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SOME ARITHMETIC OPERATIONS ON TRIANGULAR FUZZY NUMBERS AND ITS APPLICATION IN SOLVING LINEAR PROGRAMMING PROBLEM BY DUAL-SIMPLEX ALGORITHM

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ABSTRACT

The fuzzy logic and fuzzy numbers have been applied in many fields such as operation research, differential equations, fuzzy system reliability, control theory and management sciences etc. The fuzzy logic and fuzzy numbers are widely used in engineering applications also. In this paper we first describe Triangular Fuzzy Number (TFN) with arithmetic operations and solve a linear programming problem by Triangular Fuzzy Number (TFN) using Dual-simplex algorithm.

KEYWORDS: Fuzzy set, Triangular Fuzzy Number (TFN), Dual-Simplex algorithm.

INTRODUCTION

A fuzzy set in a universe X is defined by its membership function which maps X to the interval[1] and therefore implies a linear, i.e. total ordering of the[27] elements of X, one could argue that this makes them inadequate to deal with incomparable information. A possible solution, however, was already implicit in Zadeh's[29-31] seminal paper in a footnote; he mentioned that "in a more general setting, the range of the membership function can be taken to be a suitable partially ordered set P." In every sector of our life,[1-3][21-22] there arise several problems which can be formulated mathematically as optimization problem with the goal to maximize the profit or to minimize the cost to formulate the problem mathematically, some constraints or restrictions are to be considered. Linear programming is a one of the most important operational research technique and it is applied in many sector especially related to the optimization problem. Linear programming was first introduced by George Dantzig in 1947. Linear programming is a technique that is to optimize the use of limited resources. Formulation of fuzzy linear programming was first introduced by Zimmermann. Deldago[23] makes a general model of fuzzy linear programming within the limits of technical coefficients fuzzy and fuzzy right side. Fung and Hu[28] introduced the linear programming with the technique coefficients based on fuzzy numbers. Verdegay defined the dual problem through parametric linear program and shows that the problem of primal - dual fuzzy linear program has the same solution. In this paper we consider the linear programming problem in its standard form to find out it's feasible and optimal solution. We use dual simplex algorithm by triangular fuzzy number[12-16] to solve the linear programming problem.

Definition

Triangular fuzzy number A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is called triangular fuzzy number if it's membership function function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

1. Some arithmetic operations of Triangular Fuzzy Number

❖ **Properties 3.1**

If $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two TFN then $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is also TFN.

$$\tilde{A} \oplus \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

❖ **Properties 3.2**

If $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ then $\tilde{A} \Theta \tilde{B}$ is a fuzzy number

$$\tilde{A} \Theta \tilde{B} = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$$

- **Properties 3.3**

- If $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ then $\tilde{A} \oslash \tilde{B}$ is a fuzzy number $\tilde{A} \oslash \tilde{B} = (a_1/a_2, b_1/b_2, c_1/c_2)$.

- **Properties 3.4**

If $\tilde{A} = (a_1, b_1, c_1)$ and $\tilde{B} = (a_2, b_2, c_2)$ are two TFN then $\tilde{P} = \tilde{A} \otimes \tilde{B}$ is an approximated TFN.

$\tilde{A} \otimes \tilde{B} = (a_1 a_2, b_1 b_2, c_1 c_2)$.

- **Properties 3.5**

If TFN $\tilde{A} = (a_1, b_1, c_1)$ and $y = ka\,(k > 0)$, then $\tilde{Y} = k\tilde{A}$ is a TFN (ka_1, kb_1, kc_1).

If $y = ka\,(k < 0)$, then $\tilde{Y} = k\tilde{A}$ is a TFN (kc_1, kb_1, ka_1).

Construction and solution procedure of a LPP by Trapezoidal Fuzzy Number (TrFN) using simplex algorithm[7][8][9][10][11]

Consider the following steps

Let us consider a LPP in the following form which will be called the primal problrm:

Maximize $z = \sum_{j=1}^{n} c_j \tilde{x}_j$

Subject to $\sum_{j=1}^{n} a_{ij} \tilde{x}_j \le b_i$, $i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, m$

$\tilde{x}_j \ge 0,\ j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, n$

In which $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \ldots\ldots\ldots\ldots, \tilde{x}_n$ are the primal variables and z is the primal objective function.

The associated dual problem will be given by

Minimize $w = \sum_{i=1}^{m} b_i \tilde{v}_i$

Subject to $\sum_{i=1}^{m} a_{ji} \tilde{v}_i \ge c_j$, $j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, n$

$\tilde{v}_i \ge 0, i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, m$

In which $v_1, v_2, v_3, \ldots\ldots\ldots\ldots, v_m$ are the dual variables and w is the dual objective function.

To be more explicit, if the primal problem be

Maximize $z = c_1 x_1 + c_2 x_2 + \ldots\ldots\ldots\ldots\ldots + c_n x_n$

Subject to

$$a_{11} x_1 + a_{12} x_2 + \ldots\ldots\ldots\ldots\ldots + a_{1n} x_n \le b_1,$$
$$a_{21} x_1 + a_{22} x_2 + \ldots\ldots\ldots\ldots\ldots + a_{2n} x_n \le b_2,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1} x_1 + a_{m2} x_2 + \ldots\ldots\ldots\ldots\ldots + a_{mn} x_n \le b_m,$$
$$x_1, x_2, \ldots\ldots\ldots\ldots\ldots\ldots, x_n \ge 0$$

Then its dual is

Minimize $w = b_1 v_1 + b_2 v_2 + \ldots\ldots\ldots\ldots\ldots + b_m v_m$

Subject to

$$a_{11} v_1 + a_{21} v_2 + \ldots\ldots\ldots\ldots\ldots + a_{m1} v_m \le c_1,$$
$$a_{11} v_1 + a_{22} v_2 + \ldots\ldots\ldots\ldots\ldots + a_{m2} v_m \le c_2,$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{1n} v_1 + a_{2n} v_2 + \ldots\ldots\ldots\ldots\ldots + a_{mn} v_m \le c_n,$$
$$v_1, v_2, \ldots\ldots\ldots\ldots\ldots\ldots, v_m \ge 0$$

General formulation of the dual of an LPP is done in two stages. Firstly the problem is put in the standard maximization form and then the following steps are followed:

i. The maximization problem in the primal is transferred to a minimization problem in the dual.

ii. For a primal with n variable and m constrains the dual will be have m variable and n constrains,

iii. The less than signs of the primal constrains becomes greater than signs in the dual constrains.

iv. The prices $c_1, c_2, \ldots\ldots, c_n$ with n variables in the objective function of the primal are replaced by the prices $b_1, b_2, \ldots\ldots, b_m$ with m variables of the objective function in the dual.

v. The requirements $b_1, b_2, ..., b_m$ in the m primal constrains are replaced by the requirements $c_1, c_2, ..., c_n$ of the n dual constrains.

Application

In this paper we are going to solve a linear programming problem by triangular fuzzy number using simplex algorithm. Our problem is described below:

$Minz = 3x_1 + x_2$

Subject to constraint

$2x_1 + 3x_2 \geq 2$,

$x_1 + x_2 \geq 1$,

$x_1, x_2 \geq 0$

Dual of the above problem is

$Maxw = 2v_1 + v_2$

Subject to constraint

$2v_1 + v_2 \leq 3$,

$3v_1 + v_2 \leq 1$,

$v_1, v_2 \geq 0$

We use triangular fuzzy number to solve the dual problem by simplex method and put it in standard form by adding slack variables v_3 and v_4 .thus the problem becomes

$Maxw = (2,3,4)v_1 + (1,2,3)v_2 + (0,0,0)v_3 + (0,0,0)v_4$

Subject to constraint

$(2,3,4)v_1 + (1,2,3)v_2 + (1,1,1)v_3 = 3,$

$(3,4,5)v_1 + (1,2,3)v_2 + (1,1,1)v_4 = 1,$

$v_1, v_2, v_3, v_4 \geq 0$

TABLE AU

C_B	B	v_B	b	a_1	a_2	a_3	a_4
0	a_3	v_3	3	(2,3,4)	(1,2,3)	(1,1,1)	(0,0,0)
0	a_4	v_4	1	(3,4,5)	(1,2,3)	(0,0,0)	(1,1,1)
			Z_j-C_j	(-2,-3,-4)	(-1,-2,-3)	(0,0,0)	(0,0,0)
0	a_3	v_3	(7/3,9/4,11/5)	(0,0,0)	(1/3,1/2,3/5)	(1,1,1)	(-2/3,-3/4,-4/5)
(2,3,4)	a_1	v_1	(1/3,1/4,1/5)	(1,1,1)	(1/3,1/2,3/5)	(0,0,0)	(1/3,1/4,1/5)
			Z_j-C_j	(0,0,0)	(-1/3,-1/2,-3/5)	(0,0,0)	(2/3,3/4,4/5)
0	a_3	v_3	(2,2,2)	(-1,-1,-1)	(0,0,0)	(1,1,1)	(-1,-1,-1)
(1,2,3)	a_2	v_2	(1,1/2,1/3)	(3,2,5/3)	(1,1,1)	(0,0,0)	(1,1/2,1/3)
			Z_j-C_j	(1,1,1)	(0,0,0)	(0,0,0)	(1,1,1)

Here $Z_j - C_j \geq 0$ for all j .hence the last table gives the optimal solution of the problem. Since the slack variable V_3 added to the first constraint of the dual is present in the optimal solution of the dual, the optimal solution of the dual is $v_3 = (0,0,0), v_2 = (1,1,1)$ and $w_{\max} = 1$. The optimal solution to the primal can be read from the (Z_j-C_j) row below the vectors a_3 and a_4 corresponding to the slack variables and hence $x_1 = (0,0,0)$ and $x_2 = (1,1,1)$. Also $z_{\min} = w_{\max} = 1$. Note that this is a feasible solution of the given primal.

CONCLUSION

In this paper TFN and their arithmetic operations are described,[7,8,17,18,19] we have also solved a Dual-simplex problem using TFN. The procedure of solving Dual-simplex problem using TFN may help us to solve many optimization problems. Our approaches and computational procedures may be efficient and simple to implement for calculation in a Triangular fuzzy environment for all fields of engineering and science where impreciseness occur.

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