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FORMATIONS OF VERSIONS OF SOME DYNAMIC INEQUALITIES

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ABSTRACT

The study of dynamic equation on measure chain (time scale) goes back to its founder S. Hilger (1988) (Hilger 1988) and is a new area of still fairly theoretical exploration in mathematics. Motivating the subject is the ration that dynamic equation on measure chains can build bridge between continuous and discrete mathematics. It has been created in order to unify the study of differential and difference equations. We also present various properties / several example and application. The study of dynamic inequalities has received a lot of attention in the literature and has become a major field in pure and applied mathematics. In this article we mainly focused on Randons's Inequality, GronWall's Inequality, AM-GM Inequality, Lyapunov's Inequality, Antiderivative and integral and Nesbitts inequality via time scale respectively.

KEYWORDS: Time scale calculus, Dynamic inequalities, Nabla calculus and derivatives, Radon's Inequality, AM-GM Inequality, Lyapunov's Inequality, Antiderivatives.

INTRODUCTION

The time scale calculus has a scope for many applications in the field of dynamic inequalities. The time scale calculus was initiated by Stefan Hilger (Hilger 1988) for the sake of creation a theory which has the ability to unify continuous and discrete analysis. A time scale is a random nonempty closed subset of the real numbers. Thus, $\mathbb{R}$, $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{N}_0$ real numbers, integers, natural numbers and non-negative numbers respectively are the examples of time scales, where

$$[0,1] \cup [2,3],\ [0,1] \cup \mathbb{N}, \text{ are Cantor set,}$$

While,

$$\mathbb{Q},\ \mathbb{R} \setminus \mathbb{Q},\ \mathbb{C},\ (0,1).$$

For applying Stefan Hilger theory in this paper we will introduce the delta derivative f^{Δ} for f function defined on time scale T, that defines that (i) f^{Δ} = f' is the normal general derivative if T = ℝ and (ii) f^{Δ} = Δf is the general forward difference operator if T = ℤ.

The time scale calculus is studied as delta calculus, nabla calculus and diamond-α calculus. Basic work on dynamic inequalities is done by Ravi Agar- wal, George Anastassiou, Martin Bohner, Allan Peterson, Donal O'Regan, Samir Saker and many other authors. We will prove the following results given in theorems. Some classical inequalities such as Rado's, Bergstroms's, the weighted power mean, Schlorich's and Nesbitt's inequality.

Dynamic equations on time scale: In order to hybridize continuous and discrete analysis (Hoffacker and Tisdell 2005): Stability or instability of dynamic equation scale.

Inequalities on time scale: Bohner worked on opial inequalities (Bohner and Peterson 2001). Certain new dynamic inequalities investigated by Li (Li 2006).

Such like Radon's Inequality (Radon 1913) given below, we will describe and analyze different dynamic inequalities based on time scale T.

If $x_k, a_k > 0$ $k \in \{1,2.\dots n\} P > 0$ then,

$$\frac{x_1^{P+1}}{a_1^P} + \frac{x_2^{P+1}}{a_2^P} + \dots + \frac{x_n^{P+1}}{a_n^P} \geq \frac{(x_1+x_2+\dots+x_n)^{P+1}}{(a_1+a_2+\dots+a_n)^P}$$

$$\Rightarrow \frac{x_1}{a_1} = \frac{x_2}{a_2} = \dots = \frac{x_n}{a_n}$$

For $P = 1$, Inequality becomes that of Bergstrom.

If $a, b, c, d \in (0, \infty)$ and $abcd = 1$

$$\frac{(a+b+c)^5}{(b+c+d)^4} + \frac{(b+c+d)^5}{(c+d+a)^4} + \frac{(c+d+a)^5}{(d+a+b)^4} + \frac{(d+a+b)^5}{(a+b+c)^4} \geq 12$$

$$\geq \frac{[3(d+a+b)]^3}{[3(a+b+c)]^4} = 3(a+b+c+d) \geq 3.4(abcd)^{\frac{1}{4}} \geq 12$$

If $a_1, a_2, \dots a_n$ are non-negative and real numbers and $b_1, b_2, \dots b_n$ are positive and real numbers, then for $r \geq 0, S \geq 0$ and $r \geq S+1$

$$\frac{a_1^r}{b_1^S} + \frac{a_2^r}{b_2^S} + \dots + \frac{a_n^r}{b_n^S} \geq \frac{(a_1+a_2+\dots+a_n)^r}{n^{r-S-1}(b_1+b_2+\dots+b_n)^S}$$

By random inequality we have

$$\sum_{k=1}^{n} \frac{a_k^n}{b_k^S} = \sum_{k=1}^{n} \frac{\left(a_k^{\frac{r}{S=1}}\right)^{S+1}}{b_k^S} \geq \frac{\left(a_1^{\frac{r}{S=1}}+a_2^{\frac{r}{S=1}}+\dots+a_n^{\frac{r}{S=1}}\right)}{(b_1+b_2+\dots+b_n)^S}$$

$r \geq S+1 \geq 1$ then $\frac{r}{S+1} - 1 \geq 0$ using Radon's,

$$\sum_{k=1}^{n} a_k^{\frac{r}{S+1}} = \sum_{k=1}^{n} \frac{a_k^{\frac{r}{S+1}}}{1^{\frac{r}{S+1}-1}} \geq \frac{(a_1+a_2+\dots+a_n)^{\frac{r}{S+1}}}{(1+1+\dots+1)^{\frac{r}{S+1}-1}}$$

In this paper, it is supposed that all considerable integrals be present and are finite and T is a time scale, a, b ∈ T with a < b and an interval $[a, b]_T$ means the intersection of a real interval with the given time scale.

RESULT AND DISCUSSION

Radon's Inequality via time scales.

Theorem:

Let w, f, g ∈ C ([a,b] T,R) be -integrable functions, where w(x), g(x) ≠ 0, ∀x ∈ [a,b] T. If β ≥ γ ≥ 0, then □□

$$\frac{\left(\int_a^b |w(x)|\, \alpha^{\Diamond} x\right)^{\beta+1}}{\left(\int_a^b |w(x)||g(x)|\, \alpha^{\Diamond} x\right)^{\gamma}} \leq \int_a^b \frac{|w(x)||f(x)|^{\beta+1}}{|g(x)|^{\gamma}}\, \alpha^{\Diamond} x \qquad (1.0)$$

Equality present in (1.0) when f(x) ≡ g(x) ≡ c, while c is a nonzero real constant.

Let w, f, g ∈ C ([a,b] T, R) be $\alpha^{\Diamond}$ -integrable functions, where w(x), g(x) ≠ 0, ∀x ∈ $[a, b]_T$. If β ≥ 0, then

$$\frac{\left(\int_a^b |w(x)||f(x)|\, \alpha^{\Diamond} x\right)^{\beta+1}}{\left(\int_a^b |w(x)||g(x)|\, \alpha^{\Diamond} x\right)^{\beta}} \leq \int_a^b \frac{|w(x)||f(x)|^{\beta+1}}{|g(x)|^{\beta}}\, \alpha^{\Diamond} x \qquad (1.1)$$

Proof: If we put β = γ in (1.0), then we get (1.2), which is Radon's Inequality on dynamic time scales. Clearly the equality holds in (1.0), if f(x) = cg(x), where c is a real constant.

Corollary: Let w, f, g ∈ C ([a, b] T, ℝ − {0}) be $\alpha^{\Diamond}$ -integrable functions.

If β ≤ −1, then

$$\frac{\left(\int_a^b |w(x)||f(x)|\,\alpha^{\Diamond}x\right)^{\beta+1}}{\left(\int_a^b |w(x)||g(x)|\,\alpha^{\Diamond}x\right)^{\beta}} \leq \int_a^b \frac{|w(x)||f(x)|^{\beta+1}}{|g(x)|^{\beta}}\,\alpha^{\Diamond}x \qquad (1.2)$$

Equality holds in (1.2), when f(x) = cg(x), where c is a nonzero real constant.

Proof: By applying inequality (1.1) for $\beta \leq -1$, we obtain

$$\int_a^b \frac{|w(x)||f(x)|^{\beta+1}}{|g(x)|^{\beta}}\,\alpha^{\Diamond}x = \int_a^b \frac{|w(x)||g(x)|^{-\beta}}{|f(x)|^{-\beta-1}}\,\alpha^{\Diamond}x \geq \frac{\left(\int_a^b |w(x)||g(x)|\,\alpha^{\Diamond}x\right)^{-\beta}}{\int_a^b |w(x)||f(x)|\,\alpha^{\Diamond}x)^{-\beta-1}}$$

From above expression the equality holds in (1.2), if f(x) = cg(x), where c is a nonzero real constant.

Here we are presenting a generalized Nesbitt's Inequality on the base of dynamic scale calculus.

Theorem: Suppose w, f ∈ C([a, b]T, R − {0}) be $\alpha^{\Diamond}$ -integrable functions, c, d ∈ ℝ and

$$c\int_a^b |w(x)||f(x)|\,\alpha^{\Diamond}x - d|f(x)| > 0$$

Where x ∈ [a, b]T.

If $\beta \geq \gamma \geq 0$, then

$$\left(\int_a^b |w(x)|\,\alpha^{\Diamond}x\right)^{2\gamma-\beta} \frac{\left(\int_a^b |w(x)||f(x)|\,\alpha^{\Diamond}x\right)^{\beta-2\gamma+1}}{\left(c\int_a^b |w(x)|\,\alpha^{\Diamond}x - d\right)^{\gamma}} \leq \int_a^b \frac{|w(x)||f(x)|^{\beta-\gamma+1}}{\left(c\int_a^b |w(x)||f(x)|\,\alpha^{\Diamond}x - d|f(x)|\right)}\alpha^{\Diamond}x \qquad (1.3)$$

Remark 1. If we set α = 1, T = ℤ, w(x) = 1, β = γ = 1 and f(k) = xk ∈ (0, ∞) for k ∈ {1, 2, ..., n}, n ∈ ℕ − {1}, then discrete version of (1.3) reduces to

$$\frac{n}{cn-d} \leq \sum_{k=1}^{n} \frac{x_k}{cX_n - dx_k}, \qquad \text{where } X_n = x_1+x_2+\ldots+x_n \qquad (1.4)$$

Inequality (1.4) is called generalized Nesbitt's Inequality (Batinetu-Giurgiu, Marghidanu et al. 2011). Further if we set n = 3 and c = d, where c, d ∈ (0, ∞), then (1.4) takes the form

$$\frac{3}{2} \leq \frac{x_1}{x_2+x_3} + \frac{x_2}{x_3+x_1} + \frac{x_3}{x_1+x_2}, \qquad \text{where } x1, x2, x3 > 0 \qquad (1.5)$$

Expression (1.5) is known as Nesbitt's Inequality (Sahir 2017).

Gronwall's Inequality

Grönwall's inequality work on the principle of satisfy a definite differential or integral inequality from corresponding differential or integral equation solutions (Ozgün, Zafer et al. 1995).

Theorem: let y, f ∈ Crd and p ∈ R+. then,

$$y^{\Delta}(t) \le p(t)y(t) + f(t) \text{ for all } t \in T \text{ implies}$$

$$y(t) \le y(t_0)e_p(t,t_0) + \int_{t_0}^{t} e_p(t,\sigma(\tau))f(\tau)\Delta\tau \qquad \text{for all } t \in T$$

Proof: By using product rule and theorem we calculate,

$$[ye_{\theta p}(t,t_0)]^{\Delta}(t) = y^{\Delta}(t)e_{\theta p}(\sigma(t),(t_0) + y(t)(\theta p)(t)e_{\theta p}(t,t_0)$$

$$= y^{\Delta}(t)e_{\theta p}(\sigma(t),(t_0) + y(t)\frac{(\theta p)(t)}{1+\mu(t)(\theta p)(t)}e_{\theta p}(\sigma(t),t_0)$$

$$= [y^{\Delta}(t) - (\theta(\theta p))(t)y(t)]e_{\theta p}(\sigma(t),t_0)$$

$$= [y^{\Delta}(t) - p(t)y(t)]e_{\theta p}(\sigma(t),t_0)$$

Since p ∈ R+ and here we have θp ∈ R+, this is implement by eθp > 0.

Grönwall's inequality general expression = $u(t) \le \alpha(t) + \int_{[a,t)} \alpha(s)\exp\big(\mu(I_{s,t})\big)\mu(ds) \quad (1.6)$

Remarks: (i) On the functions α and u there are no continuity assumptions. (ii) The integral in Grönwall's inequality is allowed to give the value infinity. (iii) If α is the zero function and u is non-negative, then Grönwall's inequality implies that u is the zero function. (iv) The integrality of u with respect to μ is essential for the result. For a counterexample, let μ denote Lebesgue measure on the unit interval [0, 1], define u(0) = 0 and u(t) = 1/t for t ∈ (0, 1], and let α be the zero function (Ethier and Kurtz 2009).

AM-GM Inequality

If $a1$,$a2$,.....an are non-negative and real numbers and $\lambda1$,$\lambda2$.....λn are non-negative and real numbers such that $\lambda1$+$\lambda2$+.....λn=1

$$\prod_{k=1}^{n} a_k^{\lambda k} + \prod_{k=1}^{n} b_k^{\lambda k} \le \prod_{k=1}^{n} (a_k + b_k)^{\lambda k}$$

Weighted AM-GM inequality

$$\prod_{k=1}^{n} \left(\frac{ak}{ak+bk}\right)^{\lambda_k} \le \sum_{k=1}^{n} \lambda_k \left(\frac{ak}{ak+bk}\right)$$

Similarly

$$\prod_{k=1}^{n}\left(\frac{ak}{ak+bk}\right)^{\lambda_k} \leq \sum_{k=1}^{n}\lambda_k\left(\frac{ak}{ak+bk}\right)$$

Summing up

$$\prod_{k=1}^{n}\left(\frac{1}{(ak+bk)^{\lambda_k}}\right)\left[\prod_{k=1}^{n} a_k^{\lambda_k} + \prod_{k=1}^{n} b_k^{\lambda_k}\right] \leq \sum_{k=1}^{n}\lambda_k = 1$$

If a,b,c are the lengths of sides of a triangle and

$2s=a+b+c$

$$\frac{a^n}{b+c}+\frac{b^n}{c+a}+\frac{c^n}{a+b} \geq \left(\frac{2}{3}\right)^{n-2} S^{n-1}, n \geq 1$$

When $n=1$, result equal to Nesbitt Inequality for $n\geq 2$

$$\frac{a^n}{b+c}+\frac{b^n}{c+a}+\frac{c^n}{a+b} \geq \frac{(a+b+c)^n}{3^{n-1-1}(b+c+c+a+a+b}$$
$$= \left(\frac{2}{3}\right)^{n-2} S^{n-1}$$

Lyapunov's Inequality

Lyapunov inequalities have proved to be beneficial tools in oscillation theory, dis-conjugacy, eigenvalue problems and many other applications in the theory of differential and difference equations. A amusing summary of continuous and discrete Lyapunov inequalities and their applications can be found in the survey paper (Cheng 1991) by Chen. In this section we present several versions of Lyapunov inequalities on time scales. The results below are contained in (Bohner, Clark et al. 2002).

If $xk>0$

$yk>0$

$k=1,2.....n$

$0<\beta 1<\beta 2<\beta 3<\infty$ then,

$$\left(\sum_{k=1}^{n} x_k y_k^{\beta_2}\right)^{\beta_3-\beta_1} \leq \left(\sum_{k=1}^{n} x_k y_k^{\beta_1}\right)^{\beta_3-\beta_2}\left(\sum_{k=1}^{n} x_k y_k^{\beta_3}\right)^{\beta_2-\beta_1}$$

If $n\in N$

$xk\geq 0$

$xk\geq 0$

$\beta\geq 0$

and $\gamma \geq 0$

$$\frac{\left(\sum_{k=1}^{n} x_k y_k^{\gamma-1}\right)^{\beta+\gamma}}{\left(\sum_{k=1}^{n} y_k^{\lambda}\right)^{\beta+\gamma-1}} \leq \sum_{k=1}^{n} \frac{x_k^{\beta+\gamma}}{y_k^{\beta}}$$

If and only if $\frac{x_1}{y_1} = \frac{x_2}{y_2} = \ldots . \frac{x_n}{y_n}$

Thus (P) → real number, (ℤ Integer), (N natural number), (No non-negative integer) are example of time scale.

ℚ rational number, ℝ/ ℚ irrational number, ℂ complex number and the open interval between 0 and 1 are not time scales.

$f\Delta$ derivative with f defined on T

i $f\Delta = f'$ is the usual derivative if T = ℝ

ii $f\Delta = \Delta f$ is the usual forward difference operator if T = ℤ

Definition: Let π be a time scale for the mapping t□ □

$\sigma, p: \pi \rightarrow T$ such that

$$\begin{bmatrix} 6(t) = \inf(S \in T : S > T)\, and \\ P(t) = \sup(S \in \pi : S < t) \end{bmatrix} \text{Forward and backward jump operator}$$

If T has max t then

Put: - inf ∅ = Sup T (6(t)) = t

If T has minimum t then

Sup ∅ = inf T (i.e, ρ(t) = t ∴ ∅ null set

If 6(t) >t right scattered

at same tine one scattered

If ρ(t) < t left scattered

If t < Sup T and 6(t) = t right dense

at same tine dense

t > inf T and P(t) = t left dense

let TK = T(m) → if T right scattered with minimum m; otherwise TK = T. if T has a left scattered maximum M, then TK = T (M) else TK = T ∴ T = time scale throughout article.

Discussed jump operators help us in classification the points (Bohner and Peterson) of a time-scale as left-scattered and right-dense depending on different conditions such as σ(t) = (t) > t, ρ(t) and ρ(t) < t, respectively for any of t ∈ $\mathbb{T}$ as shown in table 1 and figure 1.

Table 1: Points classification according to above scenario.

t right-scattered	T < 6(t)
t right-dense	T = 6(t)
t left-scattered	ρ(t) < t
t left-dense	ρ(t) = t
t isolated	ρ(t) < t < ∞ (t)

Source: (Bohner and Peterson 2001)

Example: let $T=\left\{\sqrt{2n+1}:n\in N\right\}$

$$\text{if } t=\sqrt{2n+1} \text{ for some } n\in N,\text{ then } n=\frac{t^2-1}{2} \text{ and}$$

$$\sigma(t)=\inf\left\{l\in N:\sqrt{2l+1}>\sqrt{2n+1}\right\}=\sqrt{2n+3}=\sqrt{t^2+2} \qquad \text{for } n\in N,$$

$$\rho(t)=\sup\left\{l\in N:\sqrt{2l+1}<\sqrt{2n+1}\right\}=\sqrt{2n-1}=\sqrt{t^2-2} \qquad \text{for } n\in N,\ n\geq 2$$

Where N is natural numbers (ℕ)

For n =1 we have,

$$\rho(\sqrt{3})=\sup\varnothing=\inf T=\sqrt{3}$$

Since,

$$\sqrt{t^2-2}<t<\sqrt{t^2+2} \qquad \text{for } n\leq 2,$$

We determined that every point $\sqrt{2n+1}$, n ∈ ℕ n ≥ 2, is right-scattered and let-scattered, i.e, every point $\sqrt{2n+1}$, n ∈ ℕ n ≥ 2, is isolated because,

$$\sqrt{3}=\rho\sqrt{3}<\sigma(\sqrt{3})=\sqrt{5}$$

The point $\sqrt{3}$ is right-scattered.

Example:

Let $T=\left\{\frac{1}{2n}:n\in N\right\}\cup\{0\}$ and t ∈ T be arbitrarily chosen.

1- $t=\frac{1}{2}$ then,

1.1- Backward and Forward jump-operators, Graininess function,

$$\sigma\left(\frac{1}{2}\right)=\inf\left\{\frac{1}{2l},0:\frac{1}{2l},0>\frac{1}{2},l\in N\right\}=\inf\varnothing=\sup T=\frac{1}{2}$$
$$\rho\left(\frac{1}{2}\right)=\sup\left\{\frac{1}{2l},0:\frac{1}{2l},0<\frac{1}{2},l\in N\right\}=0<\frac{1}{2}$$

i.e., $\frac{1}{2}$ is left-scattered.

2- $t=\dfrac{1}{2n},n\in N$ where $n\geq 2$. Then,

$$\sigma\left(\frac{1}{2n}\right)=\inf\left\{\frac{1}{2l}:\frac{1}{2l}>\frac{1}{2n},l\in N\right\}=\frac{1}{2(n-1)}>\frac{1}{2},$$
$$\rho\left(\frac{1}{2n}\right)=\sup\left\{\frac{1}{2l},0:\frac{1}{2l},0<\frac{1}{2n},l\in N\right\}=\frac{1}{2(n+1)}<\frac{1}{2}.$$

Therefore, all the points $\frac{1}{2n},n\in N,n\geq 2,$ are right-scattered and left-scattered i.e., all points $\frac{1}{2n},n\in N,n\geq 2$ are isolated.

3- when t=0, then,

$$\sigma(0)=\inf\{s\in T:s>0\}=0,$$
$$\rho(0)=\sup\{s\in T:s<0\}=\sup\varnothing=\inf T=0.$$

Example: Let T = $\left\{\frac{n}{3}:n\in N_0\right\}$ and $t=\frac{n}{3},n\in N_o$ be arbitrarily chosen.

So,

1. n ∈ ℕ. Then,

$$\sigma\left(\frac{n}{3}\right)=\inf\left\{\frac{1}{3},0:\frac{1}{3},0>\frac{n}{3},l\in N_0\right\}=\frac{n+1}{3}>\frac{n}{3},$$
$$\rho\left(\frac{n}{3}\right)=\sup\left\{\frac{1}{3},0:\frac{1}{3},0<\frac{n}{3},l\in N_0\right\}=\frac{n-1}{3}<\frac{n}{3}.$$

Thus all points $t=\frac{n}{3},n\in N$ are right-scattered and left-scattered, i.e., all points $t=\frac{n}{3},n\in N$ are isolated.

2. When n=0 then,

$$\sigma(0) = \inf\left\{\frac{1}{3}, 0 : \frac{1}{3}, 0 > 0, 1 \in N_0\right\} = \frac{1}{3} > 0,$$

$$\rho(0) = \sup\left\{\frac{1}{3} : \frac{1}{3}, 0 < 0, 1 \in N_0\right\} = \sup\varnothing = \inf T = 0.$$

i.e., t = 0 is right-scattered

While graphical expression of table 1 is given in figure 1.

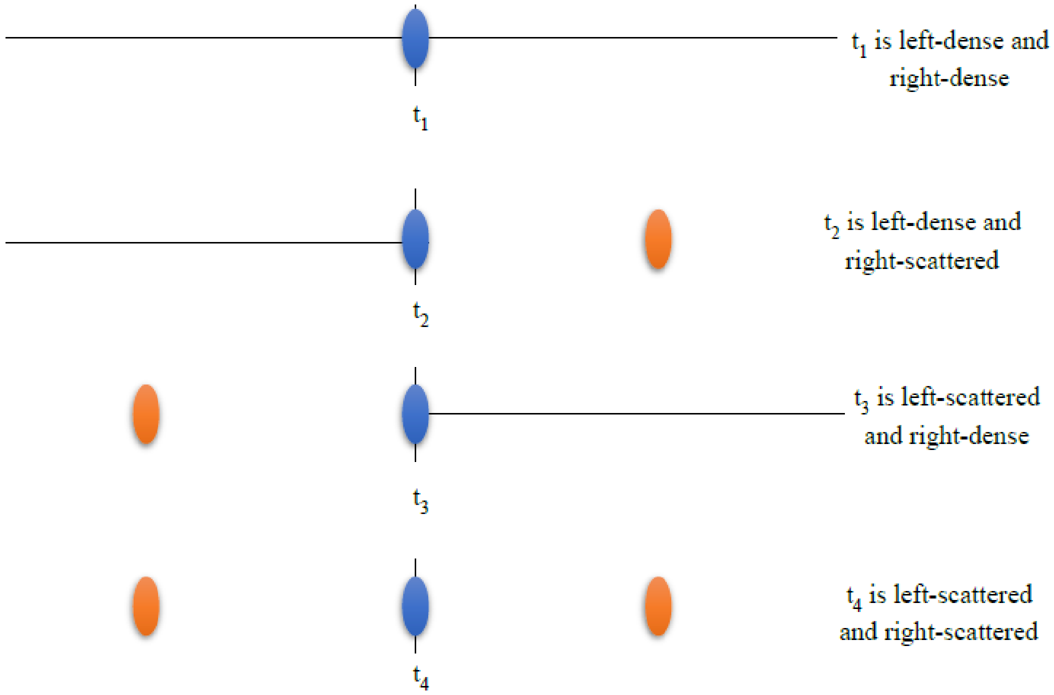

Figure 1: Points classification according to σ(t) = (t) > t, ρ(t) and ρ(t) < t.

From above predictions, both σ(t) as well as ρ(t) are in T while t ∈ T, because T is a close subset of ℝ. The observed points which are:

⟹ Left-dense and right-scattered is point 3.

⟹ Left-scattered and right-dense are points 2,4.

⟹Left-scattered and right-scattered are all those points which has 1/s where S ∈ N.

Table 2: Examples of time Scale based calculus according to T, R and Z.

T	**R**	**Z**
Backward jump operator ρ(t)	t	t-1
Forward jump ∞ (t)	t	t+1
Graininess μ (t)	0	1
Derivative fΔ (t)	f(t)	Δf(t)
Rd-continuous f	Continuous t	Any f

Assume f: T→ ℝ in delta differential on TK. Then f is nabla differential at t.

$f\nabla(t) = f\Delta\,(\rho(t)$ for t ϵ TK such that 6 (P(t)) = t

fΔ is continues on TK. The f is nabla differentials at t

$f\nabla\,(t) = f\Delta\,(\rho(t)$ hold for t ∈ TK

⇨ Assume that t, g : T→ ℝ are nabla differentiable where t ∈ TK.

→ function f: X → ℝ defined on a convex subset of ℝn is said to be convex if,

$f(\lambda x +(1-\lambda)y) \leq \lambda\, f(x) + (1-\lambda)f(y)$ for x,y ∈ X

$\lambda \in [0,1]$

f: X→ℝ strictly convex

Example:

powers = f(x) = xP ; P≥1

exponential = f(x) =eax for any a ∈ ℝ

Nabla Derivative

Nabla derivatives corresponding theory was broadly studied for the delta dynamics equations development (Atasever 2011).

For f: T → ℝ and t ∈ TK define nabla of "f" is t.

f∇ (t) for any ∈>0, there is a neighborhood

$$[\, f(\rho(t) - f(S)] - f\nabla\,(t)\,[\rho(t) - S]] < \in |\, P(t)\text{-}S \therefore S \in UT$$

⇨ Assume that t, g : T→ ℝ are nabla differentiable where t ∈ TK.

- The sum of f + g : T→ ℝ is differentiable at t

$$(f+g)\nabla\,(t) = f\,\nabla\,(t) + g\nabla\,(t).$$

- Product f g : T → ℝ differentiable at T

$$(f\,g)\nabla\,(t) = f\nabla\,(t)\,g(t) + fP\,(t)\,g\nabla\,(t) = f\,(t)g\nabla\,(t) + f\nabla\,g\rho(t)$$

- If g(t) g P(t) ≠ then f/g is differentiable

$$(f/g)^{\nabla}(t) = \frac{g(t)\,f^{\nabla}(t) - f(t)\,g^{\nabla(t)}}{g(t)g^{\nabla}(t)}$$

Antiderivative and integral

The delta integral is defined as the antiderivative with respect to the delta derivative. If F(t) has a continuous derivative f(t)=FΔ(t) one sets

$$\int_r^s f(t)^{\Delta}(t) = F(s) - F(r).$$

Let f: T! → ℝ be a delta differentiable function

Function

$$f^{\Delta} = \left\{\begin{matrix} TK \to \square \\ t \to f^{\nabla}(t) \\ \end{matrix}\right\} \text{delta differentiable of m } T^{!}$$

⇨ f: TK → ℝ → delta antiderivative of g on T! and for all t ∈ TK the condition f∇ (t) = g (t) is satisfied

⇨ for any rd-continuous mapping g: TK → ℝ exist delta antiderivative

f: t→

$$\int_{s}^{t} g(s)\ \Delta s, s, t \in T^{K}$$

Theorem: Suppose that the function g:TK → ℝ has a delta antiderivative function

f on [r,s]∈ T

then exactly integral from r to s

For T = ℝ

T = *h*ℵ *h* >0

$$\int_{r}^{s} g(t)\Delta t = \left\{\begin{matrix} \sum_{i=\frac{r}{n}}^{n^{s-1}} g(ih)h \quad s>r \\ s = r \\ \sum_{i=\frac{r}{n}}^{n^{s-1}} g(ih)(h) \quad s>r \end{matrix}\right\}$$

Theorem: function F: T → ℝ is called a nabla derivative f : T→ R

f∇ (t) = f(t) holds for all t ∈ TK

by defined integral

$$\int_{a}^{t} f(\acute{y})\nabla = F(t) - F(a) \quad t \in T$$

Suppose F and f∇ continuous

$$\left(\int_{a}^{t} f(t,s)\nabla s\right)^{\nabla} = f(P(t),t) + \int_{a}^{t} f(t,s)/s$$

Let T be a time scale a,b ∈ T with a < b and let fi (x) (I = 1,2 m)

h (x) : [a, b] T⟶ [oi+∞] be a â(delta) integral

$$\int_a^b h(x)\hat{a}\, x = 1 \qquad \text{if } 1<N_1, N_2, \ldots, N_m < \infty \qquad \text{with} \quad 1/\lambda_1 + 1/\lambda_2 +, \ldots + 1/\lambda_m < 1$$

Then,

$$\int_a^b h(x)\left(\pi_{i=1}^m f_i(x)\right)\hat{a}\ \mathrm{x} \le \pi_{i=1}^m \left(\int_a^b h(x) f_i^{\lambda}(x)\hat{a}x\right)$$

Theorem: Let T be a time scale a,b ∈ T with a < b and let,

fi (x) (i = 1,2,3, . . . , m)

h (x) : [a,b] r ⟶ ℝ be

â (delta) integral function.

1) If P >1 then,

$$\left[\int_a^b \left| h(x)\left(\sum_{i=1}^m f_i(x)^P\right)\right| \hat{a}x\right]^{1/P} \le \sum_{i=1}^m \left(\int_a^b |h(x)|\, f_i(x)^P\, \hat{a}^x\right)^{1/P}$$

2) If 0 < P < 1 then,

$$\left[\int_a^b \left| h(x)\left(\sum_{i=1}^m f_i(x)^P\right)\right| \hat{a}x\right]^{1/P} \ge \sum_{i=1}^m \left(\int_a^b |h(x)|\, f_i(x)^P\, \hat{a}^x\right)^{1/P}$$

CONCLUSION AND FUTURE PERSPECTIVE

In this article I present dynamics inequalities create bridge between continuous and discrete. Basic work on dynamic inequalities is done by Ravi Agarwal, Martin bohar and many others. This research presents extensions of Radon’s inequality, Lyapunov’s Inequality with some generalization and applications of Randons’s Inequality, GronWall’s Inequality, AM-GM Inequality, Lyapunov’s Inequality and Antiderivative and integral and Nesbitts inequality. According to outcomes of dynamic inequalities for the diamond integral which is linear combinations of delta and nabla integral, if we set,

∝ = 1 ⟶ we get delta version.

∝ = 0 ⟶ we get nabla version of diamond∝-integral

T = Z ⟶ we get discrete version.

T = ℝ ⟶ we get continuous version

In future we can consider dynamic inequalities by using n-tuple diamond integral, Quantum calculus and Riemann–Liouville integral of order α. Additionally, it is suggested that more investigation be done on the implementation of quasi-convex functions on the time scales in economics, optimization and mathematical modeling and among others.

Recently it has found that many dynamic inequalities such as Randons's Inequality, Nesbitts inequality, GronWall's Inequality, AM-GM Inequality, Lyapunov's Inequality and Antiderivative and integral are equivalent on time scales as given in (Sahir 2018), so we can find more equivalent dynamic inequalities on time scales.

T: Time scale $\mathbb{R}$: Real Number Z or $\mathbb{Z}$: Integers

N: Natural numbers

f^{Δ}: Delta derivative P(t) = ρ(t)

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