248	0.76	0.52479583
0.90	2.11456453×10^{-5}	0.73305	0.87333333	0.73304751
0.95	1.9330551×10^{-5}	0.85850313	0.935	0.85850177
1	1.77490251×10^{-5}	**1**	**1**	**1**
Here, for **p=3**, one notes that at v=c, $K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}$ J, $U_o(p,n) = -1.93739523 \times 10^{-18}$ J and $E_o(p,n) = 1.02330896 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.89322934×10^{-5}.				
0.00243235	**2.00001774**	**2.36655769**$\times 10^{-6}$	**0.25000444**	**-2.36664448**$\times 10^{-6}$
0.09	0.00147836	0.00327937	0.256075	0.00327458
0.30	0.00014711	0.04086	0.3175	0.04085476
0.40	8.82680012×10^{-5}	0.07936	0.37	0.07935450

0.60	4.4393906×10^{-5}	0.22176	0.52	0.22175435
0.80	2.75441929×10^{-5}	0.50176	0.73	0.50175568
0.90	2.2621355×10^{-5}	0.71766	0.8575	0.71765735
0.95	2.06517501×10^{-5}	0.84970375	0.926875	0.84970229
1	1.89322934×10^{-5}	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p,n) = -2.42174404 \times 10^{-18}$ J and $E_o(p,n) = 1.22796978 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.9721139×10^{-5}.

0.00243235	**2.00002366**	**1.97213724×10^{-6}**	**0.20000473**	**$-1.97222279\times 10^{-6}$**
0.09	0.00148411	0.00274374	0.20648	0.00273972
0.30	0.00015153	0.0354	0.272	0.03539533
0.40	9.18825795×10^{-5}	0.0704	0.328	0.07039492
0.60	4.66274992×10^{-5}	0.2064	0.488	0.20639445
0.80	2.88681147×10^{-5}	0.4864	0.712	0.48639555
0.90	2.36408339×10^{-5}	0.7074	0.848	0.70739723
0.95	2.15488575×10^{-5}	0.8438375	0.922	0.84383596
1	1.9721139×10^{-5}	**1**	**1**	**1**

Table 5. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=4**, the numerical results of $\mathrm{RD(v;p)}$, $\mathrm{K^*(v;p)/K_o(p)}$, $\mathrm{U^*(v;p,n)/U_o(p)}$ and $\mathrm{E^*(v;p,n)/E_o(p)}$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $\mathrm{v_N(4) = 5.46896358 \times 10^5 (\frac{m}{s}) \leq v \leq c}$. Further, $\mathrm{\frac{v_N(4)}{c} = 0.00182426}$ and $\mathrm{r_B(4) = 8.46614432 \times 10^{-10} m}$.

X(v)=v/c	$\mathrm{RD(v;p)}$	$\mathrm{K^*(v;p)/K_o(p)}$	$\mathrm{U^*(v;p,n)/U_o(p,n)}$	$\mathrm{E^*(v;p,n)/E_o(p,n)}$
Here, for **p=1**, one notes that at v=c, $\mathrm{K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}}$ J, $\mathrm{U_o(p,n) = -5.44892409 \times 10^{-19}}$ J and $\mathrm{E_o(p,n) = 6.13991552 \times 10^{-14}}$ J, suggesting that $\mathrm{E_o(p,n) \cong K_o(p)}$, with a precision of the order of $8.87451255 \times 10^{-6}$.				
0.00182426	**2.00000332**	**2.21863184** $\times 10^{-6}$	**0.50000166**	**-2.21865889** $\times 10^{-6}$
0.09	0.00082503	0.00542187	0.50405	0.00541744
0.30	7.7138905×10^{-5}	0.0627	0.545	0.06269572
0.40	4.46807056×10^{-5}	0.1152	0.58	0.11519588
0.60	2.13088578×10^{-5}	0.2832	0.68	0.28319648
0.80	1.29209877×10^{-5}	0.5632	0.82	0.56319772
0.90	1.05857834×10^{-5}	0.7587	0.905	0.75869870
0.95	9.66809688×10^{-6}	0.87316875	0.95125	0.87316806
1	8.87451255×10^{-6}	**1**	**1**	**1**
Here, for **p=2**, one notes that at v=c, $\mathrm{K_o(p) = mc^2 = 8.18662668 \times 10^{-14}}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $\mathrm{U_o(p,n) = -8.17338614 \times 10^{-19}}$ J and $\mathrm{E_o(p,n) = 8.18654494 \times 10^{-14}}$ J, suggesting that $\mathrm{E_o(p,n) \cong K_o(p)}$, with a precision of the order of 9.98382662×10^{-6}.				
0.00182426	**2.00000665**	**1.66397665** $\times 10^{-6}$	**0.33333555**	**-1.66400432** $\times 10^{-6}$
0.09	0.00082832	0.00408281	0.33873333	0.00407946
0.30	8.00605872×10^{-5}	0.04905	0.39333333	0.04904656
0.40	4.7337109×10^{-5}	0.0928	0.44	0.09279653
0.60	2.33826005×10^{-5}	0.2448	0.57333333	0.24479672
0.80	1.44582855×10^{-5}	0.5248	0.76	0.52479765
0.90	1.18944255×10^{-5}	0.73305	0.87333333	0.73304860
0.95	1.0873435×10^{-5}	0.85850313	0.935	0.85850236
1	9.98382662×10^{-6}	**1**	**1**	**1**
Here, for **p=3**, one notes that at v=c, $\mathrm{K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}}$ J, $\mathrm{U_o(p,n) = -1.08978482 \times 10^{-18}}$ J and $\mathrm{E_o(p,n) = 1.02331744 \times 10^{-13}}$ J, suggesting that $\mathrm{E_o(p,n) \cong K_o(p)}$, with a precision of the order of 1.06494151×10^{-5}.				
0.00182426	**2.00000998**	**1.33118353** $\times 10^{-6}$	**0.2500025**	**-1.33121099** $\times 10^{-6}$
0.09	0.00083158	0.00327937	0.256075	0.00327667
0.30	8.27505943×10^{-5}	0.04086	0.3175	0.04085705
0.40	4.96507507×10^{-5}	0.07936	0.37	0.07935690

0.60	2.49715721×10^{-5}	0.22176	0.52	0.22175682
0.80	1.54936085×10^{-5}	0.50176	0.73	0.50175757
0.90	1.27245122×10^{-5}	0.71766	0.8575	0.71765851
0.95	1.16166094×10^{-5}	0.84970375	0.926875	0.84970293
1	1.06494151×10^{-5}	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p,n) = -1.36223102 \times 10^{-18}$ J and $E_o(p,n) = 1.22798038 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.10931407×10^{-5}.

0.00182426	**2.0000133**	**1.10932146×10^{-6}**	**0.20000266**	**$-1.10934852\times 10^{-6}$**
0.09	0.00083481	0.00274374	0.20648	0.00274148
0.30	8.52354313×10^{-5}	0.0354	0.272	0.03539738
0.40	5.16839509×10^{-5}	0.0704	0.328	0.07039714
0.60	2.62279683×10^{-5}	0.2064	0.488	0.20639688
0.80	1.62383145×10^{-5}	0.4864	0.712	0.48639750
0.90	1.3297969×10^{-5}	0.7074	0.848	0.70739844
0.95	1.21206698×10^{-5}	0.8438375	0.922	0.84383663
1	1.10931407×10^{-5}	**1**	**1**	**1**

Table 6. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=5**, the numerical results of $RD(v; p)$, $K^*(v; p)/K_o(p)$, $U^*(v; p, n)/U_o(p)$ and $E^*(v; p, n)/E_o(p)$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $v_N(5) = 4.37517086 \times 10^5 (\frac{m}{s}) \leq v \leq c$. Further, $\frac{v_N(5)}{c} = 0.00145941$ and $r_B(5) = 1.32283505 \times 10^{-9}$m.

X(v)=v/c	$RD(v; p)$	$K^*(v; p)/K_o(p)$	$U^*(v; p, n)/U_o(p, n)$	$E^*(v; p, n)/E_o(p, n)$
Here, for **p=1**, one notes that at v=c, $K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}$ J, $U_o(p, n) = -3.48731142 \times 10^{-19}$ J and $E_o(p, n) = 6.13993513 \times 10^{-14}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of $5.67968803 \times 10^{-6}$.				
0.00145941	**2.00000212**	**1.41992353× 10^{-6}**	**0.50000106**	**-1.4199346× 10^{-6}**
0.09	0.00052802	0.00542187	0.50405	0.00541904
0.30	4.93688992× 10^{-5}	0.0627	0.545	0.06269726
0.40	2.85956516× 10^{-5}	0.1152	0.58	0.11519736
0.60	1.3637669× 10^{-5}	0.2832	0.68	0.28319775
0.80	8.26943215× 10^{-5}	0.5632	0.82	0.56319854
0.90	6.77490137× 10^{-5}	0.7587	0.905	0.75869917
0.95	6.187582× 10^{-6}	0.87316875	0.95125	0.87316831
1	5.67968803× 10^{-6}	**1**	**1**	**1**
Here, for **p=2**, one notes that at v=c, $K_o(p) = mc^2 = 8.18662668 \times 10^{-14}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $U_o(p, n) = -5.23096713 \times 10^{-19}$ J and $E_o(p, n) = 8.18657437 \times 10^{-14}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of $6.38964904 \times 10^{-6}$.				
0.00145941	**2.00000425**	**1.06494378× 10^{-6}**	**0.33333475**	**-1.06495511× 10^{-6}**
0.09	0.00053012	0.00408281	0.33873333	0.00408067
0.30	5.12387758× 10^{-5}	0.04905	0.39333333	0.04904780
0.40	3.02957497× 10^{-5}	0.0928	0.44	0.09279778
0.60	1.49648643× 10^{-5}	0.2448	0.57333333	0.24479790
0.80	9.25330272× 10^{-6}	0.5248	0.76	0.52479850
0.90	7.61243229× 10^{-6}	0.73305	0.87333333	0.73304910
0.95	6.95899837× 10^{-5}	0.85850313	0.935	0.85850264
1	6.38964904× 10^{-6}	**1**	**1**	**1**
Here, for **p=3**, one notes that at v=c, $K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}$ J, $U_o(p, n) = -6.97462284 \times 10^{-19}$ J and $E_o(p, n) = 1.02332136 \times 10^{-13}$ J, suggesting that $E_o(p, n) \cong K_o(p)$, with a precision of the order of $6.81562564 \times 10^{-6}$.				
0.00145941	**2.00000638**	**8.5195593× 10^{-7}**	**0.2500016**	**-8.51967174× 10^{-7}**
0.09	0.00053221	0.00327937	0.256075	0.00327764
0.30	5.29603803× 10^{-5}	0.04086	0.3175	0.04085811
0.40	3.17764804× 10^{-5}	0.07936	0.37	0.07935802

0.60	1.59818062×10^{-5}	0.22176	0.52	0.22175797
0.80	9.91590943×10^{-6}	0.50176	0.73	0.50175844
0.90	8.1436878×10^{-6}	0.71766	0.8575	0.71765905
0.95	7.43463003×10^{-6}	0.84970375	0.926875	0.84970322
1	6.81562564×10^{-6}	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p,n) = -8.71827854 \times 10^{-19}$ J and $E_o(p,n) = 1.22798528 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 7.09961004×10^{-6}.

0.00145941	**2.00000851**	$\mathbf{7.09964031\times 10^{-7}}$	**0.2000017**	$\mathbf{-7.09975115\times 10^{-7}}$
0.09	0.00053428	0.00274374	0.20648	0.00274229
0.30	5.4550676×10^{-5}	0.0354	0.272	0.03539832
0.40	3.30777286×10^{-5}	0.0704	0.328	0.07039817
0.60	1.67858997×10^{-5}	0.2064	0.488	0.20639800
0.80	1.03925213×10^{-5}	0.4864	0.712	0.48639840
0.90	8.51070019×10^{-6}	0.7074	0.848	0.70739900
0.95	7.75722868×10^{-5}	0.8438375	0.922	0.84383695
1	7.09961004×10^{-6}	**1**	**1**	**1**

Table 7. In the **Electron-Photon interactions in ionized Hydrogen Atom**, in which **n=35**, the numerical results of $RD(v;p)$, $K^*(v;p)/K_o(p)$, $U^*(v;p,n)/U_o(p)$ and $E^*(v;p,n)/E_o(p)$, are obtained by using Equations (7, 4, 5, 6), respectively, noting that $v_N(35) = 62502.44085714\ (\frac{m}{s}) \leq v \leq c$. Further, $\frac{v_N(35)}{c} = 0.00020849$ and $r_B(35) = 6.48189174 \times 10^{-8}m$.

X(v)=v/c	$RD(v;p)$	$K^*(v;p)/K_o(p)$	$U^*(v;p,n)/U_o(p,n)$	$E^*(v;p,n)/E_o(p,n)$
Here, for **p=1**, one notes that at v=c, $K_o(p) = \left(\frac{3}{4}\right) \times mc^2 = 6.13997001 \times 10^{-14}$ J, $U_o(p,n) = -7.11696208 \times 10^{-21}$ J and $E_o(p,n) = 6.1399693 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of $1.15912001 \times 10^{-7}$.				
0.00020849	**2.00000004**	**2.89780009**$\times 10^{-8}$	**0.50000002**	**-2.89780053**$\times 10^{-8}$
0.09	1.07758843×10^{-5}	0.00542187	0.50405	0.00542181
0.30	1.00752855×10^{-6}	0.0627	0.545	0.06269994
0.40	5.83584726×10^{-7}	0.1152	0.58	0.11519995
0.60	2.78319776×10^{-7}	0.2832	0.68	0.28319995
0.80	1.68763921×10^{-7}	0.5632	0.82	0.56319997
0.90	1.38263293×10^{-7}	0.7587	0.905	0.75869998
0.95	1.26277184×10^{-7}	0.87316875	0.95125	0.87316874
1	1.15912001×10^{-7}	**1**	**1**	**1**
Here, for **p=2**, one notes that at v=c, $K_o(p) = mc^2 = 8.18662668 \times 10^{-14}$ J, in good accordance with that proposed by Einstein in the relativistic case [1], $U_o(p,n) = -1.06754431 \times 10^{-20}$ J and $E_o(p,n) = 8.18662561 \times 10^{-14}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.30401001×10^{-7}.				
0.00020849	**2.00000008**	**2.17335012**$\times 10^{-8}$	**0.33333336**	**-2.17335057**$\times 10^{-8}$
0.09	1.08188281×10^{-5}	0.00408281	0.33873333	0.00408276
0.30	1.0456893×10^{-6}	0.04905	0.39333333	0.04904996
0.40	6.18280607×10^{-7}	0.0928	0.44	0.09279995
0.60	3.05405394×10^{-7}	0.2448	0.57333333	0.24479996
0.80	1.88842913×10^{-7}	0.5248	0.76	0.52479997
0.90	1.55355761×10^{-7}	0.73305	0.87333333	0.73304998
0.95	1.42020375×10^{-7}	0.85850313	0.935	0.85850312
1	1.30401001×10^{-7}	**1**	**1**	**1**
Here, for **p=3**, one notes that at v=c, $K_o(p) = \frac{5}{4} \times mc^2 = 1.02332833 \times 10^{-13}$ J, $U_o(p,n) = -1.42339242 \times 10^{-20}$ J and $E_o(p,n) = 1.02332819 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.39094401×10^{-7}.				
0.00020849	**2.00000012**	**1.73868013**$\times 10^{-8}$	**0.25000003**	**-1.73868059**$\times 10^{-8}$
0.09	1.08614283×10^{-5}	0.00327937	0.256075	0.00327933
0.30	1.08082409×10^{-6}	0.04086	0.3175	0.04085996

0.40	6.48499601×10^{-7}	0.07936	0.37	0.07935996
0.60	3.26159309×10^{-7}	0.22176	0.52	0.22175996
0.80	2.02365499×10^{-7}	0.50176	0.73	0.50175997
0.90	1.6619771×10^{-7}	0.71766	0.8575	0.71765998
0.95	1.51727143×10^{-7}	0.84970375	0.926875	0.84970374
1	1.39094401×10^{-7}	**1**	**1**	**1**

Here, for **p=4**, one notes that at v=c, $K_o(p) = \frac{3}{2} \times mc^2 = 1.227994 \times 10^{-13}$ J, $U_o(p,n) = -1.77924052 \times 10^{-20}$ J and $E_o(p,n) = 1.22799382 \times 10^{-13}$ J, suggesting that $E_o(p,n) \cong K_o(p)$, with a precision of the order of 1.44890001×10^{-7}.

0.00020849	**2.00000017**	**1.44890014×10^{-8}**	**0.20000003**	**$-1.44890059\times 10^{-8}$**
0.09	1.09036889×10^{-5}	0.00274374	0.20648	0.00274371
0.30	1.1132791×10^{-6}	0.0354	0.272	0.03539997
0.40	6.75055686×10^{-7}	0.0704	0.328	0.07039996
0.60	3.42569382×10^{-7}	0.2064	0.488	0.20639996
0.80	2.12092271×10^{-7}	0.4864	0.712	0.48639997
0.90	1.73687759×10^{-7}	0.7074	0.848	0.70739998
0.95	1.58310789×10^{-7}	0.84383750	0.922	0.84383749
1	1.44890001×10^{-7}	**1**	**1**	**1**