wjert, 2022, Vol. 8, Issue 7, 115-124. Review Article ISSN 2454-695X

World Journal of Engineering Research and Technology
WJERT
www.wjert.org
SJIF Impact Factor: 5.924

CORONA PRODUCT OF COMPLETE GRAPHS

James Githinji Muya[1]* and G. Shobhalatha[2]

[1]Department of Mathematics, Sri Krishnadevaraya University, 515003, Anantapuramu (A.P) India.

[2]Department of Mathematics, Sri Krishnadevaraya University, 515003, Anantapuramu (A.P) India.

Article Received on 13/05/2022 **Article Revised on 03/06/2022** **Article Accepted on 23/06/2022**

***Corresponding Author**
James Githinji Muya
Department of Mathematics, Sri Krishnadevaraya University, 515003, Anantapuramu (A.P) India.

ABSTRACT

Graph labeling has a wide range of applications such as coding theory, X-ray crystallography, network design, circuit design. It can be done by assigning numbers to edges, vertices or both. An anti-magic labeling of a graph G is a one-to-one correspondence between the edge set $E(G)$ and the set $\{1,2,3,\dots,|E|\}$ such that the vertex sums are pairwise distinct. The vertex sum is the sum of labels assigned to edges incident to a vertex. Corona product of the graphs H and T is the graph $H \odot T$ which is obtained by taking one copy of H and $|V(H)|$ copies of T and making the i^{th} vertex of H adjacent to every vertex of the i^{th} copy of T, $1 \leq i \leq |V(H)|$. In this study, we prove that the Corona product $K_n \odot K_m$ generates anti-magic graphs. We also develop a programme using MATLAB to demonstrate this anti-magic property.

Mathematics subject classification: 05C76, 05C78

KEYWORDS: Complete graph, Connectedness, corona product, anti-magic labeling.

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Graph theory and its branches have become topics of interest in various fields of Mathematics and other areas of sciences. Graph labeling is an assignment of integers to vertices, edges or both. In this paper we have assigned the integers to edges and obtained the labels of vertices by adding the labels of edges incident at each vertex. Suppose $G = (V, E)$ is a graph and $f: E \to \{1,2, \ldots, |E|$ is a bijective mapping. For each vertex n of G, the vertex sum $\varphi_f(n)$ at n is defined as $\varphi_f(n) = \sum_{e \in E(n)} f(e)$, where $E(n)$ is the set of edges incident on n. If $\varphi_f(n) \neq \varphi_f(t)$ for any two distinct vertices $n, t \in V(G)$, then f is called an anti-magic labeling of G. (Hartsfield and Ringel, 1990) introduced anti-magic labeling and they put forth the following conjecture.

Conjecture: (Hartsfield and Ringel, 1990) *Every connected graph other than K_2 is anti-magic.*

The conjecture remains open and research continues to be carried out concerning it. (Liang et al., 2014) studied the anti-magic labeling of trees. (Sugeng, 2005) looked at magic and anti-magic labeling of graphs. Other works on anti-magic labeling have been done by (Alon et al., 2004), (Wang and Hsiao, 2008) and (Berczi, 2015). For exhaustive survey of anti-magic graphs, we refer to (Gallian, 2020).

There are various products of graphs such as lexicographic product, cartesian product, strong product, direct product and corona product. In this paper we have taken an interest in Corona product. It was introduced by (Frucht and Harary, 1970). Corona product of G and H is the graph $G \odot H$, where G has n_1 vertices, e_1 edges, and H has n_2 vertices, e_2 edges. The graph $G \odot H$ is obtained by taking one copy of G and n_1 copies of H and making the i^{th} vertex of G adjacent to every vertex of the i^{th} copy of H, where $1 \leq i \leq n_1$. (Nada et al., 2017) investigated the cordiality of Corona between cycles and paths. (Dong et al., 2018) proved that Lexicographic product graphs $P_m[P_n]$ are anti-magic. (Muya and Shobhalatha, 2022) proved that the corona product of wheel graph $(W_{n,}\ 4 \leq n \leq 8)$ with complete graph $(K_m, m \geq 2)$ is anti-magic. In this study the aim is to prove the conjecture. It is proved that the Corona product of complete graphs (K_n) and complete graphs (K_m) given by $K_n \odot K_m$ is anti-magic. A programme is also developed using MATLAB to have a better demonstration of this anti-magic property.

2. Preliminaries

Definition 2.1

The degree of a vertex v_i is the number of edges incident on v_i and is denoted by $d(v_i)$.

Definition 2.2

A complete graph is a graph in which every two distinct pair of vertices are joined by an edge. A complete graph with n vertices is denoted by K_n and it is $(n-1)$ regular.

3. Main Results

(Hartsfield and Ringel, 1990) proved that complete graphs are anti-magic. In this paper it is proved that the corona product of the complete graph$\{K_n, n \geq 2\}$ with the complete graph $\{K_{m,}\ , m \geq 2\}$ is anti- magic.

Lemma 3.1: Corona product $G \odot H$ of connected graphs G and H is a connected graph

Proof

A graph is connected if there is at least a path between every pair of distinct vertices. Since graph G and graph H are each connected, then there is at least a path between every pair of distinct vertices in graph G as well as in graph H. On performance of Corona product, every i^{th} vertex of graph G is made to be adjacent to every vertex of the i^{th} copy of graph H. It then follows that there is at least a path between every pair of distinct vertices in $G \odot H$ since every i^{th} vertex of graph G is connected to every vertex of the i^{th} copy of graph H. Hence $G \odot H$ is a connected graph. In figure 1 we have graph G, figure 2 is graph H and figure 3 shows $G \odot H$.

Figure 1: Graph G.

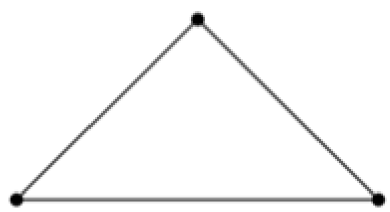

Figure 2: Graph H.

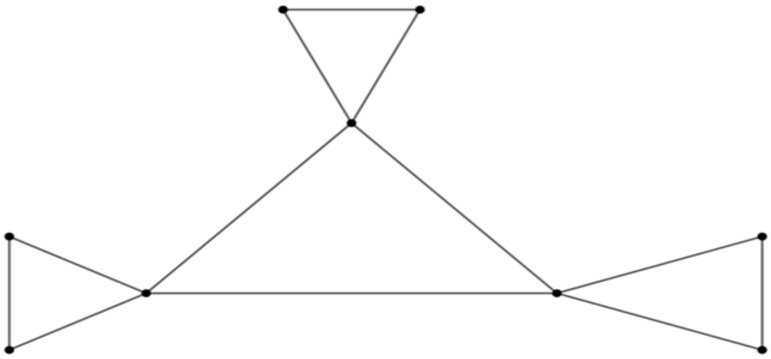

Figure 3: $G \odot H$.

It then follows that the graph $K_n \odot K_m$ is connected.

Theorem 3.2: Corona product $K_n \odot K_m$ is not commutative

Proof

By definition $K_n \odot K_m$ has $n + nm$ vertices and $\left[n\left(\frac{m(m+1)}{2}\right) + \frac{n(n-1)}{2}\right]$ edges. This is as a result of centering K_n and connecting the i^{th} vertex to every vertex of i^{th} copy of K_m. On the other hand, $K_m \odot K_n$ has $m(n+1)$ vertices and $\left[\frac{mn(n+1)}{2} + \frac{m(m-1)}{2}\right]$ edges. This is done by centering K_m and connecting the i^{th} vertex to every vertex of i^{th} copy of K_n. The vertex sets and the edge sets are not the same for any $n \neq m$, then $K_n \odot K_m \neq K_m \odot K_n$. Thus $K_n \odot K_m$ is not commutative.

Theorem 3.3: Corona product $K_n \odot K_m$ is not associative

Proof

Let G be a graph with g vertices and t edges. The corona product $G \odot (K_n \odot K_m)$ has $[g + gn(1+m)]$ vertices while corona product $(G \odot K_n) \odot K_m$ has $g(1+n)(1+m)$ vertices.. The vertex sets are not the same for any n and m. On the other hand, corona product $G \odot (K_n \odot K_m)$ has $\left\{t + g\left[n(m+1) + \frac{nm(m+1)}{2} + \frac{n(n-1)}{2}\right]\right\}$ edges while corona product $(G \odot K_n) \odot K_m$ has $\left[\frac{gm(m+1)(n+1)}{2} + t + \frac{gn(n+1)}{2}\right]$ edges. The edge sets are not the same for any n and m. Thus $G \odot (K_n \odot K_m) \neq (G \odot K_n) \odot K_m$. Hence $K_n \odot K_m$ is not associative.

Theorem 3.4: Corona product of complete graph and complete graph $K_n \odot K_m (n, m \geq 2)$ is anti-magic.

Proof

A complete graph K_n has n vertices and $\frac{n(n-1)}{2}$ edges. Let us arrange the vertices as per increasing order of vertex labels v_i, $1 \le i \le n$. Let us label the edges as $\varphi(e_i) = i,\ 1 \le i \le \frac{n(n-1)}{2}$.

Labeling technique

For the complete graph K_m, let us denote the vertices by $\{u_{1i}, u_{2i}, \dots, u_{mi}\}, 1 \le i \le n$. Now let us define the edge labels by the function ψ. For the original edges of K_m, we label the edges as follows:

For $i = 1$, we have

$\psi(u_{11}, u_{21}) = 1, \psi(u_{11}, u_{31}) = 2, \dots, \psi(u_{11}, u_{m1}) = m - 1$

$\psi(u_{21}, u_{31}) = m, \psi(u_{21}, u_{41}) = m + 1, \dots, \psi(u_{21}, u_{m1}) = 2m - 3$

$\psi(u_{31}, u_{41}) = 2m - 2, \psi(u_{31}, u_{51}) = 2m - 1, \dots, \psi(u_{31}, u_{m1}) = 3m - 6$

.

.

.

$\psi(u_{(m-1)1}, u_{m1}) = \frac{m(m-1)}{2}$.

Since the number of new edges added to each vertex of the wheel is given by $\frac{m(m+1)}{2}$, then we have:

for $i = 2, 3, 4, \dots, n$ the edge labels

$\psi(u_{1i}, u_{2i}), \dots, \psi(u_{1i}, u_{mi}), \psi(u_{2i}, u_{3i}), \dots, \psi(u_{2i}, u_{mi}), \dots, \psi(u_{(m-1)i}, u_{mi})$ are

obtained by taking the corresponding edge labels for $i = 1$, and adding them to $\frac{(i-1)m(m+1)}{2}$.

Hence the edge labels are given by;

$\psi(u_{li}, u_{ji}) = m(l-1) + j - \frac{l(l+1)}{2} + \frac{m(i-1)(m+1)}{2}$,

$1 \le l \le m - 1,\ 2 \le j \le m,\ 1 \le i \le n,\ l < j$ 1

For the edges joining K_n to K_m the edge labels are given by:

For $i = 1$, we have

$$\psi(u_{11}, v_1) = \frac{m(m-1)}{2} + 1,$$

$\psi(u_{21}, v_1) = \frac{m(m-1)}{2} + 2,$

$$\psi(u_{31}, v_1) = \frac{m(m-1)}{2} + 3,$$

.

.

.

$$\psi(u_{m1}, v_1) = \frac{m(m-1)}{2} + m.$$

for $i = 2, 3, 4, \ldots, n$ the edge labels $\psi(u_{1i}, v_i), \psi(u_{2i}, v_i), \ldots, \psi(u_{mi}, v_i)$ are obtained by taking the corresponding edge labels for $i = 1$, and adding them to $\frac{(i-1)m(m+1)}{2}$.

Hence the edge labels are given by;

$$\psi(u_{li}, v_i) = \frac{m(m-1)}{2} + l + \frac{m(i-1)(m+1)}{2}, \; 1 \le l \le m, \; 1 \le i \le n. \qquad 2$$

From the definition of ψ, we observe that the edge labels of K_m and edge labels of the edges joining K_n to K_m are distinct and the edge labels are chosen from the set $\left\{1,2,3, \ldots, \frac{nm(m+1)}{2}\right\}$.

For the original edges of K_n, since the total number of new edges is $\frac{nm(m+1)}{2}$, then the edge label for edges of K_n are given by:

$$\psi(e_i) = \frac{nm(m+1)}{2} + i, \; 1 \le i \le \frac{n(n-1)}{2}. \qquad 3$$

It is observed that from the definition of edge labels of edges of K_n the edge labels of K_n are distinct and the edge labels are chosen from the set

$$\left\{\frac{nm(m+1)}{2} + 1, \frac{nm(m+1)}{2} + 2, \ldots, \left[\frac{nm(m+1)}{2} + \frac{n(n-1)}{2}\right]\right\}.$$

We observe that the edge labels of $K_n \odot K_m$ are distinct and they are from the set $\left\{1,2,3, \ldots, \left[\frac{nm(m+1)}{2} + \frac{n(n-1)}{2}\right]\right\}$.

The vertex sum defined by φ_ψ for the vertices of $K_n \odot K_m$ are distinct.

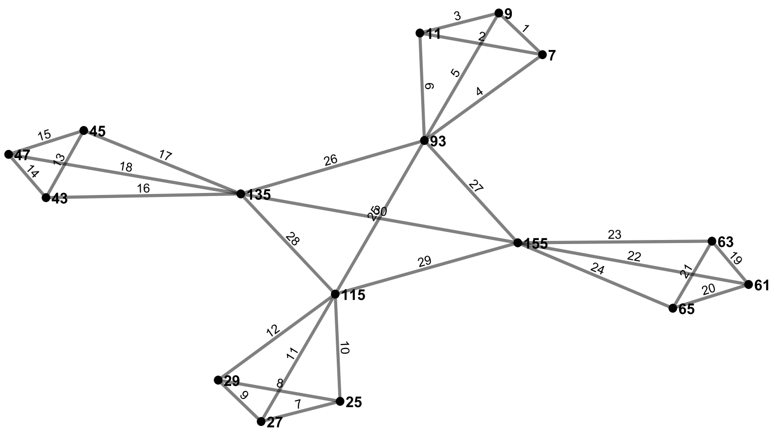

Figure 4: shows the corona product $K_4 \odot K_3$.

Figure 4: $K_4 \odot K_3$ ***is anti-magic***

The following is the programme developed using MATLAB to show the anti-magic properties of $K_n \odot K_m$. This programme has been very useful in verifying the anti-magic property for large values of n and m.

Listing 1: Matlab code for generalizing the anti-magic labeling of $K_n \odot K_m$

```
clear
n=4;
m=3;
%%
% _Plotting the complete graph_


Mat = ones(n);
Mat = Mat - diag(diag(Mat));
Kn = graph(Mat);
% plot(Kn)
for a=1:0.5*n*(n-1)
    c(a)=(m^2+m)*0.5*n+a;
end
Wf = c;
Kn.Edges.Weight =Wf';
% plot(Kn,"EdgeLabel",Kn.Edges.Weight)
Whw = Kn.Edges.Weight;
Whw = Whw';

%%
% _Plotting the complete graph_

Mat1 = ones(m);
Mat1 = Mat1 - diag(diag(Mat1));
Km = graph(Mat1);
% plot(Km)
%%
e1 = Km.Edges;
e1 = table2array(e1);
s1 = [e1(:,1)];
s1 = s1';
t1 = [e1(:,2)];
t1 = t1';
s11 = ones(length(s1));
t11 = ones(length(t1));
%%
for i = 1:n
    s2 = (n+((i-1)*m)).*[s11(1,:)];
    s = s2 + s1;
```

```
    t2 = (n+((i-1)*m)).*[t11(1,:)];
    t = t2 + t1;
    smin = min(s);
    tmax = max(t);

    weight1 = ones(length(t1));
    weight1 = [weight1(1,:)];
    weight2 = ones(length(t));
    weight2 = [weight2(1,:)];
    Kn = addedge(Kn,s,t,weight2);
end
% plot(Kn)
%%
% _Joining the complete graph to the complete graph_

for  i=1:n
    for l=1:(m-1)
        for j=2:m
            if j>l
               w4(i,j,l)=m*l-m+j-0.5*(l*(l+1))+0.5*(i-1)*m*(m+1);
            end
        end
    end
end
ww =pagectranspose(w4);
ww = nonzeros(w4);
Wkm = ww(:);
Wkm = Wkm';
Wkm = sort(Wkm);
Whwkm = [Whw,Wkm];
Kn.Edges.Weight = Whwkm';
%%
for l= 1:m
    for i= 1:n
        Wcon(i,l) = 0.5*m*(m-1) +l + 0.5*m*(m+1)*(i-1);
    end
end
Wcon = sort(Wcon(:));
Wcon = Wcon';
%%
for jj = 1:n
    s2 = (n+((jj-1)*m)).*[s11(1,:)];
    sjj = s2 + s1;
    t2 = (n+((jj-1)*m)).*[t11(1,:)];
    tjj = t2 + t1;
    swh = ones(m);
    swh = swh(1,:);
    smin2 = min(sjj);
```

tmax2 = max(tjj);
strt = 1+(jj-1)*m;
tmt = jj*m;
Wconnect = Wcon(strt:tmt);
Kn = addedge(Kn,jj.*swh,[smin2:tmax2],Wconnect);
end
%%

CONCLUSION

In this paper we have proved that the corona product of $K_n \odot K_m$ produces a graph which is anti-magic. We have also developed a programme using MATLAB to demonstrate this property. This programme is useful in obtaining the anti-magic property for large values of n and m which would otherwise be very tiresome if done manually. Further study can be extended to corona product of complete graphs with other graphs.

REFERENCES

1. Alon, N., Kaplan, G., Lev, A., Roditty, Y., & Yuster, R. *Dense Graphs Are Antimagic*, 2004; 297–309. https://doi.org/10.1002/jgt.20027.
2. Bérczi, K., Bernáth, A., & Vizer, M. Regular Graphs are Antimagic. *The Electronic Journal of Combinatorics*, 2015; *22*(3): 1–7. https://doi.org/10.37236/5465.
3. Frucht, R. and Harary, F. On the corona of two graphs. Aequationes mathematicae, 1970; 4(3): 322-325.
4. Gallian, J. A. A Dynamic Survey of Graph Labeling, 2019.
5. J. G. Muya, G. Shobhalatha Anti-magic graphs as a result of corona product of wheels and complete graphs. Journal of the Calcutta mathematical society, 2022; 18(1): 59-68.
6. Liang, Y. C., Wong, T. L., & Zhu, X. Anti-magic labeling of trees. Discrete Mathematics, 2014; 331: 9–14. https://doi.org/10.1016/j.disc.2014.04.021.
7. Ma, W., Dong, G., Lu, Y., & Wang, N. Lexicographic product graphs P m [P n] are antimagic. AKCE International Journal of Graphs and Combinatorics, 2018; 15(3): 271–283. https://doi.org/10.1016/j.akcej.2017.10.005.
8. Nada, S., Elrokh, A., Elsakhawi, E. A., & Sabra, D. E. Journal of the Egyptian Mathematical Society The corona between cycles and paths. Journal of the Egyptian Mathematical Society, 2017; 25(2): 111–118. https://doi.org/10.1016/j.joems.2016.08.004
9. N. Hartsfield, G. Ringel: *Pearls in Graph Theory*, Academic Press, INC, Boston, 1990; 180-109.
10. Sugeng, K. A. *Magic and Antimagic Labeling of Graphs*. *October*, 2005.

11. Wang, T. M., & Hsiao, C. C. On anti-magic labeling for graph products. Discrete Mathematics, 2008; 308(16): 3624–3633. https://doi.org/10.1016/J.DISC.2007.07.027.