



**NEW CRITICAL IMPURITY DENSITY IN METAL-INSULATOR
TRANSITION, OBTAINED IN N(P)- TYPE DEGENERATE NaCl-
METALLIC LIQUID (OR FERMI LIQUID), BEING JUST THAT OF
THE CARIERS, LOCALIZED IN EXPONENTIAL BAND TAILS.**

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ABSTRACT

By basing on the same physical model and treatment method, as used in our recent work^[1], we will investigate the critical impurity densities in the metal-insulator transition (**MIT**), obtained in n(p)-type degenerate **Sodium Chloride Metallic Liquid (NaCl-ML)** or **Fermi Liquid**, being due to the effects of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, and the high d(a)-density, N , assuming that all the impurities are ionized even at $T=0$ K. In such the n(p)-type **NaCl-ML**, some important remarks are given and discussed in the following. (i)- In **Table 1 in Appendix 1**, the numerical results of the pressure variation $[\Delta p(r_{d(a)})]_{n(p)}$ are given and computed, using Equation (3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing reduced effective electron (hole) mass in conduction (valence) bands, $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2] (ii)-In Tables 2-5 in Appendix1, the numerical results of the critical impurity density $N_{CDn(CDp)}(r_{d(a)})$ in the metal-insulator

transition (**MIT**), as that given in Eq. (8), by using an empirical Mott parameter $M_{n(p)} = 0.25$, are obtained and discussed. (iii)-Further, also in **Tables 2-5 in Appendix 1**, for a given $r_{d(a)}$, the numerical results of the density of electrons (holes) localized in the exponential conduction (valence)-band tails (**EBT**), $N_{CDn(CDp)}^{EBT}(r_{d(a)}, x)$, are obtained, using Eq. (26), and taking the empirical Heisenberg parameter, $\mathcal{H}_{n(p)} = 0.47137$, as that given in Eq. (15), giving raise to: $N_{CDn(CDp)}^{EBT}(r_{d(a)}) \cong N_{CDn(CDp)}(r_{d(a)})$, with a precision of the order of 2.8826×10^{-7} . In other words, such the critical d(a)-density $N_{CDn(CDp)}(r_{d(a)})$, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, respectively.

KEYWORDS: n(p)-type degenerate Sodium Chloride Metallic Liquid, critical impurity density in the metal-insulator transition (**MIT**).

INTRODUCTION

By basing on the same intrinsic energy-band-structure parameters, physical model and treatment method, as used in our recent works and also other works^[1-9], we will investigate the critical impurity density in the metal-insulator transition (**MIT**), obtained in n(p)-type degenerate **Sodium Chloride Metallic Liquid (NaCl-ML)** or **Fermi Liquid**, being due to the effects of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$ and the high d(a)-density, N , assuming that all the impurities are ionized even at $T=0$ K. In such the n(p)-type **NaCl-ML**, some important remarks are given and discussed in the following.

(i)- In **Table 1 in Appendix 1**, the numerical results of the **pressure variation** $[\Delta p(r_{d(a)})]_{n(p)}$ are given and computed, using Equation (3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing reduced effective electron (hole) mass in conduction (valence) bands, $\frac{m_{c(v)}}{m_0}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2]

(ii)- In **Tables 2-5 in Appendix 1**, the critical impurity densities $N_{CDn(CDp)}(r_{d(a)})$ in the MIT, as that given in Eq. (8), by using an empirical Mott parameter $M_{n(p)} = 0.25$, are obtained and discussed.

(iii)- Further, also in **Tables 2-5 in Appendix 1**, for a given $r_{d(a)}$, the numerical results of the density of electrons (holes) localized in the exponential conduction (valence)-band tails (**EBT**), $N_{CDn(CDp)}^{EBT}(r_{d(a)}, x)$, are obtained, using Eq. (26), and taking the empirical Heisenberg parameter, $\mathcal{H}_{n(p)} = 0.47137$, as that given in Eq. (15), giving raise to:

$N_{CDn(CDp)}^{EBT}(r_{d(a)}) \cong N_{CDn(CDp)}(r_{d(a)})$, with a precision of the order of 2.8826×10^{-7} . In other words, such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, respectively.

In the following, we will determine those functions: $N_{CDn(CDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

CRITICAL DENSITY IN THE MOTT MIT

Such the critical impurity density $N_{CDn(CDp)}(r_{d(a)})$, expressed as a function of $r_{d(a)}$, is determined as follows.

Here, the values of the intrinsic energy-band-structure parameters are found to be given by: the reduced effective electron (hole) mass in conduction (valence) bands, $\frac{m_{c(v)}}{m_0} = 3, 4, 5 \text{ and } 6$, $m_0 (= 9.109558 \times 10^{-28} \text{ g})$ being the free electron mass^[2], and the static dielectric constant $\epsilon_0 = 23$.^[3]

Therefore, the effective donor (acceptor)-ionization energy, $E_{do(ao)}$, is given by:

$$E_{do(ao)} = \frac{13600 \times [m_{c(v)}/m_0]}{[\epsilon_0]^2} \text{ meV}, \quad (1)$$

and the isothermal bulk modulus $B_{do(ao)}$, at $r_{d(a)} = r_{do(ao)}$, by:

$$B_{do(ao)} \equiv \frac{E_{do(ao)}}{(4\pi/3) \times (r_{do(ao)})^3} \left(\frac{\text{meV}}{\text{cm}^3} \right). \quad (2)$$

Effects of impurity size

Here, one shows that the effect of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, strongly affects the changes in all the energy-band-structure parameters, which can be represented by the effective relative static dielectric constant $\epsilon(r_{d(a)})$, in the following.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be given, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$ and $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, by:

(i) the pressure p , as: $p_0(r_{do(ao)}) = 0$, according to the **p-pressure variation**,

$$[\Delta p(r_{d(a)})]_{n(p)} \equiv p(r_{d(a)}) - p_0(r_{do(ao)}) = p(r_{d(a)}), \text{ being equal to 0 at } r_{d(a)} = r_{do(ao)}, \text{ and}$$

(ii) the deformation potential energy (or the strain energy) σ , as: $\sigma_0(r_{do(ao)}) = 0$, according

$$\text{to the } \sigma \text{-pressure variation, } [\Delta \sigma(r_{d(a)})]_{n(p)} \equiv \sigma(r_{d(a)}) - \sigma_0(r_{do(ao)}) = \sigma(r_{d(a)}), \text{ being}$$

equal to 0 at $r_{d(a)} = r_{do(ao)}$.

Further, the two important equations, used to determine $[\Delta p(r_{d(a)})]_{n(p)}$ and $[\Delta \sigma(r_{d(a)})]_{n(p)}$, are defined by: $\frac{dp}{dV} = -\frac{B_{do(ao)}}{V}$ and $p = -\frac{d\sigma}{dV}$, giving: $\frac{d}{dV}(\frac{d\sigma}{dV}) = \frac{B_{do(ao)}}{V}$.

Then, by an integration, one gets:

$$[\Delta p(r_{d(a)})]_{n(p)} = -B_{do(ao)} \times \ln\left(\frac{V}{V_{do(ao)}}\right) = -B_{do(ao)} \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \text{ and}$$

$$[\Delta \sigma(r_{d(a)})]_{n(p)} = B_{do(ao)} \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (3)$$

Then, in **Tables 1 in Appendix 1**, the numerical results of $[\Delta p(r_{d(a)})]_{n(p)}$ are given and computed, using Equation (3). In particular, it should be noted here that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid.^[2]

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to: the increase (the decrease) in the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)})$ in the absolute values, being obtained from the effective Bohr model, and then such the compression (dilatation) is represented respectively by: $\pm [\Delta \sigma(r_{d(a)})]_{n(p)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\epsilon_o}{\epsilon(r_{d(a)})}\right)^2 - 1\right] = + [\Delta \sigma(r_{d(a)})]_{n(p)} = + [\Delta \sigma(r_{d(a)})]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\epsilon_o}{\epsilon(r_{d(a)})}\right)^2 - 1\right] = - [\Delta \sigma(r_{d(a)})]_{n(p)}. \quad (4)$$

Therefore, from above Equations (3) and (4), one obtains the expressions for $\epsilon(r_{d(a)})$ and $E_{d(a)}(r_{d(a)})$ as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\epsilon(r_{d(a)}) = \frac{\epsilon_o}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \epsilon_o$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (5)$$

According to the increase in $E_{d(a)}(r_{d(a)})$, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3}} \geq \varepsilon_o$, with a condition,

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 < 1$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = -E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \leq 0, \quad (6)$$

corresponding to the decrease in $E_{d(a)}(r_{d(a)})$.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)})$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}) \equiv \frac{\varepsilon(r_{d(a)}) \times \hbar^2}{m_{c(v)} \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)})}{m_{c(v)}/m_o}, \quad (7)$$

where $-q$ is the electron charge; $q > 0$.

Then, the critical donor (acceptor)-density in the Mott MIT, $N_{CDn(NDp)}(r_{d(a)})$, is determined, using an empirical Mott parameter, $M_{n(p)} = 0.25$, for each the conduction (valence) band, as:

$$\left[N_{CDn(NDp)}(r_{d(a)}) \right]^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = M_{n(p)} = 0.25, \quad (8)$$

Noting that $M_{n(p)}$ could be chosen in general case so that the obtained numerical $N_{CDn(NDp)}(r_{d(a)})$ -results, being found to be in good agreement with the corresponding experimental ones.

In the following, these obtained numerical results can also be justified by calculating the numerical results of the density of electrons (holes) localized in exponential conduction (valence)-band (EBT) tails, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

Eq. (8) can be explained by using the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp),M}$, in the Mott's criterium, being characteristic of interactions, by:

$$r_{sn(sp),M}(N = N_{CDn(CDp)}(r_{d(a)}), r_{d(a)}) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)})} \right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = \left(\frac{3}{4\pi} \right)^{1/3} \times 4 \cong 2.4814,$$

for any $(r_{d(a)}, x)$ -values. Therefore, one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = 1/4 = 0.25 = M_{n(p)},$$

Explaining thus the existence of the Mott's criterium.

$N_{CDn(CDp)}^{EBT}(r_{d(a)})$ - Expression

In order to determine $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, we first present our physical model and also our mathematical methods.

Physical model

In n(p)-type degenerate X-alkali metals, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N) \equiv (3\pi^2 N)^{1/3}$, N being the total impurity density, assuming that all the impurities are ionized even at 0 K, the effective reduced Wigner-Seitz radius $r_{sn(sp)}$, characteristic of interactions, is defined by:

$$r_{sn(sp)}(N, r_{d(a)}) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = 1.1723 \times 10^8 \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}/m_o}{\varepsilon(r_{d(a)})}. \quad (9)$$

So, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ is defined by:

$$R_{sn(sp)}(N, r_{d(a)}) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} < 1. \quad (10)$$

These ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, are determined in the following.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)})$, according to the Thomas-Fermi (TF)-approximation, the ratio $R_{snTF(spTF)}$ is reduced to

$$R_{snTF(spTF)}(N, r_{d(a)}) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}(N, r_{d(a)})}{\pi}} \ll 1, \gamma \equiv \left(\frac{4}{9\pi}\right)^{\frac{1}{3}}, \quad (11)$$

being proportional to $N^{-1/6}$.

Secondly, $N < N_{CDn(NDp)}(r_{d(a)})$, according to the Wigner-Seitz (WS)-approximation, the ratio $R_{snWS(spWS)}$ is reduced to^[3]:

$$R_{snWS(spWS)}(N, r_{d(a)}) \equiv \frac{k_{snWS(spWS)}}{k_{Fn(Fp)}} = \left(\frac{3}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N, r_{d(a)})]}{dr_{sn(sp)}}\right) \times 0.5, \quad (12)$$

where $E_{CE}(N, r_{d(a)})$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N, r_{d(a)}) \equiv \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - \ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

So, n(p)-type degenerate X- alkali metals, the physical conditions are found to be given by :

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)}(N, r_{d(a)}) < 1, \quad A_{n(p)}(N, r_{d(a)}) \equiv \frac{\pm E_{Fno(Fpo)}}{\eta_{n(p)}}. \quad (13)$$

Here, $\pm E_{Fno(Fpo)}$ is the Fermi energy at 0 K, and $\eta_{n(p)}$ is defined in next Eq. (15),

$$\text{as:} \quad \pm E_{Fno(Fpo)}(N) = \frac{\hbar^2 \times k_{Fn(Fp)}(N)^2}{2 \times m_{c(v)}} \geq 0, \quad \eta_{n(p)}(N, r_{d(a)}) = \frac{\sqrt{2\pi N}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}.$$

Then, the total screened Coulomb impurity potential energy due to the attractive interaction between an electron (hole) charge, $-q(+q)$, at position $\vec{r}$, and an ionized donor (ionized acceptor) charge: $+q(-q)$ at position $\vec{R}_j$, randomly distributed throughout X(x)- crystalline alloys, is defined by:

$$V(r) \equiv \sum_{j=1}^N v_j(r) + V_o, \quad (14)$$

Where N is the total number of ionized donors (acceptors), V_o is a constant potential energy, and the screened Coulomb potential energy $v_j(r)$ is defined as:

$$v_j(r) \equiv - \frac{q^2 \times \exp(-k_{sn(sp)} \times |\vec{r} - \vec{R}_j|)}{\varepsilon(r_{d(a)}) \times |\vec{r} - \vec{R}_j|},$$

where $k_{sn(sp)}$ is the inverse screening length determined in Eq. (11).

Further, using a Fourier transform, the v_j -representation in wave vector $\vec{k}$ -space is given by

$$v_j(\vec{k}) = - \frac{q^2}{\varepsilon(r_{d(a)})} \times \frac{4\pi}{\Omega} \times \frac{1}{k^2 + k_{sn(sp)}^2},$$

where Ω is the total X- alkali metal volume.

Then, the effective auto-correlation function for potential fluctuations, $W_{n(p)}(v_{n(p)}, N, r_{d(a)}) \equiv \langle V(r)V(r') \rangle$, was determined, [4, 5] as :

$$W_{n(p)}(v_{n(p)}, N, r_{d(a)}) \equiv \eta_{n(p)}^2 \times \exp\left(\frac{-\mathcal{H}_{n(p)} \times R_{sn(sp)}(N, r_{d(a)})}{2\sqrt{|v_{n(p)}|}}\right), \quad \eta_{n(p)}(N, r_{d(a)}) \equiv \frac{\sqrt{2\pi N}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2},$$

$$v_{n(p)}(E, N) \equiv \frac{\mp E}{\pm E_{Fno(Fpo)}(N)}, \quad \mathcal{H}_{n(p)} = 0.47137. \quad (15)$$

Here, E is the total electron energy, and the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$ was chosen above such that the determination of the density of electrons localized

in the conduction(valence)-band tails will be accurate, noting that as $E \rightarrow \pm\infty$, $|v_{n(p)}| \rightarrow \infty$, and therefore, $W_{n(p)} \rightarrow \eta_{n(p)}^2$.

In the following, we will calculate the ensemble average of the function: $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$, for $a \geq 1$, $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{c(v)}}$ being the kinetic energy of the electron (hole), and $V(r)$ determined in Eq. (16), by using the two following integration methods, which strongly depend on $W_{n(p)}(v_{n(p)}, N, r_{d(a)})$.

MATHEMATICAL METHODS

Kane integration method (KIM)

Here, the effective Gaussian distribution probability is defined by:

$$P(V) \equiv \frac{1}{\sqrt{2\pi W_{n(p)}}} \times \exp\left[\frac{-V^2}{2W_{n(p)}}\right]. \quad (16)$$

So, in the Kane integration method, the Gaussian average of $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$ is defined by

$$\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{KIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} = \int_{-\infty}^E (E - V)^{a-\frac{1}{2}} \times P(V) dV, \quad \text{for } a \geq 1.$$

Then, by variable changes: $s = (E - V)/\sqrt{W_{n(p)}}$ and $y = \mp E/\sqrt{W_{n(p)}} \equiv \frac{\pm E_{\text{Fno}}(F_{\text{po}})}{\eta_{n(p)}} \times v_{n(p)} \times$

$$\exp\left(\frac{\mathcal{H}_{n(p)} \times R_{\text{sn(sp)}}}{4 \times \sqrt{|v_{n(p)}|}}\right), \text{ and using an identity:}$$

$$\int_0^\infty s^{a-\frac{1}{2}} \times \exp(-ys - \frac{s^2}{2}) ds \equiv \Gamma(a + \frac{1}{2}) \times \exp(y^2/4) \times D_{-a-\frac{1}{2}}(y),$$

where $D_{-a-\frac{1}{2}}(y)$ is the parabolic cylinder function and $\Gamma(a + \frac{1}{2})$ is the Gamma function, one thus has:

$$\begin{aligned} \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} &= \frac{\exp(-y^2/4) \times W_{n(p)}^{\frac{2a-1}{4}}}{\sqrt{2\pi}} \times \Gamma(a + \frac{1}{2}) \times D_{-a-\frac{1}{2}}(y) = \\ &= \frac{\exp(-y^2/4) \times \eta_{n(p)}^{a-\frac{1}{2}}}{\sqrt{2\pi}} \times \exp\left(-\frac{\mathcal{H}_{n(p)} \times R_{\text{sn(sp)}} \times (2a-1)}{8 \times \sqrt{|v_{n(p)}|}}\right) \times \Gamma(a + \frac{1}{2}) \times D_{-a-\frac{1}{2}}(y). \end{aligned} \quad (16)$$

Feynman path-integral method (FPIM)

Here, the ensemble average of $(E - V)^{a-\frac{1}{2}} \equiv E_k^{a-\frac{1}{2}}$ is defined by

$$\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \frac{\hbar^{a-\frac{1}{2}}}{2^{3/2} \times \sqrt{2\pi}} \times \frac{\Gamma(a+\frac{1}{2})}{\Gamma(\frac{3}{2})} \times \int_{-\infty}^{\infty} (it)^{-a-\frac{1}{2}} \times \exp \left\{ \frac{iEt}{\hbar} - \frac{(t\sqrt{W_{n(p)}})^2}{2\hbar^2} \right\} dt, \quad i^2 = -1,$$

noting that as $a=1$, $(it)^{-\frac{3}{2}} \times \exp \left\{ -\frac{(t\sqrt{W_p})^2}{2\hbar^2} \right\}$ is found to be proportional to the averaged Feynman propagator given the dense donors (acceptors). Then, by variable changes: $t =$

$$\frac{\hbar}{\sqrt{W_{n(p)}}} \quad \text{and} \quad y = \mp E / \sqrt{W_{n(p)}} \equiv \frac{\pm E_{\text{Fno(Fpo)}}}{\eta_{n(p)}} \times v_{n(p)} \times \exp \left(\frac{\mathcal{H}_{n(p)} \times R_{sn(sp)}}{4 \times \sqrt{|v_{n(p)}|}} \right), \quad \text{for } n(p)\text{-type}$$

respectively, and then using an identity:

$$\int_{-\infty}^{\infty} (is)^{-a-\frac{1}{2}} \times \exp \left\{ iys - \frac{s^2}{2} \right\} ds \equiv 2^{3/2} \times \Gamma(3/2) \times \exp(-y^2/4) \times D_{-a-\frac{1}{2}}(y),$$

one finally obtains: $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{FPIM}} \equiv \langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}}$, $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}}$ being determined in Eq. (16).

In the following, with the use of asymptotic forms for $D_{-a-\frac{1}{2}}(y)$, those given for $\langle (E - V)^{a-\frac{1}{2}} \rangle_{\text{KIM}}$ can be obtained in the two following cases.

First case: n-type ($E \geq 0$) and p-type ($E \leq 0$)

As $E \rightarrow \pm\infty$, one has: $v_{n(p)} \rightarrow \mp\infty$ and $y \rightarrow \mp\infty$. In this case, one gets: $D_{-a-\frac{1}{2}}(y \rightarrow \mp\infty) \approx$

$$\frac{\sqrt{2\pi}}{\Gamma(a+\frac{1}{2})} \times e^{\frac{y^2}{4}} \times (\mp y)^{a-\frac{1}{2}}, \quad \text{and therefore from Eq. (16), one gets:}$$

$$\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \approx E^{a-\frac{1}{2}}. \quad (17)$$

Further, as $E \rightarrow \pm 0$, one has: $v_{n(p)} \rightarrow \mp 0$ and $y \rightarrow \mp 0$. So, one obtains:

$$D_{-a-\frac{1}{2}}(y \rightarrow \mp 0) \simeq \beta(a) \times \exp \left(\left(\sqrt{a} + \frac{1}{16a^2} \right) y - \frac{y^2}{16a} + \frac{y^3}{24\sqrt{a}} \right) \rightarrow \beta(a), \quad \beta(a) = \frac{\sqrt{\pi}}{2^{\frac{2a+1}{4}} \Gamma(\frac{a}{2} + \frac{3}{4})}. \quad (18)$$

Therefore, as $E \rightarrow \pm 0$, from Eq. (16), one gets: $\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \rightarrow 0$.

Thus, in this case, one gets

$$\langle E_k^{a-\frac{1}{2}} \rangle_{\text{KIM}} \cong E^{a-\frac{1}{2}}. \quad (19)$$

Second case: n-type-case ($E \leq 0$) and p-type-case ($E \geq 0$)

As $E \rightarrow \mp 0$, one has: $(y, v_{n(p)}) \rightarrow \pm 0$, and by putting $f(a) \equiv \frac{\eta_{n(p)}^{a-\frac{1}{2}}}{\sqrt{2\pi}} \times \Gamma(a + \frac{1}{2}) \times \beta(a)$, Eq. (18) yields:

$$H_{n(p)}(v_{n(p)} \rightarrow \pm 0, N, r_{d(a)}, a) = \frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)} = \exp \left[-\frac{\mathcal{H}_{n(p)} \times R_{sn(sp)} \times (2a-1)}{8 \times \sqrt{|v_{n(p)}|}} - \left(\sqrt{a} + \frac{1}{16a^2} \right) y - \left(\frac{1}{4} + \frac{1}{16a} \right) y^2 - \frac{y^3}{24\sqrt{a}} \right] \rightarrow 0. \quad (20)$$

Further, as $E \rightarrow \mp \infty$, one has: $(y, v_{n(p)}) \rightarrow \pm \infty$. Thus, one gets: $D_{-a-\frac{1}{2}}(y \rightarrow \pm \infty) \approx y^{-a-\frac{1}{2}} \times e^{-\frac{y^2}{4}} \rightarrow 0$.

Therefore, from Eq. (16), one gets:

$$K_{n(p)}(v_{n(p)} \rightarrow \pm \infty, N, r_{d(a)}, a) \equiv \frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)} \simeq \frac{1}{\beta(a)} \times \exp \left(-\frac{(A_{n(p)} \times v_{n(p)})^2}{2} \right) \times (A_{n(p)} \times v_{n(p)})^{-a-\frac{1}{2}} \rightarrow 0, \quad (21)$$

noting that $\beta(a) = \frac{\sqrt{\pi}}{2^{\frac{2a+1}{4}} \Gamma(\frac{a+3}{4})}$, being equal to: $\frac{\sqrt{\pi}}{2^{\frac{3}{4}} \times \Gamma(5/4)}$ for $a=1$, and $\frac{\sqrt{\pi}}{2^{3/2}}$ for $a = 5/2$.

It should be noted that those ratios: $\frac{\langle E_k^{a-\frac{1}{2}} \rangle_{KIM}}{f(a)}$, obtained in Equations (20) and (21), can be taken in an approximate form as:

$$F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) = K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) + [H_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) - K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)] \times \exp[-c_1 \times (A_{n(p)} v_{n(p)})^{c_2}], \quad (22)$$

So that: $F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) \rightarrow H_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)$ for $0 \leq v_n \leq 16$, and $F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a) \rightarrow K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a)$ for $v_{n(p)} \geq 16$. Here, the constants c_1 and c_2 may be respectively chosen as: $c_1 = 10^{-40}$ and $c_2 = 80$, as $a = 1$, being used to determine the critical density of electrons (holes) localized in the exponential conduction(valence) band-tails (EBT), $N_{CDn(CDp)}^{EBT}(N, r_{d(a)})$, given in the following.

Here, by using Eq. (18) for $a=1$, the density of states $\mathcal{D}(E)$ is defined by:

$$\langle \mathcal{D}(E_k) \rangle_{KIM} \equiv \frac{g_c(v)}{2\pi^2} \left(\frac{2m_c(v)}{\hbar^2} \right)^{\frac{3}{2}} \times \langle E_k^{\frac{1}{2}} \rangle_{KIM} = \frac{g_c(v)}{2\pi^2} \left(\frac{2m_c(v)}{\hbar^2} \right)^{\frac{3}{2}} \times \frac{\exp\left(-\frac{y^2}{4}\right) \times W_n^{\frac{1}{4}}}{\sqrt{2\pi}} \times \Gamma\left(\frac{3}{2}\right) \times D_{-\frac{3}{2}}(y) = \mathcal{D}(E). \quad (23)$$

Going back to the functions: H_n , K_n and F_n , given respectively in Equations (20-22), in

which the factor $\frac{\langle E_k^2 \rangle_{KIM}}{f(a=1)}$ is now replaced by:

$$\frac{\langle E_k^2 \rangle_{KIM}}{f(a=1)} = \frac{\mathcal{D}(E \leq 0)}{\mathcal{D}_0} = F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) \quad ,$$

$$\mathcal{D}_0(N, r_{d(a)}, a = 1) = \frac{g_c(v) \times (m_c(v) \times m_o)^{3/2} \times \sqrt{\eta_{n(p)}}}{2\pi^2 \hbar^3} \times \beta(a), \beta(a = 1) = \frac{\sqrt{\pi}}{\frac{3}{2^4} \times \Gamma(5/4)}. \quad (24)$$

Therefore, $N_{CDn(CDp)}^{EBT}(N, r_{d(a)})$ can be defined by: $N_{CDn(CDp)}^{EBT}(N, r_{d(a)}) = \int_{-\infty}^0 \mathcal{D}(E \leq 0) dE$,

$$N_{CDn(CDp)}^{EBT}(N, r_{d(a)}) =$$

$$\frac{g_c(v) \times (m_c(v))^{3/2} \sqrt{\eta_{n(p)}} \times (\pm E_{Fno(Fpo)})}{2\pi^2 \hbar^3} \times \left\{ \int_0^{16} \beta(a = 1) \times F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} + \right.$$

$$I_{n(p)} \left. \right\}, \quad (23)$$

where

$$I_{n(p)} \equiv \int_{16}^{\infty} \beta(a = 1) \times K_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} = \int_{16}^{\infty} e^{\frac{-(A_{n(p)} \times v_{n(p)})^2}{2}} \times$$

$$(A_{n(p)} v_{n(p)})^{-3/2} dv_{n(p)}.$$

Then, by another variable change: $t = [A_{n(p)} v_{n(p)} / \sqrt{2}]^2$, the integral $I_{n(p)}$ yields:

$$I_{n(p)} = \frac{1}{2^{5/4} A_{n(p)}} \times \int_{z_{n(p)}}^{\infty} t^{b-1} e^{-t} dt \equiv \frac{\Gamma(b, z_{n(p)})}{2^{5/4} \times A_{n(p)}}, \text{ where } b = -1/4, \quad z_{n(p)} = [16 A_{n(p)} / \sqrt{2}]^2,$$

and $\Gamma(b, z_{n(p)})$ is the incomplete Gamma function, defined by: $\Gamma(b, z_{n(p)}) \simeq z_{n(p)}^{b-1} \times$

$$e^{-z_{n(p)}} \left[1 + \sum_{j=1}^{16} \frac{(b-1)(b-2)\dots(b-j)}{z_{n(p)}^j} \right].$$

Finally, Eq. (25) now yields:

$$N_{CDn(CDp)}^{EBT}[N = N_{CDn(NDp)}(r_{d(a)}), r_{d(a)}] = \frac{g_c(v) \times (m_c(v))^{3/2} \sqrt{\eta_{n(p)}} \times (\pm E_{Fno(Fpo)})}{2\pi^2 \hbar^3} \times \left\{ \int_0^{16} \beta(a = \right.$$

$$1) \times F_{n(p)}(v_{n(p)}, N, r_{d(a)}, a = 1) dv_{n(p)} + \frac{\Gamma(b, z_{n(p)})}{2^{5/4} \times A_{n(p)}} \left. \right\}, \quad (26)$$

being the density of electrons (holes) localized in the EBT.

In $n(p)$ -type degenerate X - alkali metals, the numerical results of $N_{CDn(CDp)}^{EBT}[N = N_{CDn(NDp)}(r_{d(a)}), r_{d(a)}] \equiv N_{CDn(CDp)}^{EBT}(r_{d(a)})$, for a simplicity of presentation, evaluated using Eq. (26), are given in following **Tables 2-5 in Appendix 1**, in which those of other results such as: $B_{do(ao)}$, ϵ , and $N_{CDn(CDp)}$ are also given and computed, using Equations (2), (5), (6),

and (8), respectively, noting that the relative deviations in absolute values are defined by:

$|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$. It should be noted here that the maximal value of $|RD|$ is found to be given by: 2.8826×10^{-7} , meaning that $N_{CDn}^{EBT} \cong N_{CDn}$, according an important conclusion that such the obtained critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes), being localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

CONCLUDING REMARKS

In Tables 1-5 in Appendix 1, some concluding remarks are given and discussed as follows.

(1)-In Table 1, the numerical results of the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, are given and computed, using Equation (3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with **Landau's remarks on the Fermi Liquid**.^[2]

(2)-In the n(p) type degenerate **NaCl – ML**, while $\varepsilon(r_{d(a)})$ decreases ($\searrow$), the functions: $N_{CDn(CDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ increase ($\nearrow$), with increasing ($\nearrow$) $r_{d(a)}$, due to the impurity size effect.

(3)- In those Tables (2-5), one also notes that the maximal value of $|RD|$ is found to be given by: 2.8826×10^{-7} , meaning that $N_{CDn}^{EBT} \cong N_{CDn}$. In other words, such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, is just the density of electrons (holes), being localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$.

(4) Finally, once $N_{CDn(CDp)}$ is determined, the effective density of free electrons (holes), N^* , given in the parabolic conduction (valence) bands of the n(p)-type degenerate **NaCl – ML** can thus be defined by:

$$N^*(N, r_{d(a)}) \equiv N - N_{CDn(NDp)} \cong N - N_{CDn(CDp)}^{EBT},$$

Being equivalent to the “compensation effect”.

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APPENDIX 1

Table 1: In the $n(p)$ type degenerate $\text{NaCl} - \text{ML}$, the numerical results of the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, are given and computed, using Equation (3). It should be noted that, for a given $r_{d(a)}$, those increase with increasing $\frac{m_{c(v)}}{m_o}$, in good agreement with Landau's remarks on the Fermi Liquid [2].

Donor					
$\frac{m_c}{m_o}$	$\nearrow$	3	4	5	6
For $r_d = r_p = 0.110 \text{ nm}$					
$[\Delta p(r_d)]_n \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	7.960733	10.614311	13.267888	15.921466
For $r_d = r_{As} = 0.118 \text{ nm}$					
$[\Delta p(r_d)]_n \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	6.6026108	8.8034811	11.004351	13.205222
For $r_d = r_{sb} = 0.136 \text{ nm}$					
$[\Delta p(r_d)]_n \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	3.856154	5.1415387	6.4269234	7.7123081
For $r_d = r_{Sn} = 0.140 \text{ nm}$					
$[\Delta p(r_d)]_n \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	3.2953816	4.3938421	5.4923026	6.5907631
Acceptor					
$\frac{m_v}{m_o}$	$\nearrow$	3	4	5	6
For $r_a = r_{Ga} = 0.126 \text{ nm}$					
$[\Delta p(r_a)]_p \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	5.3336119	7.1114826	8.8893532	10.667224
For $r_a = r_{Mg} = 0.141 \text{ nm}$					
$[\Delta p(r_a)]_p \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	3.1576920	4.2102561	5.2628201	6.3153841
For $r_a = r_{In} = 0.144 \text{ nm}$					
$[\Delta p(r_a)]_p \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	2.7504076	3.6672101	4.5840127	5.5008152
For $r_a = r_{Cd} = 0.148 \text{ nm}$					
$[\Delta p(r_a)]_p \left(\frac{10^8 \text{ meV}}{\text{cm}^3} \right)$	$\nearrow$	2.2203663	2.9604884	3.7006105	4.4407327

Table 2: In the n(p) type degenerate **NaCl – ML**, in which $r_{do(ao)}=r_{Na}=0.166$ nm, $m_{c(v)}/m_o = 3$ and $\epsilon_o = 23$, the numerical results of $B_{do(ao)} = 6.4484321 \times 10^8$ ($\frac{meV}{cm^3}$), ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (2), (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 2.8826×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	Sn	Na
r_d (nm)	↗	0.110	0.118	0.136	0.140	166
$\epsilon(r_d)$	↘	65.1343	39.2223	26.9038	25.7871	23
$N_{CDn}(r_d)$ in 10^{20} cm^{-3}	↗	0.10254862	0.46963489	1.4551766	1.6525444	2.3290329
$N_{CDn}^{EBT}(r_d)$ in 10^{20} cm^{-3}	↗	0.10254859	0.46963476	1.4551762	1.6525439	2.3290323
$ RD $ in 10^{-7}		2.8826	2.6643	2.8629	2.7682	2.5929
Acceptor		Ga	Mg	In	Cd	Na
r_a (nm)	↗	0.126	0.141	0.144	0.148	166
$\epsilon(r_a)$	↘	31.4571	25.5492	24.9192	24.2482	23
$N_{CDp}(r_a)$ in 10^{20} cm^{-3}	↗	0.91034037	1.6991366	1.8312967	1.9875584	2.3290329
$N_{CDp}^{EBT}(r_a)$ in 10^{20} cm^{-3}	↗	0.91034013	1.6991361	1.8312962	1.9875579	2.3290323
$ RD $ in 10^{-7}		2.6910	2.7331	2.7596	2.7071	2.5914

Table 3: In the n(p) type degenerate **NaCl – ML**, in which $r_{do(ao)}=r_{Na}=0.166$ nm, $m_{c(v)}/m_o = 4$ and $\epsilon_o = 23$, the numerical results of $B_{do(ao)} = 8.5979094 \times 10^8$ ($\frac{meV}{cm^3}$), ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (2), (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 2.7234×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	Sn	Na
r_d (nm)	↗	0.110	0.118	0.136	0.140	166
$\epsilon(r_d)$	↘	65.1343	39.2223	26.9038	25.7871	23
$N_{CDn}(r_d)$ in 10^{20} cm^{-3}	↗	0.24307820	1.1132086	3.4493074	3.9171422	5.5206707
$N_{CDn}^{EBT}(r_d)$ in 10^{20} cm^{-3}	↗	0.24307814	1.1132083	3.4493065	3.9171412	5.5206692
$ RD $ in 10^{-7}		2.6312	2.5153	2.6984	2.6456	2.7234
Acceptor		Ga	Mg	In	Cd	Na
r_a (nm)	↗	0.126	0.141	0.144	0.148	166
$\epsilon(r_a)$	↘	31.4571	25.5492	24.9192	24.2482	23
$N_{CDp}(r_a)$ in 10^{20} cm^{-3}	↗	2.15784380	4.0275830	4.3408514	4.7112495	5.5206707
$N_{CDp}^{EBT}(r_a)$ in 10^{20} cm^{-3}	↗	2.15784320	4.0275819	4.3408502	4.7112482	5.5206692
$ RD $ in 10^{-7}		2.5818	2.6572	2.7093	2.6561	2.7218

Table 4: In the n(p) type degenerate **NaCl – ML**, in which $r_{do(ao)}=r_{Na}=0.166$ nm, $m_{c(v)}/m_o = 5$ and $\epsilon_o = 23$, the numerical results of $B_{do(ao)} = 1.0747387 \times 10^9$ ($\frac{meV}{cm^3}$), ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (2), (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 2.7447×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	Sn	Na
r_d (nm)	↗	0.110	0.118	0.136	0.140	166
$\epsilon(r_d)$	↘	65.1343	39.2223	26.9038	25.7871	23
$N_{CDn}(r_d)$ in 10^{20} cm^{-3}	↗	0.47476211	2.1742356	6.7369285	7.6506684	10.782560
$N_{CDn}^{EBT}(r_d)$ in 10^{20} cm^{-3}	↗	0.47476198	2.1742350	6.7369267	7.6506664	10.782557
$ RD $ in 10^{-7}		2.6390	2.6593	2.6847	2.6769	2.7447
Acceptor		Ga	Mg	In	Cd	Na
r_a (nm)	↗	0.126	0.141	0.144	0.148	166
$\epsilon(r_a)$	↘	31.4571	25.5492	24.9192	24.2482	23
$N_{CDp}(r_a)$ in 10^{20} cm^{-3}	↗	4.21453870	7.8663730	8.4782254	9.2016593	10.782560
$N_{CDp}^{EBT}(r_a)$ in 10^{20} cm^{-3}	↗	4.21453760	7.8663709	8.4782231	9.2016558	10.782557
$ RD $ in 10^{-7}		2.6211	2.6221	2.7158	2.7332	2.7432

Table 5: In the n(p) type degenerate **NaCl – ML**, in which $r_{do(ao)}=r_{Na}=0.166$ nm, $m_{c(v)}/m_o = 6$ and $\epsilon_o = 23$, the numerical results of $B_{do(ao)} = 1.2896864 \times 10^9$ ($\frac{meV}{cm^3}$), ϵ , $N_{CDn(CDp)}$, and $N_{CDn(CDp)}^{EBT}$ are computed, using Equations (2), (5), (6), (8), and (26), respectively. It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 2.8198×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$, determined in Eq. (26), respectively.

Donor		P	As	Sb	Sn	Na
r_d (nm)	↗	0.110	0.118	0.136	0.140	166
$\epsilon(r_d)$	↘	65.1343	39.2223	26.9038	25.7871	23
$N_{CDn}(r_d)$ in 10^{20} cm^{-3}	↗	0.8238893	3.7570791	11.641412	13.220355	18.632263
$N_{CDn}^{EBT}(r_d)$ in 10^{20} cm^{-3}	↗	0.8238871	3.7570781	11.641409	13.220351	18.632258
$ RD $ in 10^{-7}		2.6671	2.6330	2.4579	2.6791	2.5297
Acceptor		Ga	Mg	In	Cd	Na
r_a (nm)	↗	0.126	0.141	0.144	0.148	166
$\epsilon(r_a)$	↘	31.4571	25.5492	24.9192	24.2482	23
$N_{CDp}(r_a)$ in 10^{21} cm^{-3}	↗	0.28272290	1.3593093	1.4650373	1.5900467	1.8632263
$N_{CDp}^{EBT}(r_a)$ in 10^{21} cm^{-3}	↗	0.28272100	1.3593089	1.4650369	1.5900463	1.8632258
$ RD $ in 10^{-7}		2.6425	2.8198	2.5183	2.6330	2.5281