

## CHIRPED AND DIPOLE SOLITON IN NONLINEAR NEGATIVE- INDEX MATERIAL

**Shazia Perveen\*, Muhammad Kamran\*, Rukhsana Ahmed, Maira Mukhlis,  
Ammarah Marriyam and Muhammad Aamir Hayat**

Virtual University of Pakistan.

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**\*Corresponding Author**

**Shazia Perveen &  
Muhammad Kamran**  
Virtual University of  
Pakistan.

### ABSTRACT

Due to the vital applications of short light pulses to telecommunication and ultrafast research, the soliton propagation in nonlinear optical fiber has become a vast research topic now a days. It is a type of signal emerged when frequency changes with time. Chirped pulses have vast application in the field of communication and pulse compression. Bouzida et al. used dual power law in non optical fiber to find chirped soliton.

**KEYWORDS:** Chirped And Dipole.

### 1. INTRODUCTION

The soliton propagation in nonlinear optical fiber is a topic of current research because of the vital applications of short light pulses to telecommunication and ultrafast signal routing systems Chirped soliton is a type of signal emerges when frequency changes with time.

Chirped pulses are used in solitary wave-based communications, design of fiber optic amplifier and optical pulse compressors due to their applications in amplification or pulse compression. It used dual power law in Nano optical fibers to find chirped soliton. Dipole soliton or dark in the bright soliton were first observed by Choudhuri and Porsezian. Dipole soliton are composed of product of bright and dark soliton. In this paper, we find the chirped soliton with sub-ODEs method and dipole soliton under the ansatz method for nonlinear negative index materials under quadratic-cubic nonlinearity.

## Preliminaries

We usually come across of three types of PDEs,

These are classified as

- i. Elliptic
- ii. Hyperbolic
- iii. Parabolic

## Types of boundary condition

There are three types of boundary conditions commonly encountered in the solution of partial differential equations.

1. **Dirichlet boundary conditions** specify the value of the function on a surface  $T=f(r,t)$
2. **Neumann boundary conditions** specify the normal derivative of the function on a surface,  $(\partial T)/(\partial n)=n^{\wedge} \text{del } T=f(r,t)$ .

**Robin boundary conditions.** For an elliptic partial differential equation in a region  $\Omega$ , Robin boundary conditions specify the sum of  $\alpha u$  and the normal derivative of  $u=f$  at all points of the boundary of  $\Omega$ , with  $\alpha$  and  $f$  being prescribed.

**Soliton:** A soliton or solitary wave is a self-reinforcing wave packet that maintains its shape while it propagates at a constant velocity. Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium.

**Optical Soliton:** An optical soliton is a pulse that travels without distortion due to dispersion or other effects. They are non-linear phenomenon caused by self-phase modulation which means that electric field of wave changes the index of refraction seen by the wave (Kerr effect).

In optics term soliton is used to refer to any optical field that does not change during the propagation because of a delicate balance.

## 2. Literature Review

In theoretical physics the (one-dimensional) nonlinear Schrödinger equation (NLSE) is a nonlinear variation of the Schrodinger's. It is a classical field equation whose principal applications are to the propagation of light in nonlinear optical fibers and planar waveguides and to Bose-Einstein condensates confined to highly anisotropic cigar-shaped

traps, in the mean-field regime. Additionally, the equation appears in the studies of small-amplitude gravity waves on the surface of deep in viscid (zero-viscosity) water the Langmuir waves in hot plasmas. the propagation of plane-diffracted wave beams in the focusing regions of the ionosphere.

The propagation of Davidson's alpha-helix solution's, which are responsible for energy transport along molecular chains and many others.

A localized surface soliton that causes a temporary increase in in an associated wave amplitude.

The terms light and dark solitons are borrowed from the optics where they manifest as bright spots and dark shadows in optical fibers. And yet it was in water that solitons were first observed in the 1830. Dark soliton showing the intensity and phase of black and grey. Dark soliton emerges into the light. Dark solitons are the solution s of non-linear PDEs in the space of dimensions with non zero boundary conditions and non zero phase shift. They are represented by a family of travelling wave solution extending from the limit of zero speed(so-called black soliton) to the limit of sound speed (so-called grey soliton). These higher order solitons show frequency chirp defined as the time derivative of the soliton phase . In other words dark solitons are generally chirped . The soliton propagation in nonlinear optical fiber is a topic of current research because of the vital applications of short light pulses to telecommunication and ultrafast signal routing systems . Chirped soliton is a type of signal emerges when frequency changes with time. Chirped pulses are used in solitary wave-base communications, design of fiber optic amplifier and optical pulse compressors due to their applications in amplification or pulse compression . Bouzida et al. used dual power law in nan optical fibers to find chirped soliton. Dipole soliton or dark in the bright soliton were first observed by Choudhuri and Porsezian nonlinearity. In mathematics and physics, a soliton or solitary wave is a self-reinforcing wave packet that maintains its shape while it propagates at a constant velocity.

Solitons are caused by a cancellation of nonlinear and dispersive effects in the medium.

The ansatz involving exact traveling wave solutions to nonlinear partial differential equations. To obtain wave solutions using direct method, the choice of an appropriate ansatz is of great importance. We apply this ansatz to examine new and further general traveling

wave solutions to the (1+1)-dimensional modified Benjamin–Bona–Mahony equation. Abundant traveling wave solutions are derived including solitons, singular solitons, periodic solutions and general solitary wave solutions.

### 3. METHODOLOGY

The model studies the dynamics of soliton propagation through optical material.

$iq_t + aq_{xx} + (b_1 |q| + b_2 |q|^2)q = i\{aq_x + \beta(|q|^2 q)_x + v(|q|^2)_x q\} + \theta_1(|q|^2 q)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx}$  [1] where wave profile is represented by  $q(x, t)$ , group velocity dispersion is represented by the coefficient of  $a$ . While quadratic-cubic nonlinearity is shown by  $b_1$  and  $b_2$ . On other side of mathematical model, inter modal dispersion, self-steepening and nonlinear dispersion are represented by  $\alpha$ ,  $\beta$  and  $v$  respectively. In the following subsection, we find the chirped soliton for Eq. (1)

#### Chirped Soliton

$$q = p(\xi)e^{i(x(\xi) - \Omega t)}$$

$$aq_x = a(\xi)e^{i(x(\xi) - \Omega t)}[\rho ix' + \rho']$$

Taking again derivative

$$q_{xx} = e^{i(x(\xi) - \Omega t)}[i\rho^3 x'' + 2i\rho'x' + \rho'' - \rho^3(x')^2]$$

$$iq_t = e^{i(x(\xi) - \Omega t)}[\rho x'u + \Omega - i\rho'u]$$

$$|q| = |p(\xi)| = p(\xi)$$

$$b_1 |q(x, t)| = b_1 \rho$$

$$b_2 |q(x, y)|^2 = b_2 \rho^2$$

$$aq(x, t)_x = a(\xi)e^{i(x(\xi) - \Omega t)}[\rho ix' + \rho']$$

$$b_1 |q(x, t) + b_2 |q(x, y)|^2 = b_1 \rho + b_2 \rho^2$$

$$|q(x, t)q|_x^2 \cdot q(x, t) = 2\rho^2 \rho' e^{i(x(\xi) - \Omega t)}$$

$$= iq_t + aq_{xx} + (b_1 |q(x, t)| + b_2 (q(x, y)|)$$

$$= e^{i(x(\xi) - \Omega t)}[\rho x'u + \Omega - i\rho'u] + a(\xi)e^{i(x(\xi) - \Omega t)}[i\rho x'' + 2i\rho'x' + \rho'' - \rho^3(x')^2 + b_1 \rho + b_2 \rho^2]$$

$$= e^{i(x(\xi) - \Omega t)}[\rho x'u + \Omega - i\rho'u]$$

$$= i\{aq_x + (b_1 |q|^2 q)_x\} + v(|q|^2)_x q + \theta_1(|q|^2 q)_{xx} + \theta_2 |q|^2 q_{xx} + \theta_3 q^2 q^*_{xx}$$

$$aq_x = a(\xi)e^{i(x(\xi) - \Omega t)}[\rho ix' + \rho']$$

$$(\beta |q|^2 q)_x = \beta e^{i(x(\xi) - \Omega t)}[3\rho^2 \rho' + i\rho^3 x']$$

$$v |q|^2 |q|_x = 2v\rho^2 (e^i) e^{i(x(\xi) - \Omega t)}$$

$$\theta_1(|q|^2 q)_{xx} = \theta_1 e^{i(x(\xi) - \Omega t)}[i\rho^3 x'' + 2i\rho'x' + \rho'' - \rho^3(x')^2]$$

The values are put in equation 2

$$= ia[\rho ix' + \rho'] e^{i(x(\rho)-\Omega t)} + \beta e^{i(x(\rho)-\Omega t)} [3\rho^2 \rho' + i\rho^3 x] + 2v\rho^2(x) e^{i(x(\rho)-\Omega t)}$$

$$\theta_1 e^{i(x(\rho)-\Omega t)} [i\rho^3 x'' + 2i\rho' x' + \rho'' - \rho^3(x')^2] + \theta_2 \rho^2(\xi) e^{i(x(\rho)-\Omega t)} [i\rho x'' + 2i\rho' x' + \rho'' - \rho^3(x')^2]$$

$$+ \theta_3 e^{i(x(\rho)-\Omega t)} [i\rho^2 x'' - 2i\rho' x' - i\rho^3 \rho^{11} - \rho^3(x')^2]$$

Now substituting  $q(x, t)$  and its derivatives into Eq, we get the following real and imaginary part.

$$= \rho' x' \mu + \rho \Omega + a \rho'' - a \rho(x')^2 + (b_1 \rho^2 + b_2 \rho^3) + a \rho x' + \rho^3 x' \beta + \theta_1 (-3\rho'' \rho^2 - 6\rho(\rho')^2 + \rho^3(x')^2) + \theta_2 \rho^3(x')^2 - \rho'' \rho^2 = 0$$

And

$$= -\rho' \mu + a \rho x'' + a \rho' x'' + 2a \rho' x' - a \rho' - 3\beta \rho^2 \rho' - 2v\rho^2 \rho' + \theta_1 (\rho^2 \rho' x' - 2\rho^3 x'') + \theta_2 (\rho^3 x'' - 2\rho' \rho^2 x') + \theta_3 (2\rho' \rho^2 x' + \rho^3 x'')$$

okay

$$= \rho x' \mu + \rho \Omega + a \rho'' - a \rho(x')^2 + (b_1 \rho^2 + b_2 \rho^3) + a \rho x' + \rho^3 x' \beta + \theta_1 (-3\rho'' \rho^2 - 6\rho(\rho')^2) + \rho(x')^2 + \theta_3 (\rho(x)^3 - \rho \rho) = 0$$

Where

$$x = \delta \rho^2 + \eta$$

$$\delta = \frac{3\beta + 2v}{4a}$$

$$= \rho(\delta \rho^2 + \eta) \mu + \rho \Omega + a \rho'' - a \rho(\delta \rho^2 + \eta)^2 + ((b_1 \rho^2 + b_2 \rho^3) + a \rho(\delta \rho^2 + \eta) + a \rho(\delta \rho^2 + \eta) + \rho^3 x' \beta + \theta_1 (-3\rho'' \rho^2 - 6\rho(\rho')^2) + \rho^3(\delta \rho^2 + \eta)^2 + \rho^3(\delta \rho^2 + \eta)^2 + \theta_2(\rho^3(\delta^2 \rho^4 + \eta^2 + 2\delta \rho^2)) + \theta_3(\rho(x)^3 - \rho^3 \rho'') = 0$$

$$= \rho^3 \delta + \rho \eta + \rho \Omega + a \rho'' - a \rho(\delta^2 \rho^4 + \eta^2 + 2\delta \rho^2)^2 + (b_1 \rho^2 + b_2 \rho^3) + a \rho(\delta \rho^2 + \eta) + \rho^3 x' \beta + \theta_1 (-3\rho'' \rho^2 - 6\rho(\rho')^2) + (\rho^3(\delta^2 \rho^4 + \eta^2 + 2\delta \rho^2)) + \theta_2(\rho^3(\delta^2 \rho^4 + \eta^2 + 2\delta \rho^2)) + \theta_3(\rho(x)^3 - \rho^3 \rho'') = 0$$

$$= \rho^3 \delta + \rho \eta + \rho \Omega + a \rho'' - a \rho^4 - a \rho^4 \delta^2 - a \eta^2 - 2a \delta \rho^2 + b_1 \rho^2 + b_2 \rho^3 + a \delta \rho^3 + \beta \rho^3 \delta + \eta \beta \rho^3 - 3\delta'' \delta^2 \theta_1 - 6\rho \rho' \beta_1 + \beta_1 \delta^2 \rho^7 + \beta_1 \delta^3 \eta^2 + 2\delta \rho^5 \eta \beta_1 + \theta_2 \rho^7 \delta + \theta_2 \delta^3 \eta^2 + 2\delta \rho^5 \eta \theta_2 - \theta_2 \rho^2 \delta'' + 3\delta^2 \rho^{10} \eta \rho'' \rho^3 + 3\delta^8 \rho'' \eta^2 = 0$$

$$= A \rho^7 + B \rho^5 + C \rho^3 + (b_1 + \rho'' D) \rho^2 + (E - 6\theta(\rho')^2) \rho + a \rho''$$

Multiplying Equation with  $\rho'$

$$= \rho' [A \rho^7 + B \rho^5 + C \rho^3 + (b_1 + \rho'' D) \rho^2 + \rho \rho' [(E - 6\theta_1(\rho'))^2 + a \rho'']]$$

$$= A \rho^7 \rho' + B \rho^5 \rho' + C \rho^3 \rho' + b_1 \rho^2 \rho' + \rho^2 \rho'' D + E \rho_1 \rho' - 6\theta_1(\rho') \rho + \rho'' \rho$$

Taking Intigration

$$= A \frac{\rho^8}{8} + B \frac{\rho^6}{6} + C \frac{\rho^4}{4} + b_1 \frac{\rho^3}{3} + E \rho' \frac{\rho^2}{2} - 6\theta \left[ (\rho^3)' \rho - 3\rho^3 \rho \right] + a \rho'' \frac{\rho^2}{2}$$

$$a_1 = \frac{-A}{4a}$$

$$a_2 = \frac{-B}{3a}$$

$$a_3 = \frac{-C}{2a}$$

$$a_4 = \frac{-E}{a}$$

So,

$$a_1 \rho^8 + a_2 \rho^6 + a_3 \rho^4 + a_4 \rho^2 - (\rho')^2$$

$$a_1 = \frac{-\delta^2(\theta_1 + \theta_2 + \theta_3)}{4a}$$

$$a_2 = \frac{-2\delta \eta(\theta_1 + \theta_2 + \theta_3) - a \delta^2 + \beta \delta}{3a}$$

$$a_3 = \frac{-b_2 + \delta(\mu + a) + \eta(-2a\delta + \beta) + \eta^2(\theta_1 + \theta_2 + \theta_3)}{2a}$$

$$a_4 = \frac{-\Omega + \eta(a - a\eta + \mu)}{a}$$

$$a_4 = \frac{-E}{a}$$

So,

$$a_1 \rho^8 + a_2 \rho^6 + a_3 \rho^4 + a_4 \rho^2 - (\rho')^2$$

$$a_1 = \frac{-\delta^2(\theta_1 + \theta_2 + \theta_3)}{4a}$$

$$a_2 = \frac{-2\delta\eta(\theta_1 + \theta_2 + \theta_3) - a\delta^2 + \beta\delta}{3a}$$

$$a_3 = \frac{-b_2 + \delta(\mu + a) + \eta(-2a\delta + \beta) + \eta^2(\theta_1 + \theta_2 + \theta_3)}{2a}$$

$$a_4 = \frac{-\Omega + \eta(a - a\eta + \mu)}{a}$$

By considering the following cases we get the following solutions,

### Case 1

Bell type solitary wave solutions

$$a_1 > 0, a_2 > 2a_1, a_3 = \frac{a_2^2}{4a_1} - a_1$$

Equation (3.1.25) gives the positive solution.

From eq (3.1.37) we have

$$\rho(\xi)^2 - \operatorname{sech}(2\sqrt{a_1}\xi)$$

$$q(x, t) = \rho(\xi)e^{i(x(\xi) - \Omega t)}$$

Put the value of  $\rho(\xi)$  in equation

$$\rho(\xi)^2 - \operatorname{sech}(2\sqrt{a_1}\xi)$$

$$q(x, t) = \sec(2\sqrt{a_1}\xi)^{\frac{1}{2}} e^{i(x(\xi) - \Omega t)}$$

**Case 2**

Kink type solitary wave solutions

$$a_1 > 0, a_3 < 0, \quad a_2 = -2\sqrt{(a_1 a_3)}$$

$$\frac{a\delta^2 - \beta\delta}{3a} = -2\sqrt{(a_1 a_3)}$$

$$\beta = \frac{6a\sqrt{(a_1 a_3)} + a\delta^2}{\delta}$$

$$\rho(\xi) = \left( \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right) \right)^{\frac{1}{2}}$$

$$\rho(\xi)^2 = \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right)$$

$$q(x, t) = \rho(\xi) e^{i(x(\xi) - \Omega t)}$$

Put the value of  $\rho(\xi)$ 

$$q(x, t) = \left( \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right) \right)^{\frac{1}{2}} e^{i(x(\xi) - \Omega t)}$$

**Case 3**

Algebraic solitary wave solution:

$$a_1 > 0, a_3 < 0, \quad a_2 = -2\sqrt{(a_1 a_3)}$$

$$\frac{a\delta^2 - \beta\delta}{3a} = -2\sqrt{(a_1 a_3)}$$

$$\beta = \frac{6a\sqrt{(a_1 a_3)} + a\delta^2}{\delta}$$

$$\rho(\xi) = \left( \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right) \right)^{\frac{1}{2}}$$

$$\rho(\xi)^2 = \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right)$$

$$q(x, t) = \rho(\xi) e^{i(x(\xi) - \Omega t)}$$

Put the value of  $\rho(\xi)$ 

$$q(x, t) = \left( \sqrt{\frac{a_1}{a_2}} \left( \frac{1}{2} + \frac{1}{2} \tanh(\sqrt{a_1 a_2}) \right) \right)^{\frac{1}{2}} e^{i(x(\xi) - \Omega t)}$$

$$a_1 = 0, a_2 = 1, a_3 < 0$$

By putting the equation  $a_2 = 1$ , we have

$$a\delta^2 - \beta\delta - 3a$$

$$\beta = \frac{a\delta^2 - 1}{\delta}$$

$$\rho(\xi) = \sqrt{\frac{1}{\xi^2 - c}}$$

$$q(x, t) = \rho(\xi) e^{i(x(\xi) - \Omega t)}$$

$$q(x, t) = \sqrt{\frac{1}{\xi^2 - c}}$$

$$q(x, t) = \sqrt{\frac{1}{\xi^2 - c}} e^{i(x(\xi) - \Omega t)}$$

**Case 4**

Sinusoidal wave solutions

$$a_1 = -1, a_2 = 2\sigma, a_3 = 1 - \sigma^2$$

$$a_2 = 2\sigma$$

$$a\delta^2 - \beta\delta = 6a\sigma$$

and we get  $\beta$

$$\beta = \frac{a\delta^2 - 6a\sigma}{\delta}$$

$$\rho(\xi) = \sqrt{\frac{1}{\sigma + \sin 2\xi}}$$

$$\delta\omega(\xi) = (\delta\rho^2 + \eta)$$

$$\rho(\xi) = \sqrt{\frac{1}{\sigma + \sin 2\xi}} e^{i(x(\xi) - \Omega t)}$$

**4. CONCLUSIONS**

In this article, we obtained solitary wave solutions for nonlinear negative-index materials under quadratic-cubic nonlinearity. We Obtained two types of soliton solutions; one is chirped soliton and the other is dipole soliton solution. We used sub-ODE method to get Chirped soliton and studied dipole soliton with the help of ansatz method of Choudhuri. The obtained results will be used in field of Telecommunication.

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