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MODELLING FLOW CHARACTERISTICS IN UNSTEADY MHD TWO--PHASE NANOFLUID FLOW AND HEAT TRANSFER BETWEEN PARALLEL PLATES USING SEMI-ANALYTICAL SCHEME

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ABSTRACT

The aim of this article is to investigate heat transfer and flow of unsteady MHD two-phase nanofluid transversed between two parallel plates. Mathematical formulation and analysis have been performed in the presence of magnetic field. The governing flow partial differential equations corresponding to the mass conservation, momentum, energy, and concentration are reduced to a set of nonlinear differential equations using self-similar transformations. Resulting nonlinear system is solved for the approximate analytical solution using semi-analytical iteration method. Embedded parameters of interest like Eckert number, Squeeze number, Prandtl number, Schmidt number, Brownian motion parameter, thermophoresis parameter and Hartmann number are examined in detail and given physical interpretation using graphs and tables. Analysis of flow characteristics when compared with available results in literature showed excellent agreement. Furthermore, the presence of Hartmann and Squeeze number decreased the velocity profile of the fluid. Temperature profile is increased with increase in Eckert and Prandtl numbers but decelerate with increased Squeeze and Hartmann numbers, whereas concentration profile and Schmidt vary correspondingly.

KEYWORDS: Unsteady, Two-phase Nanofluid, Heat Transfer, Semi-analytical Scheme, TAM.

INTRODUCTION

Heat transfer equations are one of the most significant engineering issues, and because they are distinctively nonlinear problems, some of them can be solved numerically and others analytically. Applications in engineering provide advantages and opportunities in a variety of businesses and fields. Numerical and analytical solutions for solving nonlinear equations in heat transfer is a cutting-edge academic source for study on the most effective methods for deriving insights from heat transfer equations and the use of these techniques in a variety of sectors.

Due to the rise in energy prices, studies on heat transfer enhancement in diverse energy systems is significant. The technology of nanofluids was proposed and investigated computationally or experimentally by many researchers in the last ten years to manage heat transfer in diverse applications. The nanofluid can be utilized in a variety of engineering operations, including cooling electronic equipment and heat exchangers. Most researchers assumed that nanofluids are treated like regular pure fluids. Sheikholeslami et al.[1] investigated the flow characteristics of nanofluids between two parallel plates in a rotating device. They showed that Nusselt number rises with rising Reynolds number and volume fraction of nanoparticles and falls with increasing magnetic, rotational, and Eckert numbers. An electrically conducting incompressible nanofluid flowing across a porous rotating disk was subjected to an investigation of the second law of thermodynamics by Rashidi et al.[2] Using the control volume finite element approach, Ganji et al.[3] examined magnetohydrodynamic natural convection in an inclined enclosure filled with nanofluid and having sinusoidal walls. Heat transfer and nanofluid flow with suction and blowing between parallel disks in the presence of varying magnetic fields have been examined by Hatami and Ganji.[4] Brownian motion and thermophoresis effects on the slip flow of an alumina/water nanofluid in a circular microchannel in the presence of a magnetic field have been studied by Malvandi and Ganji.[5] Flow of a nanofluid via a stretched sheet at the boundary-layer has been studied by Khan and Pop.[6]

All the above analyses used the assumptions that there are no slip velocities at all between fluid molecules and nanoparticles and that the concentration of nanoparticles is uniform. According to this theory, different concentrations of nanoparticles will exist in a mixture and the volume fraction of nanofluids may not be uniform because the nanoparticles could not attend fluid molecules due to some slip mechanisms like thermophoresis and Brownian motion.

Sheikholeslami et al.[7] researched the free convection heat transport in a nanofluid-filled container. In a rotating system with a permeable sheet, they investigated two phase modeling of nanofluid. They demonstrated that Nusselt number depends directly on injection parameter and Reynolds number, but that Schmidt number, rotation parameter, Brownian parameter, and thermophoretic parameter have the opposite relationship. Heat and mass transport in the squeezing flow between parallel plates have been studied by Mustafa et al.[8] Thermal instability has been studied by Nield and Kuznetsov.[9] in a porous medium layer saturated with a nanofluid. The Galerkin weighted residual technique was employed by Hedayati and Ramiar.[10] to investigate two-phase unsteady nanofluid flow and heat transfer between moving parallel plates under the influence of magnetic field. Usman et al.[11] have examined unsteady nanofluid flow and heat transfer between two infinitely long parallel plates using differential transformation method. Similarly, the same problem has been discussed using homotopy perturbation method by Ganji et al.[12]

Temimi and Ansari developed the semi-analytical iterative scheme referred to as (SAIM). The main benefit of this novel technique is that it does not use small parameters, contrary to other iterative methods like HAM and HPM, large computational work, or restrictive assumptions for nonlinear terms, which are required by the Adomian decomposition method (ADM). SAIM has been used to address problems involving nonlinear thin flows, linear and nonlinear PDEs, linear and nonlinear ODEs, the Duffing equation, the Korteweg-De Vries equation, chemical difficulties, and the Duffing equation. Extensive treatment of this approach can be found in.[13-22]

In the next section, a brief introduction about the study is explored. Section two is devoted to the basic governing equations and the problem formulation accompanied by its coordinate system and prescribed boundary conditions. The description of the solution technique is presented in section three and section four examined the approximate analytical solution for the flow distributions using the solution method. In section five, the results and discussion of the plots for different values of the control parameters are discussed and the last section six consists of the conclusion and the major findings of the study.

MATHEMATICAL FORMULATIONS

We consider an unsteady two-dimensional squeezing flow and heat transfer of two-phase nanofluid between the infinite parallel plates under the influence of uniform magnetic field as shown in figure 1. The plates are placed at a distance of $\zeta(1-\gamma t)^{1/2} = h(t)$. If $\gamma > 0$, the plates are squeezed and touched when $t = 1/\gamma$ and separated for $\gamma < 0$. Taking the heat generation due to friction and viscous dissipation into account, the uniform magnetic field applied along the lower plate becomes $\vec{B} = B\vec{e}_y$, where $\vec{e}_y$ denote the unit vector. The electromagnetic force due to Lorentz is given $\vec{F} = \vec{J} \times \vec{B} = \sigma(\vec{V} \times \vec{B}) \times \vec{B}$. The equations governing the model problem are mass conservation, conservation of momentum, energy and mass transfer given by.

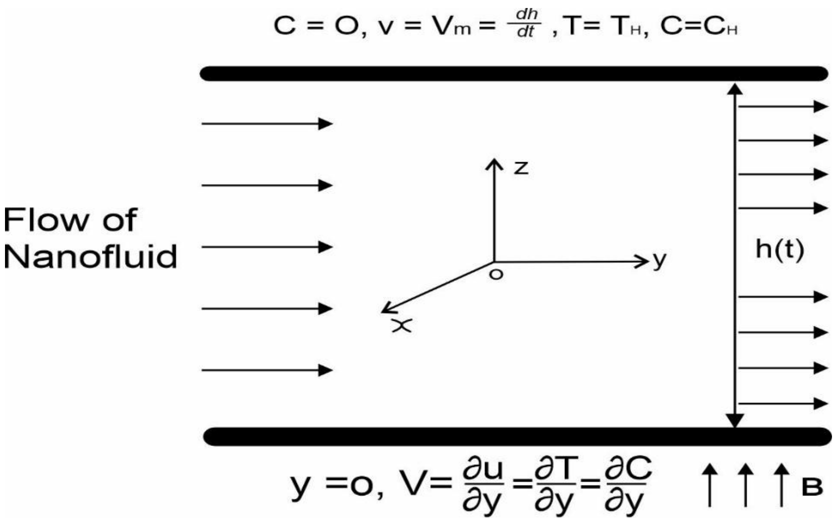

Figure 1: Configuration of the coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\rho_f\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma B^2 u \qquad (2)$$

$$\rho_f\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\mu}{(\rho C_p)_f}\left(4\left(\frac{\partial u}{\partial x}\right)^2\right) +$$

$$\frac{(\rho C_p)_p}{(\rho C_p)_f}\left[D_B\left\{\frac{\partial C}{\partial x}.\frac{\partial T}{\partial x} + \frac{\partial C}{\partial y}.\frac{\partial T}{\partial y}\right\} + \left(\frac{D_T}{T_c}\left\{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right\}\right)\right] \qquad (4)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) + \left(\frac{D_T}{T_c}\right)\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right\} \qquad (5)$$

The resulting boundary conditions are.

$C = 0, v = V_w = \frac{dh}{dt}, T = T_H, C = C_H$ at $y = h(t)$

$v = \frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C}{\partial y} = 0$ at $y = 0$ (6)

where (u, v) are the velocities in the x and y axes, T denotes the temperature, C is the concentration, P is the pressure, ρ_f represent the density of the base fluid, μ is the dynamic viscosity, κ denote thermal conductivity, c_p is the specific heat capacity of the nanofluid and D_B is the coefficient of diffusion.

To reduce the Eqs. (1-5) subject to systems of ordinary differential equations, we invoke the following dimensionless parameters of the form.

$$\eta = \frac{y}{l(1-\zeta t)^{1/2}} = \frac{y}{h(t)}, u = \frac{\zeta x}{2(1-\zeta t)} f'(\eta), v = \frac{-\zeta l}{2(1-\zeta t)^{1/2}} f(\eta), \theta = \frac{T}{T_H}, \phi = \frac{C}{C_h} \quad (7)$$

Substituting Eq. (7) into the governing equations in (1-5) and eliminating the pressure gradients results in the following set of nonlinear ODEs given by

$$f^{iv}(\eta) - S\big(\eta f'''(\eta) + 3f''(\eta) + f'(\eta)f''(\eta) - f(\eta)f'''(\eta)\big) - Ha^2 f''(\eta) = 0 \quad (8)$$

$$\theta''(\eta) + PrS\big(f(\eta)\theta'(\eta) - \eta\theta'(\eta)\big) + PrEc\big(f''^2(\eta)\big) + Nb\phi'(\eta)\theta'(\eta) + Nt\theta'^2(\eta) = 0 \quad (9)$$

$$\phi''(\eta) + S.Sc\big(f(\eta)\phi'(\eta) - \eta\phi'(\eta)\big) + \frac{Nt}{Nb}\theta''(\eta) = 0 \quad (10)$$

Subject to the boundary conditions as given as

$f(0) = 0, f''(0) = 0, \theta'(0) = 0, \phi'(0) = 0$

$f(1) = 1, f'(1) = 0, \theta(1) = \phi(1) = 1$ (11)

where S is the squeeze number, Pr is the Prandtl number, Ec is the Eckert number, Sc is the Schmidt number, Ha is the Hartmann umber of nanofluid, Nb is the Brownian motion parameter and Nt is the thermophoresis parameter all which are defined as

$$S = \frac{\beta l^2}{2\mu}\rho_f, Pr = \frac{\mu}{\rho_f \alpha}, Ec = \frac{1}{C_P}\left(\frac{\beta x}{2(1-\beta t)}\right)^2, Sc = \frac{\mu}{\rho_f D}, Ha = l\beta\sqrt{\frac{\sigma}{\mu}(1-\beta t)}$$

$$Nb = \frac{(\rho C)_p D_B (C_h)}{(\rho C)_p \alpha}, Nt = \frac{(\rho C)_p D_T (T_H)}{(\rho C)_p \alpha T_c} \quad (12)$$

Fundamentals OF Temimi-Ansari Method (TAM)

Following Temimi[13-14] and Ebiwareme[20-22], we consider the general differential equation in operator form as follows.

$$L\big(y(x)\big) + N\big(y(x)\big) + f(x) = 0, \quad x\epsilon D \qquad (10)$$

$$B\left(y, \frac{dy}{dx}\right) = 0, \; x\epsilon\mu \qquad (11)$$

where x is the independent variable, $y(x)$ is an unknown function, $f(x)$ is a given known function, L is a linear operator, N is a nonlinear operator and B is a boundary operator.

To implement TAM, we first assume an initial guess of the form, $y_0(x)$ satisfy the equation in the form.

$$L\big(y_0(x)\big) + f(x) = 0, \;\; B\left(y_0, \frac{dy_0}{dx}\right) = 0 \qquad (12)$$

The second iteration as follows.

$$L\big(y_1(x)\big) + N\big(y_0(x)\big) + f(x) = 0,, B\left(y_1, \frac{dy_1}{dx}\right) = 0 \qquad (13)$$

We consider the next iteration as follows.

$$L\big(y_2(x)\big) + N\big(y_1(x)\big) + f(x) = 0,, B\left(y_2, \frac{dy_2}{dx}\right) = 0 \qquad (14)$$

Continuing the same way, we obtain nth iterative procedure to give the subsequent iterates as

$$L\big(y_{n+1}(x)\big) + N\big(y_n(x)\big) + f(x) = 0, \;\; B\left(y_{n+1}, \frac{dy_{n+1}}{dx}\right) = 0 \qquad (15)$$

From Eq. (6), each $y_i(x)$ is considered alone as a solution of Eq. (10). This method is easy to implement and straightforward. The method gives better approximate solution which converges to the exact solution with only few members.

Solution Procedure Using Temimi-Ansari Method (TAM)

Writing Eqs. (8) - (10) in operator form, we have the expression

$$f^{iv}(\eta) = S\big(\eta f'''(\eta) + 3f''(\eta) + f'(\eta)f''(\eta) - f(\eta)f'''(\eta)\big) + Ha^2 f''(\eta) \qquad (13)$$

$$\theta''(\eta) = -\big[PrS\big(f(\eta)\theta'(\eta) - \eta\theta'(\eta)\big) + PrEc\big(f''^2(\eta)\big) + Nb\phi'(\eta)\theta'(\eta) + Nt\theta'^2(\eta)\big] \; (14)$$

$$\phi''(\eta) = S.Sc\big(\eta\phi'(\eta) - f(\eta)\phi'(\eta)\big) - \frac{Nt}{Nb}\theta''(\eta) \qquad (15)$$

The first problem to be solved is given as follows.

$$L(f_0(\eta)) = 0, f_0(0) = 0, f_0''(0) = 0, f_0'(0) = 0 \quad (16)$$

$$L(\theta_0(\eta)) = 0, \theta_0'(0) = 0, \theta_0(1) = 1 \quad (17)$$

$$L(\phi_0(\eta)) = 0, \phi_0'(0) = 0, \phi_0(1) = 1 \quad (18)$$

where $N_1(f(\eta)) = S(\eta f'''(\eta) + 3f''(\eta) + f'(\eta)f''(\eta) - f(\eta)f'''(\eta)) + Ha^2 f''(\eta)$

$$N_2(\theta(\eta)) = -[PrS(f(\eta)\theta'(\eta) - \eta\theta'(\eta)) + PrEc(f''^2(\eta)) + Nb\phi'(\eta)\theta'(\eta) + Nt\theta'^2(\eta)]$$

$$N_3(\phi(\eta)) = S.Sc(\eta\phi'(\eta) - f(\eta)\phi'(\eta)) - \frac{Nt}{Nb}\theta''(\eta)$$

Integrating Eq. (14) from 0 to η four times and Eqs. (15)- (16) twice, we obtain the expression for the first iterates as

$$f_0(\eta) = \tfrac{1}{2}\eta(3 - \eta^2), \theta_0(\eta) = 1, \phi_0(\eta) = 1 \quad (19)$$

The second problem to be solved subject to the prescribed boundary conditions is of the form.

$$f_1^{iv}(\eta) = N_1(f_0(\eta), \theta_0(\eta)), f_1(0) = 0, f_1''(0) = 0, f_1'(1) = 0 \quad (20)$$

$$\theta_1''(\eta) = N_2(f_0(\eta), \theta_0(\eta), \phi_0(\eta)), \theta_1'(0) = 0, \theta_1(1) = 1 \quad (21)$$

$$\phi_1''(\eta) = N_3(f_0(\eta), \theta_0(\eta), \phi_0(\eta)), \phi_1'(0) = 0, \phi_1(1) = 1 \quad (22)$$

Integrating Eqs. (20) – (22) from 0 to η gives the equivalent expression for the first iterates as

$$f_1(\eta) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta \left(S\left(\eta f_0'''(\eta) + 3f_0''(\eta) + f_0'(\eta)f_0''(\eta) - f_0(\eta)f_0'''(\eta)\right) + Ha^2 f_0''(\eta)\right) d\eta d\eta d\eta d\eta \quad (23)$$

$$\theta_1(\eta) = -\int_0^\eta \int_0^\eta \left(PrS(f_0(\eta)\theta_0'(\eta) - \eta\theta_0'(\eta)) + PrEc\left(f_0''^2(\eta)\right) + Nb\phi_0'(\eta)\theta_0'(\eta) + Nt\theta_0'^2(\eta)\right) d\eta d\eta \quad (24)$$

$$\phi_1(\eta) = \int_0^\eta \int_0^\eta \left(S.Sc(\eta\phi_0'(\eta) - f_0(\eta)\phi_0'(\eta)) - \frac{Nt}{Nb}\theta_0''(\eta)\right) d\eta d\eta \quad (25)$$

Similarly, the third problem to be solved is given by the expression.

$$f_2^{iv}(\eta) = N_1(f_1(\eta), \theta_1(\eta)), f_2(0) = 0, f_2''(0) = 0, f_2'(1) = 0 \quad (26)$$

$$\theta_2''(\eta) = N_2(f_1(\eta), \theta_1(\eta), \phi_1(\eta)), \theta_2'(0) = 0, \theta_2(1) = 1 \quad (27)$$

$$\phi_2''(\eta) = N_3(f_1(\eta), \theta_1(\eta), \phi_1(\eta)), \phi_2'(0) = 0, \phi_2(1) = 1 \quad (28)$$

Integrating Eqs. (26) – (28) from 0 to η four times for Eq. (26) and twice for Eqs. (27) and (28) gives

$$f_2(\eta) = \int_0^\eta \int_0^\eta \int_0^\eta \int_0^\eta \left(S\left(\eta f_1'''(\eta) + 3f_1''(\eta) + f_1'(\eta) f_1''(\eta) - f_1(\eta) f_1'''(\eta) \right) + Ha^2 f_1''(\eta) \right) d\eta d\eta d\eta d\eta \quad (29)$$

$$\theta_2(\eta) = -\int_0^\eta \int_0^\eta \left(PrS\left(f_1(\eta)\theta_1'(\eta) - \eta\theta_1'(\eta)\right) + PrEc\left(f_1''^2(\eta)\right) + Nb\phi_1'(\eta)\theta_1'(\eta) + Nt\theta_1'^2(\eta) \right) d\eta d\eta \quad (30)$$

$$\phi_2(\eta) = \int_0^\eta \int_0^\eta \left(S.Sc\left(\eta\phi_1'(\eta) - f_1(\eta)\phi_1'(\eta)\right) - \frac{Nt}{Nb}\theta_1''(\eta) \right) d\eta d\eta \quad (31)$$

The second and third iterative solution takes the expression.

$$f_1(\eta) = -\frac{1}{40}M^2\eta^5 - \frac{S\eta^5}{10} + \frac{S\eta^7}{280},\ \theta_1(\eta) = -\frac{3}{4}\mathrm{EcPr}\eta^4,\ \phi_1(\eta) = 0 \quad (32)$$

$$f_2(\eta) = \frac{1}{1009008000}\eta^7\left(455M^4(-1320 + 7S\eta^4) - 70M^2S(85800 - 715\eta^2 - 364S\eta^4 + 6S\eta^6) + S^2(-14414400 + 400400\eta^2 + 50960S\eta^4 - 1680S\eta^6 + 33S\eta^8)\right) \quad (33)$$

$$\theta_2(\eta) = -\frac{1}{369600}\mathrm{EcPr}\eta^6(\eta^2(1650M^4 - 88M^2S(-150 + 7\eta^2) + S^2(26400 - 2464\eta^2 + 63\eta^4)) + \mathrm{Pr}(59400\mathrm{EcNt}\eta^2 + 308S(120 + M^2\eta^4) + S^2(1232\eta^4 - 30\eta^6))) \quad (34)$$

$$\phi_2(\eta) = \frac{3NtPrEc}{4Nb}\eta^4 + \left(\frac{90\mathrm{EcNtPr}^2S}{\mathrm{Nb}} + \frac{135\mathrm{EcNtPr}S^2}{2\mathrm{Nb}} + \frac{3\mathrm{EcNtPr}S\mathrm{Sc}}{\mathrm{Nb}}\right)\eta^6 + \left(\frac{63\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{378\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{2\mathrm{Nb}}\right)\eta^7 + \left(\frac{14\mathrm{EcHa}^4\mathrm{NtPr}}{\mathrm{Nb}} + \frac{504\mathrm{Ec}^2\mathrm{Nt}^2\mathrm{Pr}^2}{\mathrm{Nb}} + \frac{168\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{504\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{3\mathrm{EcHa}^2\mathrm{NtPr}S\mathrm{Sc}}{2\mathrm{Nb}} + \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{\mathrm{Nb}}\right)\eta^8 + \left(-\frac{324\mathrm{EcNtPr}S^2}{5\mathrm{Nb}} + \frac{27\mathrm{EcNtPr}^2S^2}{\mathrm{Nb}}\right)\eta^9 + \left(-\frac{27\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{27\mathrm{EcHa}^2\mathrm{NtPr}^2S}{4\mathrm{Nb}} - \frac{162\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{81\mathrm{EcNtPr}^2S^2}{2\mathrm{Nb}} - \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{10\mathrm{Nb}}\right)\eta^{10} + \left(\frac{297\mathrm{EcNtPr}S^2}{25\mathrm{Nb}} - \frac{99\mathrm{EcNtPr}^2S^2}{35\mathrm{Nb}}\right)\eta^{12} \quad (35)$$

The terms $f_n(\eta), \theta_n(\eta)$ and $\phi_n(\eta)$ for $n \geq 2$ are too large to be presented graphically so are omitted for brevity. Hence, the three-term solution for the velocity, temperature and concentration gradients are given by.

$$f(\eta) = \sum_{n=0}^{\infty} f_n(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \cdots$$

$$\theta(\eta) = \sum_{n=0}^{\infty} \theta_n(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \cdots \quad (36)$$

$$\phi(\eta) = \sum_{n=0}^{\infty} \phi_n(\eta) = \phi_0(\eta) + \phi_1(\eta) + \phi_2(\eta) + \cdots$$

$$f(\eta) = \frac{3\eta}{2} - \frac{\eta^3}{2} + \left(-\frac{M^2}{40} - \frac{S}{10}\right)\eta^5 + \left(-\frac{M^4}{1680} + \frac{S}{280} - \frac{M^2S}{168} - \frac{S^2}{70}\right)\eta^7 + \left(\frac{M^2S}{20160} + \frac{S^2}{2520}\right)\eta^9 + \left(\frac{M^4S}{316800} + \frac{M^2S^2}{39600} + \frac{S^3}{19800}\right)\eta^{11} + \left(-\frac{M^2S^2}{2402400} - \frac{S^3}{600600}\right)\eta^{13} + \frac{S^3\eta^{15}}{30576000} + \cdots \quad (37)$$

$$\theta(\eta) = 1 - \frac{3}{4}\mathrm{EcPr}\eta^4 - \frac{1}{369600}\mathrm{EcPr}\eta^6(\eta^2(1650M^4 - 88M^2S(-150 + 7\eta^2) +$$

$$S^2(26400 - 2464\eta^2 + 63\eta^4)) + \mathrm{Pr}(59400\mathrm{EcNt}\eta^2 + 308S(120 + M^2\eta^4) + S^2(1232\eta^4 - 30\eta^6))) + \cdots \quad (38)$$

$$\phi(\eta) = 1 + \frac{3NtPrEc}{4Nb}\eta^4 + \left(\frac{90\mathrm{EcNtPr}^2S}{\mathrm{Nb}} + \frac{135\mathrm{EcNtPr}S^2}{2\mathrm{Nb}} + \frac{3\mathrm{EcNtPr}S\mathrm{Sc}}{\mathrm{Nb}}\right)\eta^6 + \left(\frac{63\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{378\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{2\mathrm{Nb}}\right)\eta^7 + \left(\frac{14\mathrm{EcHa}^4\mathrm{NtPr}}{\mathrm{Nb}} + \frac{504\mathrm{Ec}^2\mathrm{Nt}^2\mathrm{Pr}^2}{\mathrm{Nb}} + \frac{168\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{504\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{3\mathrm{EcHa}^2\mathrm{NtPr}S\mathrm{Sc}}{2\mathrm{Nb}} + \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{\mathrm{Nb}}\right)\eta^8 + \left(-\frac{324\mathrm{EcNtPr}S^2}{5\mathrm{Nb}} + \frac{27\mathrm{EcNtPr}^2S^2}{\mathrm{Nb}}\right)\eta^9 + \left(-\frac{27\mathrm{EcHa}^2\mathrm{NtPr}S}{\mathrm{Nb}} + \frac{27\mathrm{EcHa}^2\mathrm{NtPr}^2S}{4\mathrm{Nb}} - \frac{162\mathrm{EcNtPr}S^2}{\mathrm{Nb}} + \frac{81\mathrm{EcNtPr}^2S^2}{2\mathrm{Nb}} - \frac{9\mathrm{EcNtPr}S^2\mathrm{Sc}}{10\mathrm{Nb}}\right)\eta^{10} + \left(\frac{297\mathrm{EcNtPr}S^2}{25\mathrm{Nb}} - \frac{99\mathrm{EcNtPr}^2S^2}{35\mathrm{Nb}}\right)\eta^{12} \ldots \quad (39)$$

RESULTS

In this paper, we have investigated an unsteady two-phase nanofluid flow and heat transfer between two parallel plates under the influence of magnetic field. To get better physical insight into the problem, the dimensionless velocity, temperature, and concentration gradients are obtained as approximate analytical solutions from the nonlinear governing differential equations upon similarity transformation. The influence of several parameters such as Prandtl number, Eckert number, Squeeze number and others on the flow distributions are depicted graphically in figures (1-7), Tables (1-3) and discussed exhaustively.

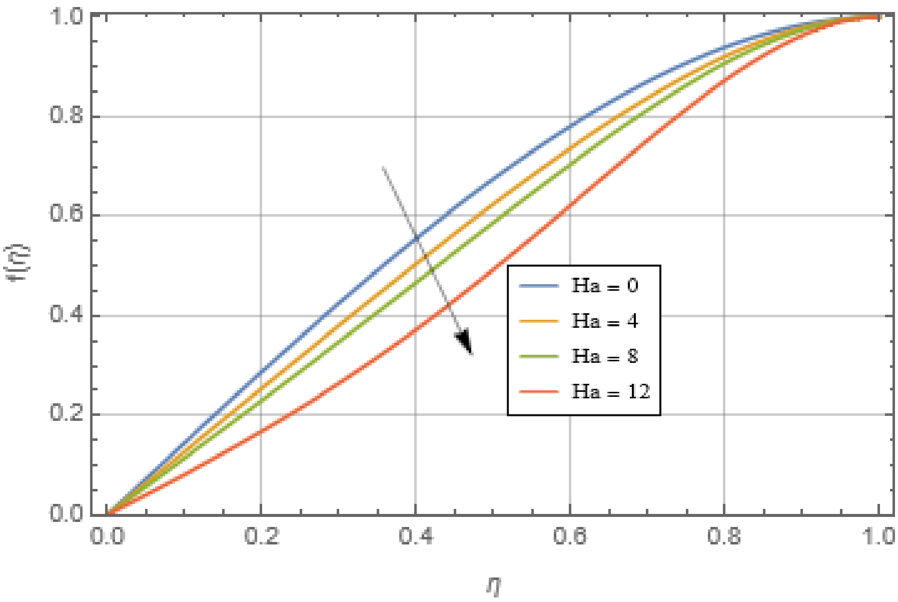

Figure 1: Effect of Hartmann number on velocity profile.

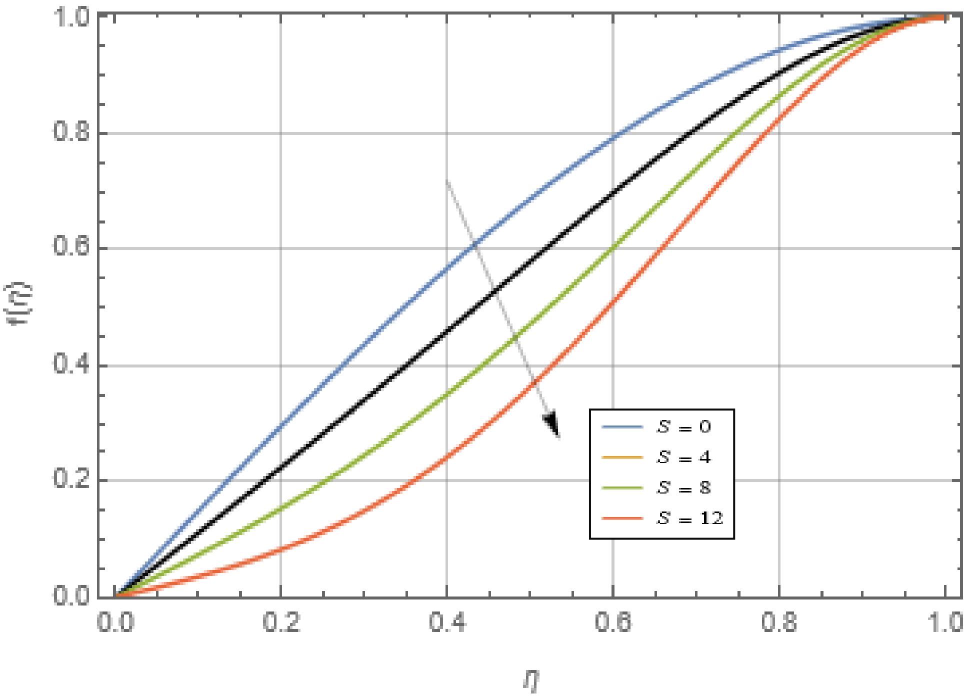

Figure 2: Velocity profile for variation in Squeeze number.

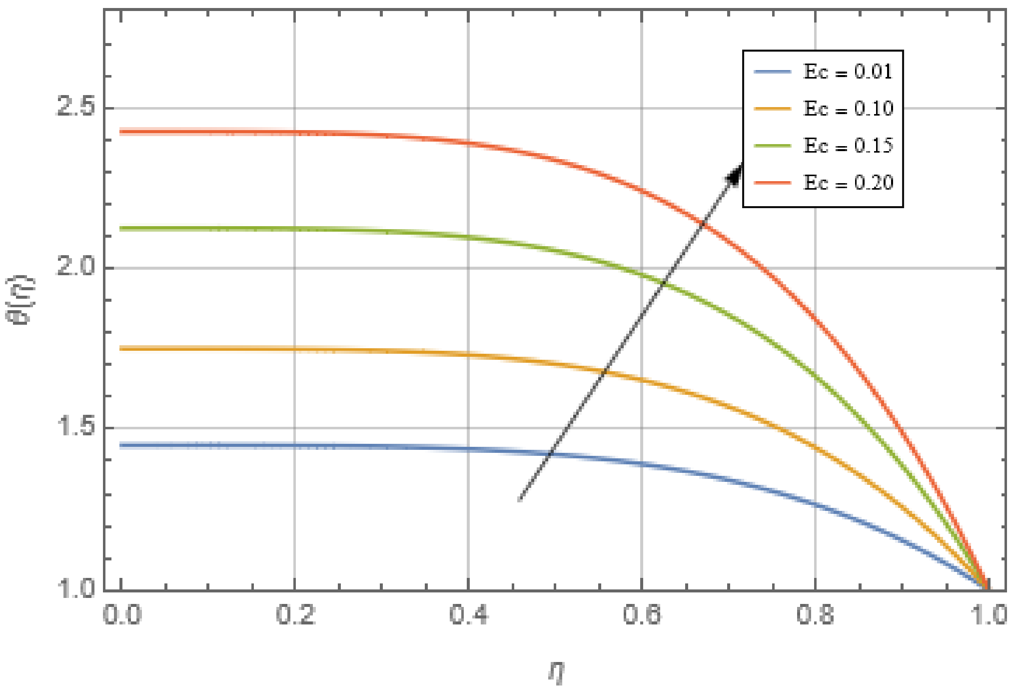

Figure 3: Effect of Eckert number on Temperature profile.

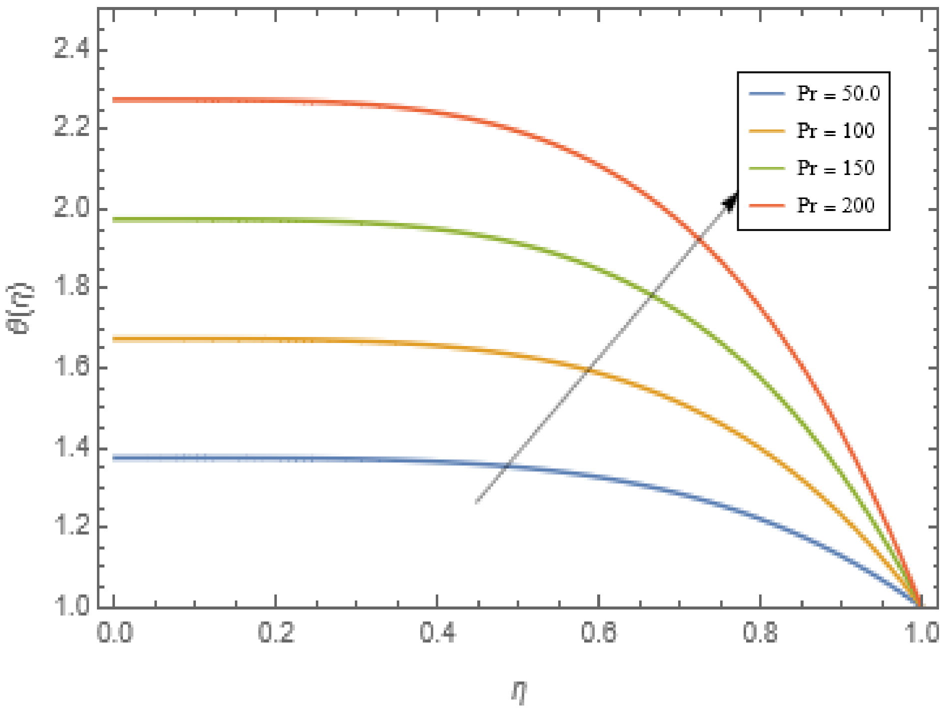

Figure 4: Influence of Prandtl number on Temperature profile.

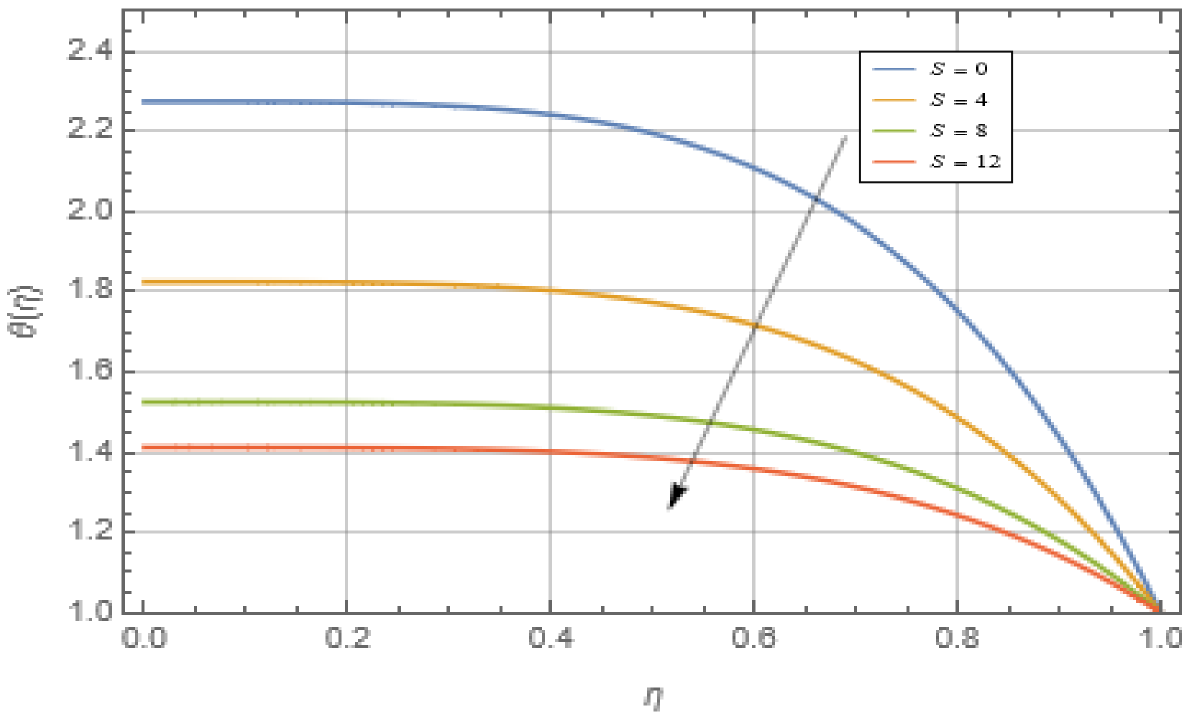

Figure 5: Impact of Squeeze number on Temperature profile.

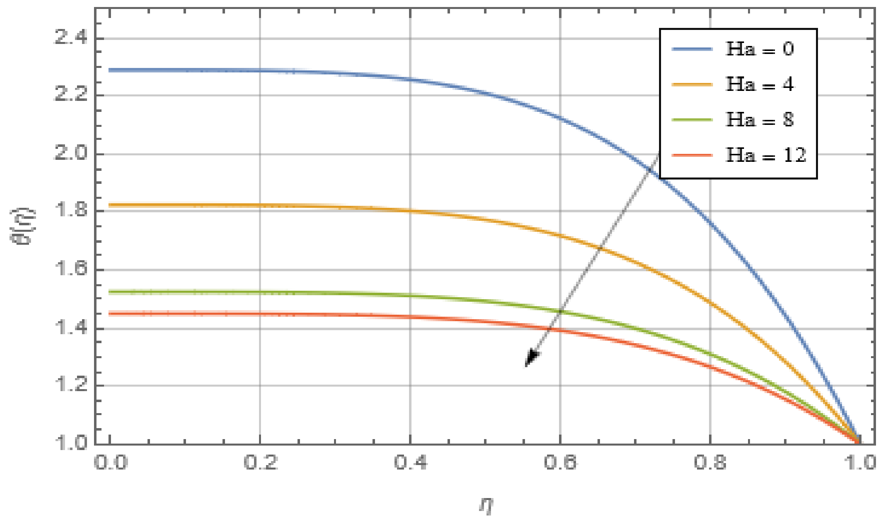

Figure 6: Temperature profile for variation in Hartmann number.

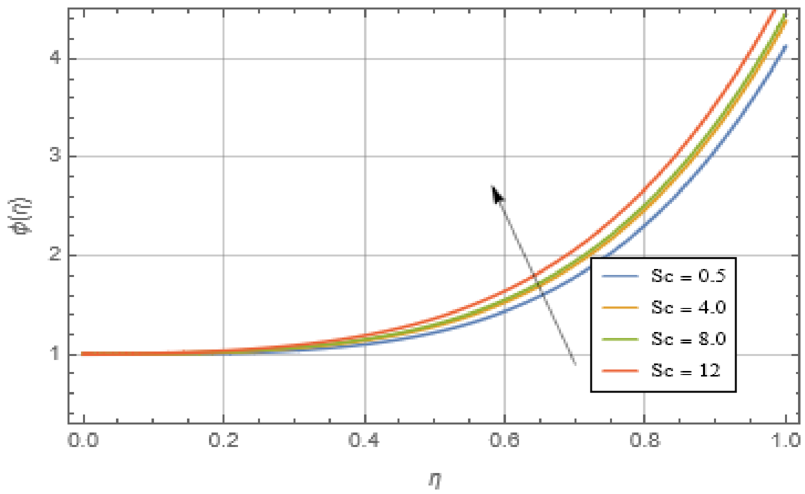

Figure 7: Effect of Schmidt number on concentration profile.

Table 1: Comparison Analysis between DTM, ADM, HPM and TAM Solutions for Velocity profile.

η	$f(\eta)$			
	DTM[11]	ADM[12]	HPM[12]	Present Study [TAM]
0.00	0.00000	0.00000	0.00000	0.00000
0.10	0.11890	0.11897	0.11889	0.11898
0.20	0.23760	0.23773	0.23778	0.23789
0.30	0.35578	0.35597	0.35589	0.35588
0.40	0.47297	0.47321	0.47290	0.47289
0.50	0.58827	0.58855	0.58856	0.58858
0.60	0.70007	0.70037	0.70008	0.70009
0.70	0.80538	0.80566	0.80540	0.80560
0.80	0.89869	0.89889	0.89885	0.89851
0.90	0.96982	0.96991	0.96934	0.96989
1.00	1.00000	1.00000	1.00000	1.00000

Table 2: Comparison Analysis between DTM, ADM, HPM and TAM Solution for Temperature profile.

η	$\theta(\eta)$			
	DTM[11]	ADM[12]	HPM[12]	Present Study [TAM]
0.00	4.76574	5.13796	5.12790	5.13800
0.10	4.72338	5.09548	5.09512	5.09632
0.20	4.59724	4.96820	4.97221	4.96993
0.30	4.39011	4.75650	4.74826	4.75672
0.40	4.10665	4.46107	4.36108	4.46109
0.50	3.75328	4.08283	4.08328	4.08512
0.60	3.33793	3.62298	3.33820	3.61993
0.70	2.86889	3.08296	3.08280	3.08992
0.80	2.35008	2.46447	2.46449	2.46512
0.90	1.76303	1.76943	1.76752	1.76789
1.00	1.00000	1.00000	1.00000	1.00000

Table 3: Comparison Analysis between DTM, ADM, HPM and TAM Solution for Temperature profile.

η	$\phi(\eta)$			
	DTM[11]	ADM[12]	HPM[12]	Present Study [TAM]
0.00	-2.74191	-3.13796	-3.33721	-3.32816
0.10	-2.69956	-3.09548	-2.89925	-2.89942
0.20	-2.57346	-2.96820	-2.97820	-2.97856
0.30	-2.36655	-2.75650	-2.36640	-2.36689
0.40	-2.08365	-2.46107	-2.51280	-2.51283
0.50	-1.73142	-2.08283	-2.19520	-2.09622
0.60	-1.31806	-1.62298	-1.67889	-1.66782
0.70	-0.85211	-1.08296	-1.08312	-1.08521
0.80	-0.33766	-0.46447	-0.46553	-0.45510
0.90	0.24370	0.23057	0.24370	0.24379
1.00	1.00000	1.00000	1.00000	1.00000

CONCLUSIONS

In this study, the effect of the magnetic field on the heat transfer between two parallel plates and the unsteady two-phase nanofluid flow is investigated. Temimi-Ansari method (TAM) is employed to solve the nonlinear differential equations that arise on performing a self-similar transformation to the governing model equations. Prandtl number, Eckert number, Squeeze number, and other relevant factors influence are shown graphically as variations in flow distributions. Here is an overview of the findings from our investigation.

1. Increase in the Hartmann and Squeeze numbers cause a decrease in the velocity profile of the fluid.
2. The influence of Eckert and Prandtl numbers is to decrease the temperature distribution.

3. We observe a declination of the temperature gradient in the presence of Squeeze and Hartmann numbers.
4. Positive correlation is witnessed between the Schmidt number and concentration profile of the fluid.

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