

**REGARDING STABILITY ANALYSIS OF THE VIBRATION
MOVEMENT OF THE SPINDLE IN LONGITUDINAL DIRECTION AT
CNC LATHE**

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ABSTRACT

The paper presents a stability analysis of CNC lathe processing using Nyquist criteria starting from the vibrations in the longitudinal direction on a model of the main shaft of the CNC machine. The model determines stability domains corresponding to analyzed shaft segments

which match real work conditions.

KEYWORDS: CNC lathe, stability, Nyquist criteria, main spindle, stability domains.

INTRODUCTION

CNC machine tools are precision cutting devices with a high degree of stability and surface finish quality. They are designed for processing complex surfaces of standard or special materials in special work conditions (gas environment, high temperature, moisture, corrosive factors, etc.).



Figure 1: CNC lathe model.

Due to the real working conditions, variable processing loads, superficial hardening of the work surfaces and non-homogeneities, the dynamic work regimes are constantly changing, which leads to appearance of disturbing factors that cause the need for continuous adjustment of the working regimes (speed, advance, etc.) in order to obtain a superior roughness of the workpiece.^[1]

In order to determine the requirements for a stable cutting processing it is required to analyze the longitudinal vibration movements of the main shaft of the CNC lathe.^[2]

Essentially, the equations that describe the vibration movement of the main shaft of the CNC lathe in longitudinal direction are defined as a closed loop system:^[3]

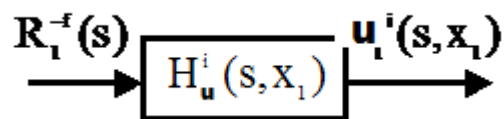


Figure 2: System for defining the vibration movement.

ESTABLISHING THE MATHEMATICAL MODEL

The analysis presented in what follows is based on using the Nyquist stability criteria and consists in determining the transfer functions on three segments of the main shaft of the CNC lathe.^[4,5]

In this case it is necessary to determine the closed loop transfer functions $H_u^{(i)}(s, x_1); i = \overline{0,2}$, in which the input is the axial force R_1^f and the outputs are the displacements $U_1^{(i)}(s, x_1); i = \overline{0,2}$.

We have:

$$H_U^0(s, x_1) = \frac{\overline{U}_1^0(s, x_1)}{\overline{R}_1^f(s)} = h_1^0(s)e^{s\sqrt{\frac{\rho}{E}}x_1} + h_2^0(s)e^{-s\sqrt{\frac{\rho}{E}}x_1}, x_1 \in (0, l_1) \quad (1)$$

$$h_1^0(s) = \frac{4EA^2\rho s^2}{m} \quad (2)$$

$$h_2^0(s) = \frac{4EA^2\rho s^2}{m}$$

$$H_U^1(s, x_1) = \frac{\overline{U}_1^1(s, x_1)}{\overline{R}_1^f(s)} = h_1^1(s)e^{s\sqrt{\frac{\rho}{E}}x_1} + h_2^1(s)e^{-s\sqrt{\frac{\rho}{E}}x_1}, x_1 \in (l_1, l_2) \quad (3)$$

$$h_1^1(s) = \frac{2sEA\sqrt{\frac{\rho}{E}} \left[k_{11}e^{-2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} + 2EA\sqrt{\frac{\rho}{E}}s \right]}{m}$$

$$h_2^1(s) = \frac{-2sEA\sqrt{\frac{\rho}{E}} \left[k_{11}e^{2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} - 2EA\sqrt{\frac{\rho}{E}}s \right]}{m}$$
(4)

$$H_U^2(s, x_1) = \frac{\overline{U}_1^2(s, x_1)}{\overline{R}_1^f(s)} = h_1^2(s)e^{s\sqrt{\frac{\rho}{E}x_1}} + h_2^2(s)e^{-s\sqrt{\frac{\rho}{E}x_1}}, \quad x_1 \in (l_2, l)$$
(5)

$$h_1^2(s) = \frac{-k_{11}^2 \left(e^{-2s\sqrt{\frac{\rho}{E}l_2}} - e^{-2s\sqrt{\frac{\rho}{E}l_1}} + e^{2s\sqrt{\frac{\rho}{E}(l_1-l_2)}} - 1 \right)}{m}$$

$$+ \frac{2k_{11}EAs\sqrt{\frac{\rho}{E}} \left(e^{-2s\sqrt{\frac{\rho}{E}l_1}} + e^{-2s\sqrt{\frac{\rho}{E}l_2}} \right) + 4k_{11}EAs\sqrt{\frac{\rho}{E}}}{m}$$

$$+ \frac{4EA^2\rho s^2}{m}$$

$$h_2^2(s) = \frac{k_{11}^2 \left(1 + e^{2s\sqrt{\frac{\rho}{E}l_1}} - e^{2s\sqrt{\frac{\rho}{E}l_2}} + e^{2s\sqrt{\frac{\rho}{E}(l_2-l_1)}} \right)}{m}$$
(6)

$$+ \frac{2k_{11}EAs\sqrt{\frac{\rho}{E}} \left(e^{-2s\sqrt{\frac{\rho}{E}l_1}} + e^{2s\sqrt{\frac{\rho}{E}l_2}} \right) + 4k_{11}EAs\sqrt{\frac{\rho}{E}}}{m}$$

$$H_i^d(s, x_1); i = \overline{0, 2}$$

Using the Nyquist stability criteria, the open loop transfer functions $H_i^d(s, x_1); i = \overline{0, 2}$ are determined using the following relations:

$$H_i^d(s, x_1) = \frac{H_U^{(i)}(s, x_1)}{1 - H_U^{(i)}(s, x_1)}; i = \overline{0, 2}$$
(7)

$$H_0^d(s, x_1) = \frac{4EA^2\rho s^2 \left(e^{s\sqrt{\frac{\rho}{E}x_1}} + e^{-s\sqrt{\frac{\rho}{E}x_1}} \right)}{m - 4EA^2\rho s^2 \left(e^{s\sqrt{\frac{\rho}{E}x_1}} + e^{-s\sqrt{\frac{\rho}{E}x_1}} \right)}$$
(8)

$$\begin{aligned}
 H_1^d(s, x_1) = & \left\{ 2EA\sqrt{\frac{\rho}{E}}s \left[\left(k_{11}e^{-2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} + 2EA\sqrt{\frac{\rho}{E}}s \right) \right] \right. \\
 & \cdot e^{2s\sqrt{\frac{\rho}{E}x_1}} - \left. \left(k_{11}e^{2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} - 2EA\sqrt{\frac{\rho}{E}}s \right) e^{-2s\sqrt{\frac{\rho}{E}x_1}} \right\} / \\
 & / \left\{ m - 2EA\sqrt{\frac{\rho}{E}}s \left[\left(k_{11}e^{-2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} + 2EA\sqrt{\frac{\rho}{E}}s \right) e^{2s\sqrt{\frac{\rho}{E}x_1}} - \right. \right. \\
 & \left. \left. - \left(k_{11}e^{2s\sqrt{\frac{\rho}{E}l_1}} + k_{11} - 2EA\sqrt{\frac{\rho}{E}}s \right) e^{-2s\sqrt{\frac{\rho}{E}x_1}} \right] \right\}
 \end{aligned} \tag{9}$$

$$H_2^d(s, x_1) = \frac{m_5 e^{s\sqrt{\frac{\rho}{E}x_1}} + m_6 e^{-s\sqrt{\frac{\rho}{E}x_1}}}{1 - \left(m_5 e^{s\sqrt{\frac{\rho}{E}x_1}} + m_6 e^{-s\sqrt{\frac{\rho}{E}x_1}} \right)} \tag{10}$$

Modelling of these functions in the $[\text{Re } H^d(j\omega), j\text{Im } H^d(j\omega)]$ coordinate system is presented in the following figures.^[6]

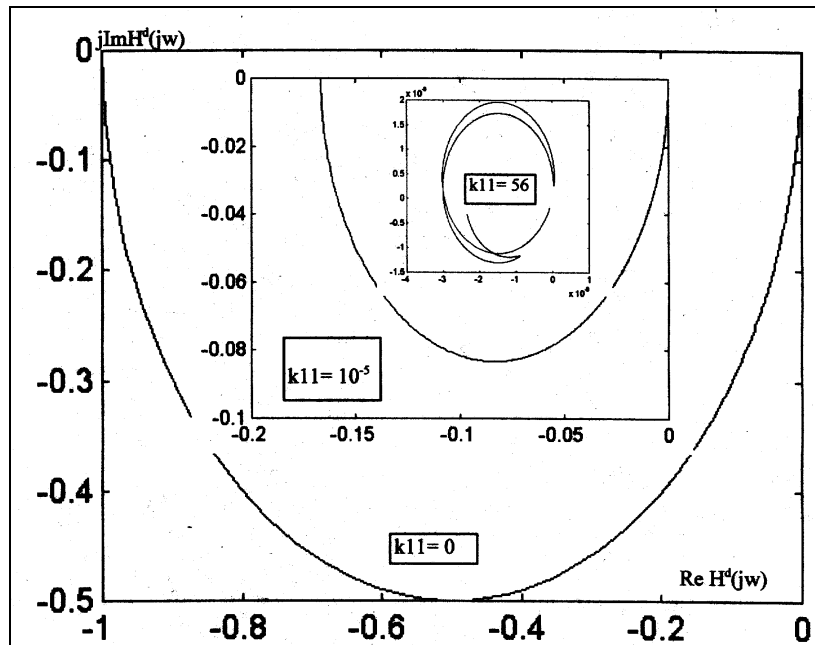


Figure-3.

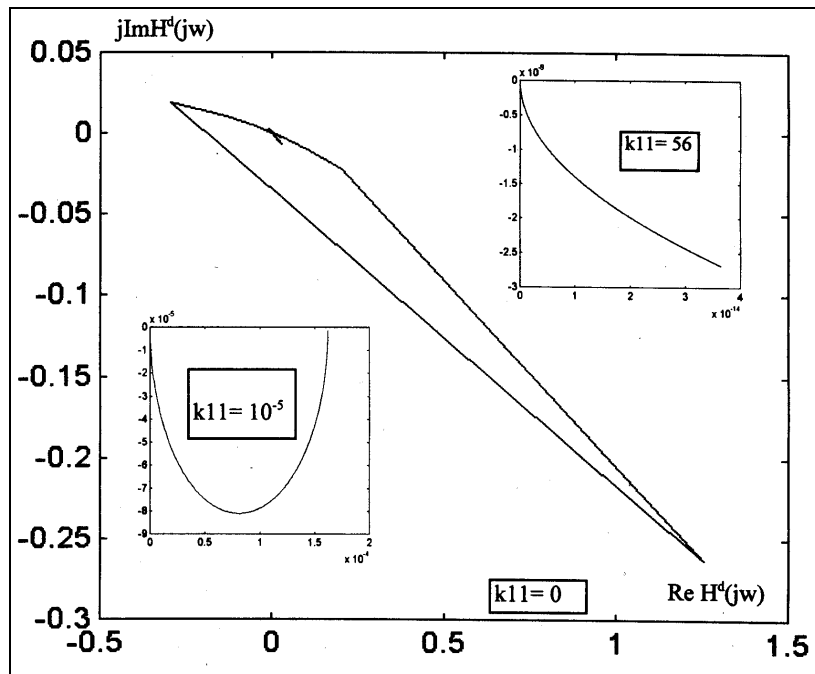


Figure-4.

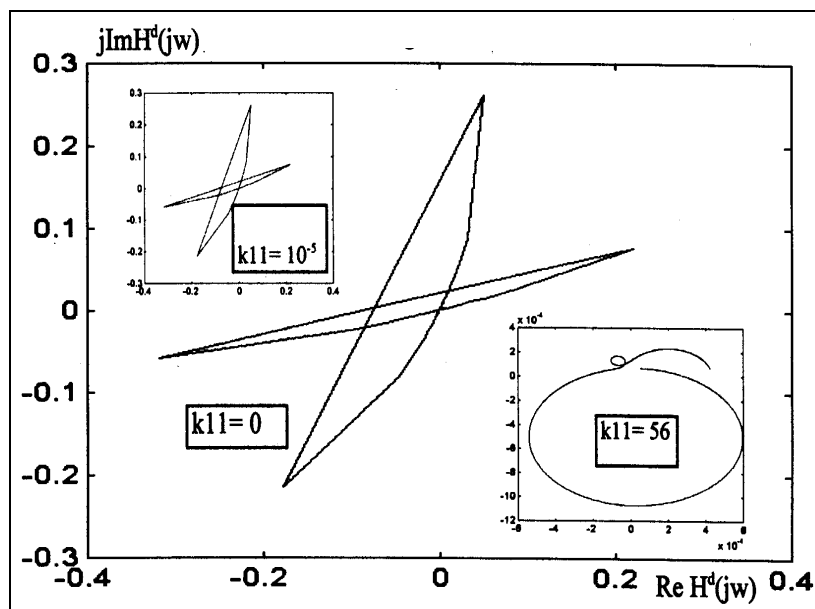


Figure-5.

CONCLUSIONS

The following conclusions can be drawn from the three shaft segment models:

- The characteristic measures for the entire model are:

$$\rho = 7800 \text{ kg/m}^3; E = 2,1 \cdot 10^{11} \text{ N/m}^2; A = 1,539 \cdot 10^{-4} \text{ m}^2; l_1 = 0,030 \text{ m}; l_2 = 0,170 \text{ m}; l = 0,295 \text{ m};$$

$$m_p = 0,0204 \text{ kg}.$$

- On the first analyzed segment (Figure 3) of the main shaft with $x_1 = 0,01\text{m}$ for $k_{11} = 0$ the system marginally stable in the -1.0 domain, with variation of the oscillation domain $\omega = 10^{-7} \div 10^{-1} \text{sec}^{-1}$ with $p = 10^{-5}$.
- o The situation appears for three rotation speeds: $n_{s1} = 15100\text{r}/\text{min}$; $n_{s2} = 17500\text{r}/\text{min}$; $n_{s3} = 19000\text{r}/\text{min}$.
- o Rigidity $k_{11} = 0$ is not true in the actual situation.
- o Increasing rigidity to $k_{11} = 10^{-5} \text{N/m}$ produces an increase of the stability reserve in the domain from the left side of the plane.
- o The dynamic rigidity k_{11} of the main shaft corresponding to the three rotation speeds is $k_{11} = 56$ for $n_s = 15100\text{r}/\text{min}$; $k_{11} = 51$ for $n_s = 17500\text{r}/\text{min}$; $k_{11} = 48$ for $n_s = 19000\text{r}/\text{min}$.
- o In all cases the frequency domain variation is $\omega = 1000 \div 10000 \text{sec}^{-1}$, $p = 10$.
- o The stability reserve increases considerably near the origin.
- The second analyzed segment (Figure 4) oscillates near the origin for $k_{11} = 0$ in the right half of the plane and has stability domain $\omega = 10^{-4} \div 10^5 \text{sec}^{-1}$ with $p = 10$.
- o For $k_{11} = 10^{-5}$ $k_{11} = 56, 51, 48$ there is an increase in the stability reserve for the variation domain $\omega = 10^{-7} \div 10^{-1} \text{sec}^{-1}$ with $p = 10^{-5}$.
- Finally, in the third analyzed segment (Figure 5) of the main shaft $x_1 = 0,23\text{m}$, for $k_{11} = 0$, $k_{11} = 10^{-5}$ and $k_{11} = 56, 51, \text{ and } 48$; the system oscillates around the coordinate axes being stable in the domain $\omega = 10^3 \div 10^5 \text{sec}^{-1}$ with $p = 10$.

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