

**MATHEMATICAL ANALYSIS OF MAGNETIC HYDRODYNAMICS
IN UNSTEADY FREE CONVECTION IN FLOW PAST AN
EXPONENTIALLY ACCELERATED INFINITE NON-CONDUCTING
VERTICAL PLATE**

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ABSTRACT

Buoyancy driven convection is the most significant behavior experienced by any system undergoing fluid motion as a result of heating in a steady and unchanging gravitational field. It is well known that when a fluid is heated it receives an intake of internal energy and therefore it undergoes thermal expansion and a decrease in density. As a result, heat is transferred at a rate significantly faster than if it was just through thermal diffusion alone. These phenomena occur on the

surface of our planet effecting many natural and technological processes. In this paper, variable natural convection and mass transfer flow past an exponentially accelerated infinite non-conducting vertical plate through a porous medium have been analysed in the presence of uniform transverse magnetic field using Boussinesq's approximation flow model.

KEYWORDS: Boussinesq's flow, Magnetohydrodynamics (MHD), Prandtl Number (Pr), Reynold Number (Re), Schmidt Number(Sc), Velocity field distribution, Skin Factor.

1. INTRODUCTION

Magnetohydrodynamics is a branch of fluid dynamics which studies the movement of an electrically-conducting fluid in a magnetic field. Magnetohydrodynamics (MHD) is a study about fluid flow movement that affected by magnetic fields and can conduct electric

current.^{[1],[23]} The basic concept of MHD is that magnetic fields can induce currents electricity in the conductive fluid that moves, so it produces the forces in the fluid and also alters the magnetic field itself.^[2] It is related to application in engineering and industry. The examples of MHD application are the power station, the nuclear reactor, the refrigerators, the generator, and the magnetohydrodynamic reactor.^[24]

Faraday first pointed out an interaction of sea flows with the Earth is magnetic field (1832). In the beginning of the 20th century the first proposals for applying electromagnetic induction phenomenon in technical devices with electrically-conducting liquids and gases appeared. Magneto hydrodynamics is characterized by dimensional parameters which include, in addition to the conventional hydrodynamic parameters (Re, Pr, etc.). Boundary conditions used in MHD equations are formulated by traditional hydrodynamics and electrodynamics methods. External conditions represent the paths of electrical current and magnetic field lines outside the flow volume, and in particular, the configuration of external electrical circuit and the type of magnet system. These properties of MHD flows are analogous to those of conventional hydrodynamics derived from theorems on vortex and circulation of velocity.

The transport of heat and fluid moving through a porous media is a phenomenon of great interest from both application and theory points of view. Mass transfer in isothermal conditions has been studied under the generic name Hydrodynamic dispersion with application to problems of mixing of fresh and salt waters in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils, etc. Heat transfer, in case of a homogeneous fluid, has been studied systematically, but rather extensively, with relation to different applications, like dynamics of hot underground springs, terrestrial heat flow through aquifers, hot fluid and ignition front displacements in reservoir engineering, heat exchange (with evaporation and condensation) between surface soil and atmosphere, flow of moisture through industrial material s, heat exchanger with fluidized beds etc.

Literature Review

Over the last few decades, the free convection in the cavity occurs in a variety of practical applications, e.g., semiconductor industry, solar energy collectors, electronic device cooling, etc.^[3,4] In some engineering disciplines, dealing with the convective heat transfer has become one of the focuses in thermal management and optimizing efficiency. There are several approaches to intensify heat transfer, including but not limited to the increase in contact area

between liquid and solid surface, the change of mass flow rate of working fluid, the high thermally conductive material, and so on.^[5,6] The interaction between flow and magnetic field, so-called Magnetohydrodynamics (MHD), has been attracting considerable attention in many areas of applied science. In early 1980, Oreper and Szekely^[7] explored the influence of an applied magnetic field on the crystal growth by numerical simulations. They indicated that the natural convective flow was retarded by the magnetic field and the crystal formation was affected apparently by the magnetic field intensity. Garandet et al.^[8] derived analytical expressions to study the free convective flow within a 2-dimensional cavity under the transverse magnetic field, and showed that the distributions of velocity and temperature in the core region followed the power law. A global approximating function involving a series expansion is used to examine the flow circulation near the vertical walls of an enclosure. Rudraiah et al.^[9] performed a numerical investigation of MHD free convection within the rectangular enclosure filled with electrically conducting fluid and subjected to magnetic field parallel to the gravity. In the conditions of low Ha number and high Grashoff number, the convection is dominant in the center of a rectangular enclosure, leading to the predominant vertical temperature stratification. Recently, Geridonmez and Oztop^[10] investigated the influence of magnetic field incident from either the left bottom corner of hot wall or the right bottom corner of cold wall on the MHD convection in the square cavity. The flow of an incompressible viscous fluid past an impulsively started, infinite horizontal flat plate, in its own plane, was studied first by Stokes.^[11] Soundalgekar^[12] has derived an exact solution for the flow past an impulsively started infinite vertical plate in its own plane known as the Stokes problem for the vertical plates. Recently Soundalgekar^[13], Raptis^[14] and Georgantopoulos et al.^[15] have studied the free convection flow past an accelerated vertical infinite plate. The effect of mass transfer on the flow past a uniformly accelerated vertical plate which is either at uniform temperature or it is supplied heat at constant rate is studied by Soundalgekar.^[16] Yamamoto and Iwamura^[17] investigated the flow with convective, acceleration through a porous medium. Gulab Ram and Mishra^[18] and Varshney^[19] applied these equations to study the MHD flow of conducting fluid through porous medium. Effects of free convection and mass transfer flow through a porous medium are studied by Raptis et al.^[20,21] and Raptis.^[22]

2. Mathematical Analysis

An unsteady motion of an electrically conducting viscous incompressible fluid along an infinite non-conducting vertical flat plate through a porous medium has been considered. The x-axis is taken along the infinite plate in the upward direction and y'-axis normal to the plate. A magnetic field of uniform strength is applied in the direction of flow.

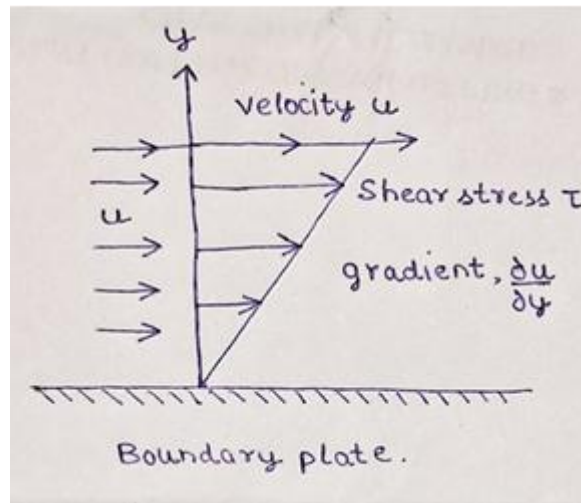


Figure 1: Schematic diagram showing Boussinesq's flow onto Accelerated moving vertical plate.

The magnetic Reynolds Number of the flow is taken to be small enough so that the induced magnetic field can be neglected. At time $t' > 0$, the plate is exponentially accelerated with a velocity u' which is given by expression.

$$u' = Ue^{at'}$$

where a' and t' are acceleration and corresponding time in its own plane and its temperature and concentration instantaneously rises or falls to T'_w , C'_w respectively from its original values of T and C which is thereafter maintained constant.

Nomenclature:

u' = velocity of exponentially accelerated vertical plate

U = Velocity of flow

Sc = Schmidt Number

K = Permeability Parameter

Gr = Grashoff Number

M = Magnetic Number

Gm = Modified Grashoff Number

Pr = Prandtl Number

t = time duration of flow

τ = Skin friction factor

a = Distance from origin

ν = Kinematic viscosity

θ = Non dimensional temperature parameter

Boussinesq's Flow Model

The Boussinesq equations for stratified flow (e.g. of the atmosphere or ocean) assume that fluid flow is incompressible yet convects a diffusive quantity that endows the fluid with positive or negative buoyancy. This buoyancy quantity is identified with a linear function of the deviation of temperature or density from adiabatic hydrostatic balance. Then under usual Boussinesq's Approximation the flow can be shown to be governed by the system of following non dimensional equation.

$$\frac{\delta u}{\delta t} = \frac{\delta^2 u}{\delta y^2} + G_r \theta + G_m \theta^* - \left[M + \frac{1}{k} \right] u Pr \frac{\partial \theta}{\delta t} = \frac{\delta^2 \theta}{\delta t^2} \quad (1)$$

$$Pr \frac{\delta \theta}{\delta t} = \frac{\delta^2 \theta}{\delta t^2} \quad (2)$$

$$Sc \frac{\delta \theta^*}{\delta t} = \frac{\delta^2 \theta^*}{\delta y^2} \quad (3)$$

With initial and final boundary conditions

$$t \leq 0: u(y, t) = 0, \theta(y, t) = 0, \theta^*(y, t) = 0 \quad (4)$$

$$t > 0 \{ u(y, t) = e^{at}, \theta(0, t) = 1, \theta^*(y, t) = 1 \quad (5)$$

$$t > 0: u(\infty, t) = 0, \theta(\infty, t) = 0, \theta^*(\infty, t) = 0 \quad (6)$$

Introducing non dimensional parameters

$$r = \frac{r^1}{\nu}, t = t^* \frac{U^2}{\nu}, u = \frac{u'}{U}, a = a' \frac{\nu}{U^2}$$

$$Sc = \frac{\nu}{D}, \theta = \frac{(T' - T'_\infty)}{(T'_\omega - T'_\infty)}, \theta^* = \frac{(C' - C'_\infty)}{(C'_\omega - C'_\infty)}$$

$$K = U^2 K' / \nu^2$$

Magnetic Reynold Number M

$$M = \frac{\sigma B_0^2 \nu}{\rho u^2}$$

Prandtl Number Pr

$$Pr = \frac{vC_p\rho}{k}$$

Grashoff Number Gr

$$Gr = \frac{vg\beta(T'_\omega - T'_\infty)}{U^3}$$

Magnetic Grashoff Number Gm

$$Gm = \frac{vg\beta^*(C'_\omega - C'_\infty)}{U^3}$$

Solving (1), (2) and (3) under boundary conditions (4), (5) and (6) using Laplace transforms we get

Case 1: For $Pr = Sc \neq 1$

$$\begin{aligned} U = & \frac{1}{2} e^{at} \left\{ e^{y\sqrt{L+a}} \operatorname{erf} \left(\frac{u}{2\sqrt{t}} \right) + \sqrt{(L+a)t} + e^{-y\sqrt{L+a}} \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \sqrt{(L+a)t} - \right. \\ & \left. \left(\frac{Gm+Gr}{2} \right) \left\{ e^{y\sqrt{L}} \operatorname{erf} \left(\frac{y}{2\sqrt{t}} + \sqrt{Lt} \right) - e^{-y\sqrt{L}} \operatorname{erf} \left(\frac{y}{2\sqrt{t}} - \sqrt{Lt} \right) + \frac{Gr}{2L} e^{\left(\frac{Lt}{Pr-1} \right)} \left[e^{-y\sqrt{\frac{LPr}{Pr-1}}} \left\{ \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \right. \right. \right. \right. \\ & \left. \left. \left. \sqrt{\frac{LPr}{Pr-1}} \right\} - \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Pr}{t}} \right) - \sqrt{\frac{LPr}{Pr-1}} \right\} + e^{-y\sqrt{\frac{LPr}{Pr-1}}} \left\{ \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \sqrt{\frac{LPr}{Pr-1}} \right\} - \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Pr}{t}} \right) - \sqrt{\frac{Lt}{Pr-1}} \right\} + \\ & \frac{Gm}{2L} e^{\left(\frac{Lt}{Sc-1} \right)} \left[e^{-y\sqrt{\frac{LSc}{Sc-1}}} \left\{ \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \sqrt{\frac{LSc}{Sc-1}} \right\} - \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Sc}{t}} \right) - \sqrt{\frac{Lt}{Sc-1}} \right\} + e^{y\sqrt{\frac{LSc}{Sc-1}}} \left\{ \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \sqrt{\frac{LSc}{Sc-1}} - \right. \\ & \left. \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Sc}{t}} \right) + \sqrt{\frac{Lt}{Sc-1}} \right\} + \frac{Gr}{L} \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Pr}{t}} \right) + \frac{Gm}{L} \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Sc}{t}} \right) \end{aligned} \quad (7)$$

Where

$$\theta(y, t) = \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Sc}{t}} \right) \quad (8)$$

$$\theta^*(y, t) = \operatorname{erf} \left(\frac{y}{2} \sqrt{\frac{Pr}{t}} \right) \quad (9)$$

and the parameter taken as L is expressed as

$$L = M + \frac{1}{K}$$

Case 2: For $Pr = Sc=1$

$$\begin{aligned} U = & \frac{1}{2} e^{at} \left\{ e^{y\sqrt{L+a}} \operatorname{erf} \left(\frac{u}{2\sqrt{t}} \right) + \sqrt{(L+a)t} + e^{-y\sqrt{L+a}} \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) - \sqrt{(L+a)t} \right\} - \\ & \frac{1}{2} \left(\frac{Gr+Gm}{L} \right) \left[e^{y\sqrt{t}} \left[\operatorname{erf} \left(\frac{y}{2\sqrt{t}} + \sqrt{Lt} \right) + e^{-y\sqrt{t}} \left[\operatorname{erf} \left(\frac{y}{2\sqrt{t}} - \sqrt{Lt} \right) \right] + \left(\frac{Gr+Gm}{L} \right) \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) \right] \end{aligned} \quad (10)$$

$$\theta(y, t) = \theta^*(y, t) = \operatorname{erf} \left(\frac{y}{2\sqrt{t}} \right) \quad (11)$$

Skin Function (τ) at the plate:

Case 1: For $Pr = Sc \neq 1$

For the given plate the skin factor is expressed as:

$$\tau = \frac{\delta u}{\delta y} \text{ at } y = 0$$

$$\tau = e^{at} \operatorname{erf}(\sqrt{(L+a)t}) (\sqrt{L+a}) + \frac{1}{\sqrt{\pi t}} e^{-Lt} - \left(\frac{Gr+Gm}{L}\right) \sqrt{t} \operatorname{erf}(\sqrt{Lt}) + \frac{Gr}{L} \left(\sqrt{\frac{LPr}{Pr-1}}\right) e^{\left(\frac{Lt}{Pr-1}\right)} \left[\operatorname{erf}\left(\sqrt{\frac{LPr}{Pr-1}}\right) - \operatorname{erf}\left(\sqrt{\frac{Lt}{Pr-1}}\right) \right] + \frac{Gm}{L} \left(\sqrt{\frac{LSc}{Sc-1}}\right) e^{\left(\frac{Lt}{Sc-1}\right)} \left[\operatorname{erf}\left(\sqrt{\frac{LSc}{Sc-1}}\right) - \operatorname{erf}\left(\sqrt{\frac{Lt}{Sc-1}}\right) \right] \quad (12)$$

Case 2: For $Pr = Sc=1$

$$\tau = \frac{\delta u}{\delta y} \text{ at } y = L$$

$$\tau = e^{at} \operatorname{erf}(\sqrt{(L+a)t}) (\sqrt{L+a}) + \frac{2}{\sqrt{\pi t}} e^{-Lt} - \left(\frac{Gr+Gm}{L}\right) \left[\frac{1}{\sqrt{\pi t}} - \sqrt{L} \operatorname{erf}(\sqrt{Lt})\right] \quad (13)$$

OBSERVATION

For finding the values pertaining to the velocity field distribution during the flow, the values of y/Gr , y/Gm , y/Sc , y/t , y/a , y/M and y/K are plotted in tabular form as below.

Table 1: Velocity field distribution for Gr at $t=2$, $M=1$, $k=2$, $a=0.5$, $Gm=0.5$ and $Sc=0.22$.

y/Gr	-2	2	4
0	1.105	1.105	1.105
0.5	0.824	1.139	1.296
1.0	0.567	0.778	0.884
1.5	0.567	0.440	0.477
2.0	0.209	0.224	0.231
2.5	0.102	0.104	0.105
3.0	0.0429	0.0430	0.0431

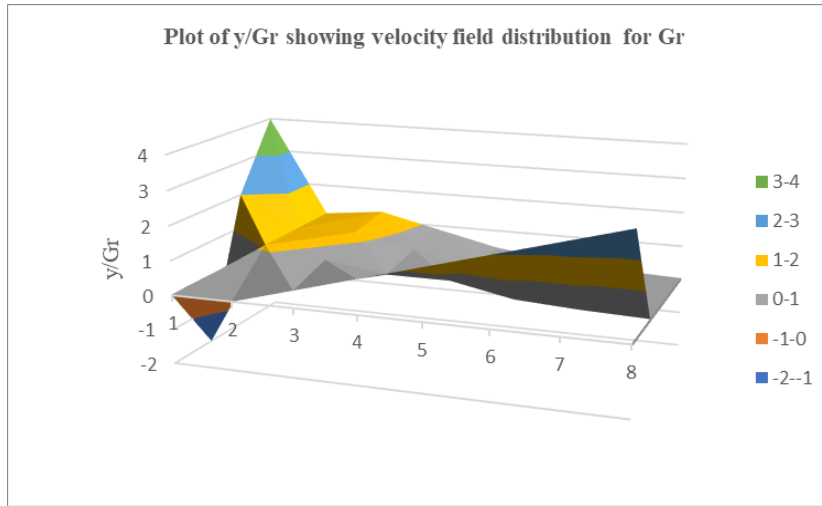


Figure 2: Velocity field distribution plot for Gr.

Table 2: Velocity field distribution for Gm at Gr=2, t=0.4, M=1, Pr=0.71, k=2, a=0.5, Gm=0.5 and Sc=0.75.

y/Gm	0	0.5	5
0	1.221	1.221	1.221
0.5	0.761	0.812	1.279
1.0	0.415	0.462	0.889
1.5	0.192	0.219	0.471
2.0	0.074	0.086	0.196
2.5	0.0233	0.0275	0.064
3.0	0.0061	0.0071	0.0166

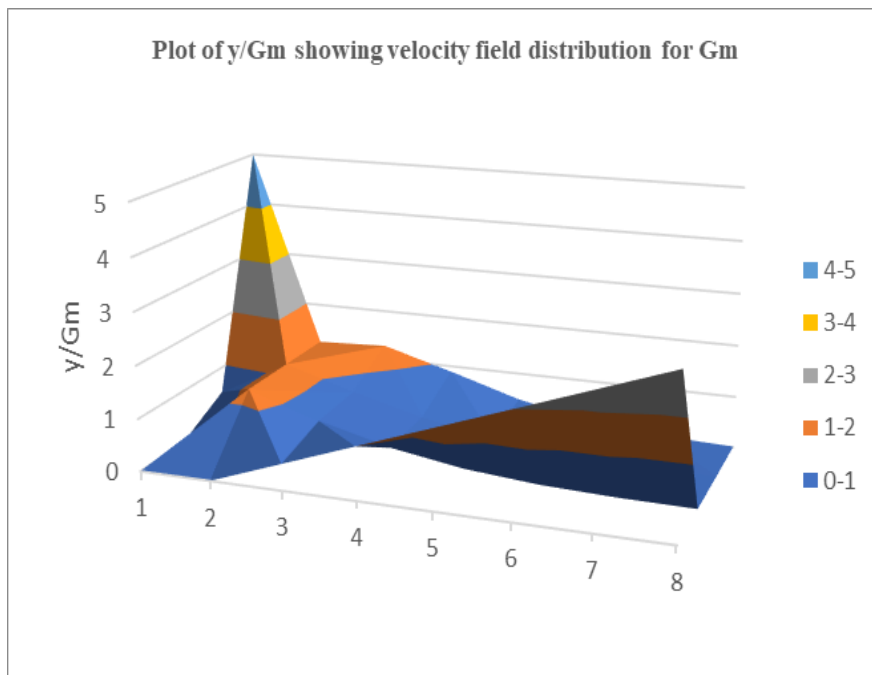


Figure 3: Velocity field distribution plot for Gm.

Table 3: Velocity field distribution for Sc at $Gr=2$, $t=0.4$, $M=1$, $Pr=0.71$, $k=2$, $a=0.5$, $Gr=4.0$ and $Gm=5.0$.

y/Sc	0.22	0.75	1.5
0	1.221	1.221	1.221
0.5	1.814	1.498	1.264
1.0	1.628	1.094	0.793
1.5	1.197	0.596	0.377
2.0	0.788	0.253	0.145
2.5	0.487	0.084	0.0458
3.0	0.289	0.022	0.119

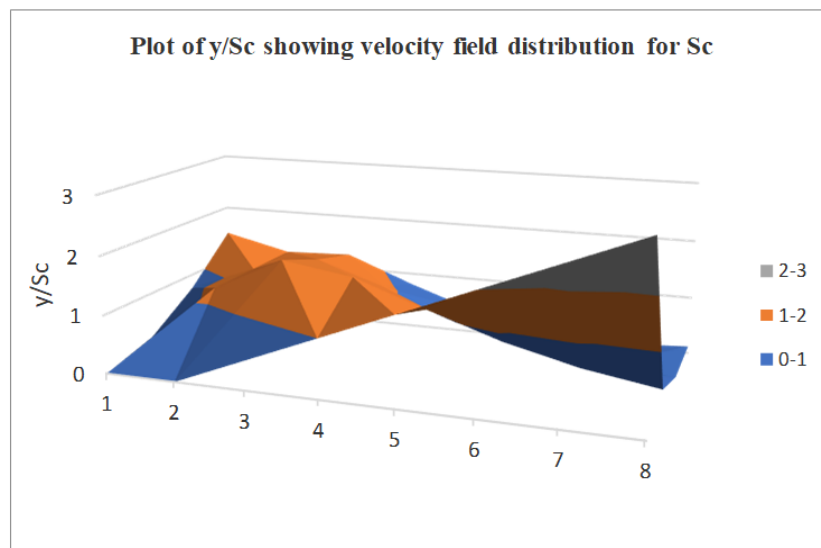


Figure 4: Velocity field distribution plot for Sc

Table 4: Velocity field distribution for t at $Gr=4.0$, $t=0.4$, $M=1$, $Pr=0.71$, $k=1$, $a=0.5$, $Gm=5.0$ and $Sc=0.22$.

y/t	0.2	0.4
0	1.105	1.221
0.5	1.205	1.648
1.0	0.802	1.425
1.5	0.431	1.027
2.0	0.209	0.6671
2.5	0.095	0.413
3.0	0.039	0.245

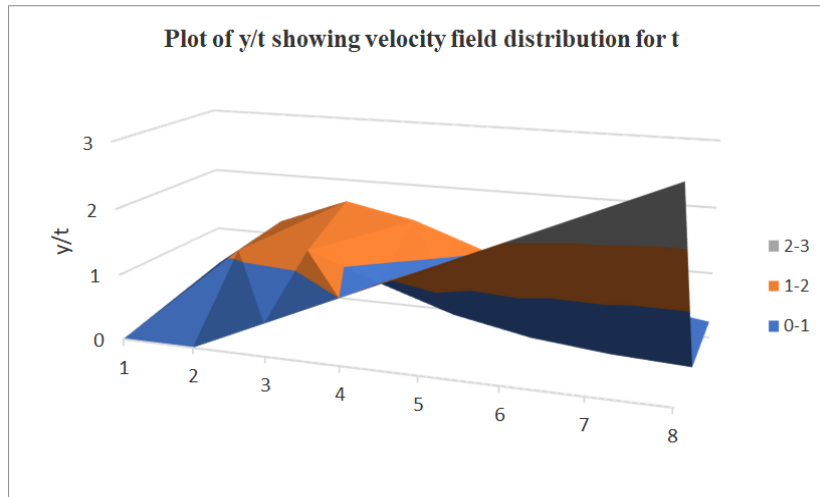


Figure 5: Velocity field distribution plot for t.

Table 5: Velocity field distribution for a at Gr=4.0, t=0.4, Pr=0.71, k=2, a=0.5, Gm=5.0, Sc=1.5 and t=0.4.

y/a	0	0.5
0	1.00	1.221
0.5	1.292	1.358
1.0	0.924	0.942
1.5	0.489	0.493
2.0	0.2025	0.2032
2.5	0.06617	0.6627
3.0	0.01716	0.1717

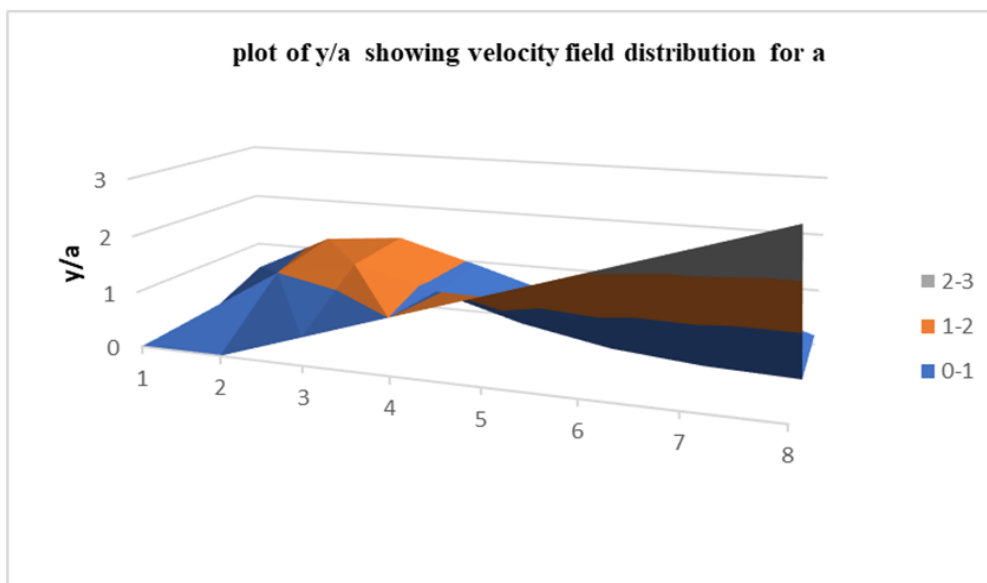


Figure 6: Velocity field distribution plot for a.

Table 6: Velocity field distribution for M at Gr=4.0, t=0.4, Pr=0.71, k=1, a=0.5, Gm=5.0, Sc=1.5 and t=0.4.

y/M	0.5	1.0
0	1.221	1.221
0.5	1.264	1.169
1.0	0.793	0.0699
1.5	0.377	0.319
2.0	0.145	0.119
2.5	0.046	0.036
3.0	0.119	0.0093

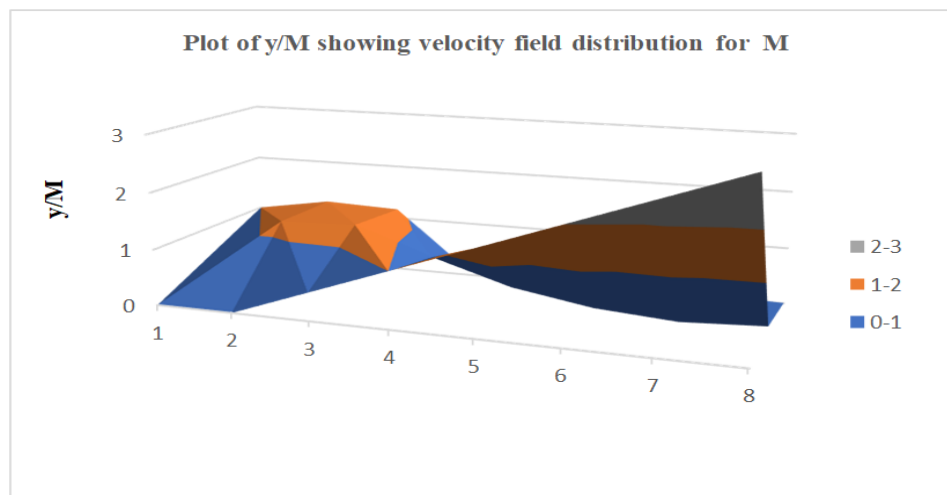


Figure 7: Velocity field distribution plot for M.

Table 7: Velocity field distribution for K at Gr=4.0, t=0.4, Pr=0.71, M=1, k=2, a=0.5, Gm=5.0, Sc=1.5 and t=0.2.

y/K	1	2
0	1.1052	1.1052
0.5	0.7929	0.8495
1.0	0.3103	0.3494
1.5	0.0795	0.0932
2.0	0.0137	0.0166
2.5	0.0015	0.00193
3.0	0.00012	0.00015

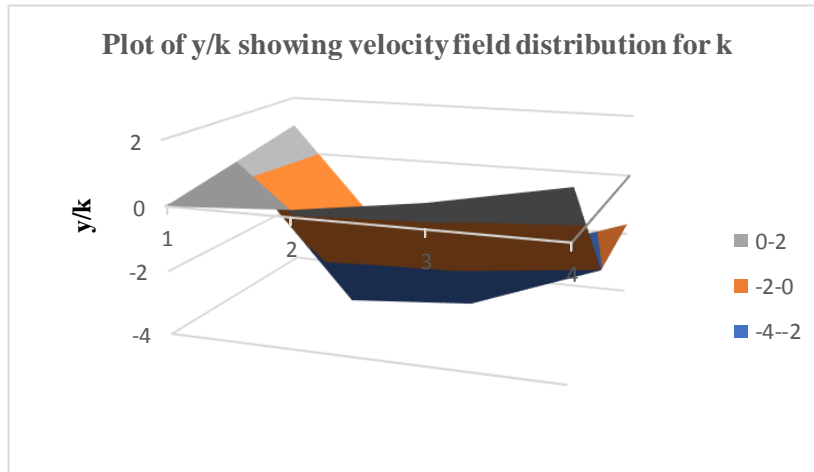


Figure 8: Velocity field distribution plot for k.

Also, skin factor distribution is observed for various indices in the form of tables as under.

Table 8: Skin Factor distribution for Sc and M at t=0.4, Gr=4, Gm=5, Pr=0.71, a=0.5 and K=1.

M/Sc	0.22	0.75	1.5
0.5	-3.855	-3.526	-2.015
1	-3.269	-3.038	-1.663

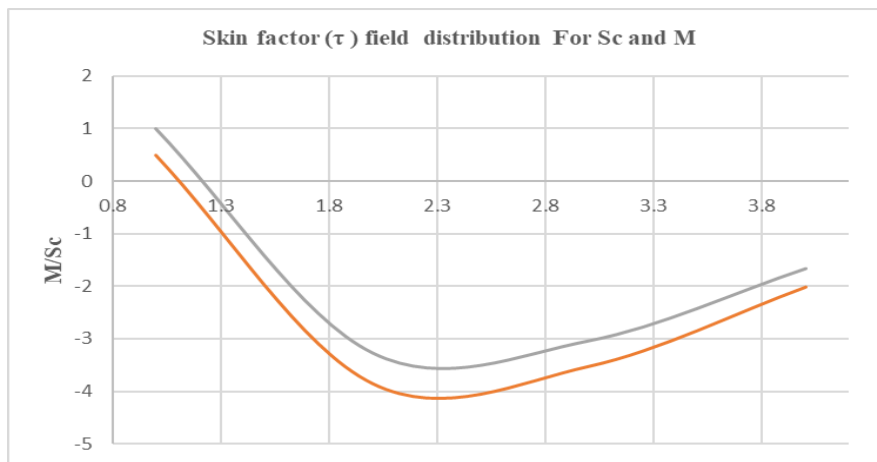


Figure 9: Skin factor field distribution plot for M/Sc.

Table 9: Skin Factor distribution for Sc and k at t=0.2, Gr=2, Gm=5, Pr=0.5, a=0.5 and M=1.

K/Sc	0.22	0.75	1.5
1	-1.543	-1.238	-0.026
2	-1.883	-1.564	-0.233

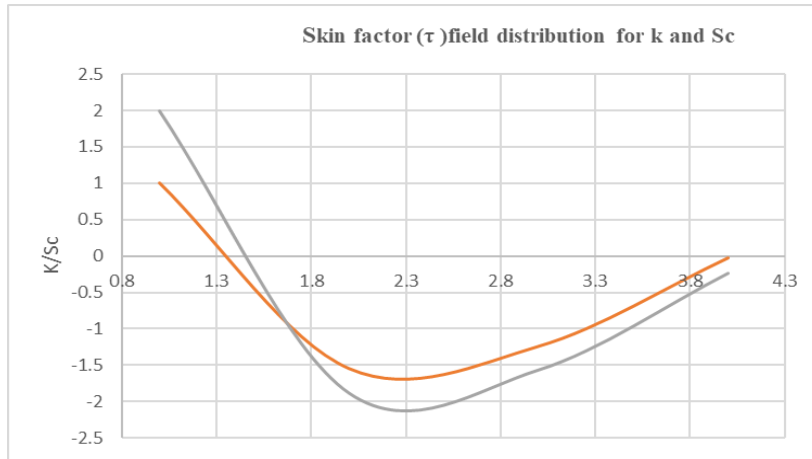


Figure 10: Skin factor field distribution plot for k/Sc .

Table 10: Skin Factor distribution for Gr , Gm and t at $Pr=0.5$, $a=0.5$, $K=2$, $Sc =0.22$ and $M=1$.

Gr	Gm/t	0.2	0.4
2.0	0.0	0.853	0.599
	0.5	0.579	0.269
	5.0	-1.883	-2.719
4.0	0.0	-0.212	-0.662
	0.5	-0.486	-0.993
	5.0	-2.498	-3.976

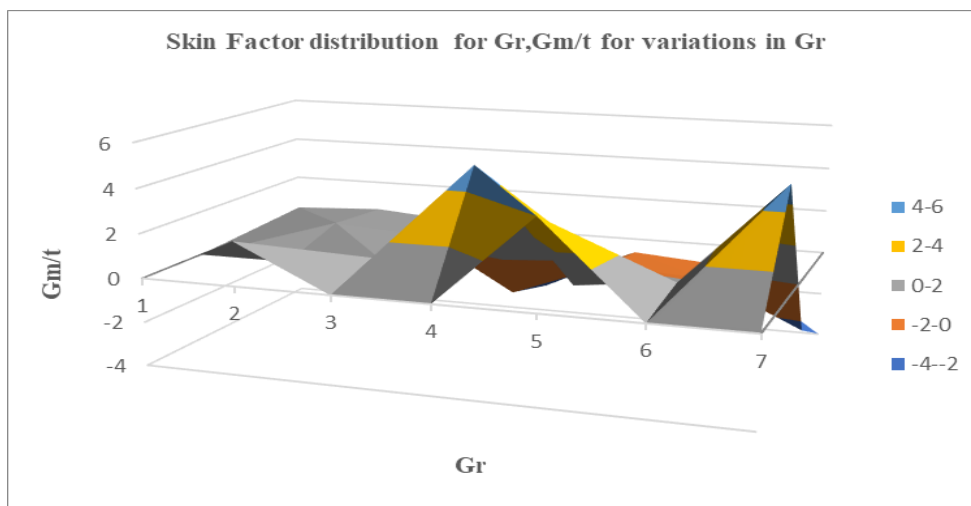


Figure 10: Skin factor field distribution plot for Gm/t .

3. DISCUSSION AND RESULTS

1. From Tables 1,2 and3 it is clear that the velocity at any point increases with increase in Gm and Gr but it decreases with increase in Sc for fixed value s of Pr , t , M , K and a .
2. From Table 4 we observed that the velocity increases with increase in t for fixed values of M and K .

3. It is also seen from Table 5 that velocity increases with increase in a for fixed values of M and K .
4. From Table 6 it is clear that velocity decreases with increase in M
5. From Table 7 it is clear that velocity field increases with increase in K .
6. In order to study the effect of various parameters on skin friction we observed that from Tables 8 and 9 for fixed value of t , Gr , Gm and Pr , the skin friction at the plate increases as magnetic parameter (M) increases for fixed values of K and Sc .
7. It can also be seen that skin friction at the plate decreases with increase in permeability number (K) for fixed values of M and Sc but increases with increases in Sc for fixed values of M and K .
8. From Table 10 it is clear that for fixed values of Pr , Sc , M and K skin friction at the plate decreases as Gm and Gr increases for the fixed values of t and also decreases with increase in t for fixed values of Gr and Gm .
9. If we take $a = 0$ then the problem becomes the case of flow past over impulsive started plate.

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Declaration of competing interest

The authors declare of having no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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