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GLOBAL EINSTEIN -AND- VAN CONG RELATIONS, AND NEW ELECRICAL THERMOELECTRIC LAWS IN N(P)-TYPE DEGENERATE, COMPENSATED AND VISCOUS Na-ALKALI METAL, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW, CONDUCTIVITY MODEL, AND ESPECIALLY ACCURATE FERMI ENERGY EXPRESSION (II)

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ABSTRACT

In degenerate, compensated and viscous **n(p) – Na- alkali metal (Na-AM)**, the Global Einstein -and- Van Cong relations, as those given in Equations (25, 32), and electrical thermoelectric laws, enhanced by: our static dielectric constant law given in Equations (5, 6), our electrical conductivity model in Eq. (19), and especially our accurate Fermi energy expression in Eq. (17), are now investigated, by basing on the same physical model and the mathematical treatment method, as those used in our recent works.[1-5] Their numerical results are obtained and given in Tables (2-7), suggesting an equivalence between degeneracy-compensation-viscosity concept, given in this Na-AM. Here, the well-known "local" Einstein relation, being valid only at the lowest degeneracy, lowest viscosity and lowest compensation, is now globalized by our "Global" Einstein -and- Van Cong relations, valid at any degeneracy, viscosity and compensation, as showed in Equations.[25,32]

KEYWORDS: Conductivity, Mobility, Viscosity coefficient, Diffusion coefficient, Activation energy, Fermi energy.

INTRODUCTION

In the **degenerate, compensated and viscous $\mathbf{n(p)-Na}$- alkali metal ($\equiv$Na-AM)**, the Global Einstein -and- Van Cong relations, as those given in Equations (25, 32), and various optical-electrical-thermoelectric laws, enhanced by : (i) our static dielectric constant law given in Equations (5, 6), (ii) our electrical conductivity (σ)-model obtained in Eq. (19), and especially (iii) our accurate Fermi energy expression in Eq. (17), with a precision of the order of $\mathbf{2.11\times10^{-4}}$, affecting strongly all the expressions of electrical coefficients, are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in References.[1-18]

Then, some important remarks can be summarized as follows.

(1) As observed in our recent work[1], the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{CDn(CDp)}^{EBT}$, being obtained with a precision of the order of $\mathbf{3.4931\times10^{-7}}$, as given in Table 2 in Appendix 1, and also in our recent work.[1] Therefore, the effective electron (hole)-density can be defined as: $N^{*}\equiv N-N_{CDn(CDp)}\simeq N-N_{CDn(CDp)}^{EBT}$, N being the total impurity density, as that observed in the compensated metals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^{*})$, defined in Eq. (13), is valid at any N^{*}.

Therefore, by basing on this σ-expression, the Global Einstein -and- Van Cong relations defined in Equations (25, 32) and various electrical thermoelectric laws are determined. Finally, the numerical results of the electrical thermoelectric coefficients are obtained, discussed and reported in Tables (3-7) in Appendix 1, suggesting an equivalence between degeneracy-compensation-viscosity concept, given in this Na-AM.

Our Static Dielectric Constant Law and Generalized Mott Criterium in the Metal-Insulator Transition (MIT)

First of all, in the Na-AM[1], we denote : the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $\mathbf{r_{do(ao)}}$=$\mathbf{r_{Na}}=0.166$ nm, the unperturbed **reduced** effective

electron (hole) mass in conduction (valence) bands by: $m_{c(v)} = 1.06$, the unperturbed effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)} \times m_o$, m_o ($= 9.109558 \times 10^{-28}$ g) being the free electron mass, and the vacuum dielectric constant by: $\varepsilon_o = 1$.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$$E_{do(ao)} = \frac{13600 \times [m_{c(v)}]}{[\varepsilon_o]^2} \text{ meV}, \quad (1)$$

and then, the isothermal bulk modulus, by :

$$B_{do(ao)} \equiv \frac{E_{do(ao)}}{\left(\frac{4\pi}{3}\right) \times (r_{do(ao)})^3}. \quad (2)$$

Effects of impurity size

Here, one shows that the effect of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, strongly affects the changes in all the energy-band-structure parameters, which can be represented by the effective relative static dielectric constant $\varepsilon(r_{d(a)})$, as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p, as: $p_o = 0$, and for the deformation potential energy (or the strain energy) σ, as: $\sigma_o = 0$. Further, the two important equations, used to determine the σ-variation: $\Delta\sigma \equiv \sigma - \sigma_o = \sigma$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\sigma(r_{d(a)})]_{n(p)} = B_{do(ao)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (3)$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to: the increase (the decrease) in the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)})$ in the absolute values, being obtained from the effective Bohr model, and then such the compression (dilatation) is represented respectively by: $\pm [\Delta\sigma(r_{d(a)})]_{n(p)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = + [\Delta\sigma(r_{d(a)})]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\left[\Delta\sigma(r_{d(a)})\right]_{n(p)}. \quad (4)$$

Therefore, from above Equations (3) and (4), one obtains the expressions for $\varepsilon(r_{d(a)})$ and $E_{d(a)}(r_{d(a)})$ as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (5)$$

according to the increase in $E_{d(a)}(r_{d(a)})$, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = -E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (6)$$

corresponding to the decrease in $E_{d(a)}(r_{d(a)})$.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)})$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}) \equiv \frac{\varepsilon(r_{d(a)}) \times \hbar^2}{m_{c(v)} \times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)})}{m_{c(v)}/m_o}, \quad (7)$$

where $-q$ is the electron charge; $q > 0$.

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(CDp)}(r_{d(a)})$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as[1] :

$$N_{CDn(CDp)}(r_{d(a)})^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = M_{n(p)},\ M_{n(p)} = 0.25, \quad (8)$$

depending thus on our **new $\varepsilon(r_{d(a)})$-law**.

This result can be explained by using the definition of the reduced effective Wigner-Seitz (**WS**) radius $\mathbf{r_{sn(sp),M}}$, in the Mott's criterium, being characteristic of interactions, by:

$$\mathbf{r_{sn(sp),M}}(N = N_{CDn(CDp)}(r_{d(a)}), r_{d(a)}) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)})}\right)^{\frac{1}{3}} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \times \mathbf{4} \cong 2.4814, \quad (9)$$

for any $r_{d(a)}$-value. Therefore, one also has :

$$N_{CDn(CDp)}(r_{d(a)})^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = \mathbf{1/4 = 0.25} = M_{n(p)}, \quad (10)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\boldsymbol{\mathcal{H}_{n(p)} = 0.47137}$, as that given in our recent work[1], we have showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $\mathbf{N^{EBT}_{CDn(CDp)}}$, with a precision of the order of $\mathbf{3.4931 \times 10^{-7}}$, as also showed in **Table 2 in Appendix 1**. So,

$$N_{CDn(NDp)}(r_{d(a)}) \cong N^{EBT}_{CDn(CDp)}(r_{d(a)}). \quad (11)$$

Here, it shoud be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen such that those of $N_{CDn(CDp)}$ and $N^{EBT}_{CDn(CDp)}$ are found to be in good agreement with experimental results.

Therefore, the effective density of electrons (holes) given in parabolic conduction (valence) bands, N^*, can be defined, as that given in compensated materials:

$$N^*(N, r_{d(a)}) \equiv N - N_{CDn(NDp)}(r_{d(a)}) \cong N - N^{EBT}_{CDn(CDp)}(r_{d(a)}) \geq 0. \quad (12)$$

One notes here that, with increasing $r_{d(a)}$ and for a given N, $N_{CDn(NDp)}(r_{d(a)})$ increases, as observed in Table 2 in Appendix 1, and therefore, $N^*(r_{d(a)})$ decreases, according the **increasing compensation**. Numerically, we will show that for given N and T, and for such the increasing compensation, the viscosity coefficient, $\mathbb{V}_{O[E]}(r_{d(a)})$, increases, suggesting an **equivalence between the compensation- viscosity concept.**

In summary, as observed in our recent paper[1], for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, while $N_{CDn(NDp)}(r_{d(a)})$ increases, affecting strongly all the electrical-and thermoelectric coefficients, as those observed in following Sections.

Physical model

In the Na-AM, the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (9), in which N is replaced by N^*, is now defined by:

$\gamma \times r_{sn(sp)}(N^*, r_{d(a)}) \equiv \frac{k^{-1}_{Fn(Fp)}}{a_{Bn(Bp)}} < 1$, $r_{sn(sp)}(N^*, r_{d(a)}) \equiv \left(\frac{3}{4\pi N^*}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})}$, being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{Fn(Fp)}(N^*) \equiv (3\pi^2 N^*)^{\frac{1}{3}}$ is the Fermi wave number.

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to $k_{Fn(kp)}$ is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + \left[R_{snTF(spTF)} - R_{snWS(spWS)}\right]e^{-r_{sn(sp)}} < 1, \quad (13)$$

being valid at any N^*.

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)})$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to:

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (14)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(snWS)}$ is respectively reduced to:

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d\left[r_{sn(sp)}^2 \times E_{CE}(N^*)\right]}{dr_{sn(sp)}}\right), \quad (15)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\mathbb{U}_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \mathbb{U}_{n(p)}(N^*, r_{d(a)}) \equiv \frac{\sqrt{2\pi \times N^*}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (16)$$

which gives: $A_{n(p)}(N^*, r_{d(a)}) = \frac{E_{Fno(Fpo)}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{d(a)})}$, $E_{Fno(Fpo)}(N^*, r_{d(a)}) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)} \times m_o}$.

Here, one remarks that: (i) the generalized Thomas-Fermi energy $\mathbb{U}_{n(p)}(N^*, r_{d(a)})$ can thus be approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{d(a)})}$, C being a constant, and (ii) $\mathbb{U}_{n(p)}(r_{d(a)})$ increases with increasing $r_{d(a)}$ and for a given N, since $\varepsilon(r_{d(a)})$ decreases, as given in Ref.[3]

Fermi Energy and Fermi-Dirac Distribution Function

Accurate Fermi Energy Expression

Here, for presentation simplicity, we change the sign of all various parameters, given in the Na-AM, in order to obtain the same one, as given in its degenerate n-type , according to the

reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N^*, r_{d(a)}, T) \equiv \frac{E_{Fn(Fp)}(N^*, r_{d(a)}, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N^*, r_{d(a)}, T)$, the reduced Fermi energy $\xi_{n(p)}(N^*, r_{d(a)}, T)$ or the Fermi energy $E_{Fn(Fp)}(N^*, r_{d(a)}, T)$, obtained in our previous paper[9], obtained with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u)+Au^B F(u)}{1+Au^B} \equiv \frac{V(u)}{W(u)}, A = 0.0005372 \text{ and } B = 4.82842262, \quad (17)$$

where u is the reduced electron density, $u(N^*, r_{d(a)}, T) \equiv \frac{N^*}{N_{c(v)}(T)}$, $N_{c(v)}(T) = 2 \times \left(\frac{m_{c(v)} \times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ (cm^{-3}), $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as $\mathbf{u \longrightarrow 0}$**, the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)} \times m_o}$ as $\mathbf{u \longrightarrow \infty}$**, the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, as $T \to 0$ K, since $u^{-1} \to 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)} \times m_o}$, being proportional to $(N^*)^{2/3}$. Further, at T=0 K and $N^* = 0$, being physical conditions, given for the metal-insulator transition (**MIT**).

In the following, it should be noted that all the optical and electrical-and-thermoelectric properties strongly depend on such an accurate expression of $\xi_{n(p)}(N^*, r_{d(a)}, T)$.[9]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

Thus, the average of E^p, calculated using the FDDF-method, as developed in our previous works[1, 6] is found to be given by:

$\langle E^p\rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}.$

Further, it should be noted that, at T=0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p(E_{Fn(Fp)}) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E^{-\beta}_{Fn(Fp)} \times I_{\beta}$, where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{\left(e^{\gamma/2}+e^{-\gamma/2}\right)^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains:

$I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma$.

Now, using the identity $(1 + e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t}\, dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. From the above Eq. of $\langle E^p\rangle_{FDDF}$, we get in the degenerate case ($\xi_{n(p)} > 1$) the following ratio:

$G_{p>1}\left(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}} < 1\right) \equiv \frac{\langle E^p\rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\ldots(p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n}$,

which can thus be approximated by:

$G_{p>1}(y < 1) \simeq 1 + p(p-1) \times |B_2| \times y^2, \; B_2 = \frac{1}{6}.$ (18)

Then, some usual results of $G_{p>1}(y < 1)$ are given in the **Table 1 in Appendix 1**, suggesting that, with increasing T (or decreasing T) and for given (N, r_d), since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases).

Electrical Properties

Now, the electrical conductivity σ can be defined and expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m^*_{n(p)}(x) \times m_o}$, k being the wave number, as:

$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times \left[k \times a_{Bn(Bp)}\right] \times \left(\frac{E_k}{\mathbb{U}_{n(p)}}\right)^{\frac{1}{2}} \left(\frac{1}{\Omega \times cm}\right)$, which is thus proportional to $E_k{}^2$, where $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}$ ohm^{-1} and $\mathbb{U}_{n(p)}(N^*, r_{d(a)})$ is determined in Eq. (16).

Then, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E^2_{Fn(Fp)}$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N^*, r_{d(a)}, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N^*, r_{d(a)}, T)$ for simplicity. Therefore, for $E_{Fno(Fpo)} > 0$ and T =0K, $G_2(y = 0) = 1$.

Therefore, if denoting the function $H(N^*, r_{d(a)}, T)$ by:

$H(N^*, r_{d(a)}, T) =$

$\left[\frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times \left[k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})\right] \times \sqrt{A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{d(a)})}}\right] \times$

$G_2(N^*, r_{d(a)}, T)$, which can be approximately expressed in terms of: $E^2_{Fno(Fpo)}(N^*) \times G_2(N^*, r_{d(a)}, T) \times \frac{\sqrt{\varepsilon(r_{d(a)})}}{(N^*)^{\frac{1}{4}}}$. Thus, with increasing $r_{d(a)}$ and for given T and N, the function $H(r_{d(a)})$ therefore decreases since $\varepsilon(r_{d(a)})$ decreases, as noted in Ref.[3]

Therefore, the electrical conductivity σ may be determined by:

$$\sigma(N^*, r_{d(a)}, T) = \frac{q^2}{\pi \times \hbar} \times H(N^*, r_{d(a)}, T) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{\Omega \times cm}\right), \quad (19)$$

Noting that, at T=0K, it can thus be approximately expressed in terms of $E^2_{Fno(Fpo)}(N^*, r_{d(a)}, x) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{1}{4}}}$, being proportional to: $\sqrt{\varepsilon(r_{d(a)}, x)} \times (N^*)^{\frac{13}{12}}$, for $E_{Fno(Fpo)} > 0$ and T =0K.

Electrical Properties

Here, if denoting, for majority electrons (holes), the thermal conductivity by: $\sigma_{Th.}(N^*, r_{d(a)}, T)$ in $\frac{W}{cm \times K}$, and the Lorenz number L by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8} (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\sigma_{Th.}}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\sigma_{Th.}(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} = L \times T. \quad (20)$$

Further, the resistivity is found to be given by: $\rho(N^*, r_{d(a)}, T) \equiv 1/\sigma(N^*, r_{d(a)}, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$.

New Electrical Coefficients

The relaxation time τ is related to by[3]:

$\tau(N^*, r_{d(a)}, T) \equiv \sigma(N^*, r_{d(a)}, T) \times \frac{m_{c(v)} \times m_o}{q^2 \times N^*}$. Therefore, the mobility $\mu_{O[E]}$ is given by:

$$\mu\left(N^*, r_{d(a)}, T\right) = \frac{q \times \tau\left(N^*, r_{d(a)}, T\right)}{m^*_{n(p)}(x) \times m_o} = \frac{\sigma\left(N^*, r_{d(a)}, T\right)}{q \times N^*} \left(\frac{cm^2}{V \times s}\right), \quad (21)$$

Being expressed in terms of $\frac{\sigma\left(N^*, r_{d(a)}, T\right)}{N^*}$. Further, as noted in above Eq. (19) for $\sigma\left(N^*, r_{d(a)}, T\right)$ and at T=0K, $\mu\left(N^*, r_{d(a)}, T\right)$ can thus be expressed in terms of:

$$E^2_{Fno(Fpo)}\left(N^*, r_{d(a)}, x\right) \times \frac{\sqrt{\varepsilon\left(r_{d(a)}\right)}}{N^{*\frac{5}{4}}}.$$

Then, from the well-known idea of Stokes, Einstein, Sutherland and Reynolds, we can define our viscosity coefficient, $\mathbb{V}(N^*, r_{d(a)}, T)$, and its reduced one, $R\mathbb{V}(N^*, r_{d(a)}, T)$, by:

$$\frac{\mathbb{V}(N^*, r_{d(a)}, T)}{q} \equiv \frac{1}{6\pi \times \mu\left(N^*, r_{d(a)}, T\right) \times R_{WS}(N^*)} \left(\frac{V}{cm} \times \frac{s}{cm^2}\right), R\mathbb{V}(N^*, r_{d(a)}, T) \equiv \frac{\mathbb{V}(N^*, r_{d(a)}, T)}{\mathbb{V}(N^*, r_{d(a)}, T=0K)}, \quad (22)$$

where $R_{WS}(N^*) \equiv \left(\frac{3}{4\pi N^*}\right)^{1/3}$ is the effective Wigner-Seitz radius, decreasing with increasing N^*.

Further, as noted above for$\mu\left(N^*, r_{d(a)}, T\right)$, $\mathbb{V}(N^*, r_{d(a)}, T)$ can thus be expressed in terms of $\frac{(N^*)^{\frac{19}{12}} \times \left[N_{CDn(NDp)}(r_{d(a)}, x)\right]^{\frac{1}{6}}}{E^2_{Fn(Fp)}\left(N^*, r_{d(a)}, T\right) \times G_2\left(N^*, r_{d(a)}, T\right)}$, giving raise to some concluding remarks as follows.

$-$**With increasing $r_{d(a)}$** and for given low T and high $N(N^* \simeq N)$,

$-$**with decreasing T** and for given high N and high $r_{d(a)}$, and

$-$**with increasing N** and for low T and high $r_{d(a)}$,

$\mathbb{V}(N)$ **thus increases**, **suggesting an equivalence between degeneracy-compensation-viscosity concept,** and **being used to explain the numerical results of $\mathbb{V}$, given next Tables.**

Then, the activation energy, $AE(N^*, r_{d(a)}, T)$, can be defined as[17]:

$$AE\left(N^*, r_{d(a)}, T\right) \equiv k_B T \times Ln\left(R\mathbb{V}\left(N^*, r_{d(a)}, T\right)\right) \leq 0 \text{ eV}, \quad (23)$$

according to the reduced activation energy, $RAE\left(N^*, r_{d(a)}, T\right)$, given by:

$$RAE\left(N^*, r_{d(a)}, T\right) \equiv \frac{AE\left(N^*, r_{d(a)}, T\right)}{k_B T} \equiv Ln\left(R\mathbb{V}\left(N^*, r_{d(a)}, T\right)\right) \leq 0.$$

Furthermore, the Hall factor, $r(N^*, r_{d(a)}, T)$, is defined by:

$r(N^*, r_{d(a)}, T) \equiv \frac{\langle\tau^2\rangle_{FDDF}}{[\langle\tau\rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N^*, r_{d(a)}, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, r_{d(a)}, T)}$, and the Hall mobility is found to be given by:

$$\mu_H\left(N^*, r_{d(a)}, T\right) \equiv \mu\left(N^*, r_{d(a)}, T\right) \times r(N^*, r_{d(a)}, T)\ \left(\frac{cm^2}{V \times s}\right). \quad (24)$$

Global Einstein -and- Van Cong relations

By taking into account Equations (21, 22) and for **any N^*, $r_{d(a)}$, and T, our global relations** are found to be defined by:

$$\mathbb{R}\left(N^*, r_{d(a)}, T\right) \equiv \frac{D\left(N^*, r_{d(a)}, T\right)}{\mu\left(N^*, r_{d(a)}, T\right)} \equiv \frac{D\left(N^*, r_{d(a)}, T\right)}{\sigma\left(N^*, r_{d(a)}, T\right)} \times q \times N^* \equiv D\left(N^*, r_{d(a)}, T\right) \times \frac{\mathbb{V}\left(N^*, r_{d(a)}, T\right)}{q} \times$$

$$6\pi \times R_{WS}(N^*) \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right),\ \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \quad (25)$$

Where $D\left(N^*, r_{d(a)}, T\right)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (17), the mobility $\mu\left(N^*, r_{d(a)}, T\right)$ is determined in Eq. (21), the conductivity $\sigma\left(N^*, r_{d(a)}, T\right)$ is determined in Eq. (19), and the viscosity coefficient $\mathbb{V}(N^*, r_{d(a)}, T)$ is defined in Eq. (22). By differentiating this function $\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}$, with respect to u, being defined in Eq. (11), one obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (24) can also be rewritten as: $\mathbb{R}(u) = \frac{k_B \times T}{q} \times u\frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)}$ where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}}e^{-du}(1 - du) + \frac{2}{3}Au^{B-1}F(u)\left[\left(1 + \frac{3B}{2}\right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}}\right]$. One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] = 1$, and therefore: $\mathbb{R}(u \to 0) = \frac{k_B \times T}{q}$, being a **well-known local ($u \to 0$) relation given by Einstein**, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2u^{2B}$ and $u[V' \times W - V \times W'] = \frac{2}{3}au^{2/3}A^2u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, our relation (24) is reduced to a correct result: $\mathbb{R}\left(N^*, r_{d(a)}, x, T = 0K\right) = \frac{2}{3}E_{Fno(Fpo)}(N^*)/q$.

It should be noted that the well-known "local" Einstein relation, being valid only at the lowest degeneracy (or $u \to 0$), is now globalized by our "Global" Einstein -and- Van Cong relations (25), valid at any degeneracy (or any u).

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (25) can be rewritten as:

$$\mathbb{R}_{(VC)}\left(N^*, r_{d(a)}, x, T = 0K\right) \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}\right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)}\right],$$

Where $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

Then, in **Tables 3n and 3p in Appendix 1,** for given $r_{d(a)}$, and T=(4.2 K and 300 K), the numerical results of $\mathbb{V}, \mu$ and D, expressed as functions of N, are obtained by using Equations (22, 21 and 25).

Further, in **Tables 4n and 4p in Appendix 1**, for given $r_{d(a)}$ and N, the numerical results of Fermi energy $E_{Fn(Fp)(T=0K)}(eV)$ [or reduced Fermi energy $\xi_{n(p)}(N^*, r_d, T)$], and viscosity coefficient $\mathbb{V}\left(N^*, r_{d(a)}, T\right)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using Equations (17, 22), respectively.

Thermoelectric Coefficients

Then, from Eq. (19), obtained for $\sigma(N^*, r_{d(a)}, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, $S_{E[O]}$, is found to be given by:

$$S\left(N^*, r_{d(a)}, T\right) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \frac{\partial \ln\sigma}{\partial E}\Big]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln\sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

$$= -2\sqrt{L} \times \frac{\sqrt{ZT_{Mott}}}{1 + ZT_{Mott}}\left(\frac{V}{K}\right) < 0, \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2},$$

Then, using Eq. (17), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$Y_{Sb}\left(N^*, r_{d(a)}, T\right) \equiv \left[1 - \frac{y^2}{3 \times G_2\left(y = \frac{\pi}{\xi_{n(p)}}\right)}\right],$$

$$S\left(N^*, r_{d(a)}, T\right) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2Y_{Sb}}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)} = -2\sqrt{L} \times \frac{\sqrt{ZT_{Mott}}}{1 + ZT_{Mott}}\left(\frac{V}{K}\right) <$$

$0, \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}$, (26) according to:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{ZT_{Mott} \times [1 - ZT_{Mott}]}{[1 + ZT_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \to +\infty$ or $\xi_{n(p)} \to +0$, one has a same limiting value of $S_{O[E]}$: $S_{O[E]} \to -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit is found to be defined by:

$$ZT\left(N^*, r_{d(a)}, T\right) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times ZT_{Mott}}{[1 + ZT_{Mott}]^2}. \quad (27)$$

Here, one notes that: (i) $\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{max.} = 1$, $ZT_{Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $ZT_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient can be defined by:

$$VC1\left(N^*, r_{d(a)}, T\right) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (28)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

and the second Van-Cong coefficient as:

$$VC2\left(N^*, r_{d(a)}, T\right) \equiv T \times VC1\left(N^*, r_{d(a)}, T\right) \equiv T \times N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (29)$$

The Thomson coefficient, Ts , by:

$$Ts\left(N^*, r_{d(a)}, T\right) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \quad (30)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

And the Peltier coefficient, Pt, as:

$$Pt\left(N^*, r_{d(a)}, x, T\right) \equiv T \times S\ (V). \quad (31)$$

Then, in **Tables 5n and 5p in Appendix1,** for given r_d and N, the numerical results of $\xi_{n(p)}$, $\mathbb{V}$, AE, $\sigma_{Th.}$, S, VC1, VC2, Ts, Pt and ZT are obtained, as functions of T=(4.2 K, 300 K), by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively.

Further, in **Tables 6n and 6p in Appendix1**, for given $r_{d(a)}$ and T=0K, the numerical results of Fermi energy $E_{Fn(Fp)}(eV)$, $\mathbb{V}(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^5}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and $D\left(\frac{cm^2}{s}\right)$, are obtained, as functions of $\mathbf{N=[\cong N_{CDn}, N1=2\times 10^{23}\ cm^{-3}, N2=3\times 10^{23}\ cm^{-3}, N3=4\times 10^{23}\ cm^{-3}}]$, by using Equations (17, 22, 19, 21, 25), respectively.

Finally, **in Table 7 in Appendix 1**, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that **(i)** for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $ZT_{max.}=1$, **(ii)** for $\xi_{n(p)}=1$, those of S, ZT, ZT_{Mott}, VC1, and Ts present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and **(iii)** for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{Mott}=1$.

Furthermore, it is interesting to remark that the VC2-coefficient, defined in Eq. (29), is related to our Global Einstein -and- Van Cong relations (25), by:

$$\frac{k_B}{q}\times VC2\left(N^*, r_{d(a)}, T\right) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D\left(N^*, r_{d(a)}, T\right)}{\mu\left(N^*, r_{d(a)}, T\right)}\left(\frac{V^2}{K}\right), \quad \frac{k_B}{q}=\sqrt{\frac{3\times L}{\pi^2}},$$

According, in this work, with the use of our Eq. (26), to: $VC2\left(N, r_{d(a)}, T\right) \equiv T \times VC1\left(N, r_{d(a)}, T\right) \equiv -\frac{D\left(N^*, r_{d(a)}, T\right)}{\mu\left(N^*, r_{d(a)}, T\right)} \times 2 \times \frac{ZT_{Mott}\times[1-ZT_{OMott}]}{[1+ZT_{Mott}]^2}$ (V), $ZT_{Mott}=\frac{\pi^2}{3\times \xi_{n(p)}^2}$.

Therefore, we obtain in this work:

$$\mathbb{R}\left(N^*, r_{d(a)}, T\right) \equiv \frac{D\left(N^*, r_{d(a)}, T\right)}{\mu\left(N^*, r_{d(a)}, T\right)} \equiv \frac{D\left(N^*, r_{d(a)}, T\right)}{\sigma\left(N^*, r_{d(a)}, T\right)} \times q \times \left(N^*/g_{c(v)}\right) \equiv D\left(N^*, r_{d(a)}, T\right) \times \frac{\mathbb{V}\left(N^*, r_{d(a)}, T\right)}{q} \times 6\pi \times R_{WS}(N^*) \equiv -\frac{VC2\left(N, r_{d(a)}, T\right)}{2} \times \frac{[1+ZT_{Mott}]^2}{ZT_{Mott}\times[1-ZT_{Mott}]} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*}, \quad (32)$$

being new results.

Concluding Remarks

In the $n(p)-$type degenerate $Na-AM$, the electrical and thermoelectric coefficients, enhanced by : **(i)** our static dielectric constant law, $\varepsilon(r_{d(a)})$, given in Equations (5, 6), **(ii)** our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq. (17), being accurate with a precision of the order of $\mathbf{2.11\times 10^{-4}}$ [9], affecting all those coefficients, and finally **(iii)** our electrical

conductivity model, given in Eq. (19), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.[2-5]

Finally, the numerical results of those coefficients are given and discussed in Tables 3n, 3p, 4n, 4p, 5n, 5p, 6n, 6p, and 7 in Appendix 1.

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APPENDIX 1

Table 1: Expressions for $G_{p>1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, are used to determine the electrical-and-thermoelectric coefficients, suggesting that, with decreasing T and for given (high N, r_d, x), the function $G_{p>1}(y)$ decreases as: $G_{p>1}(y \to 0) \to 1$.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}\right)$	$(1+y^2)$	$\left(1+\frac{35y^2}{24}\right)$	$(1+2y^2)$	$\left(1+\frac{21y^2}{8}\right)$

Table 2. In the n(p) type degenerate **Na** – Metal, the numerical results of $B_{do(ao)} = 1.2052979 \times 10^{11}\ (\frac{N}{m^2})$, ε, $N_{CDn(CDp)}$, are computed, using Equations (2), (5), (6), and (8), and $N^{EBT}_{CDn(CDp)}$, given in our recent work [1], respectively, noting that the relative deviations in absolute values are defined by: $|RD| \equiv \left|1 - \frac{N^{EBT}_{CDn(CDp)}}{N_{CDn(Cdp)}}\right|$, giving rise to their maximal value equal to $\mathbf{3.4931 \times 10^{-7}}$. It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N^{EBT}_{CDn(CDp)}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor	P	As	Sb	Sn	**Na**
r_d (nm) ↗	0.110	0.118	0.136	0.140	**0.166**
$\varepsilon(r_d)$ ↘	2.8319	1.7053	1.1697	1.1212	1
$N_{CDn}(r_d)$ in 10^{21} cm^{-3} ↗	5.5038570	25.205638	78.100359	88.693230	125.00085
$N^{EBT}_{CDn}(r_d)$ in 10^{21} cm^{-3} ↗	5.5038554	25.205631	78.100343	88.693201	125.00082
|RD| in 10^{-7}	2.8489	2.8590	2.0438	3.2447	2.6347
Acceptor	Ga	Mg	In	Cd	**Na**
r_a (nm) ↗	0.126	0.141	0.144	0.148	**0.166**
$\varepsilon(r_a)$ ↘	1.3677	1.1212	1.0834	1.0543	1
$N_{CDp}(r_a)$ in 10^{22} cm^{-3} ↗	4.8858614	8.8693230	9.8286993	10.667367	42.187787
$N^{EBT}_{CDp}(r_a)$ in 10^{22} cm^{-3} ↗	4.8858598	8.8693201	9.8286972	10.667363	42.187776
|RD| in 10^{-7}	3.3206	3.2431	2.1367	**3.4931**	2.6505

Table 3n: For a given r_d, and T=(4.2 K and 300 K), the numerical results of $\mathbb{V}, \mu$ and D, expressed respectively in $\left(\frac{10^6 \times eV}{cm} \times \frac{s}{cm^2}; \frac{cm^2}{V \times s}; \frac{cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22, 21 and 25).

Donor	P	As	Sb	Sn
r_d (nm) ↗	0.110	0.118	0.136	0.140
At T=4.2 K,				
N (10^{23} cm^{-3})				
2	(1.530 ; 3.239 ; 24.93)	(3.042 ; 1.571 ; 11.26)	(4.102 ; 1.034 ; 5.827)	(4.113 ; 1.000 ; 5.306)
3	(1.866 ; 3.049 ; 30.94)	(3.870 ; 1.436 ; 13.92)	(5.761 ; 0.899 ; 7.553)	(5.923 ; 0.860 ; 6.995)
4	(2.140 ; 2.931 ; 36.14)	(4.547 ; 1.356 ; 16.16)	(7.092 ; 0.826 ; 8.900)	(7.365 ; 0.787 ; 8.288)
5	(2.375 ; 2.847 ; 40.82)	(5.130 ; 1.300 ; 18.15)	(8.231 ; 0.779 ; 10.05)	(8.596 ; 0.740 ; 9.381)

At T=300 K,				
N (10^{23} cm^{-3})				
2	(1.530 ; 3.239 ; 24.93)	(3.042 ; 1.571 ; 11.26)	(4.101 ; 1.034 ; 5.828)	(4.113 ; 1.000 ; 5.306)
3	(1.866 ; 3.049 ; 30.94)	(3.870 ; 1.436 ; 13.92)	(5.761 ; 0.899 ; 7.553)	(5.923 ; 0.860 ; 6.995)
4	(2.140 ; 2.931 ; 36.14)	(4.547 ; 1.356 ; 16.16)	(7.092 ; 0.826 ; 8.900)	(7.365 ; 0.787 ; 8.288)
5	(2.375 ; 2.847 ; 40.82)	(5.130 ; 1.300 ; 18.15)	(8.231 ; 0.779 ; 10.05)	(8.596 ; 0.740 ; 9.382)

Table 3p: For a given r_a, and T=(4.2 K and 300 K), the numerical results of $\mathbb{V}, \mu$ and D, expressed respectively in $\left(\frac{10^6 \times eV}{cm} \times \frac{s}{cm^2}; \frac{cm^2}{V \times s}; \frac{cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22, 21 and 25).

Acceptor	Ga	Mg	In	Cd
r_d (nm) ↗	0.126	0.141	0.144	0.148
At T=4.2 K,				
N (10^{23} cm^{-3})				
3	(5.011 ; 1.077 ; 9.827)	(5.923 ; 0.860 ; 6.995)	(6.032 ; 0.831 ; 6.557)	(6.100 ; 0.811 ; 6.214)
4	(6.012 ; 1.003 ; 11.45)	(7.365 ; 0.787 ; 8.288)	(7.572 ; 0.757 ; 7.813)	(7.725 ; 0.735 ; 7.445)
5	(6.873 ; 0.954 ; 12.87)	(8.696 ; 0.740 ; 9.382)	(8.884 ; 0.710 ; 8.866)	(9.105 ; 0.688 ; 8.470)
At T=300 K,				
N (10^{23} cm^{-3})				
3	(5.011 ; 1.077 ; 9.827)	(5.923 ; 0.860 ; 6.995)	(6.032 ; 0.831 ; 6.557)	(6.100 ; 0.811 ; 6.215)
4	(6.012 ; 1.003 ; 11.45)	(7.365 ; 0.787 ; 8.288)	(7.572 ; 0.757 ; 7.813)	(7.725 ; 0.735 ; 7.445)
5	(6.873 ; 0.954 ; 12.87)	(8.696 ; 0.740 ; 9.382)	(8.884 ; 0.710 ; 8.867)	(9.105 ; 0.688 ; 8.470)

Table 4n: For given r_d and N, the numerical results of Fermi energy $E_{Fn(T=0K)}$(eV) [or reduced Fermi energy $\xi_n(N^*, r_d, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_d, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using Equations (17, 22), respectively. In particular, from these numerical results of $\mathbb{V}(T)$, one observes that, for such given (r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring [17] and complex fluids by Wenhao [18], and from those of $\mathbb{V}(r_d)$, one observes that, for given (T and N), they increase with increasing r_d, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Donor	As	Sb	Sn
r_d (nm) [4] ↗	0.118	0.136	0.140
For **N=2×10^{23} cm^{-3},**			
N_{CDn} in 10^{22} cm^{-3} ↗	**2.52056**	**7.81003**	**8.86932**
$E_{Fn(T=0K)}$(eV) ↘	**11.76490843**	**11.76490842**	**11.76490841**
$\mathbb{V}_{(0K)}$ ↗	$\mathbf{3.0425668 \times 10^6}$	$\mathbf{4.1016576 \times 10^6}$	$\mathbf{4.1133874 \times 10^6}$
$\xi_{n(T=4.2K)}$ ↘	2.9707337×10^4	2.3362199×10^4	2.1988376×10^4

$\mathbb{V}_{(4.2K)}$ ↗	3.0425667×10^6	4.1016575×10^6	4.1133873×10^6
$\xi_{n(T=77K)}$ ↘	1.620401×10^3	1.2743027×10^3	1.1993670×10^3
$\mathbb{V}_{(77K)}$ ↗	3.0425602×10^6	4.1016430×10^6	4.1133710×10^6
$\xi_{n(T=100K)}$ ↘	1.2477091×10^3	981.213628	923.51314
$\mathbb{V}_{(100K)}$ ↗	3.0425556×10^6	4.1016330×10^6	4.1133597×10^6
$\xi_{n(T=150K)}$ ↘	831.8069	654.1435	615.6765
$\mathbb{V}_{(150K)}$ ↗	3.0425415×10^6	4.1016024×10^6	4.1133250×10^6
$\xi_{n(T=200K)}$ ↘	623.8562	490.6087	461.7586
$\mathbb{V}_{(200K)}$ ↗	3.0425218×10^6	4.1015595×10^6	4.1132764×10^6
$\xi_{n(T=250K)}$ ↘	499.0857	392.4881	369.4081
$\mathbb{V}_{(250K)}$ ↗	3.0422496×10^6	4.1015043×10^6	4.1132139×10^6
$\xi_{n(T=300K)}$ ↘	415.9057	327.0746	307.8413
$\mathbb{V}_{(300K)}$ ↗	3.0424656×10^6	4.1014369×10^6	4.1131376×10^6

Table 4p: For given r_a and N, the numerical results of Fermi energy $E_{Fp(T=0K)}$(eV) [or reduced Fermi energy $\xi_p(N^*, r_a, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_a, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T, by using Equations (17, 22), respectively. In particular, from these numerical results of $\mathbb{V}(T)$, one observes that, for such given (r_a and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring [17] and complex fluids by Wenhao [18], and from those of $\mathbb{V}(r_a)$, one observes that, for given (T and N), they increase with increasing r_a, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor	**Ga**	Mg	**Cd**
r_a (nm) ↗	**0.126**	0.141	**0.148**
For **N=3×10^{23} cm^{-3},**			
N_{CDp} in 10^{22} cm^{-3} ↗	**4.8858614**	**8.869323**	**10.667367**
$E_{Fp(T=0K)}$(eV) ↘	**13.6934878**	**12.2042425**	**11.5017138**
$\mathbb{V}_{(0K)}$ ↗	$\mathbf{5.0113095 \times 10^6}$	$\mathbf{5.92341781 \times 10^6}$	$\mathbf{6.09981190 \times 10^6}$
$\xi_{p(T=4.2K)}$ ↘	3.7826048×10^4	3.3712248×10^4	3.1771626×10^4
$\mathbb{V}_{(4.2K)}$ ↗	5.0113094×10^6	5.9234178×10^6	6.0998118×10^6
$\xi_{p(T=77K)}$ ↘	2.0632396×10^3	1.8388506×10^3	1.7329985×10^3
$\mathbb{V}_{(77K)}$ ↗	5.0113027×10^6	5.9234077×10^6	6.0998002×10^6
$\xi_{p(T=100K)}$ ↘	1.5886948×10^3	1.4159153×10^3	1.3344092×10^3
$\mathbb{V}_{(100K)}$ ↗	5.0112980×10^6	5.9234008×10^6	6.0997922×10^6
$\xi_{p(T=150K)}$ ↘	1.0591305×10^3	943.944254	889.606921
$\mathbb{V}_{(150K)}$ ↗	5.0112837×10^6	5.9233795×10^6	6.0997675×10^6
$\xi_{p(T=200K)}$ ↘	794.3485	707.9589529	667.2059996
$\mathbb{V}_{(200K)}$ ↗	5.0112637×10^6	5.9233498×10^6	6.0997330×10^6
$\xi_{p(T=250K)}$ ↘	635.4795	566.3679465	533.7656318

$\mathbb{V}_{(250K)}$	↗	5.0112380×10^{6}	5.9233115×10^{6}	6.0996886×10^{6}
$\xi_{p(T=300K)}$	↘	529.5670	471.9740875	444.8055407
$\mathbb{V}_{(300K)}$	↗	5.0112066×10^{6}	5.9232647×10^{6}	6.0996344×10^{6}

Table 5n: For given r_d and N, the numerical results of ξ_n, $\mathbb{V}$, AE, $\sigma_{Th.}$, S, VC1, VC2, Ts, Pt and ZT are obtained, as functions of T=(4.2 K, 300 K), by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $\mathbb{V}$(T), one observes that, for such given (r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring [17] and complex fluids by Wenhao [18], and from those of $\mathbb{V}(r_d)$, one observes that, for given (T and N), they increase with increasing r_d, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Donor		As	Sb	Sn
r_d (nm) [4]	↗	0.118	0.136	0.140
For **N=2 × 10^{23} cm^{-3},**				
N_{CDn} in 10^{22} cm^{-3}	↗	**2.52056**	**7.81003**	**8.86932**
$\xi_{n(T=4.2K)} \times 10^{-4}$	↘	**2.9707337**	**2.3362199**	**2.1988376**
$\xi_{n(T=300K)}$	↘	415.905682	327.074562	307.8412757
$\mathbb{V}_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2}\right) \times 10^{-6}$	↗	3.0425668	4.1016575	4.1133874
$\mathbb{V}_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2}\right) \times 10^{-6}$	↗	3.0424656	4.1014369	4.1131376
$-AE_{(4.2K)}$ (eV) $\times 10^{12}$	↗	2.3604581	3.8167716	4.3086107
$-AE_{(300K)}$ (eV) $\times 10^{7}$	↗	8.6022403	13.9094961	15.7019096
$\sigma_{Th.\,(4.2K)} (\frac{W}{cm \times K}) \times 10^{3}$	↘	4.5158397	2.0716637	1.8299444
$\sigma_{Th.\,(300K)} (\frac{W}{cm \times K}) \times 10$	↘	3.2257071	1.4983940	1.3071826
$-S_{(4.2K)} (\frac{V}{K}) \times 10^{8}$	↗	1.9085961	2.4269679	2.5786036
$-S_{(300K)} (\frac{V}{K}) \times 10^{6}$	↗	1.3632473	1.7334752	1.8417718
$-VC1_{(4.2K)} (\frac{V}{K}) \times 10^{8}$	↗	1.2723974	1.6179785	1.7190690
$-VC1_{(300K)} \left(\frac{V}{K}\right) \times 10^{7}$	↗	9.0878398	11. 5555239	12.2773067
$-VC2_{(4.2K)} (\frac{V}{K}) \times 10^{8}$	↗	5.3440690	6.7955099	7.22009000
$-VC2_{(300K)} \left(\frac{V}{K}\right) \times 10^{4}$	↗	2.7263520	3.4666572	3.68319200
$-Ts_{(4.2K)} (\frac{V}{K}) \times 10^{8}$	↗	1.9085961	2.4269678	2.57860360
$-Ts_{(300K)} \left(\frac{V}{K}\right) \times 10^{6}$	↗	1.3631760	1.7333286	1.84159600
$-Pt_{(4.2K)}$(V) $\times 10^{8}$	↗	8.0161036	10.1932650	10.83013520
$-Pt_{(300K)}$(V) $\times 10^{4}$	↗	4.0897418	5.2004255	5.52531550
$ZT_{(4.2K)} \times 10^{8}$	↗	1.4911146	2.4110767	2.72177460
$ZT_{(300K)} \times 10^{5}$	↗	7.6073300	12.3003716	13.88527960

Table 5p: For given r_a and N, the numerical results of ξ_p, $\mathbb{V}$, AE, $\sigma_{Th.}$, S, VC1, VC2, Ts, Pt and ZT are obtained, as functions of T=(4.2 K, 300 K), by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $\mathbb{V}(T)$, one observes that, for such given (r_a and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring [17] and complex fluids by Wenhao [18], and from those of $\mathbb{V}(r_a)$, one observes that, for given (T and N), they increase with increasing r_d, suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

Acceptor	**Ga**	Mg	Cd
r_a (nm) ↗	**0.126**	0.141	0.148
For **x=0** and **N=3 × 10^{23} cm^{-3}**,			
N_{CDp} in 10^{22} cm^{-3} ↗	**4.8858614**	**8.869323**	**10.667367**
$\xi_{p(T=4.2K)} \times 10^{-4}$ ↘	**3.782605**	**3.3712248**	**3.1771626**
$\xi_{p(T=300K)}$ ↘	529.5670	471.97409	444.805541
$\mathbb{V}_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2}\right) \times 10^{-6}$ ↗	5.0113094	5.9234178	6.0998118
$\mathbb{V}_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2}\right) \times 10^{-6}$ ↗	5.0112066	5.9232647	6.0996344
$-AE_{(4.2K)}$ (eV) $\times 10^{12}$ ↗	1.4559350	1.8329407	2.0636921
$-AE_{(300K)}$ (eV) $\times 10^{7}$ ↗	5.3058770	6.6798027	7.5207327
$\sigma_{Th.\,(4.2K)} \left(\frac{W}{cm \times K}\right) \times 10^{3}$ ↘	4.4451038	2.9871309	2.5764021
$\sigma_{Th.\,(300K)} \left(\frac{W}{cm \times K}\right) \times 10$ ↘	3.1751393	2.1337201	1.8403407
$-S_{(4.2K)} \left(\frac{V}{K}\right) \times 10^{8}$ ↗	1.4989488	1.6818608	1.7845894
$-S_{(300K)} \left(\frac{V}{K}\right) \times 10^{6}$ ↗	1.0706604	1.2013047	1.2746776
$-VC1_{(4.2K)} \left(\frac{V}{K}\right) \times 10^{8}$ ↗	0.9992992	1.1212405	1.1897262
$-VC1_{(300K)} \left(\frac{V}{K}\right) \times 10^{7}$ ↗	7.1375058	8.0083729	8.4974619
$-VC2_{(4.2K)} \left(\frac{V}{K}\right) \times 10^{8}$ ↗	4.1970565	4.7092101	4.9968502
$-VC2_{(300K)} \left(\frac{V}{K}\right) \times 10^{4}$ ↗	2.1412518	2.4025119	2.5492386
$-Ts_{(4.2K)} \left(\frac{V}{K}\right) \times 10^{8}$ ↗	1.4989487	1.6818607	1.7845894
$-Ts_{(300K)} \left(\frac{V}{K}\right) \times 10^{6}$ ↗	1.0706259	1.2012559	1.2746193
$-Pt_{(4.2K)}(V) \times 10^{8}$ ↗	6.2955848	7.0638152	7.4952754
$-Pt_{(300K)}(V) \times 10^{4}$ ↗	3.2119812	3.6039142	3.8240327
$ZT_{(4.2K)} \times 10^{8}$ ↗	0.9197219	1.1578787	1.3036457
$ZT_{(300K)} \times 10^{5}$ ↗	4.6923076	5.9073043	6.6509497

Table 6n: For given r_d and T=0K, the numerical results of Fermi energy E_{Fn}(eV), $\mathbb{V}(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^5}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and D $\left(\frac{cm^2}{s}\right)$, are obtained, as functions of **N=[≅ N_{CDn}, N1=2×10^{23} cm^{-3}, N2 = 3×10^{23} cm^{-3}, N3 = 4×10^{23} cm^{-3}]**, by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions , (T=0K, N=N_{CDn} or $N^* = 0$), and (ii) those values of $E_{Fn} \gtrsim 0$, $\mathbb{V} \gtrsim 0, \sigma \gtrsim 0, \mu \gtrsim 0$, and D ≳ 0, obtained for N ≳ N_{CDn} , thus define the properties of **the degenerate (or viscous) Na-alkali metal**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}$, one observes that, for such given (r_d and T=0K), they **increase with increasing N (or increasing E_{Fn})**, in good agreement with those, obtained in complex fluids by Wenhao [18], suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

N ≳ N_{CDn}	≅ N_{CDn}	N1	N2	N3
For $r_d = r_P$ and N_{CDn}=5.503857×10^{21} cm^{-3},				
E_{Fn} ↗	≅0 [0]	11.548	15.227	18.504
$\mathbb{V}$ ↗	≅0 [0]	1.5299	1.8662	2.1401
σ ↗	≅0 [0]	1.0092	1.4385	1.8523
μ ↘	≅0 [0]	3.2386	3.0488	2.9306
D ↗	≅0 [0]	24.927	30.942	36.144
For $r_d = r_{As}$ and N_{CDn}=25.205638×10^{21} cm^{-3},				
E_{Fn} ↗	≅0 [0]	10.754	14.540	17.883
$\mathbb{V}$ ↗	≅0 [0]	3.0426	3.8704	4.5471
σ ↗	≅0 [0]	0.4401	0.6324	0.8143
μ ↘	≅0 [0]	1.5715	1.4365	1.3560
D ↗	≅0 [0]	11.265	13.922	16.162
For $r_d = r_{Sb}$ and N_{CDn}=78.100359×10^{21} cm^{-3},				
E_{Fn} ↗	≅0 [0]	8.4574	12.609	16.158
$\mathbb{V}$ ↗	≅0 [0]	4.1016	5.7610	7.0921
σ ↗	≅0 [0]	0.2019	0.3195	0.4262
μ ↘	≅0 [0]	1.0338	0.8987	0.8264
D ↗	≅0 [0]	5.8275	7.5526	8.8998
For $r_d = r_{Sn}$ and N_{CDn}=88.69323×10^{21} cm^{-3},				
E_{Fn} ↗	≅0 [0]	7.9600	12.204	15.801
$\mathbb{V}$ ↗	≅0 [0]	4.1134	5.9234	7.3654
σ ↗	≅0 [0]	0.1783	0.2911	0.3925

μ ↘	≅0 [0]	1.0001	0.8599	0.7869
D ↗	≅0 [0]	5.3059	6.9949	8.2876

Table 6p: For given r_a and T=0K, the numerical results of Fermi energy $E_{Fp}(eV)$, $\mathbb{V}(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^4}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and D $\left(\frac{cm^2}{s}\right)$, are obtained, as functions of **N=[≅ N_{CDp}, N1=3×10^{23} cm^{-3}, N2 = 4×10^{23} cm^{-3}, N3 = 5×10^{23} cm^{-3}]**, by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions , (T=0K, N=N_{CDp} or $N^* = 0$), and (ii) those values of $E_{Fp} \gtrsim 0$, $\mathbb{V} \gtrsim 0$, $\sigma \gtrsim 0$, $\mu \gtrsim 0$, and D ≳ 0, obtained for N ≳ N_{CDp} , thus define the properties of **the degenerate (or viscous) Na-alkali metal**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}$, one observes that, for such given (r_d and T=0K), they **increase with increasing N (or increasing E_{Fp})**, in good agreement with those, obtained in complex fluids by Wenhao [18], suggesting **an equivalence between degeneracy-compensation-viscosity concept**.

N ≳ N_{CDp}	≅ N_{CDp}	N1	N2	N3
For $r_a = r_{Ga}$ and N_{CDp}=4.8858614×10^{22} cm^{-3},				
E_{Fp} ↗	≅0 [0]	13.693	17.122	20.235
$\mathbb{V}$ ↗	≅0 [0]	5.0113	6.0123	6.8733
σ ↗	≅0 [0]	4.3323	5.6456	6.8975
μ ↘	≅0 [0]	1.0767	1.0035	0.9543
D ↗	≅0 [0]	9.8266	11.452	12.870
For $r_a = r_{Mg}$ and N_{CDp}=8.869323×10^{22} cm^{-3},				
E_{Fp} ↗	≅0 [0]	12.204	15.801	19.026
$\mathbb{V}$ ↗	≅0 [0]	5.9234	7.3654	8.5965
σ ↗	≅0 [0]	2.9113	3.9249	4.8753
μ ↘	≅0 [0]	0.8599	0.7869	0.7398
D ↗	≅0 [0]	6.9949	8.2576	9.3817
For $r_a = r_{In}$ and N_{CDp}=9.8286993×10^{22} cm^{-3},				
E_{Fp} ↗	≅0 [0]	11.832	15.475	18.729
$\mathbb{V}$ ↗	≅0 [0]	6.0319	7.5722	8.8838
σ ↗	≅0 [0]	2.6872	3.6617	4.5716
μ ↘	≅0 [0]	0.8315	0.7575	0.7103
D ↗	≅0 [0]	6.5572	7.8128	8.8666

For $r_a = r_{Cd}$ and N_{CDp}=10.667367×10^{22} cm^{-3},

E_{Fp}	↗	≅0 [0]	11.502	15.187	18.467
$\mathbb{V}$	↗	≅0 [0]	6.0998	7.7250	9.1054
σ	↗	≅0 [0]	2.5110	3.4568	4.3366
μ	↘	≅0 [0]	0.8107	0.7355	0.6881
D	↗	≅0 [0]	6.2146	7.4454	8.4702

Table 7: In the n(p) type degenerate **Na** – Metal, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{min.}$ $\left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of ZT show a same maximum $ZT_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, those of S, ZT, ZT_{Mott}, VC1, and Ts present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{Mott} = 1$.

$\xi_{n(p)}$	↘	1.880 [1.880]		**1.8138 [1.8138]**		1.750 [1.750]		**1 [1]**	0.998 [0.998]
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562 [−1.562]	↘	**−1.563** [**−1.563**]	↗	−1.562 [−1.562]	↗	**−1.322** [**−1.322**]	↗ −1.320 [−1.320]
ZT		0.999 [0.999]	↗	**1** [1]	↘	0.999 [0.999]	↘	**0.715** [**0.715**]	↘ 0.713 [0.713]
$(ZT)_{Mott}$	↗	0.931 [0.931]		**1** [1]		1.074 [1.074]		**3.290 [3.290]**	3.306 [3.306]
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061[−0.061]	↗	0 [0]	↗	0.063 [0.063]	↗	**1.105 [1.105]**	↗ 1.109 [1.109]
$Ts\left(10^{-4}\frac{V}{K}\right)$		−0.092 [−0.092]	↗	0 [0]	↗	0.094 [0.094]	↗	**1.657 [1.657]**	↗ 1.663 [1.663]