



GLOBAL EINSTEIN -AND- VAN CONG RELATIONS, AND NEW ELECTRICAL THERMOELECTRIC LAWS IN N(P)-TYPE DEGENERATE, COMPENSATED AND VISCOUS Ca- ALKALI METAL, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW, CONDUCTIVITY MODEL, AND ESPECIALLY ACCURATE FERMI ENERGY EXPRESSION (V)

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ABSTRACT

In degenerate, compensated and viscous $n(p) - \text{Ca- alkali metal (Ca-AM)}$, the Global Einstein -and- Van Cong relations, as those given in Equations (25, 32), and electrical thermoelectric laws, enhanced by: our static dielectric constant law given in Equations (5, 6), our electrical conductivity model in Eq. (19), and especially our accurate Fermi energy expression in Eq. (17), are now investigated, by basing on the same physical model and the mathematical treatment method, as those used in our recent works.^[1-5] Their numerical results are obtained and given in next Tables in Appendix 1, suggesting an equivalence between degeneracy-and-viscosity concept, given in this Mg-AM. Here, the well-known “local” Einstein relation, being valid only at the lowest degeneracy and lowest viscosity, is now globalized by our “Global” Einstein -and- Van Cong relations, valid at any degeneracy and viscosity, as showed in Equations (25, 32).

KEYWORDS: Conductivity, Mobility, Viscosity coefficient, Diffusion coefficient, Activation energy, Fermi energy.

INTRODUCTION

In the **degenerate, compensated and viscous $n(p) - \text{Ca-alkali metal}$ ($\equiv \text{Ca-AM}$)**, the Global Einstein -and- Van Cong relations, as those given in Equations (25, 32), and various optical-electrical-thermoelectric laws, enhanced by : (i) our static dielectric constant law given in Equations (5, 6), (ii) our electrical conductivity (σ)-model obtained in Eq. (19), and especially (iii) our accurate Fermi energy expression in Eq. (17), with a precision of the order of 2.11×10^{-4} , affecting strongly all the expressions of electrical coefficients, are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in References.^[1-18]

Then, some important remarks can be summarized as follows.

(1) As observed in our recent work.^[1], the critical impurity density $N_{\text{CDn(CDp)}}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{\text{CDn(CDp)}}^{\text{EBT}}$, being obtained with a precision of the order of 3.2253×10^{-7} , as given in **Table 1 in Appendix 1**, and also in our recent work.^[1] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{\text{CDn(CDp)}} \simeq N - N_{\text{CDn(CDp)}}^{\text{EBT}}$, N being the total impurity density, as that observed in the compensated metals.

(2) The ratio of the inverse effective screening length $k_{\text{Sn(sp)}}$ to Fermi wave number $k_{\text{Fn(kp)}}$ at 0 K, $R_{\text{Sn(sp)}}(N^*)$, defined in Eq. (13), is valid at any N^* .

Therefore, by basing on this σ -expression, the Global Einstein -and- Van Cong relations defined in Equations (25, 32) and various electrical thermoelectric laws are determined. Finally, the numerical results of the electrical thermoelectric coefficients are obtained, discussed and reported in Tables (3-7) in Appendix 1, suggesting an equivalence between degeneracy-and-viscosity concept, given in this **Ca-AM**.

Our Static Dielectric Constant Law and Generalized Mott Criterium in the Metal-Insulator Transition (MIT)

First of all, in the **Ca-AM**^[1], we denote : the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{Ca} = 0.176 \text{ nm}$, the unperturbed **reduced** effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)} = 1$, the unperturbed effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)} \times m_o$, $m_o (= 9.109558 \times 10^{-28} \text{ g})$ being the free electron mass, and the vacuum dielectric constant by: $\epsilon_o = 1$.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$$E_{do(ao)} = \frac{13600 \times [m_{c(v)}]}{[\epsilon_o]^2} \text{ meV}, \quad (1)$$

and then, the isothermal bulk modulus, by :

$$B_{do(ao)} \equiv \frac{E_{do(ao)}}{\left(\frac{4\pi}{3}\right) \times (r_{do(ao)})^3}. \quad (2)$$

Effects of impurity size

Here, one shows that the effect of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, strongly affects the changes in all the energy-band-structure parameters, which can be represented by the effective relative static dielectric constant $\epsilon(r_{d(a)})$, as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p , as: $p_o = 0$, and for the deformation potential energy (or the strain energy) σ , as: $\sigma_o = 0$. Further, the two important equations, used to determine the σ -variation: $\Delta\sigma \equiv \sigma - \sigma_o = \sigma$, are defined by:

$\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\sigma}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\sigma}{dV}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$\begin{aligned} [\Delta\sigma(r_{d(a)})]_{n(p)} &= B_{do(ao)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \\ \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 &\geq 0. \end{aligned} \quad (3)$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to: the increase (the decrease) in the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)})$ in the absolute values, being obtained from the effective Bohr

model, and then such the compression (dilatation) is represented respectively by:
 $\pm [\Delta\sigma(r_{d(a)})]_{n(p)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})} \right)^2 - 1 \right] = + [\Delta\sigma(r_{d(a)})]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})} \right)^2 - 1 \right] = - [\Delta\sigma(r_{d(a)})]_{n(p)}. \quad (4) \text{ €}$$

Therefore, from above Equations (3) and (4), one obtains the expressions for $\varepsilon(r_{d(a)})$ and

$E_{d(a)}(r_{d(a)})$ as:

$$(i)\text{-for } r_{d(a)} \geq r_{do(ao)}, \text{ since } \varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3}} \leq \varepsilon_o, \text{ and}$$

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \geq 0, \quad (5)$$

according to the increase in $E_{d(a)}(r_{d(a)})$, and

$$(ii)\text{-for } r_{d(a)} \leq r_{do(ao)}, \text{ since } \varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3}} \geq \varepsilon_o, \text{ with a condition,}$$

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 < 1$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = -E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \leq 0, \quad (6)$$

corresponding to the decrease in $E_{d(a)}(r_{d(a)})$.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)})$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}) \equiv \frac{\varepsilon(r_{d(a)}) \times \hbar^2}{(m_{c(v)} \times m_o) \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)})}{m_{c(v)}}, \quad (7)$$

where $-q$ is the electron charge; $q > 0$. €

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at $T=0$ K, $N_{CDn(CDp)}(r_{d(a)})$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as^[1] :

$$N_{CDn(CDp)}(r_{d(a)})^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = M_{n(p)}, M_{n(p)} = 0.25, \quad (8)$$

depending thus on our **new $\varepsilon(r_{d(a)})$ -law**.

This result can be explained by using the definition of the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp),M}$, in the Mott's criterium, being characteristic of interactions, by:

$$r_{sn(sp),M}(N = N_{CDn(CDp)}(r_{d(a)}), r_{d(a)}) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)})} \right)^{\frac{1}{3}} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = \left(\frac{3}{4\pi} \right)^{\frac{1}{3}} \times 4 \cong 2.4814, \quad (9)$$

for any $r_{d(a)}$ -value. Therefore, one also has :

$$N_{CDn(CDp)}(r_{d(a)})^{\frac{1}{3}} \times a_{Bn(Bp)}(r_{d(a)}) = 1/4 = 0.25 = M_{n(p)}, \quad (10)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$, as that given in our recent work.^[1], we have showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $N_{CDn(CDp)}^{EBT}$, with a precision of the order of 3.2253×10^{-7} , as also showed in **Table 1 in Appendix 1**. So,

$$N_{CDn(NDp)}(r_{d(a)}) \cong N_{CDn(CDp)}^{EBT}(r_{d(a)}). \quad (11)$$

Here, it should be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen such that those of $N_{CDn(CDp)}$ and $N_{CDn(CDp)}^{EBT}$ are found to be in good agreement with experimental results.

Therefore, the effective density of electrons (holes) given in parabolic conduction (valence) bands, N^* , can be defined, as that given in compensated materials:

$$N^*(N, r_{d(a)}) \equiv N - N_{CDn(NDp)}(r_{d(a)}) \cong N - N_{CDn(CDp)}^{EBT}(r_{d(a)}) \geq 0. \quad (12)$$

One notes here that, with increasing $r_{d(a)}$ and for a given N , $N_{CDn(NDp)}(r_{d(a)})$ increases, as observed in **Table 1 in Appendix 1**, and therefore, $N^*(r_{d(a)})$ decreases, according to the **increasing compensation**.

In summary, as observed in our recent paper.^[1], for an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)})$ decreases, while $N_{CDn(NDp)}(r_{d(a)})$ increases, affecting strongly all the electrical-and-thermoelectric coefficients, as those observed in following Sections.

Physical model

In the **Ca-AM**, the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (9), in which N is replaced by N^* , is now defined by:

$\gamma \times r_{\text{sn(sp)}}(N^*, r_{\text{d(a)}}) \equiv \frac{k_{\text{Fn(Fp)}}^{-1}}{a_{\text{Bn(Bp)}}} < 1$, $r_{\text{sn(sp)}}(N^*, r_{\text{d(a)}}) \equiv \left(\frac{3}{4\pi N^*}\right)^{1/3} \times \frac{1}{a_{\text{Bn(Bp)}}(r_{\text{d(a)}})}$, being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{\text{Fn(Fp)}}(N^*) \equiv (3\pi^2 N^*)^{1/3}$ is the Fermi wave number.

Then, the ratio of the inverse effective screening length $k_{\text{sn(sp)}}$ to $k_{\text{Fn(kp)}}$ is defined by:

$$R_{\text{sn(sp)}}(N^*) \equiv \frac{k_{\text{sn(sp)}}}{k_{\text{Fn(Fp)}}} = \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{sn(sp)}}^{-1}} = R_{\text{snWS(spWS)}} + [R_{\text{snTF(spTF)}} - R_{\text{snWS(spWS)}}]e^{-r_{\text{sn(sp)}}} < 1, \quad (13)$$

being valid at any N^* .

Here, these ratios, $R_{\text{snTF(spTF)}}$ and $R_{\text{snWS(spWS)}}$, can be determined as follows.

First, for $N \gg N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{\text{snTF(spTF)}}(N^*)$ is reduced to:

$$R_{\text{snTF(spTF)}}(N^*) \equiv \frac{k_{\text{snTF(spTF)}}}{k_{\text{Fn(Fp)}}} = \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{snTF(spTF)}}^{-1}} = \sqrt{\frac{4\gamma r_{\text{sn(sp)}}}{\pi}} \ll 1, \quad (14)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{\text{snWS(spWS)}}$ is respectively reduced to:

$$R_{\text{sn(sp)WS}}(N^*) \equiv \frac{k_{\text{sn(sp)WS}}}{k_{\text{Fn}}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d[r_{\text{sn(sp)}}^2 \times E_{\text{CE}}(N^*)]}{dr_{\text{sn(sp)}}}\right), \quad (15)$$

where $E_{\text{CE}}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{\text{CE}}(N^*) = \frac{-0.87553}{0.0908 + r_{\text{sn(sp)}}} + \frac{\frac{0.87553}{0.0908 + r_{\text{sn(sp)}}} + \left(\frac{2[1 - \ln(2)]}{\pi^2}\right) \times \ln(r_{\text{sn(sp)}}) - 0.093288}{1 + 0.03847728 \times r_{\text{sn(sp)}}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{\text{Fn(Fp)}}^{-1}}{a_{\text{Bn(Bp)}}} < \frac{\mathbb{U}_{n(p)}}{E_{\text{Fno(Fpo)}}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{sn(sp)}}^{-1}} \equiv R_{\text{sn(sp)}} < 1, \quad \mathbb{U}_{n(p)}(N^*, r_{\text{d(a)}}) \equiv \frac{\sqrt{2\pi \times N^*}}{\varepsilon(r_{\text{d(a)}})} \times q^2 k_{\text{sn(sp)}}^{-1/2}, \quad (16)$$

$$\text{which gives: } A_{n(p)}(N^*, r_{\text{d(a)}}) = \frac{E_{\text{Fno(Fpo)}}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{\text{d(a)}})}, \quad E_{\text{Fno(Fpo)}}(N^*, r_{\text{d(a)}}) \equiv \frac{\hbar^2 \times k_{\text{Fn(Fp)}}^2(N^*)}{2 \times m_{\text{c(v)}} \times m_0}.$$

Here, one remarks that: (i) the generalized Thomas-Fermi energy $\mathbb{U}_{n(p)}(N^*, r_{d(a)})$ can thus be approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{d(a)})}$, C being a constant, and (ii) $\mathbb{U}_{n(p)}(r_{d(a)})$ increases with increasing $r_{d(a)}$ and for a given N , since $\varepsilon(r_{d(a)})$ decreases, as given in Ref.^[3]

Fermi Energy and Fermi-Dirac Distribution Function

Accurate Fermi Energy Expression

Here, for presentation simplicity, we change the sign of all various parameters, given in the Mg-AM, in order to obtain the same one, as given in its degenerate n-type, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N^*, r_{d(a)}, T) \equiv \frac{E_{Fn(Fp)}(N^*, r_{d(a)}, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N^*, r_{d(a)}, T)$, the reduced Fermi energy $\xi_{n(p)}(N^*, r_{d(a)}, T)$ or the Fermi energy $E_{Fn(Fp)}(N^*, r_{d(a)}, T)$, obtained in our previous paper [9], obtained with a precision of the order of 2.11×10^{-4} , is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, A = 0.0005372 \text{ and } B = 4.82842262, (17)$$

where u is the reduced electron density, $u(N^*, r_{d(a)}, T) \equiv \frac{N^*}{N_{c(v)}(T)}$, $N_{c(v)}(T) = 2 \times$

$$\left(\frac{m_{c(v)} \times m_0 \times k_B T}{2\pi\hbar^2} \right)^{\frac{3}{2}} (\text{cm}^{-3}), F(u) = au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}}, a = [3\sqrt{\pi}/4]^{2/3}, b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2, c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4, \text{ and } G(u) \simeq \ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}; d = 2^{3/2} \left[\frac{1}{\sqrt{27}} - \frac{3}{16} \right] > 0.$$

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times \ln(u)$ as $u \rightarrow 0$, **the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$),

$$\text{one gets: } E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)} \times m_0}$$

as $u \rightarrow \infty$, **the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, as $T \rightarrow 0 \text{ K}$, since $u^{-1} \rightarrow 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)} \times m_0}$, being proportional to $(N^*)^{2/3}$. Further, at $T=0 \text{ K}$ and $N^* = 0$, being physical conditions, given for the metal-insulator transition (**MIT**).

In the following, it should be noted that all the optical and electrical-and-thermoelectric properties strongly depend on such an accurate expression of $\xi_{n(p)}(N^*, r_{d(a)}, T)^{[9]}$

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^\gamma)^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

Thus, the average of E^p , calculated using the FDDF-method, as developed in our previous works^[1, 6] is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E_{Fn(Fp)}^p \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E} \right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^\gamma}{(1+e^\gamma)^2}.$$

Further, it should be noted that, at $T=0$ K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fn(Fp)})$, $\delta(E - E_{Fn(Fp)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fn(Fp)}) = 1$.

Then, at low T , by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$$G_p(E_{Fn(Fp)}) \equiv 1 + E_{Fn(Fp)}^{-p} \times \int_{-\infty}^{\infty} \frac{e^\gamma}{(1+e^\gamma)^2} \times (k_B T \gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\dots}^p C_p^\beta \times (k_B T)^\beta \times E_{Fn(Fp)}^{-\beta} \times I_\beta, \text{ where } C_p^\beta \equiv p(p-1) \dots (p-\beta+1)/\beta! \text{ and the integral } I_\beta \text{ is given by:}$$

$$I_\beta = \int_{-\infty}^{\infty} \frac{\gamma^\beta \times e^\gamma}{(1+e^\gamma)^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^\beta}{(e^{\gamma/2} + e^{-\gamma/2})^2} d\gamma, \text{ vanishing for odd values of } \beta. \text{ Then, for even values of } \beta = 2n, \text{ with } n=1, 2, \dots, \text{ one obtains:}$$

$$I_{2n} = 2 \int_0^{\infty} \frac{\gamma^{2n} \times e^\gamma}{(1+e^\gamma)^2} d\gamma.$$

Now, using the identity $(1 + e^\gamma)^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1} \pi^{2n} |B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. From the above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case ($\xi_{n(p)} > 1$) the following ratio:

$$G_{p>1} \left(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}} < 1 \right) \equiv \frac{\langle E^p \rangle_{FDDF}}{E_{Fn(Fp)}^p} = 1 + \sum_{n=1}^p \frac{p(p-1) \dots (p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n},$$

which can thus be approximated by:

$$G_{p>1}(y < 1) \simeq 1 + p(p-1) \times |B_2| \times y^2, B_2 = \frac{1}{6}. \quad (18)$$

Then, some usual results of $G_{p>1}(y < 1)$ are given in the **Table 2 in Appendix 1**, suggesting that, with increasing T (or decreasing T) and for given (N, r_d) , since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases).

Electrical Properties

Now, the electrical conductivity σ can be defined and expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{n(p)}^*(x) \times m_0}$, k being the wave number, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{U_{n(p)}} \right)^{\frac{1}{2}} \left(\frac{1}{\Omega \times cm} \right), \text{ which is thus proportional to } E_k^2,$$

where $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5} \text{ ohm}^{-1}$ and $U_{n(p)}(N^*, r_{d(a)})$ is determined in Eq. (16).

Then, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E_{Fn(Fp)}^2$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N^*, r_{d(a)}, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N^*, r_{d(a)}, T)$ for simplicity. Therefore, for $E_{Fno(Fpo)} > 0$ and $T = 0K$, $G_2(y = 0) = 1$.

Therefore, if denoting the function $H(N^*, r_{d(a)}, T)$ by:

$$H(N^*, r_{d(a)}, T) =$$

$$\left[\frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{U_{n(p)}(N^*, r_{d(a)})}} \right] \times$$

$$G_2(N^*, r_{d(a)}, T), \text{ which can be approximately expressed in terms of: } E_{Fno(Fpo)}^2(N^*) \times$$

$$G_2(N^*, r_{d(a)}, T) \times \frac{\sqrt{\varepsilon(r_{d(a)})}}{(N^*)^{\frac{1}{4}}}. \text{ Thus, with increasing } r_{d(a)} \text{ and for given } T \text{ and } N, \text{ the function}$$

$$H(r_{d(a)}) \text{ therefore decreases since } \varepsilon(r_{d(a)}) \text{ decreases, as noted in Ref.}^{[3]}$$

Therefore, the electrical conductivity σ may be determined by:

$$\sigma(N^*, r_{d(a)}, T) = \frac{q^2}{\pi \times \hbar} \times H(N^*, r_{d(a)}, T) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}} \right)^2 \left(\frac{1}{\Omega \times cm} \right), \quad (19)$$

noting that, at $T = 0K$, it can thus be approximately expressed in terms of

$$E_{Fno(Fpo)}^2(N^*, r_{d(a)}, x) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{1}{4}}}, \text{ being proportional to: } \sqrt{\varepsilon(r_{d(a)}, x)} \times (N^*)^{\frac{13}{12}}, \text{ for}$$

$$E_{Fno(Fpo)} > 0 \text{ and } T = 0K.$$

Electrical Properties

Here, if denoting, for majority electrons (holes), the thermal conductivity by: $\sigma_{Th.}(N^*, r_{d(a)}, T)$ in $\frac{W}{cm \times K}$, and the Lorenz number L by:

$L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8} (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\sigma_{Th.}}{\sigma}$, is proportional to the temperature $T(K)$, as:

$$\frac{\sigma_{Th.}(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} = L \times T. \quad (20)$$

Further, the resistivity is found to be given by: $\rho(N^*, r_{d(a)}, T) \equiv 1/\sigma(N^*, r_{d(a)}, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$.

New Electrical Coefficients

The relaxation time τ is related to by [3]:

$\tau(N^*, r_{d(a)}, T) \equiv \sigma(N^*, r_{d(a)}, T) \times \frac{m_{c(v)} \times m_0}{q^2 \times N^*}$. Therefore, the mobility μ is given by:

$$\mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N^*, r_{d(a)}, T)}{m_{c(v)} \times m_0} = \frac{\sigma(N^*, r_{d(a)}, T)}{q \times N^*} \left(\frac{cm^2}{V \times s}\right), \quad (21)$$

being expressed in terms of $\frac{\sigma(N^*, r_{d(a)}, T)}{N^*}$. Further, as noted in above Eq. (19) for $\sigma(N^*, r_{d(a)}, T)$

and at $T=0K$, $\mu(N^*, r_{d(a)}, T)$ can thus be expressed in terms of:

$$E_{Fno(Fpo)}^2(N^*, r_{d(a)}, x) \times \frac{\sqrt{\epsilon(r_{d(a)})}}{N^{*4/5}}.$$

Then, from the well-known idea of Stokes, Einstein, Sutherland and Reynolds, we can define our viscosity coefficient, $V(N^*, r_{d(a)}, T)$, and its reduced one, $RV(N^*, r_{d(a)}, T)$, by:

$$\frac{V(N^*, r_{d(a)}, T)}{q} \equiv \frac{1}{6\pi \times \mu(N^*, r_{d(a)}, T) \times R_{WS}(N^*)} \left(\frac{V}{cm} \times \frac{s}{cm^2}\right), RV(N^*, r_{d(a)}, T) \equiv \frac{V(N^*, r_{d(a)}, T)}{V(N^*, r_{d(a)}, T=0K)}, \quad (22)$$

where $R_{WS}(N^*) \equiv \left(\frac{3}{4\pi N^*}\right)^{1/3}$ is the effective Wigner-Seitz radius, decreasing with increasing N^* .

Further, as noted above for $\mu(N^*, r_{d(a)}, T)$, $V(N^*, r_{d(a)}, T)$ can thus be expressed in terms of

$$\frac{(N^*)^{19/12} \times [N_{CDn(NDp)}(r_{d(a)}, x)]^{1/6}}{E_{Fno(Fpo)}^2(N^*, r_{d(a)}, T) \times G_2(N^*, r_{d(a)}, T)}, \text{ giving raise to some concluding remarks as follows.}$$

—With decreasing T and for given N and $r_{d(a)}$, and

– **with increasing N** and for given T and $r_{d(a)}$, corresponding to the high degeneracy conditions,

$\mathbb{V}(N)$ thus increases, suggesting an equivalence between degeneracy-and-viscosity concept, and being used to explain the numerical results of $\mathbb{V}$, given next Tables.

Then, the activation energy, $AE(N^*, r_{d(a)}, T)$, can be defined as^[17]:

$$AE(N^*, r_{d(a)}, T) \equiv k_B T \times \text{Ln} \left(RW(N^*, r_{d(a)}, T) \right) \leq 0 \text{ eV}, \quad (23)$$

according to the reduced activation energy, $RAE(N^*, r_{d(a)}, T)$, given by:

$$RAE(N^*, r_{d(a)}, T) \equiv \frac{AE(N^*, r_{d(a)}, T)}{k_B T} \equiv \text{Ln} \left(RW(N^*, r_{d(a)}, T) \right) \leq 0.$$

Furthermore, the Hall factor, $r(N^*, r_{d(a)}, T)$, is defined by:

$$r(N^*, r_{d(a)}, T) \equiv \frac{\langle \tau^2 \rangle_{FDDF}}{[\langle \tau \rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}, \quad y \equiv \frac{\pi}{\xi_{n(p)}(N^*, r_{d(a)}, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, r_{d(a)}, T)}, \quad \text{and the Hall mobility is found to be given by:}$$

$$\mu_H(N^*, r_{d(a)}, T) \equiv \mu(N^*, r_{d(a)}, T) \times r(N^*, r_{d(a)}, T) \left(\frac{\text{cm}^2}{\text{V} \times \text{s}} \right). \quad (24)$$

Global Einstein -and- Van Cong relations

By taking into account Equations (21, 22) and for **any N^* , $r_{d(a)}$, and T**, our **global relations** are found to be defined by:

$$\begin{aligned} \mathbb{R}(N^*, r_{d(a)}, T) &\equiv \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \equiv \frac{D(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} \times q \times N^* \equiv D(N^*, r_{d(a)}, T) \times \frac{\mathbb{V}(N^*, r_{d(a)}, T)}{q} \times \\ 6\pi \times R_{WS}(N^*) &\equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}} \end{aligned} \quad (25)$$

where $D(N^*, r_{d(a)}, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (17), the mobility $\mu(N^*, r_{d(a)}, T)$ is determined in Eq. (21), the conductivity $\sigma(N^*, r_{d(a)}, T)$ is determined in Eq. (19), and the viscosity coefficient $\mathbb{V}(N^*, r_{d(a)}, T)$ is defined in Eq. (22). By differentiating this

function $\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}$, with respect to u, being defined in Eq.

(11), one obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (24) can also be rewritten as: $\mathbb{R}(u) = \frac{k_B \times T}{q} \times$

$u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)}$ where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}} e^{-du} (1 - du) +$

$\frac{2}{3} Au^{B-1} F(u) \left[\left(1 + \frac{3B}{2} \right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}} \right]$. One remarks that: (i) as $u \rightarrow 0$, one has: $W^2 \simeq 1$ and

$u[V' \times W - V \times W'] = 1$, and therefore: $\mathbb{R}(u \rightarrow 0) = \frac{k_B \times T}{q}$, being a **well-known local**

(u → 0) relation given by Einstein, and (ii) as $u \rightarrow \infty$, one has: $W^2 \approx A^2 u^{2B}$ and $u[V' \times W - V \times W'] = \frac{2}{3} a u^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at $T=0K$, our relation (24) is reduced to a correct result: $\mathbb{R}(N^*, r_{d(a)}, x, T = 0K) = \frac{2}{3} E_{Fno(Fpo)}(N^*)/q$.

It should be noted that the well-known “local” Einstein relation, being valid only at the lowest degeneracy (or $u \rightarrow 0$), is now globalized by our “Global” Einstein -and- Van Cong relations (25), valid at any degeneracy (or any u).

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (25) can be rewritten as:

$$\mathbb{R}_{(VC)}(N^*, r_{d(a)}, x, T = 0K) \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}} \right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)} \right],$$

$$\text{where } a = [3\sqrt{\pi}/4]^{2/3}, b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2 \text{ and } c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4.$$

Then, in **Tables 3n and 3p in Appendix 1**, for given $r_{d(a)}$, and $T=(4.2 \text{ K and } 300 \text{ K})$, the numerical results of $\mathbb{V}, \mu$ and D , expressed as functions of N , are obtained by using Equations (22, 21 and 25).

Further, in **Tables 4n and 4p in Appendix 1**, for given $r_{d(a)}$ and N , the numerical results of Fermi energy $E_{Fn(Fp)}(T=0K) \text{ (eV)}$ [or reduced Fermi energy $\xi_{n(p)}(N^*, r_d, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_{d(a)}, T)$, expressed in $\left(\frac{\text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2} \right)$, are obtained, as functions of T , by using Equations (17, 22), respectively.

Thermoelectric Coefficients

Then, from Eq. (19), obtained for $\sigma(N^*, r_{d(a)}, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S , is found to be given by:

$$S(N^*, r_{d(a)}, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times k_B T \times \left. \frac{\partial \ln \sigma}{\partial E} \right]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln \sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

$$= -2\sqrt{L} \times \frac{\sqrt{ZT_{Mott}}}{1 + ZT_{Mott}} \left(\frac{V}{K} \right) < 0, \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2},$$

Then, using Eq. (17), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$Y_{Sb}(N^*, r_{d(a)}, T) \equiv \left[1 - \frac{y^2}{3 \times G_2 \left(y = \frac{\pi}{\xi_{n(p)}} \right)} \right],$$

$$S(N^*, r_{d(a)}, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2Y_{Sb}}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)} = -2\sqrt{L} \times \frac{\sqrt{ZT_{Mott}}}{1 + ZT_{Mott}} \left(\frac{V}{K}\right) <$$

$$0, \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}, \quad (26) \quad \text{according to:}$$

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{ZT_{Mott} \times [1 - ZT_{Mott}]}{[1 + ZT_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \rightarrow +\infty$ or $\xi_{n(p)} \rightarrow +0$, one has a same limiting value of S :

$S \rightarrow -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum

$(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit is found to be defined by:

$$ZT(N^*, r_{d(a)}, T) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times ZT_{Mott}}{[1 + ZT_{Mott}]^2}. \quad (27)$$

Here, one notes that: (i) $\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since

$\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{\max.} = 1$, $ZT_{Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one

obtains: $ZT \simeq 0.715$ and $ZT_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient can be defined by:

$$VC1(N^*, r_{d(a)}, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (28)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

and the second Van-Cong coefficient as:

$$VC2(N^*, r_{d(a)}, T) \equiv T \times VC1(N^*, r_{d(a)}, T) \equiv T \times N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (29)$$

the Thomson coefficient, T_s , by:

$$T_s(N^*, r_{d(a)}, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \quad (30)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

and the Peltier coefficient, P_t , as:

$$P_t(N^*, r_{d(a)}, T) \equiv T \times S. \quad (31)$$

Then, in **Tables 5n and 5p in Appendix 1**, for given r_d and N , the numerical results of $\xi_{n(p)}$, V , AE , $\sigma_{Th.}$, S , $VC1$, $VC2$, T_s , P_t and ZT are obtained, as functions of $T=(4.2 \text{ K}, 300 \text{ K})$, by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively.

Further, in **Tables 6n and 6p in Appendix 1**, for given $r_{d(a)}$ and $T=0K$, the numerical results of Fermi energy $E_{Fn(Fp)}(eV)$, $V(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^4}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and $D(\frac{cm^2}{s})$, are obtained, as functions of N , by using Equations (17, 22, 19, 21, 25), respectively.

Finally, in **Table 7 in Appendix 1**, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: $\nearrow$, decrease: $\searrow$). One notes here that **(i)** for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{min.} (\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $ZT_{max.} = 1$, **(ii)** for $\xi_{n(p)} = 1$, those of S , ZT , ZT_{Mott} , $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and **(iii)** for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{Mott} = 1$.

Furthermore, it is interesting to remark that the $VC2$ -coefficient, defined in Eq. (29), is related to our Global Einstein -and- Van Cong relations (25), by:

$$\frac{k_B}{q} \times VC2(N^*, r_{d(a)}, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \left(\frac{V^2}{K}\right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}},$$

according, in this work, with the use of our Eq. (26), to:

$$VC2(N, r_{d(a)}, T) \equiv T \times VC1(N, r_{d(a)}, T) \equiv -\frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \times 2 \times \frac{ZT_{Mott} \times [1 - ZT_{Mott}]}{[1 + ZT_{Mott}]^2} (V), \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}.$$

Therefore, we obtain in this work:

$$\begin{aligned} \mathbb{R}(N^*, r_{d(a)}, T) &\equiv \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \equiv \frac{D(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} \times q \times (N^*/g_{c(v)}) \equiv D(N^*, r_{d(a)}, T) \times \\ &\frac{V(N^*, r_{d(a)}, T)}{q} \times 6\pi \times R_{WS}(N^*) \equiv -\frac{VC2(N, r_{d(a)}, T)}{2} \times \frac{[1 + ZT_{Mott}]^2}{ZT_{Mott} \times [1 - ZT_{Mott}]} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*}, \quad (32) \end{aligned}$$

being new results.

Concluding Remarks

In the $n(p)$ – type degenerate **Ca – AM**, the electrical and thermoelectric coefficients, enhanced by : **(i)** our static dielectric constant law, $\varepsilon(r_{d(a)})$, given in Equations (5, 6), **(ii)** our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq. (17), being accurate with a precision of the order of 2.11×10^{-4} [9], affecting all those coefficients, and finally **(iii)** our electrical conductivity model, given in Eq. (19), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.^[2-5]

Finally, the numerical results of those coefficients are given and discussed in Tables 3n, 3p, 4n, 4p, 5n, 5p, 6n, 6p, and 7 in Appendix 1.

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$10^{-4} \frac{\text{V}}{\text{K}}$), at which Same $(ZT)_{\text{Mott}} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1 \right)$. *SCIREA Journal of Physics*, 2023; 8(2): 133-157.

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APPENDIX 1

Table 1. In the n(p) type degenerate **Ca – AM**, the numerical results of $B_{do(ao)} = 9.5405807 \times 10^{10} \left(\frac{N}{m^2}\right)$, ϵ , $N_{CDn(CDp)}$ and $N_{CDn(CDp)}^{EBT}$, were given and discussed in Ref.^[1] It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 3.2253×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$ given in the parabolic conduction (valence) band, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

Donor	As	Sb	Sn	Ca	
r_d (nm) ↗	0.118	0.136	0.140	0.176	
$\epsilon(r_d) \searrow$	2.4840	1.3092	1.2318	1	
$N_{CDn}(r_d)$ in 10^{22} cm^{-3} ↗	0.68478457	4.6767671	5.6148959	10.495313	
$N_{CDn}^{EBT}(r_d)$ in 10^{22} cm^{-3} ↗	0.68478439	4.6767658	5.6148941	10.495310	
$ RD $ in 10^{-7}	2.5669	2.7507	3.2253	2.9684	
Acceptor	Ga	Mg	In	Cd	Ca
r_a (nm) ↗	0.126	0.141	0.144	0.148	0.176
$\epsilon(r_a) \searrow$	1.6546	1.2155	1.1722	1.1256	1
$N_{CDp}(r_a)$ in 10^{22} cm^{-3} ↗	2.3169669	5.8443109	6.5153448	7.3594320	10.495313
$N_{CDp}^{EBT}(r_a)$ in 10^{22} cm^{-3} ↗	2.3169664	5.8443090	6.5153433	7.3594298	10.495310
$ RD $ in 10^{-7}	2.3028	3.2202	2.3519	2.9345	2.9669

Table 2: Expressions for $G_{p>1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, are used to determine the electrical-and-thermoelectric coefficients, suggesting that, with decreasing T and for given (high N, r_d), the function $G_{p>1}(y)$ decreases as: $G_{p>1}(y \rightarrow 0) \rightarrow 1$.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1 + \frac{y^2}{8}\right)$	$\left(1 + \frac{y^2}{3}\right)$	$\left(1 + \frac{5y^2}{8}\right)$	$(1 + y^2)$	$\left(1 + \frac{35y^2}{24}\right)$	$(1 + 2y^2)$	$\left(1 + \frac{21y^2}{8}\right)$

Table 3n: For a given r_d , and $T=(4.2 \text{ K and } 300 \text{ K})$, the numerical results of V, μ and D , expressed respectively in $\left(\frac{10^6 \times \text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}; \frac{\text{cm}^2}{\text{V} \times \text{s}}; \frac{\text{cm}^2}{\text{s}}\right)$, as functions of N , are obtained by using Equations (22, 21 and 25), noting that, in this very degenerate case, for same physical conductions, all the corresponding physical results are found to almost be the same.

Donor	As	Sb	Sn	Ca
$r_d \text{ (nm)}$	0.118	0.136	0.140	0.176
At $T=4.2 \text{ K}$,				
$N (10^{23} \text{ cm}^{-3})$				
3	(2.088 ; 2.721 ; 29.18)	(4.935 ; 1.096 ; 10.66)	(5.249 ; 1.018 ; 9.657)	(6.088 ; 0.815 ; 6.659)
4	(2.400 ; 2.610 ; 34.04)	(5.911 ; 1.023 ; 12.42)	(6.337 ; 0.945 ; 11.28)	(7.696 ; 0.740 ; 7.969)
5	(2.669 ; 2.532 ; 38.41)	(6.750 ; 0.973 ; 13.96)	(7.272 ; 0.897 ; 12.69)	(9.062 ; 0.692 ; 9.061)
6	(2.906 ; 2.472 ; 42.42)	(7.494 ; 0.937 ; 15.34)	(8.101 ; 0.862 ; 13.95)	(10.27 ; 0.659 ; 10.02)
At $T=300 \text{ K}$,				
$N (10^{23} \text{ cm}^{-3})$				
3	(2.088 ; 2.721 ; 29.18)	(4.935 ; 1.096 ; 10.66)	(5.248 ; 1.018 ; 9.657)	(6.088 ; 0.815 ; 6.659)
4	(2.400 ; 2.610 ; 34.04)	(5.911 ; 1.023 ; 12.42)	(6.336 ; 0.945 ; 11.28)	(7.695 ; 0.740 ; 7.969)
5	(2.668 ; 2.532 ; 38.41)	(6.750 ; 0.973 ; 13.96)	(7.271 ; 0.897 ; 12.69)	(9.062 ; 0.692 ; 9.061)
6	(2.906 ; 2.472 ; 42.42)	(7.494 ; 0.937 ; 15.34)	(8.101 ; 0.862 ; 13.95)	(10.27 ; 0.659 ; 10.02)

Table 3p: For a given r_a , and $T=(4.2 \text{ K and } 300 \text{ K})$, the numerical results of V, μ and D , expressed respectively in $\left(\frac{10^6 \times \text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}; \frac{\text{cm}^2}{\text{V} \times \text{s}}; \frac{\text{cm}^2}{\text{s}}\right)$, as functions of N , are obtained by using Equations (22, 21 and 25), noting that, in this very degenerate case, for same physical conductions, all the corresponding physical results are found to almost be the same.

Acceptor	Ga	Mg	In	Cd
$r_d \text{ (nm)}$	0.126	0.141	0.144	0.148
At $T=4.2 \text{ K}$,				
$N (10^{23} \text{ cm}^{-3})$				
3	(3.732; 1.493; 15.42)	(5.315; 1.002; 9.446)	(5.491; 0.961; 8.889)	(5.677; 0.918; 8.289)
4	(4.376; 1.411; 17.90)	(6.430; 0.930; 11.04)	(6.682; 0.889; 10.41)	(6.960; 0.846; 9.748)
5	(4.931; 1.355; 20.10)	(7.387; 0.881; 12.42)	(7.704; 0.841; 11.73)	(8.059; 0.799; 10.99)
6	(5.423; 1.313; 22.11)	(8.237; 0.846; 13.66)	(8.610; 0.806; 12.91)	(9.033; 0.764; 12.11)
At $T=300 \text{ K}$,				
$N (10^{23} \text{ cm}^{-3})$				
3	(3.732; 1.493; 15.42)	(5.315; 1.002; 9.446)	(5.491; 0.961; 8.889)	(5.677; 0.918; 8.289)
4	(4.376; 1.411; 17.90)	(6.430; 0.930; 11.04)	(6.682; 0.889; 10.41)	(6.960; 0.846; 9.748)
5	(4.931; 1.355; 20.10)	(7.387; 0.881; 12.42)	(7.704; 0.841; 11.73)	(8.059; 0.799; 10.99)
6	(5.423; 1.313; 22.11)	(8.237; 0.846; 13.66)	(8.610; 0.806; 12.91)	(9.033; 0.764; 12.11)

Table 4n: For given r_d and N , the numerical results of Fermi energy $E_{Fn(T=0K)}(eV)$ [or reduced Fermi energy $\xi_n(N^*, r_d, T)$], and viscosity coefficient $V(N^*, r_d, T)$, expressed in $(\frac{eV}{cm} \times \frac{s}{cm^2})$, are obtained, as functions of T , by using Equations (17, 22), respectively. In particular, from these numerical results of $V(T)$, one observes that, for such given (r_d and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring^[17] and complex fluids by Wenhao^[18], and from those of $V(r_d)$, one observes that, for given (T and N), they increase with increasing r_d .

Donor	As	Sb	Sn
r_d (nm) [4] ↗	0.118	0.136	0.140

For $N=4 \times 10^{23} \text{ cm}^{-3}$,			
N_{CDn} in 10^{22} cm^{-3} ↗	0.68478457	4.67676710	5.61489590
$E_{Fn(T=0K)}(eV)$ ↘	19.79616573	19.79616572	19.79616571
$V_{(0K)}$ ↗	2.4003900×10^6	5.9106739×10^6	6.3366214×10^6
$\xi_n(T=4.2K)$ ↘	5.4057806×10^4	5.0333632×10^4	4.9438452×10^4
$V_{(4.2K)}$ ↗	2.4003900×10^6	5.9106739×10^6	6.3366214×10^6
$\xi_n(T=77K)$ ↘	2.9486080×10^3	2.7454713×10^3	2.6966433×10^3
$V_{(77K)}$ ↗	2.4003884×10^6	5.9106694×10^6	6.3366164×10^6
$\xi_n(T=100K)$ ↘	2.2704284×10^3	2.1140131×10^3	2.0764156×10^3
$V_{(100K)}$ ↗	2.4003873×10^6	5.9106663×10^6	6.3366129×10^6
$\xi_n(T=150K)$ ↘	1.5136194×10^3	1.4093426×10^3	1.3842776×10^3
$V_{(150K)}$ ↗	2.400384×10^6	5.9106568×10^6	6.3366024×10^6
$\xi_n(T=200K)$ ↘	1.135215×10^3	1.0570074×10^3	1.0382087×10^3
$V_{(200K)}$ ↗	2.400379×10^6	5.9106435×10^6	6.3365876×10^6
$\xi_n(T=250K)$ ↘	908.1724921	845.6064747	830.5674865
$V_{(250K)}$ ↗	2.4003733×10^6	5.9106263×10^6	6.3365685×10^6
$\xi_n(T=300K)$ ↘	756.8109081	704.6725972	692.1401167
$V_{(300K)}$ ↗	2.4003659×10^6	5.9106054×10^6	6.3365453×10^6

Table 4p: For given r_a and N , the numerical results of Fermi energy $E_{Fp(T=0K)}(eV)$ [or reduced Fermi energy $\xi_p(N^*, r_a, T)$], and viscosity coefficient $V(N^*, r_a, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T , by using Equations (17, 22), respectively. In particular, from these numerical results of $V(T)$, one observes that, for such given (r_a and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring^[17] and complex fluids by Wenhao^[18], and from those of $V(r_a)$, one observes that, for given (T and N), they increase with increasing r_a .

Acceptor		Ga	Mg	Cd
r_a (nm)	↗	0.126	0.141	0.148

For $N=4 \times 10^{23} \text{ cm}^{-3}$,				
N_{CDp} in 10^{22} cm^{-3}	↗	2.3169669	5.8443109	7.3594320
$E_{Fp(T=0K)}(eV)$	↘	19.0241380	17.8176239	17.2867317
$V_{(0K)}$	↗	4.3762876×10^6	6.4299834×10^6	6.9597435×10^6
$\xi_{p(T=4.2K)}$	↘	5.2551108×10^4	4.9218307×10^4	4.7751803×10^4
$V_{(4.2K)}$	↗	4.3762876×10^6	6.4299833×10^6	6.9597435×10^6
$\xi_{p(T=77K)}$	↘	2.8664245×10^3	2.6846354×10^3	2.6046443×10^3
$V_{(77K)}$	↗	4.3762845×10^6	6.4299782×10^6	6.9597376×10^6
$\xi_{p(T=100K)}$	↘	2.2071471×10^3	2.0671695×10^3	2.0055764×10^3
$V_{(100K)}$	↗	4.3762824×10^6	6.4299747×10^6	6.9597335×10^6
$\xi_{p(T=150K)}$	↘	1.4714318×10^3	1.3781135×10^3	1.3370514×10^3
$V_{(150K)}$	↗	4.3762760×10^6	6.4299639×10^6	6.9597211×10^6
$\xi_{p(T=200K)}$	↘	1.1035744×10^3	1.0335856×10^3	1.0027891×10^3
$V_{(200K)}$	↗	4.3762669×10^6	6.4299487×10^6	6.9597037×10^6
$\xi_{p(T=250K)}$	↘	882.8600038	826.8690542	802.2318337
$V_{(250K)}$	↗	4.3762553×10^6	6.4299292×10^6	6.9596812×10^6
$\xi_{p(T=300K)}$	↘	735.7171822	689.0580922	668.5270920
$V_{(300K)}$	↗	4.3762411×10^6	6.4299054×10^6	6.9596539×10^6

Table 5n: For given r_d and N , the numerical results of ξ_n , V , AE , $\sigma_{Th.}$, S , $VC1$, $VC2$, Ts , Pt and ZT are obtained, as functions of $T=(4.2\text{ K}, 300\text{ K})$, by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $V(T)$, one observes that, for such given (r_d and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring^[17] and complex fluids by Wenhao^[18], suggesting **an equivalence between degeneracy-and-viscosity concept**, and from those of $V(r_d)$, one observes that, for given (T and N), they increase with increasing r_d .

Donor	As	Sb	Sn
r_d (nm) [4] ↗	0.118	0.136	0.140

For $N=4 \times 10^{23}\text{ cm}^{-3}$,			
N_{CDn} in 10^{22} cm^{-3} ↗	0.68478457	4.67676710	5.61489590
$\xi_n(T=4.2K) \times 10^{-4}$ ↘	5.4057806	5.0333632	4.9438452
$\xi_n(T=300K)$ ↘	756.8109081	704.672597	692.140117
$V_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-6}$ ↗	2.4003900	5.9106739	6.3366214
$V_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-6}$ ↗	2.4003659	5.9106054	6.3365453
$-AE_{(4.2K)} (eV) \times 10^{13}$ ↗	7.1286561	8.2225759	8.5230422
$-AE_{(300K)} (eV) \times 10^7$ ↗	2.5978996	2.9965577	3.1060571
$\sigma_{Th. (4.2K)} \left(\frac{W}{cm \times K} \right) \times 10^2$ ↘	1.6868423	0.5939078	0.5344554
$\sigma_{Th. (300K)} \left(\frac{W}{cm \times K} \right)$ ↘	1.2048994	0.4242247	0.3817584
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$ ↗	1.0488644	1.1264696	1.1468665
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^7$ ↗	7.4918293	8.0461383	8.1918265
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^9$ ↗	6.9924292	7.5097974	7.6457769
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^7$ ↗	4.9944740	5.3639945	5.4611146
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$ ↗	2.9368203	3.1541149	3.2112263
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^4$ ↗	1.4983422	1.6091983	1.6383344
$-Ts_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$ ↗	1.0488644	1.1264696	1.1468665
$-Ts_{(300K)} \left(\frac{V}{K} \right) \times 10^7$ ↗	7.4917110	8.0459917	8.1916718
$-Pt_{(4.2K)} (V) \times 10^8$ ↗	4.4052304	4.7311724	4.8168395
$-Pt_{(300K)} (V) \times 10^4$ ↗	2.2475488	2.4138415	2.4575480
$ZT_{(4.2K)} \times 10^9$ ↗	4.5032044	5.1942392	5.3840458
$ZT_{(300K)} \times 10^5$ ↗	2.2975170	2.6500738	2.7469103

Table 5p: For given r_a and N , the numerical results of ξ_p , V , AE , $\sigma_{Th.}$, S , $VC1$, $VC2$, Ts , Pt and ZT are obtained, as functions of $T=(4.2\text{ K}, 300\text{ K})$, by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $V(T)$, one observes that, for such given (r_a and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring^[17] and complex fluids by Wenhao^[18], suggesting **an equivalence between degeneracy-and-viscosity concept**, and from those of $V(r_a)$, one observes that, for given (T and N), they increase with increasing r_d .

Acceptor		Ga	Mg	Cd
r_a (nm)	$\nearrow$	0.126	0.141	0.148

For $N=4 \times 10^{23}\text{ cm}^{-3}$,				
N_{CDP} in 10^{22} cm^{-3}	$\nearrow$	2.3169669	5.8443109	7.3594320
$\xi_{p(T=4.2K)} \times 10^{-4}$	$\searrow$	5.2551108	4.9218307	4.7751803
$\xi_{p(T=300K)}$	$\searrow$	735.717182	689.058092	668.527092
$V_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-6}$	$\nearrow$	4.3762876	6.4299833	6.9597435
$V_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-6}$	$\nearrow$	4.3762411	6.4299054	6.9596539
$-AE_{(4.2K)} (eV) \times 10^{13}$	$\nearrow$	7.5432878	8.5994545	9.1357602
$-AE_{(300K)} (eV) \times 10^7$	$\nearrow$	2.7490044	3.1339050	3.3293512
$\sigma_{Th. (4.2K)} \left(\frac{W}{cm \times K} \right) \times 10^3$	$\searrow$	8.7437425	5.2201498	4.5396857
$\sigma_{Th. (300K)} \left(\frac{W}{cm \times K} \right) \times 10$	$\searrow$	6.2455967	3.7287236	3.2426744
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$	$\nearrow$	1.0789365	1.1519963	1.1873752
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^7$	$\nearrow$	7.7066248	8.2284665	8.4811656
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^9$	$\nearrow$	7.1929099	7.6799752	7.9158346
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^7$	$\nearrow$	5.1376640	5.4855398	5.6539960
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$	$\nearrow$	3.0210221	3.2255896	3.3246505
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^4$	$\nearrow$	1.5412992	1.6456619	1.6961988
$-Ts_{(4.2K)} \left(\frac{V}{K} \right) \times 10^8$	$\nearrow$	1.0789365	1.1519963	1.1873752
$-Ts_{(300K)} \left(\frac{V}{K} \right) \times 10^7$	$\nearrow$	7.7064960	8.2283097	8.4809939
$-Pt_{(4.2K)} (V) \times 10^8$	$\nearrow$	4.5315332	4.8383844	4.9869758
$-Pt_{(300K)} (V) \times 10^4$	$\nearrow$	2.3119874	2.4685400	2.5443497
$ZT_{(4.2K)} \times 10^9$	$\nearrow$	4.7651298	5.4323174	5.7711043
$ZT_{(300K)} \times 10^5$	$\nearrow$	2.4311480	2.7715377	2.9443814

Table 6n: For given r_d and $T=0K$, the numerical results of Fermi energy $E_{Fn}(eV)$, $\mathbb{V}(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^4}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and $D(\frac{cm^2}{s})$, are obtained, as functions of $N=[\cong N_{CDn}, N1=4 \times 10^{23} \text{ cm}^{-3}, N2 = 5 \times 10^{23} \text{ cm}^{-3}, N3 = 6 \times 10^{23} \text{ cm}^{-3}]$, by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions, ($T=0K, N=N_{CDn}$ or $N^* = 0$), and (ii) those values of $E_{Fn} \gtrsim 0$, $\mathbb{V} \gtrsim 0$, $\sigma \gtrsim 0$, $\mu \gtrsim 0$, and $D \gtrsim 0$, obtained for $N \gtrsim N_{CDn}$, thus define the properties of **the degenerate (or viscous) Ca-AM**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}$, one observes that, for such given (r_d and $T=0K$), they **increase with increasing N (or increasing E_{Fn})**, in good agreement with those, obtained in complex fluids by Wenhao^[18], suggesting **an equivalence between degeneracy-and-viscosity concept**.

$N \gtrsim N_{CDn}$	$\cong N_{CDn}$	N1	N2	N3
For $r_d = r_{As}$ and $N_{CDn}=6.8478457 \times 10^{21} \text{ cm}^{-3}$,				
E_{Fn}	$\nearrow \cong 0 [0]$	19.569	22.761	25.742
$\mathbb{V}$	$\nearrow \cong 0 [0]$	2.4004	2.6686	2.9060
σ	$\nearrow \cong 0 [0]$	16.440	20.005	23.498
μ	$\searrow \cong 0 [0]$	2.6099	2.5318	2.4725
D	$\nearrow \cong 0 [0]$	34.043	38.410	42.423
For $r_d = r_{Sb}$ and $N_{CDn}=4.6767671 \times 10^{22} \text{ cm}^{-3}$,				
E_{Fn}	$\nearrow \cong 0 [0]$	18.221	21.516	24.574
$\mathbb{V}$	$\nearrow \cong 0 [0]$	5.9107	6.7497	7.4946
σ	$\nearrow \cong 0 [0]$	5.7883	7.0673	8.3031
μ	$\searrow \cong 0 [0]$	1.0228	0.9732	0.9367
D	$\nearrow \cong 0 [0]$	12.421	13.956	15.343
For $r_d = r_{Sn}$ and $N_{CDn}=5.6148959 \times 10^{22} \text{ cm}^{-3}$,				
E_{Fn}	$\nearrow \cong 0 [0]$	17.897	21.218	24.296
$\mathbb{V}$	$\nearrow \cong 0 [0]$	6.3366	7.2716	8.1013
σ	$\nearrow \cong 0 [0]$	5.2089	6.3796	7.5080
μ	$\searrow \cong 0 [0]$	0.9455	0.8971	0.8616
D	$\nearrow \cong 0 [0]$	11.279	12.687	13.953
For $r_d = r_{Ca}$ and $N_{CDn}=1.0495313 \times 10^{23} \text{ cm}^{-3}$,				
E_{Fn}	$\nearrow \cong 0 [0]$	16.161	19.632	22.819
$\mathbb{V}$	$\nearrow \cong 0 [0]$	7.6957	9.0620	10.268
σ	$\nearrow \cong 0 [0]$	3.4972	4.3827	5.2257

$\mu \searrow$	$\cong 0 [0]$	0.7398	0.6924	0.6588
$D \nearrow$	$\cong 0 [0]$	7.9687	9.0608	10.021

Table 6p: For given r_a and $T=0K$, the numerical results of Fermi energy $E_{Fp}(eV)$, $V(10^6 \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^4}{\Omega \times cm})$, $\mu(\frac{cm^2}{V \times s})$, and $D(\frac{cm^2}{s})$, are obtained, as functions of $N=[\cong N_{CDp}, N1=3 \times 10^{23} cm^{-3}, N2 = 4 \times 10^{23} cm^{-3}, N3 = 5 \times 10^{23} cm^{-3}]$, by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions, ($T=0K, N=N_{CDp}$ or $N^* = 0$), and (ii) those values of $E_{Fp} \gtrsim 0, V \gtrsim 0, \sigma \gtrsim 0, \mu \gtrsim 0$, and $D \gtrsim 0$, obtained for $N \gtrsim N_{CDp}$, thus define the properties of **the degenerate (or viscous) Ca-AM**, given in the Mott MIT. In particular, from these numerical results of V , one observes that, for such given (r_d and $T=0K$), they **increase with increasing N (or increasing E_{Fp})**, in good agreement with those, obtained in complex fluids by Wenhao^[18] suggesting **an equivalence between degeneracy-and-viscosity concept**.

$N \gtrsim N_{CDp}$	$\cong N_{CDp}$	N1	N2	N3
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For $r_a = r_{Ga}$ and $N_{CDp}=2.3169669 \times 10^{22} cm^{-3}$,

$E_{Fp} \nearrow$	$\cong 0 [0]$	15.489	19.024	22.256
$V \nearrow$	$\cong 0 [0]$	3.7323	4.3763	4.9308
$\sigma \nearrow$	$\cong 0 [0]$	6.6234	8.5218	10.351
$\mu \searrow$	$\cong 0 [0]$	1.4933	1.4115	1.3550
$D \nearrow$	$\cong 0 [0]$	15.416	17.897	20.099

For $r_a = r_{Mg}$ and $N_{CDp}=5.8443109 \times 10^{22} cm^{-3}$,

$E_{Fp} \nearrow$	$\cong 0 [0]$	14.143	17.818	21.144
$V \nearrow$	$\cong 0 [0]$	5.3154	6.4300	7.3875
$\sigma \nearrow$	$\cong 0 [0]$	3.8779	5.0876	6.2363
$\mu \searrow$	$\cong 0 [0]$	1.0020	0.9297	0.8815
$D \nearrow$	$\cong 0 [0]$	9.4456	11.041	12.423

For $r_a = r_{In}$ and $N_{CDp}=6.5153448 \times 10^{22} cm^{-3}$,

$E_{Fp} \nearrow$	$\cong 0 [0]$	13.880	17.583	20.930
$V \nearrow$	$\cong 0 [0]$	5.4915	6.6820	7.7037
$\sigma \nearrow$	$\cong 0 [0]$	3.6152	4.7679	5.8594
$\mu \searrow$	$\cong 0 [0]$	0.9608	0.8887	0.8410
$D \nearrow$	$\cong 0 [0]$	8.8887	10.415	11.732

For $r_a = r_{Cd}$ and $N_{CDP} = 7.359432 \times 10^{22} \text{ cm}^{-3}$,

E_{Fp}	$\nearrow$	$\cong 0 [0]$	13.546	17.287	20.658
V	$\nearrow$	$\cong 0 [0]$	5.6768	6.9597	8.0589
σ	$\nearrow$	$\cong 0 [0]$	3.3306	4.4244	5.4567
μ	$\searrow$	$\cong 0 [0]$	0.9182	0.8460	0.7987
D	$\nearrow$	$\cong 0 [0]$	8.2895	9.7478	10.997

Table 7: In the n(p) type degenerate **Ca – AM**, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: $\nearrow$, decrease: $\searrow$). One notes here that (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{\min.} (\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $ZT_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, those of S , ZT , ZT_{Mott} , $VC1$, and Ts present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$.

$\xi_{n(p)} \searrow$	1.880 [1.880]	1.8138 [1.8138]	1.750 [1.750]	1 [1]	0.998 [0.998]
$S (10^{-4} \frac{V}{K})$	-1.562 [-1.562] $\searrow$	-1.563 [-1.563] $\nearrow$	-1.562 [-1.562] $\nearrow$	-1.322 [-1.322] $\nearrow$	-1.320 [-1.320]
ZT	0.999 [0.999] $\nearrow$	1 [1] $\searrow$	0.999 [0.999] $\searrow$	0.715 [0.715] $\searrow$	0.713 [0.713]
$(ZT)_{\text{Mott}}$	0.931 [0.931] $\nearrow$	1 [1]	1.074 [1.074]	3.290 [3.290]	3.306 [3.306]
$VC1 (10^{-4} \frac{V}{K})$	-0.061 [-0.061] $\nearrow$	0 [0] $\nearrow$	0.063 [0.063] $\nearrow$	1.105 [1.105] $\nearrow$	1.109 [1.109]
$Ts (10^{-4} \frac{V}{K})$	-0.092 [-0.092] $\nearrow$	0 [0] $\nearrow$	0.094 [0.094] $\nearrow$	1.657 [1.657] $\nearrow$	1.663 [1.663]