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FUNDAMENTALS OF ELECTROMAGNETIC FIELDS AND WAVES

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ABSTRACT

This course presents a clearer view on the understanding of Electromagnetic Fields and waves, for the development of field-related appliances, such as GSM, etc. For proper understanding, it is to be noted that most of the laws and their derived mathematical expressions (equations) are highly interwoven and inter-related that their easy understanding pose some problems. To solve these problems, clearer knowledge of vectors is necessary as Electromagnetic Fields and Waves are examples of Vector Fields. Laws and their appropriate equations and their inter-relations are explained. Some examples are; $F = q_o V \times B$, is equivalent to F= iLB, Faraday's law of $\varepsilon = -\frac{d\emptyset_B}{dt}$, is also Blv. That, extension of Ampere's law, is, the Maxwell equation which is $\oint B.dl = \mu_o(\frac{d\phi_E}{dt} + \text{i})$ and Ampere's law is $\oint B.dl = \mu_o$I.

KEYWORDS: Electrostatic-field, Electromagnetic-field, Gauss' law, Ampere's law, Coulomb's law, Bio-Savart, Maxwell equation, Radiated Field.

INTRODUCTION

Both Electrostatic Fields and Electromagnetic Fields are Fields-related forces and so vector fields. For this reason, brief knowledge of vectors is very necessary. Electrostatic fields deal with stationary charges. Stationary charges do not change its magnitude with time within a confined space. It can exist as positive or negative charge and are capable of either repelling themselves when they are of the same charge (i.e all being positive charges or negative charges), or attracting themselves when they are oppositely charged (i.e positive and negative charges being close together). Examples of electrostatic fields are; Electric Power Transmission, X-rays mechanism, lighting protection, Solid-State Electronic Devices such as Bipolar and Field-effect Transistors, Touch-pads, Capacitance Keyboards, Cathod-ray tubes, Liquid Crystal Display (LCD), Electrostatic Printers, Cardiograms, Electro-encephalograms, specialized reading of the organs of eyes ,ears, nose, stomach, paints- spraying, electrode position, chemical separation of fine particles, seed-sortings, measurement of moisture content of the crops, spinning of cottons, speed baking of bread and smoking of meat, electrostatic photo-copiers.

Charges moving at steady or uniform velocity is called Electric current, while $I \triangleq \frac{dq}{dt}$ is

Magneto-Static Field. Charges under acceleration with sinusoidal pulses with respect to time is **Electromagnetic Field.**

1.1 Vectors.

Laws of vectors

Laws	Addition	Multiplication
Commutative	$\tilde{A} + \mathcal{B} = \mathcal{B} + \tilde{A}$	$K\tilde{A} = \tilde{A}K$
Associative	$\tilde{A} + (\mathcal{B} + \acute{C}) = (\tilde{A} + \mathcal{B}) + \acute{C}$	$K(L\tilde{A}) = (KL)\tilde{A}$
Distributive	$K(\tilde{A} + \mathcal{B}) = K\tilde{A} + K\mathcal{B}$	

1.1.1 Unit Vector; This has both magnitude and direction and it is defined,

$\breve{a}_A = \frac{\tilde{A}}{|\tilde{A}|}$ ________(1)

$\tilde{A} = |\tilde{A}|\breve{a}_A$ ----------(2)

$\tilde{A}$ = (Ax, Ay, Az), or $Ax\breve{a}_x + Ay\breve{a}_y + Az\breve{a}_z$,

$|\tilde{A}| = \sqrt{Ax^2 + Ay^2 + Az^2}$,

$\therefore \breve{a}_A = \frac{Ax\breve{a}_x + Ay\breve{a}_y + Az\breve{a}_z,}{\sqrt{Ax^2 + Ay^2 + Az^2,}}$.

1.1.2 Addition and Subtraction of Vectors

$\tilde{A} + \tilde{B} = Ax\breve{a}_x + Ay\breve{a}_y + Az\breve{a}_z + Bx\breve{a}_x + By\breve{a}_y + Bz\breve{a}_z$,

$\acute{C} = \tilde{A} + \tilde{B} = (Ax + Bx)\breve{a}_x + (Ay + By)\breve{a}_y + (Az + Bz)\breve{a}_z$,

$\bar{D} = \tilde{A} - \tilde{B} = (Ax - Bx)\breve{a}_x + (Ay - By)\breve{a}_y + (Az - Bz)\breve{a}_z$.

1.1.3 Position Vectors and Distance Vectors

A point P in Cartesian Cordinates may be represented by (x, y, z). The position vector $\bar{r}p$ of points P is the directed distance from the origin O to P.

i.e. $\bar{r}p = OP = X\breve{a}_x + Y\breve{a}_y + Z\breve{a}_z$,

The point (3,4,5) has position vector of $3\breve{a}_x + 4\breve{a}_y + 5\breve{a}_z$,

If two points p_1 and p_2 are given by $(x_1 + y_1 + z_1)$ *and* $(x_2 + y_2 + z_2)$, the distance vector (seperation vector) is the displacement from p_1 *to* p_2,

$\bar{r}p_1p_2 = \bar{r}p_2 - \bar{r}p_1 \quad = (x_2\breve{a}_x + y_2\breve{a}_y + z_2\breve{a}_z) - (x_1\breve{a}_x + y_1\breve{a}_y + z_1\breve{a}_z)$,

$= (x_2 - x_1)\breve{a}_x + (y_2 - y_1)\breve{a}_y + (z_2 - z_1)\breve{a}_z$.

1.1.4 Difference between a point P and the vector $\tilde{A}$

P, may be represented as (x, y, z) and not a vector, while $\bar{r}p$ is a vector. $\tilde{A}$ depends on P. Example, if P=(2, -1,4) and $\tilde{A} = x^2y\breve{a}_x + y^2\breve{a}_y + xz^2\breve{a}_z$, then $\tilde{A}$ at P would be$(2\times 2 \times -1)\breve{a}_x + (-1)^2\breve{a}_y + (2\times 4^2)\breve{a}_z, = -4\breve{a}_x + \breve{a}_y - 32\breve{a}_z$.

Constant Field Vectors; do not depend on space variables (x, y, z), e.g , $3\breve{a}_x - 2\breve{a}_y + 10\breve{a}_z$.

Non-Constant Field Vectors or Non-Uniform Field Vectors depend on space variables at different time, e.g, $x^2y\breve{a}_x + y^2\breve{a}_y + xz^2\breve{a}_z$

1.2 Vector Multiplication

When two vectors are multiplied, the result is either a scalar or a vector depending on how they are multiplied. That is either by Dot or Scalar product $\tilde{A} . \mathrm{B}$ for scalars on one hand or by Cross or Vector product $\tilde{A} \times \mathrm{B}$ for vectors on the other. Also multiplication of three vectors can result to Scalar Triple product $\tilde{A}. (\mathrm{B} \times \acute{C})$ or Vector Triple product $\tilde{A}\times (\mathrm{B} \times \acute{C})$.

1.2.1 Dot or Scalar Product

$\tilde{A} . \mathrm{B}$=(Ax$\breve{a}_x$ + Ay$\breve{a}_y$ + Az$\breve{a}_z$) . (Bx$\breve{a}_x$ + By$\breve{a}_y$ + Bz$\breve{a}_z$)

= Ax$Bx\breve{a}_x . \breve{a}_x$ +Ax By$\breve{a}_x . \breve{a}_y$ +Ax Bz$\breve{a}_x . \breve{a}_z$ +AyBx$\breve{a}_y . \breve{a}_x$+ AyBy$\breve{a}_y . \breve{a}_y$+AyBz$\breve{a}_y . \breve{a}_z$ + AzBx$\breve{a}_z . \breve{a}_x$+ AzBy $\breve{a}_z . \breve{a}_y$ + AzBz$\breve{a}_z . \breve{a}_z$

=AxBx +AyBy +AzBz, as angle between $\breve{a}_x . \breve{a}_x$ or $\breve{a}_y . \breve{a}_y$ or $\breve{a}_z . \breve{a}_z$ =0, cos o^o= 1

Also $\tilde{A} . \mathrm{B}$= $|\tilde{A}||\mathrm{B}|$ cos θ_{AB}.

Dot product obeys Commutative law, Distributive law and A.A = A^2.

1.2.2 Cross Product or Vector Product

$\tilde{A} \times \mathrm{B}$ = $|\tilde{A}||\mathrm{B}|$ sin θ_{AB}. $\breve{a}n$, where $\breve{a}n$ is a unit vector normal or perpendicular to the plane containing $\tilde{A}$ and B. The direction of $\breve{a}n$ is the direction of the right thumb when the fingers of the right hand rotate from $\tilde{A}$ to B. Alternatively, it is the direction of advancement of a right screw as $\tilde{A}$ is turned into B. Cross product is due to the cross-sign and because theresult is a vector, it is called vector product. $\tilde{A} \times \mathrm{B} = \begin{vmatrix} \breve{a}x & \breve{a}y & \breve{a}z \\ Ax & Ay & Az \\ Bx & By & Bz \end{vmatrix}$ = (AyBz-ByAz)$\breve{a}x$+ (AzBx-BzAx)$\breve{a}y$ +(AxBy-BxAy)$\breve{a}z$.

Properties of Cross-Product

(i)Anti-commutative, i.e. $\tilde{A} \times \mathrm{B} \neq \mathrm{B} \times \tilde{A}$, therefore, $\tilde{A} \times \mathrm{B} = -\mathrm{B} \times \tilde{A}$,

(ii) Not-Associative i.e. $\tilde{A} \times (\mathrm{B} \times \acute{C}) \neq (\tilde{A} \times \mathrm{B}) \times \acute{C}$.

(iii) Obeys Distributive law, $\tilde{A} \times (\mathrm{B} + \acute{C}) = (\tilde{A} \times \mathrm{B}) + (\tilde{A} \times \acute{C})$.

(iv) $\tilde{A} \times \tilde{A} = 0$, as $\breve{a}x \times \breve{a}y = \breve{a}z,\ \breve{a}y \times \breve{a}z = \breve{a}x,\ \breve{a}z \times \breve{a}x = \breve{a}y$

(v) $(\tilde{A} . \mathrm{B})\acute{C} \neq \tilde{A}\ (\mathrm{B} . \acute{C})$, but, $(\tilde{A} . \mathrm{B})\acute{C} = \acute{C}\ (\tilde{A} . \mathrm{B})$

1.2.3 Scalar Triple Product, $\tilde{A} . (\mathrm{B} \times \acute{C}) = \mathrm{B} . (\acute{C} \times \tilde{A}) = (\acute{C} . (\tilde{A} \times \mathrm{B})$.

1.2.4 Vector Triple Product: $\tilde{A} \times (\mathrm{B} \times \acute{C}) = \mathrm{B}\ (\tilde{A} . \acute{C}) - \acute{C}\ (\tilde{A} . \mathrm{B})$.

1.3 Different forms of Vectors and Their Transformations

1.3.1 Different Forms Of Vectors

Cartesian Cordinates $\tilde{A}$ = (x, y, z) =(Ax, Ay, Az), Ranges = $-\infty \leq \tilde{A} \leq \infty$

Cylindrical Cordinates $\tilde{A} = (\rho, \emptyset, z)$, = $(A\rho, A\emptyset, Az)$, Ranges, $-\infty \leq \rho, z \leq \infty$, while $0 \leq \emptyset < 2\pi$.

Spherical Cordinates: $\tilde{A} = (r, \theta, \emptyset) = (Ar, A\theta, A\emptyset)$, Ranges, $0 \leq r \leq \infty; 0 \leq \theta \leq \pi : 0 \leq \emptyset \leq 2\pi$.

1.3.2 Differential Element in Length, Area and Volume

Cartesian Cordinates,

(i) Differential Displacement, $dl = dx\breve{a}x + dy\breve{a}y + dz\breve{a}z$

(ii) Differential normal Area; $ds = dxdz\breve{a}y$, $= dydz\breve{a}x$, $= dxdy\breve{a}z$

(iii) Differential Volume (Scalar), dv= dxdydz

Cylindrical Cordinates

(i) $dl = d\rho\breve{a}_\rho + d\emptyset\breve{a}_\emptyset + dz\breve{a}_z$,

(ii) $ds = \rho d\emptyset dz\breve{a}_\rho$, $= d\rho dz\breve{a}_\emptyset$ $= \rho d\emptyset\, d\rho\breve{a}_z$

(iiI) dv = $\rho d\rho d\emptyset dz$,

The unit vectors, $\breve{a}_\rho$, $\breve{a}_\emptyset$ and $\breve{a}_z$, are mutually perpendicular

Spherical Cordinates, for degree of spherical Symmetry, where r is radius, θ is the co-latitude, while $\emptyset$ is azuthimal angle, Ranges, $0 \leq r \leq \infty$, $0 \leq \theta \leq \pi$, $0 \leq \emptyset \leq 2\pi$.

(i) $dl = dr\breve{a}_r + rd\theta\breve{a}_\theta + r\sin\theta d\emptyset\breve{a}_\emptyset$

(ii) $ds = r^2\sin\theta d\theta d\emptyset\breve{a}_r$, $= r\sin\theta dr d\emptyset\breve{a}_\theta$, $= rdrd\theta\breve{a}_\emptyset$.

(iii) $dv = r^2\sin\theta dr d\theta d\emptyset$.

The Unit Vectors, $\breve{a}_r, \breve{a}_\theta, \breve{a}_\emptyset$ are mutually orthorgonal.

1.3.3 Transformation Matrices

Cartesian to Cylindrical Cordinates:

$$\begin{bmatrix} A\rho \\ A\emptyset \\ Az \end{bmatrix} = \begin{bmatrix} \cos\emptyset & \sin\emptyset & 0 \\ -\sin\emptyset & \cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix}$$

Cylindrical to Cartesian Cordinates.

$$\begin{bmatrix} Ax \\ Ay \\ Az \end{bmatrix} = \begin{bmatrix} \cos\emptyset & -\sin\emptyset & 0 \\ \sin\emptyset & \cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A\rho \\ A\emptyset \\ Az \end{bmatrix}$$

Spherical to Cartesian Cordinates:

$$\begin{bmatrix} \breve{a}x \\ \breve{a}y \\ \breve{a}z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\emptyset & \cos\theta\cos\emptyset & -\sin\emptyset \\ \sin\theta\sin\emptyset & \cos\theta\sin\emptyset & \cos\emptyset \\ \cos\emptyset & -\sin\emptyset & 0 \end{bmatrix}$$

Cartesian to Spherical Cordinates.

$$\begin{bmatrix} \breve{a}r \\ \breve{a}\theta \\ \breve{a}\emptyset \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\emptyset & \sin\theta\sin\emptyset & \cos\emptyset \\ \cos\theta\cos\emptyset & \cos\theta\sin\emptyset & -\sin\emptyset \\ -\sin\emptyset & \cos\emptyset & 0 \end{bmatrix} \begin{bmatrix} \breve{a}x \\ \breve{a}y \\ \breve{a}z \end{bmatrix}$$

Note, the inverse of an orthogonal matrix is the transpose of that orthogonal matrix.

1.4 Distance between two points "d" is necessary in Electromagnetic Field theory and with position vectors, $\bar{r}_1$ and $\bar{r}_2$, then $d = |\bar{r}_2 - \bar{r}_1|$

In Cartesian Cordinate System, $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

In Spherical Cordinate, $d^2 = {r_2}^2 - {r_1}^2 - 2r_1r_2\cos\theta_2\cos\theta_1 - 2r_1r_2\sin\theta\sin\theta_1\cos(\emptyset_2 - \emptyset_1)$

1.5 Vector Calculus

Line Integral; $\int_L \tilde{A}.dl$ = line integral of vector Ã. If the path L of integration is closed, we have $\oint A.dl$ which is calculation of Ã around L.

Surface Integral; $\psi = \int \tilde{A}\cos\theta ds = \oint_s \tilde{A}.ds$. This is net outward flux of vector Ã from the surface s.

Note; a closed path defines an open surface while a closed surface defines a volume

Volume Integral; $\int_v \rho_v\, dv$, the volume integral of the scalar ρ_v over the volume V.

2.1 Fields

Stationary Charges constitutes— **Electrostatics**

Charges at uniform or steady velocity constitutes--- **Steady Current**

Accelerating charges which are sinusoidal pulses wrt time t constitutes ---**Electromagnetic Field**.

In the classical theory of electromagnetism the electric and magnetic fields are central concept. The space surrounding a charged-rod is Electric field and the space surrounding a bar magnet is magnetic field. Two charges close to each other, set up electric fields around themselves separately and each field acts on the other charge, which experiences a force F

Charge$\rightleftharpoons$ Field $\rightleftharpoons$ Charge.

The situation is symmetrical, each charge being immersed in the field associated with the other charge.

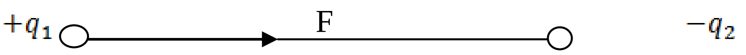
For an arbitrary point in space P, a given distribution of static charges is determined by Gauss's law for electricity as $\varepsilon_0 \oint E.ds = q$ addressed as product of electric constant to the surface integral of electric field.

Note; charge on an insulated conductor moves to its outer surface.

For an arbitrary point P in space, a given distribution of current is determined by Ampere's law for Magnetism as $\oint B.dl = \mu_0$ i, addressed as the closed line integral of magnetic field gives current within.

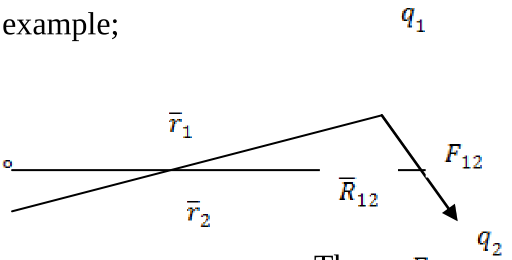
2.2 Coulomb's law and Electric Field Intensity.

Electric Force and Electric Field Intensity between two charges q_1 and q_2



F$\propto \frac{q_1 q_2}{R^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}$ Coulomb's Law

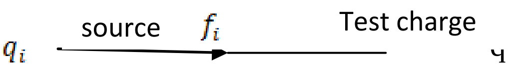
With position vector, where origin is important. If origin is zero (0), with the figure below, as example;



Then, $F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}\ \bar{a}R_{12}$, where $\bar{a}R_{12}$ is unit Vector= $\frac{\bar{R}_{12}}{R}$,

where,$\bar{R}_{12}=\overline{r2} - \overline{r1}$

R=$|R_{12}|$ $\therefore$, $F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}\frac{\bar{R}_{12}}{R} \Rightarrow, F_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2}\frac{\overline{r2}-\overline{r1}}{R} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^3}\overline{r2}-\overline{r1}$



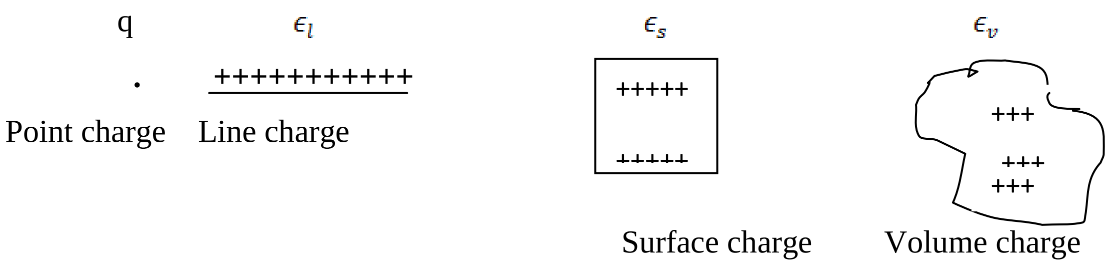
$F_{12} = \frac{q q_i}{4\pi\varepsilon_0 R^3}\overline{r2}-\overline{r1}$

Total Force $=\sum F_i$

$$= \frac{q}{4\pi\varepsilon_0}\sum\frac{q_i(\bar{r}-\bar{r_i})}{R^3},$$

Continuous Charge Distributions

So far we have only considered forces and electric fields due to point charges It is also possible to have continuous charge distribution along a line, on a surface or in a volume as in the figures below.



It is customary to denote the line charge density, surface charge density and the volume charge density by ϵ_l in C/m, ϵ_s in C/m^2 and ϵ_v in C/m^3 *respectively*. The electric field intensity due to each of the charge distribution may be regarded as the summation of the field contributions by the numerous point charges making up the charge distribution

2.2.1 Charge Densities; They are charges per unit volume: ϵ_l in C/m, ϵ_s in C/m^2 and ϵ_v in C/m^3.

2.2.2 With Charge Elements; with $\epsilon_l dl$, $\epsilon_s$ds and $\epsilon_v$dv and by replacing q with them in;

$$E = \frac{q}{4\pi\varepsilon_0}\frac{(r-r^1)}{|r-r^1|^3}$$

$$\epsilon = \int \frac{\epsilon_l \mathrm{dl},}{4\pi\varepsilon_0 R^2}\,\bar{a}R_{(line\ charge)},$$

$$\epsilon = \int \frac{\epsilon_s \mathrm{ds},}{4\pi\varepsilon_0 R^2}\,\bar{a}R_{(Surface\ charge)},$$

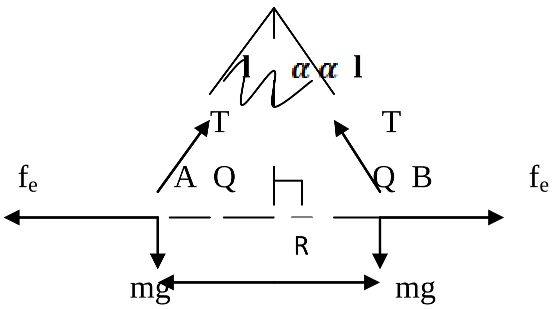
$$\epsilon = \int \frac{\epsilon_v \mathrm{dv},}{4\pi\varepsilon_0 R^2}\,\bar{a}R_{(volume\ charge)}$$

2.2.3 Coulomb's Law for Differential Electric Field on Differential Charge Element (dq); dE=$\frac{dq}{4\pi\varepsilon_0}\frac{\bar{r}}{R^3}$, where $\bar{r}$ is the displacement vector from the charge (dq) to the point P.

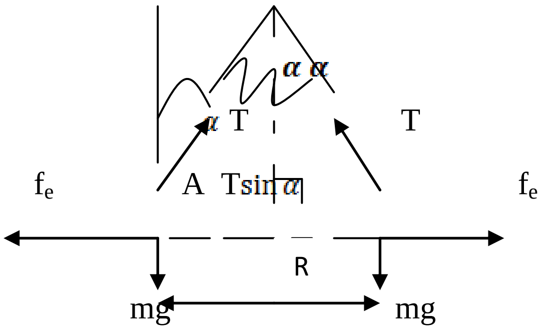
2.2.4 Bio-Savart Law for Differential Magnetic Field on Differential Current Element (di); dB=$\frac{\mu_0\, i}{4\pi}\frac{dI\times\bar{r}}{R^3}$

Exercise, Two points charges of equal mass and charge Q are suspended at a common point of negligible mass and lenght l. Show at equilibrium, the angle of inclination of each thread to the vertical is $Q^2 = 16\pi\varepsilon_0 \mathrm{mgl}^2(\sin\alpha)^2\tan\alpha$. If α is very small, show that $\alpha = \sqrt[3]{\frac{Q^2}{16\pi\varepsilon_0 \mathrm{mgl}^2}}$

Solutions.



Consider the system of charges as shown in the fig. where f_e is the coulomb force. T is the tension in the thread and mg is the weight of each charges at A and B.



$$T\sin\alpha = f_e = \frac{Q^2}{4\pi\varepsilon_0 R^2} = \frac{Q^2}{4\pi\varepsilon_0 4l^2(\sin\alpha)^2} \text{ ----(1)}$$

$$T\cos\alpha = mg\text{--------------------------------(2)}$$

$$(1)\div(2)\ \frac{T\ \sin\alpha}{T\ \cos\alpha} = \frac{Q^2}{16\pi\varepsilon_0 mgl^2(\sin\alpha)^2 \times mg}$$

$$\tan\alpha^{\cdot} = \frac{Q^2}{16\pi\varepsilon_0 mgl^2(\sin\alpha)^2}$$

$$Q^2 = 16\pi\varepsilon_0 mgl^2(\sin\alpha)^2\tan\alpha$$

When α is very small, $\sin\alpha \to \alpha$

$$\tan\alpha^{\cdot} \to \alpha$$

$$Q^2 = 16\pi\varepsilon_0 mgl^2\alpha^3$$

$$\therefore\ \alpha = \sqrt[3]{\frac{Q^2}{16\pi\varepsilon_0 mgl^2}}$$

Electric Flux Density (D)

It is also called **Electric Displacement** and it is measured in C/m^2. With Coulomb's Law for Electric Flux Linkage $\varphi = \int D.ds$, Relationship between D and E. D is obtained by multiplying E by ε_0 i.e D= ε_0E, $\therefore\ \varphi = \int D.ds = \varepsilon_0 \int E.ds$. [4].

With Gauss's Law, which is one of the fundamental laws of Electromagnetism that states that, "the total flux φ through any closed surface is equals the total charge enclosed by the surface", i.e $\varphi = q(enclosed)$, or $\varphi(total\ charge\ q\ enclosed) = \oint d\,\varphi = \oint_s\ D.ds = \int e_v dv$.

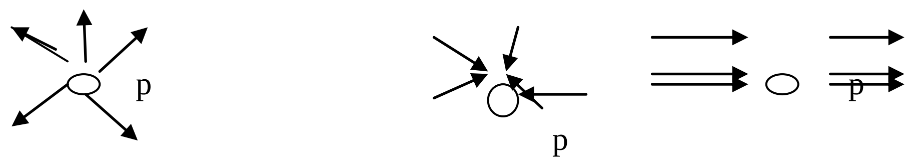
2.3.1 Applying Divergence Theorem, $\oint_s D.ds = \int_v (\nabla.\mathrm{D})\mathrm{dv} = \int e_v dv$, $\therefore$ $e_v = \nabla.D = \nabla.\varepsilon_0\mathrm{E}$,

The net outflow of the Flux of a vector field $\bar{A}$ from a closed surface is the integral $\oint \bar{A}$.ds, which is the divergence of $\bar{A}$ and which is the net outflow of flux per unit volume over a closed incremental surface. The net outflow of flux per unit volume decreases at incremental surface.

Hence, div. $\bar{A}$ =$\nabla.\bar{A} = \lim_{\Delta V\to 0} \oint \frac{\bar{A}\,\mathrm{ds}}{\Delta V}$.

Where Δ_v is the volume enclosed by the closed surface s in which P is located.

Physically we may required the divergence of the vector field $\bar{A}$ at a point P as a measure of how much the field diverges or emerges from that point.



(a) +ve divergence (b) –ve divergence (c) zero divergence

In Cartesian Cordinates, Divergence of a vector$\bar{A}$ is $\nabla.\bar{A} = \frac{\partial \bar{A}\,\mathrm{x}}{\partial x} + \frac{\partial \bar{A}\,\mathrm{y}}{\partial y} + \frac{\partial \bar{A}\,\mathrm{z}}{\partial z}$.

In Cylindrical Cordinates is $\nabla.\bar{A} = \frac{1}{\rho}\frac{\partial(\overline{\rho A\rho})}{\partial \rho} + \frac{1}{\rho}\frac{\partial \bar{A}\emptyset}{\partial \emptyset} + \frac{\partial \bar{A}\,\mathrm{z}}{\partial z}$.

In Spherical Cordinates is $\nabla.\bar{A} = \frac{1}{r^2}\frac{\partial(r^2\bar{A}r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\bar{A}\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial \bar{A}\,\emptyset}{\partial\emptyset}$,

Div. $\bar{A}$=$\nabla.\bar{A} = \oint_s \bar{A}.ds$ =$\int_v \nabla.\bar{A}$ dv-------Divergence Theorem or Gauss Theorem.

2.3.2 CURL of A Vector and Stoke's Theorem

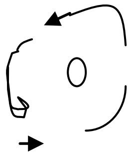
It is the maximum circulation of a vector field $\bar{A}$ around a closed path L, defined as $\oint_L \bar{A}.dl$. Its magnitude is maximum per unit area as the area tends to zero. $\therefore CurlA\bar{} = \nabla\times\bar{A}$ =$\lim_{\Delta s\to 0}\frac{\oint \bar{A}.dl\bar{a}_x}{\Delta s}$, where the surface is bounded by the curl L.

For Cartesian Cordinates; $\nabla\times\bar{A}$= $\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ Ax & Ay & Az \end{vmatrix}$

For Cylindrical Cordinates; $\nabla\times\bar{A}$= $\frac{1}{\rho}\begin{vmatrix} \bar{a}_\rho & \rho\bar{a}_\emptyset & \bar{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\emptyset} & \frac{\partial}{\partial z} \\ A\rho & A\emptyset & Az \end{vmatrix}$

For SphericalCordinates; $\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\,\bar{a}_\emptyset \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\emptyset} \\ Ar & ArA\theta & r\sin\theta\, A\emptyset \end{vmatrix}$

$\oint_L \bar{A}.dl = \int(\nabla \times \bar{A}).ds$ Stoke's Theorem.



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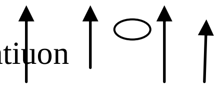
zero circulation  p

2.3.3 Laplacian of A Scalar; Laplacian of a Scalar field V, written as$\nabla^2 V$ is the divergence of the gradient of V. $\nabla.\nabla V \Longrightarrow \nabla^2 V$.

In Cartesian Cordinates; $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$,

In Cylindrical Cordinates; $\nabla^2 V = \frac{1}{\rho_v}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial\rho}\right] + \frac{1}{\rho^2}\frac{\partial}{\partial\theta}\frac{\partial^2 V}{\partial\emptyset^2} + \frac{\partial^2 V}{\partial z^2}$,

In Spherical Cordinates; $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial V}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\frac{\partial V}{\sin\theta\,\partial\theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial\emptyset^2}$,

$\nabla^2 V = \frac{-\varepsilon_v}{\epsilon}$ → Poisson's Equation

$\nabla^2 V = 0$ → Laplace's Equation

2.3.4 Applications of Gauss, s Law for calculation of electric field, symmetry must exist and a mathematical close surface of the symmetrical known as Gaussian Surface is obtained. If Electric Displacement D is normal to this surface $\bar{D}.\,ds = Dds$. While if it is tangential to the surface $\bar{D}.\,ds = 0$.

We learned how to calculate E and B at an arbitrary point P in space for any given distribution of static charges and currents. We use Gauss's law and Ampere,s law to calculate the above respectively.

Gauss's law for electricity, $\varepsilon_0 \oint E.ds = q$

For charge and electric field.

Experiment shows, like charges repel and unlike charges attract, as the inverse square of their separation. A charge on an insulated conductor moves to its outer surface.

Gauss's law for Magnetism, $\oint B.ds = 0$ (the Magnetic Field)

It is impossible to create an isolated magnetic pole.

2.4 Faraday's Law and Maxwell Extension of Ampere's Law

Whenever change occurs on charge source, electric field E at a point P learns about it through a field disturbance that moves out of the source with a speed "C" (speed of light) and so the field changes. As the field is changing, other fields are generated and these changes are described by Faraday's Law; E=cB; $\epsilon = \frac{-d\emptyset_B}{dt}$ also $\frac{-d\emptyset_B}{dt} = \oint E.dl$. i.e change in magnetic flux $\emptyset_B$ with time produces induced e.m.f or the electric effect of the changing magnetic field [6] Experiment shows that any bar magnet thrust through a closed loop of wire will set up a current in the loop. Also changing electric field may be produced by charging a parallel-plate capacitor.

To Express E and B in the equivalent Coulomb and Biot-Savart forms.

Using vector notation and considering only differential charge and current elements.

Thus, dE= $\frac{\mathrm{d}_q\mathrm{r}}{4\pi\varepsilon_0 r^3}$ and

dB=$\frac{\mu_0}{4\pi}\frac{i\ dl\times\bar{r}}{r^3}$

Where r is the displacement vector from the charge or current element to the point P. at which we wish to calculate dE and dB

Maxwell Extension of Ampere's Law

$\oint B.dl = \mu_0\varepsilon_0\frac{d\emptyset_E}{dt}$ i.e change in electric flux$\emptyset_E$ with time produces magnetic field.

The magnetic flux $\emptyset_B = \oint B.ds$.

Experiment shows that, if the lines of E are counterclockwise, those of B are clockwise and this difference requires that the minus sign is omitted in production of E.

Maxwell extension; $\oint B.dl = \mu_0\varepsilon_0\frac{d\emptyset_E}{dt} + \mu_0$I, shows that at least two ways of setting up a magnetic field: (a) by a changing electric flux and (b) by a current. If there is no changing electric flux, then B is only, the concentric circles around a wire carrying current, or, no conduction currents, then B is only from changing electric flux.

]Magnetic Field B is also produced whenever a sideways deflecting force act on a positive test charge q_0 with a velocity v through a point P i.e B=$\frac{F}{q_0 V}$ or F=$q_0 V \times B$

In another way, if a test charge q_0 is moving with a velocity v through a point and if a sideways force act on the moving in charge, a magnetic field B is present at P. The magnitude of the magnetic deflecting force f is according to the rules of vector product or cross product. F=$q_0 VB \sin\theta$, where, is the angle between v and B and if $\theta = 90^{\circ}$, F=$q_0 VB$,

Electric Field E is produced whenever a test charge q_0 is placed at a point P and if an Electric force F acts on the stationary charge, an electric field is present at P. F=q_0E, direction of E is that of F.

Unit of Magnetic Field vector is Weber per square metre(Wb/m^2) or Tesla T or Gauss G, 1T=1wbm^{-2}=10^4Gauss, unit of magnetic flux $\emptyset_B$ is weber. The fact that magnetic force is always at right angle to the motion of charge, a steady magnetic field does not work on the particle and so cannot change the kinetic energy of a moving charge, it can only deflect it sideways.

2.4.1 Lorentz Relation, whenever a charged particle moves through a region in which both electric and magnetic fields exist, the resultant force is the combination of the equations; F=q_0E+$q_0 V \times B$.

2.5 Magnetic Force on a Current; a current is an assembly of moving charges. Because Magnetic Field B exerts a sideways force on a moving charge, it is expected that it will exert a sideways force on a wire carrying-current. If a wire carrying-current is placed at right angle to B, the current density vector "j" is at right angle to B and the current in the metal wire is carried by the free (conduction) electrons, "n" being the number of such electrons per unit volume of the wire. The magnitude of average force on one such electron is, F^1=$q_0 VB \sin\theta$ $\Rightarrow q_0 V \times B$ = e$V_d B(\theta = 90^{\circ})$, V_d is drift velocity or speed and it is $\frac{j}{ne}$.

$\therefore F^1 = e\left(\frac{j}{ne}\right)B = \frac{jB}{n}$. The length L of the wire contains "nAL" free electrons, AL, being the volume of the section of the wire. Thus, the total force on the free electrons in the wire is F= (nAL)F^1= nAL$\frac{jB}{n}$, where jA is the current i. F = iLB =$q_0 V \times B$. If negative charges move to the right in the wire, an equivalent positive charge will move to the left and that is the direction of the current. For such a positive charge, the velocity v would point to the left and the force on the wire by the wire, given by the equation. F = $q_0 V \times B$ points outwardly from the page. Thus, considering sideways magnetic force on a wire carrying a current placed in a magnetic field, we cannot tell whether the current-carriers are negative charges moving in a given direction, or positive charges moving in the opposite direction. To determine the direction of B near a wire carrying a current, Right Hand Grip Rule is used, where, the Thumb points to current direction and the curled fingers around the wire as direction of B. B $\propto \frac{i}{r}$.

Ampere's Law, deals with the production of magnetic field by a current-carrying conductor, or by a moving charge; B $\propto \frac{i}{r}$. B $= \frac{\mu_0 \text{i}}{2\pi r}$. Also, B. $(2\pi r) = \mu_0$I, $\oint B.dl = \oint Bdl = B\oint dl = B\,(2\pi r) = \mu_0$i

$\oint B.dl = \mu_0$I, is true, as long as time-varying electric fields $\frac{d\emptyset_E}{dt}$, is absent for every magnetic field.

Faraday's Law; $\epsilon = \frac{-d\emptyset_B}{dt}$,

For N turns of a coil $\epsilon = \frac{-Nd\emptyset_B}{dt} = \frac{-d(N\emptyset_B)}{dt}$, N$\emptyset_B$ is the flux-linkage in the device. The flux enclosed by loop lx is $\emptyset_B$ = Blx, where lx is the area of that part of the loop in which B is not zero.

$\epsilon = \frac{-d\emptyset_B}{dt} = \frac{-d(Blx)}{dt} = \frac{-Bl(dx)}{dt}$ = -B lv.

i $= \frac{-Blv}{R}$.

2.6 Wave Pressure and Momentum

Whenever charges are accelerated, electromagnetic wave radiates from the source with a speed c. This wave can transfer energy and momentum to gross objects placed at its path.

Wave of this at a large distance is known as **Radiated Field** at the distant point and it is proportional to $\frac{1}{r}$.

But close to dipole, we cannot neglect $\frac{1}{r^2}$ and $\frac{1}{r^3}$ and at this point we called it **Near Field.**

At Radiated Field, rate of energy transported in electromagnetism is measured in watt/metre-square and it is simply called **Pointing Vector S.** S $= \frac{1}{\mu_0}\,(E \times B)$, c= $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ or $1 = c^2\mu_0\varepsilon_0$.

Example, An observer is 1.0m from a point light source, whose power output P_o is 1.0×10^3 watts. Calculate the magnitudes of the electric and the magnetic fields. Assume that the source is monochromatic, that it radiates uniformly in all directions and that at distant points, it behaves like the travelling plane wave.

Power that passes through a sphere of radius r is $(,S^1)$ $(4\pi r^2)$where S^1 is the average value of pointing vector at the surface of the sphere. This power must equal P_o, or $S^1 4\pi r^2$
Energy Stored = Pointing Vector × Area×Time.

Average Pointing Vector $S^1 = \frac{1}{2\mu_0}(E_m \times B_m)$, as E_m *and* B_m vary sinusoidally along propagation. To eliminate B leads to $S^1 = \frac{1}{\mu_0 c}E^2$, from E =cB. The average value of E^2 over one cycle is ½ E^2, since E varies sinusoidally. This leads to

$P_o = (\frac{1}{\mu_0 c}E^2)(4\pi r^2)$ or $E_m = \frac{1}{r}\sqrt{\frac{Po\ \mu_0 c}{2\pi}}$

$$E_m = \frac{1}{1.0}\sqrt{\frac{(1\times10^3)\left(4\pi\times\frac{10^7 wb}{Am}\right)(3.0\times10^8\frac{m}{s})}{2\pi}}$$

E_m = 240 Volts/metre

The relationship $E_m = cB_m$ leads to,

$B_m = E_m/c = \frac{240}{3\times10^8} = 8\times 10^{-7}$ weber/$metre^2$

E_m *is* appreciable by ordinary laboratory standard but B_m is quite small.

2.6.1 Momentum; as electromagnetic wave transports linear momentum and exert pressure called Radiation Pressure, Momentum (ρ) = $\frac{Energy\ (U)}{speed\ of\ light\ (c)}$, [7]

In a reflecting edge, the total ρ after total reflection is $\rho = \frac{2U}{c}$.

If energy is partly reflected and partly absorbed momentum (ρ) is between $\frac{U}{c}$ and $\frac{2U}{c}$.

If total absorption (ρ) = $\frac{U}{c}$. Where c is the speed of light.

The direction of ρ is the direction of incident beam.

In the same way, twice as much momentum is delivered to a heavy object when a light but perfectly elastic ball is bounced from it as when it is struck by a perfectly inelastic ball of the same mass and speed. If the energy μ is partly bounced back and partly absorbed, the delivered momentum will lie between $\frac{U}{c}$ and $\frac{2U}{c}$.

Example, A parallel beam of light with an energy flux S of 10 watts/cm^2 falls for 1.0 hr on a perfectly reflecting plane mirror of 1.0 cm^2 area,

(a) What momentum is delivered to the mirror in this time and

(b) What force acts on the mirror?

(a)The energy that falls on the mirror is,

μ = (10 watt/cm^2)(1.0cm^2)(3600s) = 3.6× 10^4J

The momentum delivered after 1.0 hr illumination is,

$\rho = \frac{2U}{c} = \frac{(2\times3.6\times10^4)J}{3.0\times10^8 m/s}$ = 2.4× 10^4kgm/s or 2.4× 10^{-4} Ns.

(b)From Newtons 2nd law, the average force on the mirror is equals the average rate at which momentum is delivered to the mirror or

F = $\frac{\rho}{t} = \frac{2.4\times10^{-4}}{3600}$ =6.7 × 10^{-8}N or Kgm/s^2

Summary

The in depth study of this simplified version of electromagnetic fields and waves theories will in no small way help students aspiring into obtaining higher knowledge in this area of Engineering to achieve it with ease.

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