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APPLICATION OF FUZZY ELZAKI TRANSFORMS FOR SOLVING FUZZY PARTIAL VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

Fuzzy partial integro-differential equation have an important part in the fields of science and building. In this paper, we propose the arrangement of fuzzy partial Volterra integro-differential equation mathematical statement with convolution type kernel utilizing fuzzy ELzaki transform technique (FETM) under differentiability.

KEYWORDS: ELzaki transform, fuzzy partial differential equation, fuzzy, fuzzy partial Volterra integro-differential equation.

1. INTRODUCTION

The theme of fuzzy integro differential equation (FIDEs) has been quickly developed late year. A standout amongst the most vital field of the fuzzy hypothesis is the fuzzy differential equation[1], fuzzy vital comparisons and fuzzy integro-differential equation (FIDEs).

In the arrangement of traditional PIDEs was examined utilizing established ELzaki transform. In the present article we examine the arrangement of various sorts of fuzzy halfway integro differential equation with convolution piece (FPIDEs) utilizing fuzzy ELzaki transform strategy. Keeping in mind the end goal to decide the lower and upper elements of the arrangement we change over the offered FPIDEs to two crisp common differential equation by utilizing FET.

2. Preliminaries

In this section we will survey a couple stray pieces definitions and speculations required all through the paper, for instance fuzzy number, fuzzy esteemed capacity and the subsidiary of the fuzzy esteemed capacities.

Definition 2.1 A fuzzy number is defined as the mapping such that $u : R \to [0,1]$ which satisfies the following four properties

1. u is upper semi-continuous.
2. u is fuzzy convex that is $u\left(\lambda x + (1-\lambda)\mathrm{y}\right) \geq \min\{u(\mathrm{x}), \mathrm{u(y)}\}. x, y \in R$ and $\lambda \in [0,1]$.
3. u is normal that is $\exists x_0 \in R$, where $u(\mathrm{x}_0) = 1$.
4. $A = \{\overline{x \in R : u(\mathrm{x}) > 0}\}$ is compact, where $\overline{A}$ is closure of A.

Definition 2.2. A fuzzy number in parametric form is given as an order pair of the form $u = (\underline{u}(\mathrm{r}), \overline{\mathrm{u}}(\mathrm{r}))$, where $0 \leq \mathrm{r} \leq 1$ satisfying the following conditions.

1. $\underline{u}(\mathrm{r})$ is a bounded left continuous increasing function in the interval [0, 1].
2. $\overline{\mathrm{u}}(\mathrm{r})$ is a bounded left continuous decreasing function in the interval [0, 1].
3. $\underline{u}(\mathrm{r}) \leq \overline{\mathrm{u}}(\mathrm{r})$

If $\underline{u}(\mathrm{r}) = \overline{\mathrm{u}}(\mathrm{r}) = \mathrm{r}$, then r is called crisp number.

Since each $y \in R$ can be regarded as a fuzzy number if

$$y(\mathrm{t}) = \begin{cases} 1, & if\ y = t \\ 0, & if\ y \neq t \end{cases}$$

For arbitrary fuzzy numbers $u = (\underline{u}(\alpha), \overline{u}(\alpha))$ and $v = (\underline{v}(\alpha), \overline{v}(\alpha))$ and arbitrary crisp number j, we define addition and scalar multiplication as:

1. $(\underline{u + v})(\alpha) = \left(\underline{u}(\alpha) + \underline{v}(\alpha)\right)$.
2. $\left(\overline{u + v}\right)(\alpha) = \left(\overline{u}(\alpha) + \overline{v}(\alpha)\right)$.
3. $\left(j\underline{u}\right)(\alpha) = \mathrm{j}\underline{u}(\alpha), \left(j\overline{u}\right)(\alpha) = j\overline{u}(\alpha), \mathrm{j} \geq 0.$
4. $\left(j\underline{u}\right)(\alpha) = \mathrm{j}\overline{u}(\alpha)\alpha, \left(j\overline{u}\right)(\alpha) = j\underline{u}(\alpha)\alpha, \mathrm{j} < 0.$

Definition 2.3. Let us suppose that $x, y \in E$, if $\exists\, z \in E$ such that $x = y + z$, then z is called the H-difference of x and y and is given by $x \ominus y$.

The Housdorff distance between the fuzzy numbers defined by

$d : E \times E \longrightarrow R^{+} \cup \{0\}$,

$d\,(\mathrm{u},\mathrm{v}) = \sup_{r\in[0,1]} \max\{|\underline{u}(\mathrm{r}) - \underline{v}(\mathrm{r})|\}$,

Where $u = (\underline{u}(r), \overline{u}(r))$ and $v = (\underline{v}(r), \overline{v}(r)) \subset \mathrm{R}$.

We know that if d is a metric in E, then it will satisfy the following properties:

1. $d(u + w, v + w) = d(u, v), \forall\, u, v, w \in E$.
2. $(k \odot u, k \odot v) = |k| d(u, v), \forall\, k \in R, and\; u, v \in E$.
3. $d(u \oplus v, w \oplus e) \leq d(u, w) + d(v, e), \forall\, u, v, w, e \in E$.

3. Two dimensional fuzzy ELzaki transform

In this section we state some definitions and theorems from which will be used in the next section.

Definition 3.1. Let $u = u(x, t)$ is a fuzzy-valued function and p is a real parameter, then FET of the function u with respect to t denoted by $U(x, p)$, is defined as follows:

$$U(\mathrm{x},\mathrm{p}) = E\left[u(\mathrm{x},\mathrm{t})\right] = p\int_0^{\infty} e^{-\frac{t}{p}} u(\mathrm{x},\mathrm{t})dt = \lim_{\tau\to\infty} p\int_0^{\tau} e^{-\frac{t}{p}} u(\mathrm{x},\mathrm{t})dt,$$

$$U(\mathrm{x},\mathrm{p}) = \left[\lim_{\tau\to\infty} p\int_0^{\tau} e^{-\frac{t}{p}} \underline{u}(\mathrm{x},\mathrm{t})dt, \lim_{\tau\to\infty} p\int_0^{\tau} e^{-\frac{t}{p}} \overline{u}(\mathrm{x},\mathrm{t})dt\right],$$

Whenever the limits exist. The r-cut representation of $U(x, p)$ is given as:

$U(x, p;\, r) = E[u(x, t;\, r)] = [E(\underline{u}(x, t;\, r)), E(\overline{u}(x, t;\, r))]$,

$$E(\underline{u}(x, t;\, r) = p\int_0^{\infty} e^{-\frac{t}{p}} \underline{u}(\mathrm{x},\mathrm{t};\mathrm{r})dt = \lim_{\tau\to\infty} p\int_0^{\tau} e^{-\frac{t}{p}} \underline{u}(\mathrm{x},\mathrm{t};\mathrm{r})dt,$$

$$E(\overline{u}\,(x, t;\, r) = p\int_0^{\infty} e^{-\frac{t}{p}} \overline{u}(\mathrm{x},\mathrm{t};\mathrm{r})dt = \lim_{\tau\to\infty} p\int_0^{\tau} e^{-\frac{t}{p}} \overline{u}(\mathrm{x},\mathrm{t};\mathrm{r})dt,$$

3.2 Elzaki Transform of common functions

Elzaki transform of some common functions is given as under Elzaki transform of exponential Function

$$E(e^{at}) = T(p) = p\int_0^{\infty} e^{at}.e^{-\frac{t}{p}}dt = \frac{p^2}{1-ap}$$

Elzaki transform of Sin function

$$E(\sin(at)) = \frac{ap^3}{a^2p^2+1}$$

Elzaki transform of Cos function

$$E(\cos(at)) = \frac{p^2}{a^2p^2+1}$$

4. Numerical examples

In this segment we will talk about the arrangement of fuzzy convolution halfway Volterra integro-differential comparisons utilizing FET to demonstrate the utility of the proposed technique in Section 4.

Example 4.1.

Let us consider the following fuzzy convolution partial Volterra integro-differential equation.

$$xu_x = u_{tt} + (\mathrm{xsinx})(\mathrm{r}-1,1-\mathrm{r}) + \int_0^t \sin(\mathrm{t}-\mathrm{s})u(\mathrm{x},\mathrm{s})\,\mathrm{ds} \qquad (4.1)$$

with initial conditions

$u(x,0;\ r) = (0,0), u_t(x,0;\ r) = ((r-1)x, (1-r)x),$

and Boundary condition

$u(1,t;\ r) = ((r-1)t, (1-r)t).$

Taking FET with respect to t on (4.1), then we get

$$xE[u_x] = E[u_{tt}] + x(r-1, 1-r)E[\sin t + E[\sin t]E[u(x,t)]. \qquad (4.2)$$

Using FET (4.2) becomes

$$x\frac{d}{dx}U(\mathrm{x},\mathrm{p}) = \mathrm{p}^{-2}U(\mathrm{x},\mathrm{p})!\ \ \mathrm{pu}(\mathrm{x},0)!\ \ pu_t(\mathrm{x},0)\,\mathrm{pu}(\mathrm{x},0) + \frac{\mathrm{xp}^3}{1+p^2}(\mathrm{r}-1,1-r) + \frac{\mathrm{p}^3}{1+p^2}U(\mathrm{x},\mathrm{p}) \qquad (4.3)$$

Also applying FET boundary condition becomes

$$\underline{U}(1,\mathrm{p};r)=(\mathrm{r}-1),(1-\mathrm{r})\,\mathrm{p} \qquad (4.4)$$

The r-cut representation of (4.3) after using initials conditions is given by

$$x\frac{d}{dx}\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r})=\mathrm{p}^{-2}\underline{U}(\mathrm{x},\mathrm{p},\mathrm{r})-(\mathrm{r}-1)\,\mathrm{x}+\frac{\mathrm{xp}^3}{1+p^2}(\mathrm{r}-1)+\frac{\mathrm{p}^3}{1+p^2}\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r}) \qquad (4.5)$$

And

$$x\frac{d}{dx}\overline{U}(\mathrm{x},\mathrm{p};\mathrm{r})=\mathrm{p}^{-2}\overline{U}(\mathrm{x},\mathrm{p},\mathrm{r})-(1-r)\,\mathrm{x}+\frac{\mathrm{xp}^3}{1+p^2}(\mathrm{r}-1)+\frac{\mathrm{p}^3}{1+p^2}\overline{U}(\mathrm{x},\mathrm{p};\mathrm{r}) \qquad (4.6)$$

From (4.5) we get:

$$x\frac{d}{dx}\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r})-\frac{p^5+p^2+1}{x(p^4+\mathrm{p}^2)}\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r})-\frac{(\mathrm{r}-1)(\mathrm{p}^3-\mathrm{p}^2-1)}{1+p^2}=0 \qquad (4.7)$$

Solving (4.7) we get

$$\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r})=\frac{(\mathrm{r}-1)(\mathrm{p}^5-\mathrm{p}^4-\mathrm{p}^2)}{p^5-p^4+1}x+cx^{\frac{p^5+p^2+1}{p^4+p^2}} \qquad (4.8)$$

On using boundary condition given in (4.4) we get, C = 0 therefore (4.8) become

$$\underline{U}(\mathrm{x},\mathrm{p};\mathrm{r})=\frac{(\mathrm{r}-1)(\mathrm{p}^5-\mathrm{p}^4-\mathrm{p}^2)}{p^5-p^4+1}x \qquad (4.9)$$

Similarly on simplifying (4.6) the following differential equation is obtained:

$$x\frac{d}{dx}\overline{U}(\mathrm{x},\mathrm{p};\mathrm{r})-\frac{p^5+p^2+1}{x(p^4+\mathrm{p}^2)}\overline{U}(\mathrm{x},\mathrm{p};\mathrm{r})-\frac{(\mathrm{r}-1)(\mathrm{p}^3-\mathrm{p}^2-1)}{1+p^2}=0 \qquad (4.10)$$

Which gives the final upper solution of (4.1) as follows

$$\overline{u}(\mathrm{x},\mathrm{p};\mathrm{r})=\frac{(\mathrm{r}-1)(\mathrm{p}^3-\mathrm{p}^2-1)}{1+p^2} \qquad (4.11)$$

Example 4.2.

Let us consider the following

$$u_x=u_{tt}+2(1+r,3-r)\,\mathrm{e}^x-2\int_0^t(\mathrm{t}-\mathrm{s})u(\mathrm{x},\mathrm{s})\,\mathrm{ds} \qquad (4.12)$$

with initial conditions

$ut(x,0;\,r) = (0,0), u(x,0,r) = e^{x}((1+r),(3-r)),$

and Boundary condition

$u(0,t;\,r) = \cos t((r+1),(3-r)).$

Applying FET on (4.11), we have

$E[u_x] = E[u_{tt}] + 2(r-1,1-r)e^{x}E[1] - 2E[t]E[u(x,t)].$ (4.13)

Using definition of FET (4.12) becomes

$$\frac{d}{dx}U(\mathrm{x,p}) = \mathrm{p}^{-2}U(\mathrm{x,p})\,!\ \ \mathrm{pu(x,o)}\,!\ \ pu_t(\mathrm{x},0)\,\mathrm{pu(x,0)} + 2p^2\,\mathrm{e}^x(r+1,3-r) + -2p^3U(\mathrm{x,p}) \qquad (4.14)$$

After using FET boundary condition gives:

$$U(0,\mathrm{p};r) = \frac{(\mathrm{r}+1,3-\mathrm{r})\,\mathrm{p}}{1+p^2} \qquad (4.15)$$

The classical form of (4.13) after using initial conditions, is

$$\frac{d}{dx}\underline{U}(\mathrm{x,p;r}) = \mathrm{p}^{-2}\underline{U}(\mathrm{x,p,r}) - e^x(\mathrm{r}+1) + \frac{2\mathrm{p}^4}{1+p^2}\mathrm{e}^x(\mathrm{r}+1) - 2p^3\underline{U}(\mathrm{x,p,r}) \qquad (4.16)$$

And

$$\frac{d}{dx}\overline{U}(\mathrm{x,p;r}) = \mathrm{p}^{-2}\overline{U}(\mathrm{x,p;r}) - e^x(3-\mathrm{r}) + \frac{2\mathrm{p}^4}{1+p^2}\mathrm{e}^x(3-\mathrm{r}) - 2p^3\overline{U}(\mathrm{x,p,r}) \qquad (4.17)$$

Now solving (4.16) and (4.17) after using boundary condition (4.15) we get:

$$\underline{U}(\mathrm{x,p;r}) = e^x(\mathrm{r}+1)\left(\frac{2\mathrm{p}^4}{1+p^2}-1\right) \qquad (4.18)$$

$$\overline{U}(\mathrm{x,p;r}) = e^x(3-\mathrm{r})\left(\frac{2\mathrm{p}^4}{1+p^2}-1\right) \qquad (4.19)$$

5. CONCLUSION

In this work we investigated the genuine nature of fuzzy ELzaki transform for the plan of under H-differentiability with crisp part. In our knowledge this is the key try toward the game plan of such numerical explanations with fuzzy conditions. We have spoken to the method by enlightening a couple tests. In future we will look at the course of action of FPVIDEs under summed up H-differentiability with both crisp and fuzzy segment.

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