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DUAL AMPLITUDE ENVELOPE SYNCHRONIZATION IN PERIODICALLY COUPLED OSCILLATORS

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ABSTRACT

Amplitude envelope synchronization is a kind of strong correlation with the coupling between the phases and the amplitude in the coupled oscillators. A new type of synchronous amplitude envelope is generated on the original synchronous amplitude envelope induced by the frequency mismatches in the periodically coupled nonidentical oscillators. With the increment of the variation amplitude of the periodic coupling strength the amplitude of the new born synchronous amplitude envelope increases accordingly while the average value of the synchronous amplitude envelope decreases to the increment of the average value of the periodic coupling strength. The frequencyof the synchronous amplitude envelope keeps equal to that of the periodic couplings. The new born synchronous amplitude envelope may enlarge or suppress the amplitude of the original frequency mismatches induced amplitude envelope in different time of a period. It is meaningful to understand the inner regimes of the correlation between the phase and amplitude coupling in the coupled neural system by exploring the dual amplitude envelope synchronization in the periodically coupled system.

KEYWORDS: periodic coupling; dual amplitude envelope synchronization; frequency mismatches.

1. INTRODUCTION

The synchronization behavior of coupled oscillator systems reflects the collective behavior arising from interactions among subunits in many systems and has drawn much attention.[1-4] It plays a crucial role in understanding the structure formation and functional implementation of many systems. Synchronization in coupled oscillator systems refers to the phenomenon where originally unrelated subsystems in the coupled oscillator system exhibit strong correlations and oscillate in unison under mutual interactions. This phenomenon is often manifested as complete synchronization[5], phase synchronization[6-7], generalized synchronization[8], antiphase synchronization[9], explosive synchronization[10], and amplitude envelope synchronization.[11] It is widely observed in various domains, including electronic communications, chemical reactions, physical systems, biological ecosystems (such as flocks of birds or fishes), nervous systems, and the heart system.

Amplitude envelope synchronization in coupled oscillator systems is another manifestation of strong correlation in the presence of strong coupling. It represents a form of strong correlation among oscillators in coupled nonidentical oscillator systems with interactions between phase and amplitude. In amplitude envelope synchronization, the correlation between the amplitudes of two coupled oscillators are strong while the phases of the two coupled oscillators are weakly correlated. Consequently, the amplitude envelopes modulated on the time series of each oscillator keep synchronous with each other. Amplitude envelope synchronization is observed in various fields, including neuroscience, communication engineering, and biomedicine. In the field of neuroscience, the study of amplitude envelope synchronization is crucial for understanding the interactions and information transfer among neurons in the brain. Adriano et.al[12] conducted a study on the mutual relationship between the phase and amplitude of neurons, discovering that phase-amplitude coupling is crucial for interpreting the significance of continuous physiological electrical signals in the brain. In order to quantify the relationship more accurately, they proposed a novel measurement method to assess the degree of phase-amplitude coupling, providing insights into the amplitude envelope synchronization of signals. In communication engineering, amplitude envelope synchronization can be applied to signal transmission. By modulating high-frequency signals with low-frequency signals, amplitude envelope signals can be generated, thereby improving the efficiency and noise resistance of signal transmission. Amplitude envelope analysis also finds widespread applications in the biomedical field, such as determining neural activity and heart rhythm more effectively through amplitude envelope analysis of electroencephalogram (EEG) and electrocardiogram (ECG) signals.[13] This contributes to understanding the interactionsamong brain neurons and the rhythmic behavior of the heart.

Currently, there is a growing focus on the existence and influencing factors of amplitude envelope synchronization in coupled oscillator systems under constant coupling. The phenomenon of amplitude envelope synchronization was initially discovered in coupled chaotic systems.[11] Later, Zhan et.al observed the amplitude envelope synchronization in coupled periodic oscillator systems, revealing that the correlation between the amplitude is strongly related to the frequency mismatch of coupled oscillators. The regimes of the emergence of amplitude envelope synchronization[14] is theoretically presented. Liu et.al further discovered two types of amplitude envelope synchronization, sinusoidal and non-sinusoidal ones, emerge with different frequency mismatches. An theoretical equation is proposed to describe the main characteristics of the synchronous amplitude envelope.[15] Additionally, a splay state base on the amplitude envelopes is observed by introducing a heterogeneous oscillator in a globally coupled oscillators.[16] However, in real-world systems, coupling interactions are not always constant and may change intermittently or periodically. Forexample, Chen et.al discovered that when the time scale of switch coupling is comparable to the time scale of the local dynamics, the parameter region of the stable synchronization expands, meanwhile the synchronous speed increases remarkably.[17] Moreover, periodic coupling, as one kind of time-varying coupling, also has significant effects on the amplitude death in coupled oscillators. The magnitude and frequency of the periodic coupling significantly affect the stability of amplitude death in the coupled oscillators.[18] However, the effects of the periodic coupling on the amplitude envelope synchronization in coupled oscillators remains unclear. The influence of the periodic coupling on the synchronization stability of the amplitude envelope in coupled oscillators need further exploration. With the notion in mind, a model of periodically coupled nonidentical Stuart-Landau oscillators is set up to reveal the amplitude envelope synchronization. A new synchronous amplitude envelope is born modulated on the original frequency mismatches induced amplitude envelope. The characteristics of the newly generated synchronous amplitude envelope are related to the periodic couplings.

2. Model of periodic coupled nonidentical oscillators

To explore the main characteristics of amplitude envelope synchronization in a coupled oscillators with periodic coupling, a model of two periodically coupled nonidentical Stuart-Landau oscillators is set up,

$$\dot{Z}_1(t)=[\beta+j\omega_1-|Z_1(t)|^2]Z_1(t)+\varepsilon(t)(Z_2(t)-Z_1(t))$$
$$\dot{Z}_2(t)=[\beta+j\omega_2-|Z_2(t)|^2]Z_2(t)+\varepsilon(t)(Z_1(t)-Z_2(t)) \quad (1)$$

Where $Z_i = x_i + jy_i$ $(i = 1,2)$ and $j = \sqrt{-1}$ are complex variables, β determines the amplitude of the single oscillators, ω_i ($\omega_1 \neq \omega_2$) represents the intrinsic angular frequency of the oscillator i, and the angular frequency difference between the two oscillators is denoted as $\Delta\omega = \omega_1 - \omega_2$.

The strength of periodic coupling[18] can be represented as

$\varepsilon(t) = \varepsilon_0 (1+\alpha \cos(\omega_0 t))$ (2)

where ε_0 is the average coupling strength, $\omega_0 = \frac{2\pi}{T_0}$ is the frequency of the periodic coupling and $\alpha \in [0,2]$ is the varying magnitude of the periodic coupling strength. When $\alpha \in (0,1]$, the coupling strength remains positive; whereas, when $\alpha \in (1,2]$, the coupling strength varies between positive and negative values. The time evolution of coupling strength is depicted in Figure 1(a). Here we mainly focus on exploring the dynamics of amplitude envelope as the periodic coupling strength is positive, i.e. $\alpha \in (0,1]$.

In the absence of coupling ($\varepsilon_0 = 0$), the amplitudes of each Stuart-Landau oscillator are identical. In the case of constant coupling strength ($\alpha = 0$), the two coupled oscillator systems exhibit three dynamic behaviors in the parameter spaces of frequency difference and coupling strength, amplitude envelope synchronization, phase-locking, and amplitude death[19] as presented in regions I, II, and III of Figure 1(b). The theoretical boundaries for regions (I, II), (I, II, III), and (II, III) were determined through theoretical analysis in reference[15] and are given by $\Delta\omega = 2\varepsilon$, $\varepsilon = \beta$, $\Delta\omega = 2\sqrt{\beta(2\varepsilon-\beta)}$. When the parameters of the coupled oscillator system are in region I, the time series of the coupled oscillator system are in a non- phase-locked state, while the amplitude envelope modulated on the time series is in synchronous state, as shown in Figure 2(a).

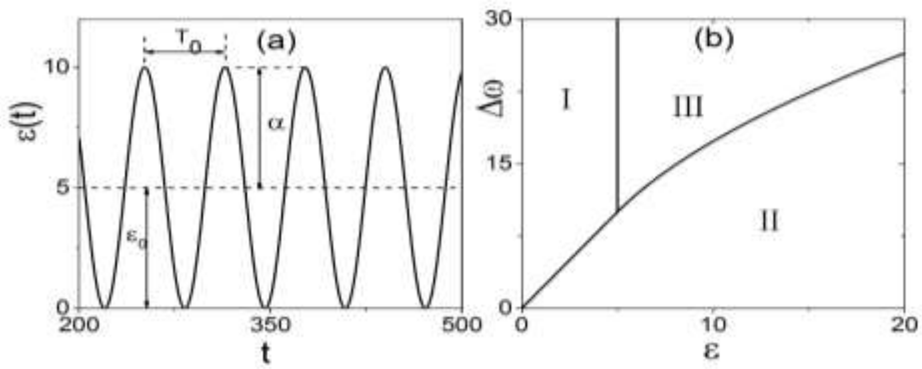

Fig. 1: (a) The periodic coupling strength $\varepsilon(t)$ versus time, ε_0, ω_0 and α are the average

coupling strength, frequency and amplitude of the coupling strength, respectively. (b) Phase diagram in parameter space $\varepsilon \sim \Delta\omega$, I, II, III are the amplitude envelope synchronization, phase locking and amplitude death, respectively.

3. The effects of periodic coupling on the dual amplitude envelope synchronization

With the constant coupling, the coupled nonidentical oscillators exhibit a kind of frequency mismatch induced synchronous amplitude envelope as discussed in reference.[15] Introducing periodic coupling and setting other parameters $\omega_0 = 3$, $\Delta\omega = 30$, $\varepsilon_0 = 2$, $\alpha = 0.1$, the coupled oscillators exhibit dual amplitude envelope synchronization where a new-born synchronous amplitude envelope x_{M1}, x_{M2} is modulating on the original frequency mismatches induced amplitude envelope x_{E1}, x_{E2} which is modulating on the time series as shown in Figure 2(b), whereas the frequencies of the frequency mismatches induced amplitude envelopes x_{E1} and x_{E2} in the time series remain equal to the angular frequency difference $\Delta\omega$ of the coupled oscillators, the frequencies of the new-born amplitude envelopes x_{M1} and x_{M2} modulated on the frequency mismatches induced amplitude envelopes precisely match the varying angular frequency ω_0 of the periodic coupling strength. For convenience, the frequencymismatches induced amplitude envelope x_E is named as the original amplitude envelope,and the coupling-induced amplitude envelope is named as the new-born amplitude envelope x_M. Both types of amplitude envelopes modulated in the time series of the two systems remain in synchronous states. As the magnitude of coupling variations α increases, the amplitude of the new-born amplitude envelope also increases. Figures 2(c)(d) show the time series and the modulated amplitude envelope of the coupled oscillator system for $\alpha = 0.5, 1$,respectively.

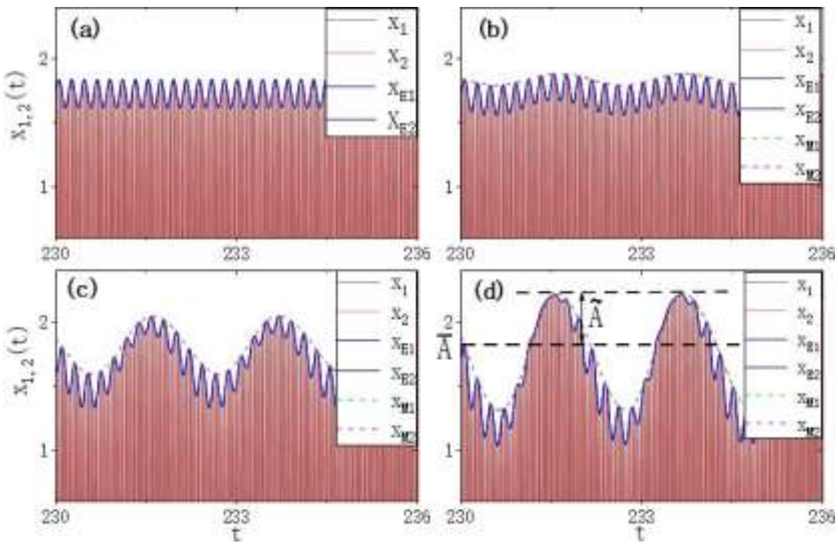

Fig. 2: The time series $x_1(t)$, $x_2(t)$ and their modulating synchronous amplitude envelope in the periodically coupled non-identical Stuart-Landau oscillators (the original amplitude envelope $x_{E1}(t)$, $x_{E2}(t)$,and the new-born amplitude envelope $x_{M1}(t)$, $x_{M2}(t)$) for (a)$\alpha = 0$ (b)$\alpha = 0.1$(c)$\alpha = 0.5$(d)$\alpha = 1$.

To further analyze the impact of the varying angular frequency ω_0 of the couplingstrength on new-born amplitude envelope, we record the time series and their modulating amplitude envelope by arbitrarily setting $\Delta\omega = 30, \alpha = 1, \varepsilon_0 = 2$ and increasing the varying angular frequency ω_0 of the coupling strength. When $\omega_0 = 3$, the angular frequency ω_M of the new-born amplitude envelope equals to 3, whereas the angular frequency ω_E of the original amplitude envelope below it right equals to the original frequency mismatches $\Delta\omega = 30$. Therefore, within one period of the new-born amplitude envelope, there are 10 periods of the original amplitude envelope, as shown in Figure 3(a). When $\omega_0 = 5,6$, the angular frequency ω_M of the new-born amplitude envelope increases accordingly, i.e., $\omega_M = 5,6$, while the angular frequency ω_E of the original amplitude envelope remains constant ($\Delta\omega = 30$). Within one period of the new-born amplitude envelope, the corresponding numbers of periods of the original amplitude envelope are 6 and 5, as shown in Figure 3(b)(c), respectively. The angular frequency ω_M of the new-born amplitude envelope keeps equal to the varying angular frequency ω_0 of the coupling strength for all varying angular frequency ω_0 as shown in Figure 3(d). Therefore, the generation of the amplitude envelope modulated on the original amplitude envelope is attributed to the periodic variations in the coupling strength.

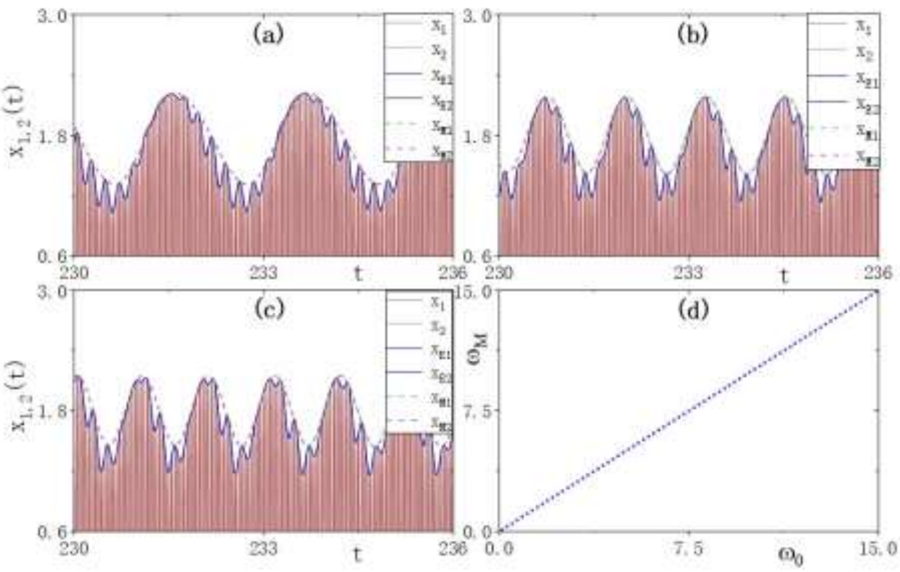

Fig. 3: The time series and their modulating amplitude envelope for (a) $\omega_0 = 3$ (b) $\omega_0 = 5$ (c) $\omega_0 = 6$.(d) The relationship between the frequency of the new-born amplitude envelope and the varying frequency of the periodic coupling.

Considering the effects of the varying magnitude α of the coupling strength on the two types of amplitude envelopes in the coupled oscillators, we record the amplitude of the new-born amplitude envelope by setting $\Delta\omega=30$, $\omega_0=3$, $\varepsilon_0=2$ and increasing varying magnitude α of the coupling strength. The results indicate that the amplitude ofthe new-born amplitude envelope linearly increases to the varying magnitude α of the coupling strength as shown in Figure 4(a). Meanwhile, the varying magnitude α of the periodic coupling strength also affects the amplitude of the original amplitude envelope. Modulated by the new-born amplitude envelope, the amplitude of the original amplitude envelope varies at different time. When the new-born amplitude envelope is at its maximum, the amplitude $\tilde{A}_1$ of the original amplitude envelope is relatively small, whereas when the new-born amplitude envelope is at its minimum, the amplitude $\tilde{A}_2$ of the original amplitude envelope is larger. Comparing with the periodic couplingstrength, it is obvious that the new-born amplitude envelope is precisely at its maximumas the coupling strength is at its minimum, and vice versa. To further determine the impact of the changing magnitude α of the periodic coupling strength on the two amplitudes $\tilde{A}_1$ and $\tilde{A}_2$ of the original amplitude envelope, Figure 4(b) records the relationship between the two amplitudes $\tilde{A}_1$ and $\tilde{A}_2$ of the original amplitude envelope and the varying magnitude α of the coupling strength. Obviously, $\tilde{A}_1$ gradually decreases, while $\tilde{A}_2$ increases slowly with the increment of α. To theoretically predict the relationship between the two amplitudes $\tilde{A}_1$ and $\tilde{A}_2$ of the original amplitude envelope and the varying magnitude α, Let's start from constant coupling strength (α = 0) where the amplitude of the original amplitude envelope is $\tilde{A} = \frac{\varepsilon\sqrt{\beta-\varepsilon}}{\Delta\omega}$ is the constant coupling strength, β is the amplitude parameter of a single oscillator and Δω is the angular frequency mismatches between the two oscillators) as presented in Ref.[15] When introducing periodic coupling strength (with $\alpha \neq 0$ in equation (2)), the maximum value of the instantaneous coupling strength is $\varepsilon_0 (1+\alpha)$, which occurs at the point where $\cos(\omega_0 t) = 1$, and the minimum value is $\varepsilon_0 (1-\alpha)$, which also occurs when $\cos(\omega_0 t) = -1$. Therefore, the theoretical value of the minimum amplitude of the original amplitude envelope is $\tilde{A}_1 = \frac{\varepsilon_0(1-\alpha)\sqrt{\beta-\varepsilon_0(1-\alpha)}}{\Delta\omega}$ and the theoretical value of the maximum amplitude of the original amplitude envelope is $\tilde{A}_2 = \frac{\varepsilon_0(1+\alpha)\sqrt{\beta-\varepsilon_0(1+\alpha)}}{\Delta\omega}$. The theoretical values of the minimum amplitude $\tilde{A}_1$ (the maximum amplitude $\tilde{A}_2$) of the original amplitude envelope versus α are presented in black solid line (red solid line) in Figure 4(b). The theoretical values closely match numerical results with minor errors owing to the fact that the numerical amplitude is obtained by recording the half peak-peak value in one period of the original amplitude envelope, while the theoretical value represents the amplitude of the amplitude envelope for a given instantaneous coupling strength.

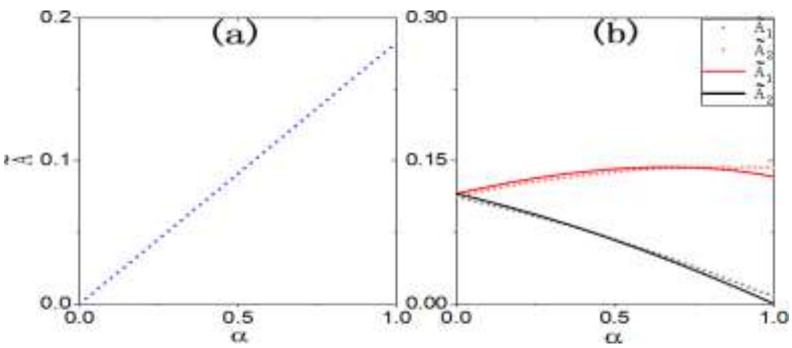

Fig. 4: (a) The relationship between the amplitude $\tilde{A}$ of the amplitude envelope and the variation amplitude α of the coupling strength; (b)The relationship between the

minimum amplitude $\tilde{A}_1$ and maximum amplitude $\tilde{A}_2$ of the original amplitude envelopeand the variation amplitudeα of the coupling strength.

To comprehensively study the impact of periodic coupling strength on the main characteristic parameters of the dual amplitude envelopes in the periodic coupled oscillators, the parameters are arbitrarily set as $\omega_0 = 0.1$, $\alpha = 1$, $\Delta\omega = 30$. Furtheranalysis is conducted on the effect of the average coupling value ε_0 of the periodic coupling strength on the average values $\bar{A}$ and amplitudes $\bar{A}$ of the new-born amplitude envelope. The results indicate that the mean value of the original amplitude envelope shifts downward overall with the increment of the average value ε_0 of the coupling strength as shown in Figure 5(a). Meanwhile, the amplitude $\bar{A}$ of the new-born amplitude envelope continuously increases, as shown in Figure 5(d). The increment of the amplitude α of the periodic coupling strength leads to a continuous increase in the amplitude of the new-born amplitude envelope, as shown in Figure 5(e). However, its impact on the mean value $\bar{A}$ of the new-born amplitude envelope is relatively small.

With the increment of α, the mean value $\bar{A}$ of the new-born amplitude envelope decreases slowly, as shown in Figure 5(b). The frequency mismatches $\Delta\omega$ between the coupled oscillators has a relatively small impact on the new-born amplitude envelope.

With the increment of the frequency mismatches $\Delta\omega$ between the two coupled oscillators, the mean value $\bar{A}$ of the new-born amplitude envelope decreases slowly, while its amplitude $\bar{A}$ increases slowly, as shown in Figures 5(c)(f). Therefore, the introduction of periodic coupling strength is the root cause of the generation of the new- born amplitude envelope, and its impact on the original amplitude envelope is relatively_ small. The introduction of periodic coupling indirectly changes the amplitude value $\bar{A}$ of the original amplitude envelope by modulating a new amplitude envelope on it. Theoscillating frequency of the new-born amplitude envelope is equal to the varying frequency of the periodic coupling strength.

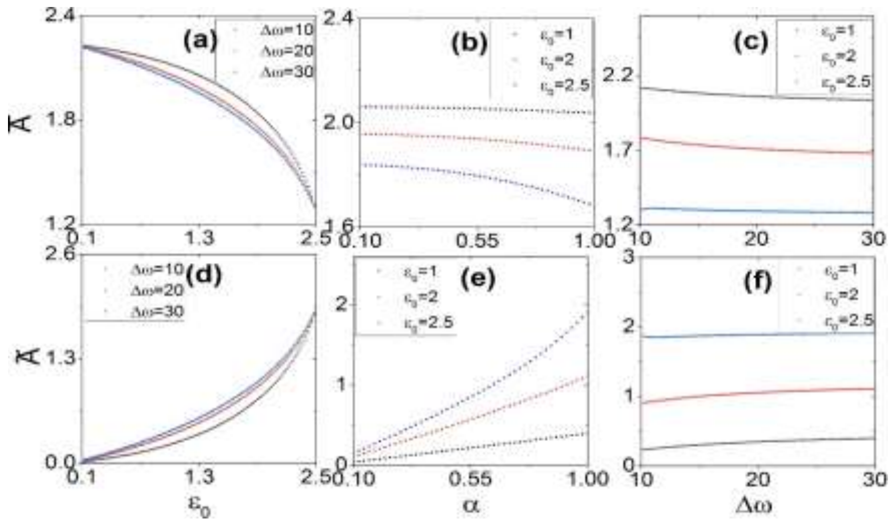

Fig. 4 (a) The average value $\bar{A}$ of the new-born amplitude envelope versus the average coupling strength ε_0;(b) The relationship between the average value $\bar{A}$ of the new-born amplitude envelope and the variation amplitude α of the coupling strength; (c) The average value $\bar{A}$ of the new-born amplitude envelope changes with the frequency mismatches $\Delta\omega$;(d)The relationship between the amplitude $\tilde{A}$ of the new-born amplitude envelope and the average coupling strength ε_0;(e) The relationship between the amplitude $\tilde{A}$ of the new-born amplitude envelope and the variation amplitude α of the coupling strength;(f) The relationship between the amplitude $\tilde{A}$ of the new-born amplitude envelope and the frequency mismatches $\Delta\omega$.

4. CONCLUSION

By introducing periodic coupling in a formally constant coupled nonidentical oscillators a newly generated amplitude envelope is superimposed on the synchronous amplitude envelope caused by the original frequency mismatches. The amplitude and frequency of this newly generated amplitude envelope are closely related to the parameters of the periodic coupling strength, such as variation amplitude and variation frequency. The periodic coupling strength changes the amplitude and frequency of the original synchronous amplitude envelope, leading to the phenomenon of dual synchronous amplitude envelope. The characteristic parameters of the original amplitude envelope are mainly influenced by the frequency mismatches of the coupled oscillators, while the new-born amplitude envelope is entirely determined by the periodic coupling strength. Since the coupling interactions in real systems are not always constant, the impact of varying coupling strength on the amplitude envelope synchronization of coupled oscillators has guiding significance for understanding the correlation between speech recognition and the dynamics of coupled neurons in the human brain.[20]

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