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ELECRICAL – AND - THERMOELECTRIC LAWS GIVEN IN N(P) - TYPE DEGENERATE InSb (1-x) P(x) - CRYSTALLINE ALLOY, DUE TO OUR STATIC DIELECTRIC CONSTANT LAW AND ELECRICAL CONDUCTIVITY FORMULA (II)

Prof. Dr. Huynh Van Cong*

Université De Perpignan Via Domitia, Laboratoire De Mathématiques et Physique (LAMPS), EA 4217, Département De Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

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***Corresponding Author**
Prof. Dr. Huynh Van Cong
Université De Perpignan Via Domitia, Laboratoire De Mathématiques et Physique (LAMPS), EA 4217, Département De Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

ABSTRACT

In the $n^+(p^+) - p(n)$ $\mathbf{InSb_{1-x}P_x}$- crystalline alloy, $0 \le x \le 1$, all the numerical results of electrical-and- thermoelectric coefficients, obtained in our previous work (Van Cong, 2018; Van Cong et al., 1984), are now revised and performed, by basing on our basic expressions, given Equations (1, 3, 5, 7, 11, 14, 19). Some remarkable results could be cited in the following. In Tables 5n (5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n (6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n\ (p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). Further, one notes in these Tables that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n\ (p)} \simeq 1.8138$, while the numerical results of the See beck coefficient S present **a same minimum** $(S)_{min.}$ $(\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of the figure of merit ZT show **a same maximum** $(ZT)_{max.} = 1$, (ii) for $\xi_{n\ (p)} = 1$, S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the Van-Cong coefficient VC, and the Thomson coefficient Ts present **the same results**: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$,

respectively, and (iii) for $\xi^{n} \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent a new law in the thermoelectric properties, obtained in the degenerate case.

KEYWORDS: Electrical conductivity, Seebeck coefficient, Figure of merit, Van-Cong coefficient, Thomson coefficient, Peltier coefficient.

INTRODUCTION

In the $\mathbf{n^+(p^+) - p(n)\ X(x) \equiv InSb_{1-x}P_x}$ - crystalline alloy, $0 \le x \le 1$, all the numerical results of electrical-and-thermoelectric coefficients, obtained in our previous work (Van Cong, 2018; Van Cong et al.,1984), are now revised and performed, by basing on our following basic expressions (Van Cong, 1980 and 2024; Van Cong and Debiais, 1993; Van Cong and Doan Khanh, 1992).

(1) The effective extrinsic static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, due to the impurity size effect, is determined in Eq. (1).

(2) The generalized Mott criterium in the metal-insulator transition is expressed in Equations (3, 5, 6), showing that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $N^{EBT}_{CDn(CDp)}$, with a precision of the order of $\mathbf{2.86} \times \mathbf{10^{-7}}$, as given in our recent work (Van Cong, 2024), and the effective electron (hole)-density: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals.

(3) The ratio of the inverse effective screening length $k_{sn\,(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any density N^*.

(4) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} (Van Cong, 1993), and it is present in all the expressions of electrical-and-thermoelectric coefficients.

(5) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions for determining the following electrical-and- thermoelectric coefficients.

OUR STATIC DIELECTRIC CONSTANT LAW-AND-GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $\mathbf{n^+(p^+) - p(n)\ X(x) \equiv InSb_{1-x}P_x}$ - crystalline alloy at T=0K, we denote the donor (acceptor) d(a) -radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{Sb(In)}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, the unperturbed relative static dielectric constant by: $\varepsilon_o(x)$. Then, their values are reported in **Table 1 in Appendix 1**.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as: $E_{do(ao)}(x) = \frac{13600\times[m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2}$ meV, and then, the isothermal bulk modulus, by: $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right)\times(r_{do(ao)})^3}$

Effect of Impurity $r_{d(a)}$ -size, with a given x

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3)\times(r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3)\times(r_{do(ao)})^3$ (for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) α, $\alpha_o = 0$. Further, the two important equations (Van Cong, 1984 and 2018), used to determine the $\boldsymbol{\alpha}$ -variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\alpha}{dV}$. giving: $\frac{d}{dV}\left(\frac{d\alpha}{dV}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\alpha(\mathbf{r}_{d(a)},\mathbf{x})]_{n(p)} = B_{do(ao)}(x)\times(V-V_{do(ao)})\times\ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x)\times\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)},x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)},x)$ in absolute values, obtained in the

effective Bohr model, which is represented respectively by: $\pm[\Delta\alpha(\mathbf{r}_{d(a)},\mathbf{x})]_{n(p)}$,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x)\times\left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +[\Delta\alpha(\mathbf{r}_{d(a)},\mathbf{x})]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x)\times\left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -[\Delta\alpha(\mathbf{r}_{d(a)},\mathbf{x})]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant ε ($r_{d(a)}$, x) and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new** $\boldsymbol{\varepsilon(r_{d(a)}, x)}$**-law,**

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \qquad (1a)$$

According to the increase in both $E_{gn(gp)}$ ($r_{d(a)}$, x) and $E_{d(a)}$ ($r_{d(a)}$, x), with increasing $r_{d(a)}$ and for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by:

$\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being a **new** $\boldsymbol{\varepsilon(r_{d(a)}, x)}$**-law,**

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \qquad (1b)$$

Corresponding to the decrease in both $E_{gn(gp)}$ ($r_{d(a)}$, x) and $E_{d(a)}$ ($r_{d(a)}$, x), with decreasing $r_{d(a)}$ and for a given x; therefore, the effective Bohr radius $a_{Bn(Bp)}$ ($r_{d(a)}$, x) is defined by

$$a_{Bn(Bp)}(r_{d(a)}, x) \equiv \frac{\varepsilon(r_{d(a)}, x) \times \hbar^2}{m_{c(v)}(x) \times m_o \times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)}, x)}{m_{c(v)}(x)}. \qquad (2)$$

Generalized Mott Criterium in the Metal-Insulator Transition

Furthermore, it is interesting to remark that the critical total donor (acceptor)-density in the metal-insulator transition (**MIT**) at T=0 K, $N_{CDn(NDp)}$ ($r_{d(a)}$, x), was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as:

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = M_{n(p)},\ M_{n(p)} = 0.25, \qquad (3)$$

Depending thus on our new $\boldsymbol{\varepsilon(r_{d(a)}, x)}$**-law.**

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, by:

$$r_{sn(sp)}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)}, x)} = 1.1723 \times 10^8 \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}(x) \times m_o}{\varepsilon(r_{d(a)}, x)}, \qquad (4)$$

Being equal to, in particular, at N= $N_{CDn(CDp)}$ ($r_{d(a)}$, x): $r_{sn(sp)}$ ($N_{CDn(CDp)}$ ($r_{d(a)}$, x), $r_{d(a)}$, x) = **2.4813963**, for any ($r_{d(a)}$, x) - values. Then, from Eq. (4), one also has:

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \times \frac{1}{2.4813963} = \mathbf{0.25} = (\mathbf{WS})_{\mathbf{n(p)}} = \mathbf{M}_{\mathbf{n(p)}} \qquad (5)$$

Explaining thus the existance of the Mott's criterium

Furthermore, by using $\mathbf{M_{n(p)}} = \mathbf{0.25}$, according to the empirical Heisenberg parameter $\mathcal{H}_{\mathbf{n(p)}} = \mathbf{0.47137}$, as those given in our previous work (Van Cong, 2024), we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail,** $\mathbf{N^{EBT}_{CDn(CDp)}}$, with a precision of the order of $\mathbf{2.86} \times \mathbf{10^{-7}}$.

It should be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $\mathbf{N^{EBT}_{CDn(CDp)}}$ are in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials, by:

$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) = N^*$ for a presentation simplicity. (6)

In summary, as observed in Table 1 of our previous paper (Van Cong, 2024), one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}$ $(r_{d(a)}, x)$, $N_{CDn(NDp)}$ $(r_{d(a)}, x)$ and $\mathbf{N^{EBT}_{CDn(CDp)}}$ $(r_{d(a)}, x)$ increase, affecting strongly all the physical properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+ (p^+) - p(n)$ $\mathbf{X(x) \equiv InSb_{1-x}P_x}$ - crystalline alloy, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{g_{c(v)}}\right)^{\frac{1}{3}}$ the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^*, is now defined by: $\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k^{-1}_{Fn(Fp)}}{a_{Bn(Bp)}} < 1,$

Being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k^{-1}_{Fn(Fp)}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}$ $(r_{d(a)}, x)$ is determined in Eq. (2). Then, **the ratio of the inverse effective screening length $\mathbf{k_{sn(sp)}}$** to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k^{-1}_{Fn(Fp)}}{k^{-1}_{sn(sp)}} = R_{snWS(spWS)} + \left[R_{snTF(spTF)} - R_{snWS(spWS)}\right]e^{-r_{sn(sp)}} < 1, \quad (7)$$

Being valid at any N*

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows. First, for $N \gg$

$N_{CDn(NDp)}$ ($r_{d(a)}$, x) , according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}$ (N^*) is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

Being proportional to $N^{*-1/6}$

Secondly, for $N \ll N_{CDn(NDp)}$ ($r_{d(a)}$), according to the **Wigner-Seitz (WS)-approximation**, the ratio $R^{snWS(snWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N^*)]}{dr_{sn(sp)}}\right), \quad (9)$$

Where E_{CE} (N^*) is the majority-carrier correlation energy (CE), being determined by (Van Cong, 2018):

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by (Van Cong, 2018)

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \eta_{n(p)}(N^*) \equiv \frac{\sqrt{2\pi N^*}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (10)$$

Which gives: $A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy and generalized Einstein relation

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$ - crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$-crystalline alloy, according to the reduced Fermi energy, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$ obtained respectively in the degenerate (non-degenerate) case.

For any (N, $r_{d(a)}$, x, T), the reduced Fermi energy $\xi_{n(p)}$ (N, $r_{d(a)}$, x, T) or the Fermi energy $E_{Fn(Fp)}$ (N, $r_{d(a)}$, x, T), obtained in our previous paper (Van Cong and Debiais, 1993), obtained with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u)+Au^B F(u)}{1+Au^B} \equiv \frac{V(u)}{W(u)}, \; A = 0.0005372 \text{ and } B = 4.82842262, \tag{11}$$

Where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T,x)}$, $N_{c(v)}(T,x) = 2g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} (cm^{-3})$, $g_{c(v)} = 1$, $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as $\boldsymbol{u \longrightarrow 0}$**, the limiting condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)}(x) \times m_o}$ as $\boldsymbol{u \longrightarrow \infty}$ **, the limiting condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions. In the following, it will be present in all the electrical-and-thermoelectric coefficients.

In particular, at T=0K, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)}(x) \times m_o}$ being proportional to $(N^*)^{2/3}$, and also equal to 0, according to the MIT. In the following, it should be noted that such the accurate expression of $\xi_{n(p)}$ (N, $r_{d(a)}$, x, T) is present in all the following electrical-and-thermoelectric.

FERMI-DIRAC DISTRIBUTION FUNCTION (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$. So, the average of E^p, calculated using the FDDF-method, as developed in our previous work (Van Cong, 2018) is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \; -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}.$$

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno}(F_{po})) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn}(F_p))/(k_B T)$, one has:

$$G_p(E_{Fn(Fp)}) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\beta=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E^{-\beta}_{Fn(Fp)} \times I_{\beta},$$ where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_\beta = \int_{-\infty}^{\infty} \frac{\gamma^\beta \times e^\gamma}{(1+e^\gamma)^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^\beta}{(e^{\gamma/2}+e^{-\gamma/2})^2}$ dγ vanishing for old values of β. Then, for even values of β=2n, with n=1, 2 one obtains: $I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^\gamma}{(1+e^\gamma)^2} d\gamma$.

Now, using an identity $(1+e^\gamma)^{-2} \equiv \sum_{s=1}^{\infty}(-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$ and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. So, from Eq. (22), we get in the degenerate case the following ratio:

$$G_p(E_{Fn(Fp)}) \equiv \frac{\langle E^p \rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\dots(p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n} \equiv G_{p\geq 1}(y), \quad (12)$$

Where $y \equiv \frac{\pi}{\xi_{n(p)}(N^*,T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*,T)}$.

Then, some usual results of $G_p \geq 1(y)$ are given **in Table 2 in Appendix 1, being important ones in this work.**

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by σ (N, $r_{d(a)}$, x, T), expressed in ohm−1×cm−1, the thermal conductivity by κ(N, $r_{d(a)}$, x, T), expressed in $\frac{W}{cm \times K}$, and Lorenz number L by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8}\ (V^2 \times K^{-2})$

then the well-known Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\kappa(N, r_{d(a)}, x, T)}{\sigma(N, r_{d(a)}, x, T)} = L \times T. \quad (13)$$

We now determine the general form of σ in the following.

First, it is expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{cn(cp)} \times m_o}$, or the wave number k, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\eta_{n(p)}}\right)^{1/2}$$

Which is thus proportional to $E_k 2$.

Then, for E≥0, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E^2_{Fn(Fp)}$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N, r_{d(a)}, x, T)$ for a presentation simplicity. Therefore, one obtains:

$$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi\times\hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times \left[k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})\right] \times \sqrt{A_{n(p)}(N^*)}\right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)}\right)^2\right] \left(\frac{1}{\text{ohm}\times\text{cm}}\right) \quad , \quad \frac{q^2}{\pi\times\hbar} = 7.7480735\times 10^{-5}\ \text{ohm}^{-1} \quad , \quad A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)} \tag{14}$$

Which also determine the resistivity as:, $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$ noting that $N^* \equiv N - N_{CDnNDp)}$ $(r_{d(a)}, x)$. **This σ(N, $r_{d(a)}$, x, T)-result is an essential one in this paper.** In Eq. (14), one notes that at T= 0 K, σ(N, $r_{d(a)}$, x, T = 0K) is proportional to $E_{Fno(Fpo)}^{\frac{3}{2}}$ or to N^*. Thus, σ(N = $N_{CDn(NDp)}$, $r_{d(a)}$, x, T = 0K) = 0 at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by:

$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_{c(v)}(x)\times m_o}{q^2\times N^*}$ Therefore, the mobility μ is given by:

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q\times\tau(N, r_{d(a)}, x, T)}{m_{c(v)}(x)\times m_o} = \frac{\sigma(N, r_{d(a)}, x, T)}{q\times N^*}\ \left(\frac{cm^2}{V\times s}\right). \tag{15}$$

Here, at T = 0K, μ (N^*, $r_{d(a)}$, T) is thus proportional to $(N^*)^{1/3}$, since σ(N^*, $r_{d(a)}$, T = 0K) is proportional to $(N^*)^{4/3}$. Thus, μ (N^* = 0, $r_{d(a)}$, T = 0K) = 0 at N^* = 0, at which the metal-insulator transition (MIT) occurs. Then, since τ and σ are both proportional to $E_{Fn(Fp)}$ $(N^*, T)^2$, as given above, the Hall factor can thus be determined by:

$r_H(N, r_{d(a)}, x, T) \equiv \frac{\langle\tau^2\rangle_{FDDF}}{[\langle\tau\rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}$, and therefore, the Hall mobility yields:

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T) \times r_H(N^*, T)\ \left(\frac{cm^2}{V\times s}\right), \tag{16}$$

noting that, at T = 0K, since r_H(N, $r_{d(a)}$, x, T) = 1, one then gets at N = $N_{CDn(NDp)}$: μ_H(N, $r_{d(a)}$, x, T) ≡ μ(N, $r_{d(a)}$, x, T) = 0 at N^* = 0, at which the metal-insulator transition (MIT) occurs. Finally, our **generalized Einstein relation** is found to be defined (Van Cong, 1980) as:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B\times T}{q} \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right), \tag{17}$$

Where D (N, $r_{d(a)}$, x, T) is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility μ (N, $r_{d(a)}$, x, T) is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u, one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$$\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} = \frac{k_B \times T}{q} \times u \frac{V'(u)\times W(u) - V(u)\times W'(u)}{W^2(u)},$$

where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}}e^{-du}(1-du) + \frac{2}{3}Au^{B-1}F(u)\left[\left(1+\frac{3B}{2}\right) + \frac{4}{3}\times\frac{bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}}{1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}}\right]$

One remarks that: (i) as u → 0, one has: W2 ≃ 1 and u [V′ × W – V × W′] ≃ 1, and therefore: $\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}$, and (ii) as u → ∞, one has: $W^2 \approx A^2u^{2B}$ and u [V′ × W – V × W′] $\approx \frac{2}{3}au^{2/3}A^2u^{2B}$, and therefore, in this **highly degenerate case** and at T = 0K, the **above generalized Einstein relation** is reduced to the **usual Einstein one**: $\frac{D(N,r_{d(a)},x,T=0\,K)}{\mu(N,r_{d(a)},x,T=0\,K)} \approx \frac{2}{3}E_{Fno(Fpo)}(N^*)/q$. In other words, **Eq. (17) verifies the correct limiting conditions.** One also notes that, for $N^* = 0$, μ ($N^* = 0$, $r_{d(a)}$, T = 0K) = 0, as remarked in above, and therefore, for any $r_{d(a)}$, D ($N^* = 0$, $r_{d(a)}$, T = 0K) = 0, according to the MIT.

Further, in the present degenerate case (u≫1), Eq. (17) gives:

$$\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}\right)}{\left(1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}\right)}\right],$$

where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

In **Tables 3n (3p) given in Appendix 1**, for given x, N > N_{CDn} and T (= 4.2 K and 77 K), and from Equations (14, 15, 16, 18), the numerical results of the coefficients: σ, μ, μH, D, expressed respectively in $\left(\frac{10^3}{ohm\times cm}, \frac{10^3\times cm^2}{V\times s}, \frac{10^3\times cm^2}{V\times s}, \frac{10\times cm^2}{s}\right)$, are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First off all, from Eq. (14), obtained for σ(N, $r_{d(a)}$, x, T), the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S, is given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \frac{\partial \ln\sigma(E)}{\partial E}\bigg]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln\sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for $\xi_{n(p)}$ (N, $r_{d(a)}$, x, T) $\gtrsim$ 1, one gets, by putting F_S(N, $r_{d(a)}$, x, T) $\equiv$ $\left[1 - \frac{y^2}{3\times G_2\left(y=\frac{\pi}{\xi_{n(p)}}\right)}\right]$,

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N^*,T)}{\xi_{n(p)}} = -\sqrt{\frac{3\times L}{\pi^2}} \times \frac{2\times\xi_{n(p)}}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)}\left(\frac{V}{K}\right), \quad (19)$$

Giving here: (i) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, one gets: $S = -\sqrt{L} \simeq -1.563\times10^{-4}\left(\frac{V}{K}\right)$ and at $\xi_{n(p)} = 1$ one Obtains: $S \simeq -1.322\times10^{-4}\left(\frac{V}{K}\right)$.

Further, the figure of merit, ZT, is found to be given by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{S^2\times\sigma\times T}{\kappa} = \frac{S^2}{L} = (ZT)_{Mott}\times[2\times F_S]^2,\ (ZT)_{Mott} = \frac{\pi^2}{3\times\xi_{n(p)}^2}, \quad (20)$$

Giving here: (i) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, one gets: $ZT = (ZT)_{Mott} = 1$, and at $\xi_{n(p)} = 1$ one obtains: $ZT \simeq 0.715$ and $(ZT)_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Furthermore, from Eq. (19), one gets:

$$\frac{\partial S}{\partial\xi_{n(p)}} = \sqrt{\frac{3\times L}{\pi^2}} \times \frac{\frac{3\times\xi_{n(p)}^2}{\pi^2}-1}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)^2},\ \frac{dS}{dT} = \frac{\partial S}{\partial\xi_{n(p)}}\times\frac{\partial\xi_{n(p)}}{\partial T},\ \text{and}\ \frac{dS}{dN^*} = \frac{\partial S}{\partial\xi_{n(p)}}\times\frac{\partial\xi_{n(p)}}{\partial N^*},\ \text{and}\ \frac{d(ZT)}{dT} = \frac{2\times S}{L}\times\frac{dS}{dT}\ \text{and}\ \frac{d(ZT)}{dN^*} = \frac{2\times S}{L}\times\frac{dS}{dN^*},$$

Noting that: (i) at given (N, $r_{d(a)}$, x), and for $\frac{\partial\xi_{n(p)}}{\partial T} > 0\ (or\ < 0)$, $\xi_{n(p)}$ increases (or decreases) for decreasing (or increasing) T, (ii) at given ($r_{d(a)}$, x, T), and for $\frac{\partial\xi_{n(p)}}{\partial T} > 0\ (or\ < 0)$, $\xi_{n(p)}$ increases (or decreases) for increasing (or decreasing) N.

Finally, the Van-Cong coefficient, VC, is defined by:

$$VC(N, r_{d(a)}, x, T) \equiv N^* \times \frac{dS}{dN}\left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial\xi_{n(p)}} \times \frac{\partial\xi_{n(p)}}{\partial N^*},\ \text{being equal to 0 for}\ \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \quad (22)$$

The Thomson coefficient, Ts, by:

$$Ts(N, r_{d(a)}, x, T) \equiv T \times \frac{dS}{dT}\left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial\xi_{n(p)}} \times \frac{\partial\xi_{n(p)}}{\partial T},\ \text{being equal to 0 for}\ \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

And then, the Peltier coefficient, Pt, as:

Pt(N, $r_{d(a)}$, x, T)$\equiv$ T × S (V). (24)

Furthermore, from Equations (17, 22), we can obtain a new electrical-and-thermoelectric law by: $\frac{k_B \times T}{q} \times VC(N, r_{d(a)}, x, T) \equiv \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \left(\frac{V^2}{K}\right)$, (25)

Where, as given in Eq. (21), $\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3\times L}{\pi^2}} \times \frac{\frac{3\times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1+\frac{3\times \xi_{n(p)}^2}{\pi^2}\right)^2}$

Now, in the lightly degenerate n(p)-type X(x)− alloy, in which **$N=5\times10^{17}$ cm^{-3}(10^{19} cm^{-3})>$N_{CDn(CDp)}$**, and for T=3K and 80K, **the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1**, noting that their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in **Tables 5n(5p) given Appendix 1 for a given N and with increasing T**, and then in **Tables 6n(6p) given Appendix 1 for a given T and with decreasing N**, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present **a same minimum** $(S)_{min.}$ $\left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$ those of ZT show **a same maximum** $(ZT)_{max.}$=1, (ii) for $\xi_{n(p)}$=1, those of S, ZT, $(ZT)_{Mott,}$ VC, and T_s present **the same results**: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these results could represent **a new law in the thermoelectric properties, obtained in the degenerate case.**

CONCLUDING REMARKS

In the $n^+(p^+) - p(n)$ X(x) − cristalline alloy, all the numerical results of electrical-and-thermoelectric coefficients, obtained in our previous work (Van Cong, 2018), were revised and performed, by basing on our following basic expressions.

1. The effective extrinsic static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, due to the impurity size effect, is determined in Eq. (1).
2. The generalized Mott criterium in the metal-insulator transition is expressed in Equations (3, 5, 6), showing that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $N^{EBT}_{CDn(CDp)}$ with a precision of the order of **2.86×10^{-7}**, as given in our recent work (Van Cong, 2024), and the effective electron

(hole)-density: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals.

3. The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0K, $R_{sn(sp)}$ (N^*), defined in Eq. (7), is valid at any density N^*.
4. The Fermi energy for any N and T, $E_{Fn(Fp),}$ determined in Eq. (11) with a precision of the order of 2.11×10^{-4} (Van Cong, 1993), and it exists in all the expressions of electrical-and-thermoelectric coefficients.
5. Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions for determining the electrical-and-thermoelectric coefficients.
6. Our new electrical-and-thermoelectric law is given in Eq. (25), by:

$$\frac{k_B \times T}{q} \times VC\left(N, r_{d(a)}, x, T\right) \equiv \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \left(\frac{V^2}{K}\right), \frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3\times L}{\pi^2}} \times \frac{\frac{3\times \xi_{n(p)}{}^2}{\pi^2} - 1}{\left(1+\frac{3\times \xi_{n(p)}{}^2}{\pi^2}\right)^2}.$$

7. Finally, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T, and then in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present **a same minimum** $(S)_{min.}$ $\left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of ZT show **a same maximum** $(ZT)_{max.}$=1, (ii) for $\xi_{n(p)}$=1, those of S, ZT, $(ZT)_{Mott,}$ VC, and T_s present **the same results**:

$$-1.322 \times 10^{-4} \frac{V}{K}, 0.715, 3.290, -1.105 \times 10^{-4} \frac{V}{K}, \text{ and } 1.657 \times 10^{-4} \frac{V}{K},$$

Respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott}$=1. It seems that these results could represent a new law in the thermoelectric properties, obtained in the degenerate case. In summary, all the numerical results of electrical-and-thermoelectric coefficients, given in our previous work (Van Cong, 2018), are now revised and performed.

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APPENDIX 1

Table 1: The values of energy-band-structure parameters are given in the following.

In $InSb_{1-x}P_x$-alloy, in which $r_{do(ao)}$=$r_{Sb(In)}$=0.136 nm (0.144 nm), we have: $g_{c(v)}(x) = 1 \times x + 1 \times (1-x)$, $m_{c(v)}(x)/m_o = 0.007\ (0.5) \times x + 0.1\ (0.4) \times (1-x)$, $\varepsilon_o(x) = 12.5 \times x + 16.8 \times (1-x)$, $E_{go}(x) = 1.424 \times x + 0.23 \times (1-x)$.

Table 2: Expressions for $G_{p\geq 1}(y \equiv \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}+\frac{7y^4}{640}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}-\frac{7y^4}{384}\right)$	$(1+y^2)$	$\left(1+\frac{35y^2}{24}+\frac{49y^4}{384}\right)$	$\left(1+2y^2+\frac{7y^4}{15}\right)$	$\left(1+\frac{21y^2}{8}+\frac{147y^4}{128}\right)$

Table 3n: Here, one notes that, for given x, N>N_{CDn} and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{\text{ohm}\times\text{cm}}, \frac{10^3\times \text{cm}^2}{\text{V}\times\text{s}}, \frac{10^3\times \text{cm}^2}{\text{V}\times\text{s}}, \frac{10^2\times\text{cm}^2}{\text{s}}\right)$, decrease with increasing r_d.

Donor	P	As	Sb	Sn
r_d (nm) ↗	0.110	0.118	0.136	0.140
For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{18} cm^{-3})				
3	1.69, 3.525, 3.526, 1.78	1.48, 3.098, 3.099, 1.56	1.33, 2.793, 2.794, 1.40	1.32, 2.779, 2.780, 1.40
10	4.39, 2.745, 2.746, 3.09	3.82, 2.387, 2.388, 2.69	3.41, 2.136, 2.136, 2.41	3.40, 2.124, 2.124, 2.39
40	14.0, 2.185, 2.185, 6.21	12.0, 1.874, 1.874, 5.33	10.6, 1.658, 1.658, 4.71	10.5, 1.648, 1.648, 4.68
70	22.7, 2.026, 2.026, 8.36	19.4, 1.730, 1.730, 7.14	17.1, 1.524, 1.524, 6.29	17.0, 1.515, 1.515, 6.25
For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{18} cm^{-3})				
3	1.36, 2.850, 2.851, 162	1.20, 2.512, 2.513, 1.43	1.08, 2.266, 2.267, 1.29	1.07, 2.255, 2.256, 1.28
10	3.47, 2.168, 2.168, 2.76	3.04, 1.899, 1.899, 2.42	2.73, 1.707, 1.708, 2.17	2.71, 1.699, 1.699, 2.16
40	10.7, 1.665, 1.665, 5.35	9.22, 1.439, 1.439, 4.62	8.20, 1.281, 1.281, 4.11	8.16, 1.274, 1.274, 4.09
70	17.1, 1.525, 1.525, 7.11	14.7, 1.311, 1.311, 6.12	13.0, 1.161, 1.161, 5.41	12.9, 1.154, 1.154, 5.38
For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**				

For **x=1**, the values of (σ, μ, μ_H, D) at **4. 2K**

$N\ (10^{18}\ cm^{-3})$				
3	1.09, 2.289, 2.289, 1.50	0.96, 2.012, 2.012, 1.31	0.86, 1.809, 1.809, 1.18	0.86, 1.799, 1.800, 1.17
10	2.76, 1.723, 1.723, 2.52	2.43, 1.517, 1.517, 2.22	2.19, 1.368, 1.369, 2.00	2.18, 1.362, 1.362, 1.99
40	8.20, 1.280, 1.280, 4.72	7.14, 1.115, 1.115, 4.11	6.40, 0.999, 0.999, 3.68	6.36, 0.993, 0.993, 3.66
70	12.9, 1.155, 1.155, 6.19	11.2, 1.000, 1.000, 5.36	10.0, 0.892, 0.892, 4.78	9.94, 0.887, 0.887, 4.75

For **x=0**, the values of (σ, μ, μ_H, D) at **77 K**

$N\ (10^{18}\ cm^{-3})$				
3	1.76, 3.683, 4.038, 1.84	1.55, 3.237, 3.550, 1.61	1.39, 2.919, 3.201, 1.45	1.38, 2.904, 3.185, 1.45
10	4.33, 2.770, 2.825, 3.11	3.85, 2.409, 2.457, 2.71	3.44, 2.155, 2.198, 2.42	3.43, 2.143, 2.186, 2.41
40	14.0, 2.188, 2.195, 6.22	12.0, 1.877, 1.883, 5.33	10.6, 1.660, 1.666, 4.72	10.6, 1.651, 1.656, 4.69
70	22.7, 2.028, 2.031, 8.37	19.4, 1.731, 1.734, 7.14	17.1, 1.525, 1.528, 6.29	17.0, 1.516, 1.518, 6.26

For **x=0.5**, the values of (σ, μ, μ_H, D) at **77 K**

$N\ (10^{18}\ cm^{-3})$				
3	1.41, 2.950, 3.175, 1.67	1.24, 2.600, 2.799, 1.47	1.12, 2.346, 2.526, 1.32	1.11, 2.334, 2.513, 1.32
10	3.49, 2.183, 2.217, 2.77	3.06, 1.912, 1.942, 2.43	2.75, 1.719, 1.746, 2.18	2.73, 1.710, 1.737, 2.17
40	10.7, 1.667, 1.671, 5.35	9.23, 1.441, 1.444, 4.62	8.21, 1.282, 1.286, 4.12	8.17, 1.275, 1.278, 4.09
70	17.1, 1.525, 1.527, 7.11	14.7, 1.311, 1.313, 6.11	13.0, 1.162, 1.163, 5.42	12.9, 1.155, 1.156, 5.39

For **x=1**, the values of (σ, μ, μ_H, D) at **77 K**

$N\ (10^{18}\ cm^{-3})$				
3	1.12, 2.349, 2.486, 1.53	0.99, 2.065, 2.186, 1.34	0.88, 1.857, 1.966, 1.20	0.88, 1.847, 1.955, 1.19
10	2.77, 1.732, 1.753, 2.53	2.44, 1.525, 1.543, 2.23	2.20, 1.376, 1.392, 2.01	2.19, 1.369, 1.385, 2.00
40	8.21, 1.281, 1.283, 4.73	7.15, 1.116, 1.118, 4.12	6.40, 0.999, 1.001, 3.69	6.37, 0.994, 0.996, 3.67
70	12.9, 1.155, 1.156, 6.19	11.2, 1.001, 1.002, 5.36	10.0, 0.892, 0.893, 4.78	9.95, 0.887, 0.888, 4.75

Table 3p: Here, one notes that, for given x, $N > N_{CDp}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm\times cm}, \frac{10^3\times cm^2}{V\times s}, \frac{10^3\times cm^2}{V\times s}, \frac{10\times cm^2}{s}\right)$, decrease with increasing r_a.

Acceptor		Ga	Mg	In	Cd
r_a (nm)	↗	0.120	0.140	0.144	0.148

For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{18} cm^{-3})				
3	0.47, 1.598, 1.613, 1.47	0.37, 1.460, 1.477, 1.21	0.37, 1.454, 1.470, 1.20	0.36, 1.448, 1.464, 1.19
10	1.92, 1.356, 1.358, 3.53	1.67, 1.210, 1.211, 3.08	1.65, 1.203, 1.204, 3.06	1.64, 1.196, 1.197, 3.04
40	7.52, 1.209, 1.209, 8.43	6.63, 1.072, 1.072, 7.44	6.58, 1.065, 1.065, 7.39	6.54, 1.058, 1.059, 7.34
70	12.8, 1.164, 1.164, 11.9	11.3, 1.031, 1.031, 10.5	11.2, 1.024, 1.024, 10.4	11.2, 1.018, 1.018, 10.4

For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{18} cm^{-3})				
8	0.84, 0.951, 0.953, 1.61	0.68, 0.858, 0.861, 1.35	0.67, 0.854, 0.856, 1.34	0.67, 0.849, 0.852, 1.33
10	1.11, 0.922, 0.923, 1.91	0.93, 0.828, 0.829, 1.63	0.92, 0.823, 0.825, 1.62	0.91, 0.819, 0.820, 1.61
40	4.85, 0.807, 0.807, 4.88	4.24, 0.716, 0.716, 4.29	4.21, 0.712, 0.712, 4.26	4.18, 0.707, 0.707, 4.24
70	8.38, 0.775, 0.775, 6.94	7.37, 0.687, 0.687, 6.12	7.32, 0.682, 0.682, 6.08	7.27, 0.678, 0.678, 6.04

For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{18} cm^{-3})				
8	0.28, 0.689, 0.695, 0.63	0.15, 0.681, 0.696, 0.40	0.14, 0.683, 0.699, 0.39	0.13, 0.686, 0.703, 0.38
10	0.47, 0.641, 0.644, 0.86	0.32, 0.596, 0.600, 0.65	0.31, 0.595, 0.599, 0.64	0.30, 0.593, 0.597, 0.63
40	2.94, 0.530, 0.531, 2.74	2.52, 0.472, 0.472, 2.38	2.50, 0.469, 0.470, 2.36	2.48, 0.467, 0.467, 2.34
70	5.24, 0.507, 0.507, 3.97	4.57, 0.450, 0.450, 3.47	4.53, 0.447, 0.447, 3.45	4.50, 0.444, 0.444, 3.43

For x=**0**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{18} cm^{-3})				
3	0.65, 2.188, 6.522, 2.73	0.54, 2.109, 6.844, 2.42	0.53, 2.108, 6.874, 2.40	0.53, 2.107, 6.907, 2.39
10	2.25, 1.586, 2.094, 3.99	1.96, 1.423, 1.894, 3.51	1.95, 1.416, 1.885, 3.49	1.93, 1.408, 1.875, 3.46
40	7.70, 1.237, 1.301, 8.58	6.78, 1.096, 1.154, 7.57	6.74, 1.090, 1.147, 7.52	6.69, 1.083, 1.140, 7.47
70	12.9, 1.177, 1.206, 12.0	11.4, 1.042, 1.068, 10.6	11.4, 1.036, 1.061, 10.5	11.3, 1.029, 1.054, 10.4

For x=**0.5**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{18} cm^{-3})				
8	1.11, 1.256, 2.082, 2.52	0.91, 1.138, 1.991, 2.31	0.90, 1.133, 1.988, 2.29	0.89, 1.127, 1.985, 2.28
10	1.40, 1.157, 1.694, 2.40	1.18, 1.056, 1.588, 2.14	1.17, 1.051, 1.584, 2.13	1.16, 1.047, 1.580, 2.11
40	5.00, 0.831, 0.888, 5.00	4.38, 0.739, 0.790, 4.40	4.35, 0.734, 0.785, 4.37	4.32, 0.730, 0.780, 4.34
70	8.50, 0.786, 0.811, 7.02	7.47, 0.696, 0.719, 6.18	7.42, 0.692, 0.714, 6.14	7.37, 0.688, 0.710, 6.10

For x=**1**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{18} cm^{-3})				
10	0.61, 0.838, 1.707, 1.60	0.42, 0.776, 1.960, 1.17	0.41, 0.775, 1.986, 1.15	0.40, 0.775, 2.015, 1.13
40	3.06, 0.553, 0.604, 2.83	2.64, 0.493, 0.541, 2.46	2.61, 0.490, 0.538, 2.44	2.59, 0.488, 0.535, 2.42
70	5.34, 0.516, 0.537, 4.02	4.65, 0.458, 0.478, 3.53	4.62, 0.456, 0.475, 3.50	4.59, 0.453, 0.472, 3.48

Table 4n: In the lightly degenerate n-type X(x) − alloy, in which $N=5\times10^{17}$ cm^{-3}, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor		P	As	Sb	Sn
For x=0,					
$\xi_{n(T=3K)}$	↘	87.31	86.776	86.209	86.178
$\xi_{n(T=80K)}$	↘	3.30	3.270	3.240	3.238
$\kappa_{(T=3K)}$ $(\frac{10^{-5}\times W}{cm\times K})$	↘	3.077	2.678	2.384	2.370
$\kappa_{(T=80K)}$ $(\frac{10^{-4}\times W}{cm\times K})$	↘	10.84	9.429	8.384	8.335
$-S_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	6.491	6.531	6.574	6.576
$-S_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	13.201	13.260	13.324	13.327
$VC_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↗	4.322	4.349	4.377	4.379
$VC_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	6.477	6.472	6.463	6.4629
$-Ts_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	6.484	6.523	6.566	6.568
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.947	1.959	1.972	1.973
$-Pt_{(T=80K)}(10^{-2}\times V)$	↘	1.056	1.061	1.066	1.0662
$ZT_{(T=3K)}$ (10^{-3})	↗	1.725	1.746	1.769	1.770
$ZT_{(T=80K)}$	↗	0.713	0.7197	0.72665	0.72704
For x=0.5,					
$\xi_{n(T=3K)}$	↘	98.57	97.941	97.272	97.235
$\xi_{n(T=80K)}$	↘	3.85	3.823	3.791	3.790
$\kappa_{(T=3K)}$ $(\frac{10^{-5}\times W}{cm\times K})$	↘	2.451	2.115	1.868	1.857
$\kappa_{(T=80K)}$ $(\frac{10^{-4}\times W}{cm\times K})$	↘	8.668	7.485	6.612	6.571
$-S_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	5.750	5.787	5.827	5.829
$-S_{(T=80K)}$ $(\frac{10^{-5}\times V}{K})$	↘	12.048	12.106	12.169	12.173
$VC_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↗	3.830	3.854	3.881	3.882
$VC_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↗	6.055	6.093	6.133	6.135
$-Ts_{(T=3K)}$ $(\frac{10^{-6}\times V}{K})$	↘	5.745	5.782	5.821	5.823
$-Ts_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	9.083	9.140	9.200	9.203

$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.725	1.736	1.748	1.749
$-Pt_{(T=80K)}(10^{-2}\times V)$	↘	0.964	0.968	0.973	0.974
$ZT_{(T=3K)}\ (10^{-3})$	↗	1.353	1.371	1.390	1.391
$ZT_{(T=80K)}$	↗	0.594	0.600	0.606	0.6065
For x=1,					
$\xi_{n(T=3K)}$	↘	113.2	112.4	111.6	111.5
$\xi_{n(T=80K)}$	↘	4.48	4.446	4.413	4.411
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	1.888	1.615	1.417	1.408
$\kappa_{(T=80K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	6.519	5.588	4.912	4.880
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	5.009	5.043	5.080	5.082
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	10.880	10.934	10.992	10.995
$VC_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↗	3.337	3.360	3.384	3.386
$VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	5.255	5.288	5.324	5.326
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	5.006	5.040	5.076	5.079
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	7.883	7.932	7.986	7.989
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.503	1.513	1.524	1.5247
$-Pt_{(T=80K)}(10^{-2}\times V)$	↘	0.870	0.875	0.8793	0.8796
$ZT_{(T=3K)}\ (10^{-3})$	↗	1.027	1.041	1.056	1.057
$ZT_{(T=80K)}$	↗	0.484	0.489	0.4946	0.4949

Table 4p: In the lightly degenerate p-type X(x) − alloy, in which $N=10^{19}\ cm^{-3}$, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Acceptor		Ga	Mg	In	Cd
For x=0,					
$\xi_{p(T=3K)}$	↘	151	147.97	147.8	147.63
$\xi_{p(T=80K)}$	↘	5.88	5.776	5.769	5.763
$\kappa_{(T=3K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	1.41	1.221	1.212	1.203
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	4.45	3.876	3.849	3.821
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	3.76	3.831	3.835	3.840
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	8.80	8.936	8.943	8.951
$VC_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↗	2.50	2.553	2.556	2.559
$VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	4.51	4.546	4.548	4.550

$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	3.75	3.8296	3.834	3.838
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	6.77	6.8193	6.822	6.825
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.13	1.1494	1.151	1.152
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	7.04	7.1486	7.155	7.161
$ZT_{(T=3K)}\ (10^{-4})$	↗	5.78	6.008	6.022	6.036
$ZT_{(T=80K)}(10^{-1})$	↗	3.17	3.268	3.274	3.280
For x=0.5,					
$\xi_{p(T=3K)}$	↘	120.52	114.57	114.23	113.88
$\xi_{p(T=80K)}$	↘	4.76	4.532	4.519	4.505
$\kappa_{(T=3K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	0.81	0.679	0.672	0.666
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	2.77	2.338	2.317	2.295
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.70	4.948	4.962	4.978
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	10.39	10.78	10.806	10.830
$VC_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↗	3.13	3.296	3.306	3.316
$VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	5.00	5.199	5.212	5.226
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.70	4.9442	4.959	4.974
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	7.50	7.7982	7.8183	7.839
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.41	1.4843	1.4887	1.4933
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	8.32	8.6265	8.645	8.664
$ZT_{(T=3K)}\ (10^{-4})$	↗	9.05	10.02	10.08	10.14
$ZT_{(T=80K)}(10^{-1})$	↗	4.42	4.7596	4.780	4.80
For x=1,					
$\xi_{p(T=3K)}$	↘	77.688	63.188	62.320	61.42
$\xi_{p(T=80K)}$	↘	2.786	2.020	1.975	1.928
$\kappa_{(T=3K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	0.34	0.234	0.229	0.223
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	1.19	0.820	0.803	0.786
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	7.29	8.966	9.090	9.224
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	14.29	15.54	15.573	15.600
$VC_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↗	4.85	5.963	6.046	6.134
$VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	5.777	1.791	1.432	1.042
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	7.28	8.945	9.069	9.201
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	8.66	2.686	2.147	1.563

$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	2.19	2.689	2.727	2.767
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	11.43	12.432	12.459	12.48
$ZT_{(T=3K)}\ (10^{-4})$	↗	21.78	32.90	33.826	34.825
$ZT_{(T=80K)}(10^{-1})$	↗	8.36	9.88	9.928	9.962

Table 5n: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}$ $\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_n=1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott}=1$.

For x=0,

In the degenerate P- X(x) − alloy, for $N = 2\times N_{CDn}(r_P)$, one gets:

T(K)	↗.	10		**10.528**		12		**14.325731**		14.5
ξ_n	↘	1.974		**1.8138**		1.442		**1**		0.972
$S\left(10^{-4}\frac{V}{K}\right)$		−1.557	↘	**−1.563**	↗	−1.523	↗	**− 1.322**	↗	− 1.301
ZT		0.993	↗	**1**	↘	0.949	↘	**0.715**	↘	0.693
$(ZT)_{Mott}$	↗	0.844		**1**		1.581		**3.290**		3.484
$VC\left(10^{-4}\frac{V}{K}\right)$		0.142	↘	0	↘	−0.423	↘	**−1.105**	↘	−1.153
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.213	↗	0	↗	0.635	↗	**1.657**	↗	1.730
$Pt\ (10^{-3}V)$		−1.557	↘	−1.645	↘	−1.827	↘	−1.893	↗	−1.887

In the degenerate As- X(x) − alloy, for $N = 2\times N_{CDn}(r_{As})$, one gets:

T(K)	↗	10		**12.813**		15		**17.435302**		17.5
ξ_n	↘	2.685		**1.8138**		1.372		**1**		0.991
$S\left(10^{-4}\frac{V}{K}\right)$		−1.450	↘	**−1.563**	↗	−1.504	↗	**− 1.322**	↗	− 1.315
ZT		0.860	↗	**1**	↘	0.926	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$	↗	0.456		**1**		1.748		**3.290**		3.348
$VC\left(10^{-4}\frac{V}{K}\right)$		0.547	↘	0	↘	−0.519	↘	**−1.105**	↘	−1.120
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.820	↗	0	↗	0.779	↗	**1.657**	↗	1.680
$Pt\ (10^{-3}V)$		−1.450	↘	−2.003	↘	−2.256	↘	−2.304	↗	−2.302

In the degenerate Sb- X(x) − alloy, for $N = 2\times N_{CDn}(r_{Sb})$, one gets:

T(K)	↗	10		**15.0335**		16		**20.4571662**		20.5
ξ_n	↘	3.382		**1.8138**		1.631		**1**		0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.302	↘	**−1.563**	↗	−1.554	↗	**− 1.322**	↗	− 1.318
ZT		0.694	↗	**1**	↘	0.989	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.287		**1**		1.236		**3.290**		3.322
$VC\left(10^{-4}\frac{V}{K}\right)$		0.647	↘	0	↘	−0.191	↘	**−1.105**	↘	−1.113
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.971	↗	0	↗	0.286	↗	**1.657**	↗	1.670
$Pt\ (10^{-3}V)$		−1.302	↘	−2.350	↘	−2.487	↘	−2.704	↗	−2.70

In the degenerate Sn- X(x) − alloy, for $N = 2 \times N_{CDn}(r_{Sn})$, one gets:

T(K)	↗	10		**15.1525**		16		**20.6188**		20.7
ξ_n	↘	3.418		**1.8138**		1.654		**1**		0.991
$S\left(10^{-4}\frac{V}{K}\right)$		−1.294	↘	**−1.563**	↗	−1.556	↗	**− 1.322**	↗	− 1.315
ZT		0.686	↗	**1**	↘	0.991	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$	↗	0.281		**1**		1.203		**3.290**		3.351
$VC\left(10^{-4}\frac{V}{K}\right)$		0.646	↘	0	↘	−0.165	↘	**−1.105**	↘	−1.121
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.969	↗	0	↗	0.248	↗	**1.657**	↗	1.681
$Pt\,(10^{-3}V)$		−1.294	↘	−2.368	↘	−2.490	↘	−2.725	↗	−2.722

For x=0.5,

In the degenerate P- X(x) − alloy, for $N = 2 \times N_{CDn}(r_P)$, one gets:

T(K)	↗.	10		**12.25241**		14		**16.672601**		16.675
ξ_n	↘	2.508		**1.8138**		1.436		**1**		0.9997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.484	↘	**−1.563**	↗	−1.521	↗	**− 1.322**	↗	− 1.3215
ZT		0.902	↗	**1**	↘	0.947	↘	**0.715**	↘	0.7148
$(ZT)_{Mott}$	↗	0.523		**1**		1.595		**3.290**		3.292
$VC\left(10^{-4}\frac{V}{K}\right)$		0.477	↘	0	↘	−0.432	↘	**−1.105**	↘	−1.1055
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.716	↗	0	↗	0.648	↗	**1.657**	↗	1.6582
$Pt\,(10^{-3}V)$		−1.484	↘	−1.915	↘	−2.130	↘	−2.204	↗	−2.203

In the degenerate As- X(x) − alloy, for $N = 2 \times N_{CDn}(r_{As})$, one gets:

T(K)	↗	12		**14.9117**		16		**20.291589**		20.4
ξ_n	↘	2.563		**1.8138**		1.608		**1**		0.987
$S\left(10^{-4}\frac{V}{K}\right)$		−1.474	↘	**−1.563**	↗	−1.552	↗	**− 1.322**	↗	− 1.313
ZT		0.889	↗	**1**	↘	0.986	↘	**0.715**	↘	0.705
$(ZT)_{Mott}$	↗	0.501		**1**		1.272		**3.290**		3.374
$VC\left(10^{-4}\frac{V}{K}\right)$		0.501	↘	0	↘	−0.217	↘	**−1.105**	↘	−1.126
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.851	↗	0	↗	0.325	↗	**1.657**	↗	1.689
$Pt\,(10^{-3}V)$		−1.769	↘	−2.331	↘	−2.483	↘	−2.682	↗	−2.678

In the degenerate Sb- X(x) − alloy, for $N = 2 \times N_{CDn}(r_{Sb})$, one gets:

T(K)	↗	15		**17.4965**		20		**23.808502**		23.9
ξ_n	↘	2.324		**1.8138**		1.435		**1**		0.991
$S\left(10^{-4}\frac{V}{K}\right)$		−1.516	↘	**−1.563**	↗	−1.521	↗	**− 1.322**	↗	− 1.315
ZT		0.941	↗	**1**	↘	0.947	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$	↗	0.609		**1**		1.598		**3.290**		3.350
$VC\left(10^{-4}\frac{V}{K}\right)$		0.384	↘	0	↘	−0.433	↘	**−1.105**	↘	−1.120
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.576	↗	0	↗.	0.650	↗	**1.657**	↗	1.680
$Pt\,(10^{-3}V)$		−2.274	↘	−2.735	↘	−3.042	↘	−3.147	↗	−3.14

In the degenerate Sn- X(x) − alloy, for $N = 2 \times N_{CDn}(r_{Sn})$, one gets:

T(K)	↗	15		**17.6345**		20		**23.996614**		24
ξ_n	↘	2.352		**1.8138**		1.456		**1**		0.9997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.511	↘	**−1.563**	↗	−1.526	↗	**− 1.322**	↗	− 1.321
ZT		0.935	↗	**1**	↘	0.953	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.594		**1**		1.552		**3.290**		3.292
$VC\left(10^{-4}\frac{V}{K}\right)$		0.400	↘	0	↘	−0.405	↘	**−1.105**	↘	−1.1055
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.600	↗	0	↗	0.608	↗	**1.657**	↗	1.658
$Pt\,(10^{-3}V)$		−2.267	↘	−2.756	↘	−3.052	↘	−3.172	↗	−3.171

For x=1,

In the degenerate P- X(x) − alloy, for $N = 2 \times N_{CDn}(r_P)$, one gets:

T(K)	↗.	10		**14.6428**		17		**19.925354**		20
ξ_n	↘	3.263		**1.8138**		1.394		**1**		0.991
$S\left(10^{-4}\frac{V}{K}\right)$		−1.327	↘	**−1.563**	↗	−1.510	↗	**− 1.322**	↗	− 1.315
ZT		0.721	↗	**1**	↘	0.934	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$	↗	0.309		**1**		1.693		**3.290**		3.348

In the degenerate Mg- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets:

T(K)	↗.	30		**59.722**		70		**81.26823**		81.3
ξ_p	↘	4.658		**1.8138**		1.369		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.057	↘	**−1.563**	↗	−1.503	↗	**−1.322**	↗	−1.321
ZT		0.457	↗	**1**	↘	0.925	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.152		**1**		1.756		**3.290**		3.296
$VC\left(10^{-4}\frac{V}{K}\right)$		0.508	↘	0	↘	−0.524	↘	**−1.105**	↘	−1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.762	↗	0	↗	0.786	↗	**1.657**	↗	1.660
Pt $(10^{-2}V)$		−0.317	↘	−0.934	↘	−1.052	↘	−1.07412	↗	−1.07402

In the degenerate In- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K)	↗.	30		**60.135**		70		**81.828665**		81.9
ξ_p	↘	4.689		**1.8138**		1.387		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.052	↘	**−1.563**	↗	−1.508	↗	**−1.322**	↗	−1.320
ZT		0.453	↗	**1**	↘	0.931	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.150		**1**		1.710		**3.290**		3.303
$VC\left(10^{-4}\frac{V}{K}\right)$		0.506	↘	0	↘	−0.499	↘	**−1.105**	↘	−1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.759	↗	0	↗	0.748	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−0.315	↘	−0.940	↘	−1.056	↘	−1.08153	↗	−1.0812

In the degenerate Cd- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K)	↗.	30		**60.558**		70		**82.404885**		82.5
ξ_p	↘	4.721		**1.8138**		1.405		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.046	↘	**−1.563**	↗	−1.513	↗	**−1.322**	↗	−1.320
ZT		0.448	↗	**1**	↘	0.938	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.147		**1**		1.666		**3.290**		3.308
$VC\left(10^{-4}\frac{V}{K}\right)$		0.503	↘	0	↘	−0.473	↘	**−1.105**	↘	−1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.755	↗	0	↗	0.710	↗	**1.657**	↗	1.664
Pt $(10^{-2}V)$		−0.314	↘	−0.9465	↘	−1.059	↘	−1.08915	↗	−1.0888

For x=0.5,

In the degenerate Ga- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Ga})$, one gets:

T(K)	↗.	70		**77.2013**		80		**105.052709**		105.1
ξ_p	↘	2.127		**1.8138**		1.708		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.543	↘	**−1.563**	↗	−1.560	↗	**−1.322**	↗	−1.321
ZT		0.975	↗	**1**	↘	0.996	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.727		**1**		1.128		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		0.259	↘	0	↘	−0.106	↘	**−1.105**	↘	−1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.388	↗	0	↗	0.160	↗	**1.657**	↗	1.660
Pt $(10^{-2}V)$		−1.080	↘	−1.207	↘	−1.248	↘	−1.3884	↗	−1.3883

In the degenerate Mg- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets:

T(K)	↗.	80		**88.3555**		90		**120.231045**		120.5
ξ_p	↘	2.132		**1.8138**		1.758		**1**		0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.543	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.318
ZT		0.974	↗	**1**	↘	0.999	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.724		**1**		1.064		**3.290**		3.325
$VC\left(10^{-4}\frac{V}{K}\right)$		0.262	↘	0	↘	−0.054	↘	**−1.105**	↘	−1.114
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.394	↗	0	↗	0.081	↗	**1.657**	↗	1.671
$Pt\,(10^{-2}V)$		−1.234	↘	−1.381	↘	−1.406	↘	−1.589	↗	−1.588

In the degenerate In- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K)	↗.	80		**88.964**		95		**121.06018**		121.1
ξ_p	↘	2.155		**1.8138**		1.622		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.540	↘	**−1.563**	↗	−1.553	↗	**−1.322**	↗	−1.321
ZT		0.970	↗	**1**	↘	0.987	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.708		**1**		1.251		**3.290**		3.295
$VC\left(10^{-4}\frac{V}{K}\right)$		0.279	↘	0	↘	−0.201	↘	**−1.105**	↘	−1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.418	↗	0	↗	0.302	↗	**1.657**	↗	1.659
$Pt\,(10^{-2}V)$		−1.232	↘	−1.390	↘	−1.475	↘	−1.600	↗	−1.599

In the degenerate Cd- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K)	↗.	80		**89.5919**		95		**121.912654**		122.2
ξ_p	↘	2.179		**1.8138**		1.642		**1**		0.994
$S\left(10^{-4}\frac{V}{K}\right)$		−1.540	↘	**−1.563**	↗	−1.555	↗	**−1.322**	↗	−1.318
ZT		0.967	↗	**1**	↘	0.990	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.692		**1**		1.221		**3.290**		3.326
$VC\left(10^{-4}\frac{V}{K}\right)$		0.295	↘	0	↘	−0.179	↘	**−1.105**	↘	−1.114
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.442	↗	0	↗	0.268	↗	**1.657**	↗	1.671
$Pt\,(10^{-2}V)$		−1.229	↘	−1.400	↘	−1.477	↘	−1.611	↗	−1.610

For x=1,

In the degenerate Ga- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Ga})$, one gets:

T(K)	↗.	100		**117.825**		120		**160.33191**		160.5
ξ_p	↘	2.361		**1.8138**		1.759		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.510	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.934	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.590		**1**		1.063		**3.290**		3.306
$VC\left(10^{-4}\frac{V}{K}\right)$		0.404	↘	0	↘	−0.054	↘	**−1.105**	↘	−1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.606	↗	0	↗	0.081	↗	**1.657**	↗	1.664
$Pt\,(10^{-2}V)$		−1.510	↘	−1.842	↘	−1.875	↘	−2.119	↗	−2.118

In the degenerate Mg- X(x) − alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets:

T(K)	↗.	120		**134.8488**		140		**183.49716**		183.6
ξ_p	↘	2.191		**1.8138**		1.702		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.535	↘	**−1.563**	↗	−1.560	↗	**−1.322**	↗	−1.321
ZT		0.965	↗	**1**	↘	0.996	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.685		**1**		1.135		**3.290**		3.298
$VC\left(10^{-4}\frac{V}{K}\right)$		0.303	↘	0	↘	−0.112	↘	**−1.105**	↘	−1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.454	↗	0	↗	0.168	↗	**1.657**	↗	1.661
$Pt\,(10^{-2}V)$		−1.842	↘	−2.108	↘	−2.184	↘	−2.119	↗	−2.42

In the degenerate In- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K)	↗.	120		**135.779**		140		**184.76258**		185
ξ_p	↘	2.215		**1.8138**		1.723		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.532	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.319
ZT		0.961	↗	**1**	↘	0.997	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.670		**1**		1.109		**3.290**		3.310
$VC\left(10^{-4}\frac{V}{K}\right)$		0.318	↘	0	↘	−0.091	↘	**−1.105**	↘	−1.110
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.477	↗	0	↗	0.137	↗	**1.657**	↗	1.665
$Pt\,(10^{-2}V)$		−1.839	↘	−2.122	↘	−2.185	↘	−2.442	↗	−2.441

In the degenerate Cd- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K)	↗.	130		**136.735**		140		**186.06364**		186.5
ξ_p	↘	1.971		**1.8138**		1.743		**1**		0.994
$S\left(10^{-4}\frac{V}{K}\right)$		−1.558	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.318
ZT		0.993	↗	**1**	↘	0.998	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.847		**1**		1.082		**3.290**		3.326
$VC\left(10^{-4}\frac{V}{K}\right)$		0.140	↘	0	↘	−0.070	↘	**−1.105**	↘	−1.114
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.210	↗	0	↗	0.105	↗	**1.657**	↗	1.671
$Pt\,(10^{-2}V)$		−2.025	↘	−2.137	↘	−2.186	↘	−2.459	↗	−2.458

Table 6n: Here, for a given T and with decreasing N, the reduced Fermi-energy ξn decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}$ $\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, **those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_n=1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results:** $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, **respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott}=1$.**

For x=0,

In the degenerate P- X(x) – alloy, for T= 10.528 K, one gets:

$N(10^{16}cm^{-3})$	↘	3		**2.5942062**		2.3		**2.11428385**		2.11
ξ_n	↘	2.42542		**1.8138**		1.336		**1**		0.992
$S\left(10^{-4}\frac{V}{K}\right)$		−1.499	↘	**−1.563**	↗	−1.493	↗	**− 1.322**	↗	− 1.316
ZT		0.920	↗	**1**	↘	0.912	↘	**0.715**	↘	0.709
$(ZT)_{Mott}$	↗	0.559		**1**		1.844		**3.290**		3.345
$VC\left(10^{-4}\frac{V}{K}\right)$		0.438	↘	0	↘	−0.571	↘	**−1.105**	↘	−1.119
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.657	↗	0	↗	0.856	↗	**1.657**	↗	1.678
$Pt\,(10^{-3}V)$		−1.578	↘	−1.645	↗	−1.571	↗	−1.391	↗	−1.385

In the degenerate As- X(x) – alloy, for T= 12.813 K, one gets:

$N(10^{16}cm^{-3})$	↘	4		**3.4831608**		3		**2.8387545**		2.838
ξ_n	↘	2.395		**1.8138**		1.221		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.504	↘	**−1.563**	↗	−1.448	↗	**− 1.322**	↗	− 1.3209
ZT		0.927	↗	**1**	↘	0.858	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.574		**1**		2.206		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		0.422	↘	0	↘	−0.742	↘	**−1.105**	↘	−1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.633	↗	0	↗	1.114	↗	**1.657**	↗	1.660
$Pt\,(10^{-3}V)$		−1.928	↘	−2.003	↗	−1.856	↗	−1.693	↗	−1.692

In the degenerate Sb- X(x) – alloy, for T= 15.0335 K, one gets:

$N(10^{16}cm^{-3})$ ↘	4.5		**4.4268778**		4		**3.6078456**		3.607
ξ_n ↘	1.880		**1.8138**		1411		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.514	↗	**−1.322**	↗	−1.321
ZT	0.999	↗	**1**	↘	0.939	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.930		**1**		1.653		**3.290**		3.296
$VC\left(10^{-4}\frac{V}{K}\right)$	0.062	↘	0	↘	−0.466	↘	**−1.105**	↘	−1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.699	↗	**1.657**	↗	1.660
$Pt\,(10^{-3}V)$	−2.348	↘	−2.350	↗	−2.277	↗	−1.987	↗	−1.986

In the degenerate Sn- X(x) – alloy, for T=15.1525 K, one gets:

$N(10^{16}cm^{-3})$ ↘	5		**4.4794468**		4		**3.6507193**		3.65
ξ_n ↘	2.271		**1.8138**		1.364		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$	−1.524	↘	**−1.563**	↗	−1.502	↗	**−1.322**	↗	−1.321
ZT	0.951	↗	**1**	↘	0.923	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.638		**1**		1.768		**3.290**		3.295
$VC\left(10^{-4}\frac{V}{K}\right)$	0.353	↘	0	↘	−0.530	↘	**−1.105**	↘	−1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.529	↗	0	↗	0.796	↗	**1.657**	↗	1.659
$Pt\,(10^{-3}V)$	−2.310	↘	−2.368	↗	−2.275	↗	−2.003	↗	−2.002

For x=0.5,

In the degenerate P- X(x) – alloy, for T=12.25241 K, one gets:

$N(10^{16}cm^{-3})$ ↘	3		**2.7117476**		2.5		**2.2100481**		2.2
ξ_n ↘	2.233		**1.8138**		1.490		**1**		0.981
$S\left(10^{-4}\frac{V}{K}\right)$	−1.530	↘	**−1.563**	↗	−1.533	↗	**−1.322**	↗	−1.308
ZT	0.958	↗	**1**	↘	0.962	↘	**0.715**	↘	0.701
$(ZT)_{Mott}$ ↗	0.660		**1**		1.482		**3.290**		3.415
$VC\left(10^{-4}\frac{V}{K}\right)$	0.329	↘	0	↘	−0.361	↘	**−1.105**	↘	−1.136
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.494	↗	0	↗	0.541	↗	**1.657**	↗	1.705
$Pt\,(10^{-3}V)$	−1.874	↘	−1.915	↗	−1.879	↗	−1.619	↗	−1.603

In the degenerate As- X(x) – alloy, for T=14.9117 K, one gets:

$N(10^{16}cm^{-3})$ ↘	4		**3.64098**		3.5		**2.9673356**		2.96
ξ_n ↘	2.203		**1.8138**		1.656		**1**		0.990
$S\left(10^{-4}\frac{V}{K}\right)$	−1.534	↘	**−1.563**	↗	−1.556	↗	**−1.322**	↗	−1.314
ZT	0.963	↗	**1**	↘	0.992	↘	**0.715**	↘	0.707
$(ZT)_{Mott}$ ↗	0.678		**1**		1.200		**3.290**		3.357
$VC\left(10^{-4}\frac{V}{K}\right)$	0.310	↘	0	↘	−0.163	↘	**−1.105**	↘	−1.122
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.466	↗	0	↗	0.245	↗	**1.657**	↗	1.683
$Pt\,(10^{-3}V)$	−2.2.287	↘	−2.331	↗	−2.321	↗	−1.971	↗	−1.960

In the degenerate Sb- X(x) – alloy, for T=17.4965 K, one gets:

$N(10^{16}cm^{-3})$ ↘	5		**4.6274562**		4		**3.7713369**		3.75
ξ_n ↘	2.133		**1.8138**		1.235		**1**		0.977
$S\left(10^{-4}\frac{V}{K}\right)$	−1.543	↘	**−1.563**	↗	−1.454	↗	**−1.322**	↗	−1.305
ZT	0.974	↗	**1**	↘	0.866	↘	**0.715**	↘	0.697
$(ZT)_{Mott}$ ↗	0.723		**1**		2.155		**3.290**		3.447
$VC\left(10^{-4}\frac{V}{K}\right)$	0.263	↘	0	↘	−0.720	↘	**−1.105**	↘	−1.144
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.394	↗	0	↗	1.080	↗	**1.657**	↗	1.716
$Pt\,(10^{-3}V)$	−2.699	↘	−2.735	↗	−2.545	↗	−2.312	↗	−2.283

In the degenerate Sn- X(x) – alloy, for T=17.6345 K one gets:

$N(10^{16}cm^{-3})$ ↘	5		**4.682407**		4		**3.8160912**		3.81
ξ_n ↘	2.083		**1.8138**		1.889		**1**		0.993
$S\left(10^{-4}\frac{V}{K}\right)$	−1.548	↘	**−1.563**	↗	−1.433	↗	**− 1.322**	↗	− 1.317
ZT	0.981	↗	**1**	↘	0.841	↘	**0.715**	↘	0.710
$(ZT)_{Mott}$ ↗	0.758		**1**		2.328		**3.290**		3.333
$VC\left(10^{-4}\frac{V}{K}\right)$	0.227	↘	**0**	↘	−0.793	↘	**−1.105**	↘	−1.116
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.341	↗	**0**	↗	1.190	↗	**1.657**	↗	1.674
$Pt\ (10^{-3}V)$	−2.730	↘	**−2.756**	↗	−2.527	↗	**−2.331**	↗	−2.323

For x=1,

In the degenerate P- X(x) – alloy, for T=14.6428 K, one gets:

$N(10^{16}cm^{-3})$ ↘	3		**2.8752424**		2.5		**2.3432946**		2.34
ξ_n ↘	1.987		**1.8138**		1.259		**1**		0.994
$S\left(10^{-4}\frac{V}{K}\right)$	−1.556	↘	**−1.563**	↗	−1.464	↗	**− 1.322**	↗	− 1.318
ZT	0.992	↗	**1**	↘	0.878	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$ ↗	0.833		**1**		2.076		**3.290**		3.328
$VC\left(10^{-4}\frac{V}{K}\right)$	0.153	↘	**0**	↘	−0.685	↘	**−1.105**	↘	−1.115
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.229	↗	**0**	↗	1.027	↗	**1.657**	↗	1.672
$Pt\ (10^{-3}V)$	−2.279	↘	**−2.289**	↘	−2.144	↗	**−1.935**	↗	−1.929

In the degenerate As- X(x) – alloy, for T=17.821 K, one gets:

$N(10^{16}cm^{-3})$ ↘	4		**3.8604994**		3.5		**3.1462495**		3.145
ξ_n ↘	1.907		**1.8138**		1.375		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.561	↘	**−1.563**	↗	−1.505	↗	**− 1.322**	↗	− 1.320
ZT	0.997	↗	**1**	↘	0.927	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.904		**1**		1.740		**3.290**		3.300
$VC\left(10^{-4}\frac{V}{K}\right)$	0.086	↘	**0**	↘	−0.515	↘	**−1.105**	↘	−1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.129	↗	**0**	↗	0.773	↗	**1.657**	↗	1.661
$Pt\ (10^{-3}V)$	−2.400	↘	**−2.785**	↗	−2.313	↗	**−2.355**	↗	−2.353

In the degenerate Sb- X(x) – alloy, for T=20.91 K, one gets:

$N(10^{16}cm^{-3})$ ↘	5.5		**4.9064514**		4.5		**3.9987166**		3.995
ξ_n ↘	2.236		**1.8138**		1.420		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$	−1.529	↘	**−1.563**	↗	−1.517	↗	**− 1.322**	↗	− 1.319
ZT	0.957	↗	**1**	↘	0.942	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$ ↗	0.658		**1**		1.630		**3.290**		3.314
$VC\left(10^{-4}\frac{V}{K}\right)$	0.331	↘	**0**	↘	−0.452	↘	**−1.105**	↘	−1.111
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.497	↗	**0**	↗	0.679	↗	**1.657**	↗	1.667
$Pt\ (10^{-3}V)$	−2.759	↘	−3.268	↗	−2.737	↗	−2.764	↗	−2.758

In the degenerate Sn- X(x) – alloy, for T=21.0748 K, one gets:

$N(10^{16}cm^{-3})$ ↘	5.5		**4.9647154**		4.5		**4.0461555**		4.045
ξ_n ↘	2.186		**1.8138**		1.374		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$	−1.536	↘	**−1.563**	↗	−1.505	↗	**− 1.322**	↗	− 1.321
ZT	0.966	↗	**1**	↘	0.927	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.689		**1**		1.742		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$	0.299	↘	**0**	↘	−0.516	↘	**−1.105**	↘	−1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.448	↗	**0**	↗	0.775	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$	−2.793	↘	−3.294	↗	−2.735	↗	−2.785	↗	−2.784

Table 6p: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}$ $\left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_p=1$, those of S, ZT, $(ZT)_{Mott,}$ VC, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott}=1$.

For x=0,

In the degenerate Ga- X(x) − alloy, for T=52.184 K, one gets:

$N(10^{18} cm^{-3})$ ↘	2.5		**2.2902686**		2		**1.8665662**		1.866
ξ_p ↘	2.176		**1.8138**		1.276		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$	−1.537	↘	**−1.563**	↗	−1.471	↗	**−1.322**	↗	−1.321
ZT	0.968	↗	**1**	↘	0.886	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.695		**1**		2.021		**3.290**		3.298
$VC\left(10^{-4}\frac{V}{K}\right)$	0.292	↘	0	↘	−0.659	↘	**−1.105**	↘	−1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.438	↗	0	↗	0.988	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$	−8.023	↘	−8.156	↗	−7.676	↗	−6.897	↗	−6.893

In the degenerate Mg- X(x) − alloy, for T=59.722 K, one gets:

$N(10^{18} cm^{-3})$ ↘	3		**2.8041452**		2.5		**2.2853372**		2.28
ξ_p ↘	2.091		**1.8138**		1.358		**1**		0.990
$S\left(10^{-4}\frac{V}{K}\right)$	−1.547	↘	**−1.563**	↗	−1.500	↗	**−1.322**	↗	−1.315
ZT	0.980	↗	**1**	↘	0.921	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$ ↗	0.752		**1**		1.784		**3.290**		3.353
$VC\left(10^{-4}\frac{V}{K}\right)$	0.233	↘	0	↘	−0.539	↘	**−1.105**	↘	−1.121
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.350	↗	0	↗	0.809	↗	**1.657**	↗	1.682
$Pt\ (10^{-3}V)$	−9.241	↘	−9.334	↗	−8.956	↗	−7.893	↗	−7.853

In the degenerate In- X(x) − alloy, for T=60.135 K, one gets:

$N(10^{18} cm^{-3})$ ↘	3		**2.8332018**		2.5		**2.30904345**		2.305
ξ_p ↘	2.048		**1.8138**		1.317		**1**		0.993
$S\left(10^{-4}\frac{V}{K}\right)$	−1.551	↘	**−1.563**	↗	−1.486	↗	**−1.322**	↗	−1.317
ZT	0.985	↗	**1**	↘	0.904	↘	**0.715**	↘	0.710
$(ZT)_{Mott}$ ↗	0.784		**1**		1.896		**3.290**		3.337
$VC\left(10^{-4}\frac{V}{K}\right)$	0.201	↘	0	↘	−0.598	↘	**−1.105**	↘	−1.117
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.301	↗	0	↗	0.897	↗	**1.657**	↗	1.676
$Pt\ (10^{-3}V)$	−9.330	↘	−9.400	↗	−8.937	↗	−7.948	↗	−7.918

In the degenerate Cd- X(x) − alloy, for T=60.558 K, one gets:

$N(10^{18} cm^{-3})$ ↘	3		**2.8631806**		2.5		**2.3334658**		2.33
ξ_p ↘	2.004		**1.8138**		1.275		**1**		0.994
$S\left(10^{-4}\frac{V}{K}\right)$	−1.555	↘	**−1.563**	↗	−1.471	↗	**−1.322**	↗	−1.317
ZT	0.990	↗	**1**	↘	0.885	↘	**0.715**	↘	0.710
$(ZT)_{Mott}$ ↗	0.819		**1**		2.022		**3.290**		3.330
$VC\left(10^{-4}\frac{V}{K}\right)$	0.167	↘	0	↘	−0.659	↘	**−1.105**	↘	−1.115
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.250	↗	0	↗	0.989	↗	**1.657**	↗	1.673
$Pt\ (10^{-3}V)$	−9.418	↘	−9.465	↗	−8.907	↗	−8.004	↗	−7.978

For x=0.5,

In the degenerate Ga- X(x) − alloy, for T=**77.2013** K, one gets:

$N(10^{18}cm^{-3})$	↘	5		**4.9176652**		4.5		**4.0078454**		4
ξ_p	↘	1.881		**1.8138**		1.461		**1**		0.992
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.527	↗	**−1.322**	↗	−1.316
ZT		0.999	↗	**1**	↘	0.954	↘	**0.715**	↘	0.709
$(ZT)_{Mott}$	↗	0.930		**1**		1.542		**3.290**		3.343
$VC\left(10^{-4}\frac{V}{K}\right)$		0.062	↘	0	↘	−0.399	↘	**−1.105**	↘	−1.118
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.094	↗	0	↗	0.599	↗	**1.657**	↗	1.678
$Pt\ (10^{-2}V)$		−1.206	↘	−1.207	↗	−1.179	↗	−1.020	↗	−1.016

In the degenerate Mg- X(x) − alloy, for T=**88.3555** K, one gets:

$N(10^{18}cm^{-3})$	↘	6.5		**6.021061**		5.9		**4.9070987**		4.89
ξ_p	↘	2.129		**1.8138**		1.732		**1**		0.986
$S\left(10^{-4}\frac{V}{K}\right)$		−1.543	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.311
ZT		0.975	↗	**1**	↘	0.998	↘	**0.715**	↘	0.704
$(ZT)_{Mott}$	↗	0.726		**1**		1.096		**3.290**		3.385
$VC\left(10^{-4}\frac{V}{K}\right)$		0.260	↘	0	↘	−0.081	↘	**−1.105**	↘	−1.129
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.390	↗	0	↗	0.122	↗	**1.657**	↗	1.694
$Pt\ (10^{-2}V)$		−1.363	↘	−1.381	↗	−1.379	↗	−1.168	↗	−1.159

In the degenerate In- X(x) − alloyalloy, for T=**88.964**K, one gets:

$N(10^{18}cm^{-3})$	↘	6.5		**6.0834514**		5.9		**4.95792**		4.95
ξ_p	↘	2.086		**1.8138**		1.691		**1**		0.993
$S\left(10^{-4}\frac{V}{K}\right)$		−1.548	↘	**−1.563**	↗	−1.559	↗	**−1.322**	↗	−1.317
ZT		0.981	↗	**1**	↘	0.995	↘	**0.715**	↘	0.710
$(ZT)_{Mott}$	↗	0.756		**1**		1.150		**3.290**		3.333
$VC\left(10^{-4}\frac{V}{K}\right)$		0.229	↘	0	↘	−0.125	↘	**−1.105**	↘	−1.116
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.344	↗	0	↗	0.187	↗	**1.657**	↗	1.674
$Pt\ (10^{-2}V)$		−1.377	↘	−1.390	↗	−1.387	↗	−1.176	↗	−1.172

In the degenerate Cd- X(x) − alloy, for T=**89.5919**, one gets:

$N(10^{18}cm^{-3})$	↘	6.5		**6.1478218**		6		**5.0104274**		5
ξ_p	↘	2.042		**1.8138**		1.716		**1**		0.991
$S\left(10^{-4}\frac{V}{K}\right)$		−1.552	↘	**−1.563**	↗	−1.560	↗	**−1.322**	↗	−1.316
ZT		0.986	↗	**1**	↘	0.997	↘	**0.715**	↘	0.708
$(ZT)_{Mott}$	↗	0.789		**1**		1.117		**3.290**		3.346
$VC\left(10^{-4}\frac{V}{K}\right)$		0.196	↘	0	↘	−0.098	↘	**−1.105**	↘	−1.119
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.294	↗	0	↗	0.147	↗	**1.657**	↗	1.679
$Pt\ (10^{-2}V)$		−1.390	↘	−1.400	↗	−1.398	↗	−1.184	↗	−1.179

For x=1,

In the degenerate Ga- X(x) − alloy, for T=**117.825** K, one gets:

$N(10^{19}cm^{-3})$	↘	1.1		**1.0859611**		0.89		**0.885047**		0.884
ξ_p	↘	1.866		**1.8138**		1.022		**1**		0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.337	↗	**−1.322**	↗	−1.318
ZT		0.999	↗	**1**	↘	0.732	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.945		**1**		3.146		**3.290**		3.322
$VC\left(10^{-4}\frac{V}{K}\right)$		0.049	↘	0	↘	−1.066	↘	**−1.105**	↘	−1.113
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.073	↗	0	↗	1.600	↗	**1.657**	↗	1.670
$Pt\ (10^{-2}V)$		−1.841	↘	−1.842	↗	−1.576	↗	−1.557	↗	−1.553

In the degenerate Mg- X(x) – alloy, for T=**134.8488** K, one gets:

$N(10^{19}cm^{-3})$	↘	1.35		**1.3296224**		1.3		**1.08362885**		1.08
ξ_p	↘	1.875		**1.8138**		1.723		**1**		0.986
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.312
ZT		0.999	↗	**1**	↘	0.997	↘	**0.715**	↘	0.704
$(ZT)_{Mott}$	↗	0.935		**1**		1.108		**3.290**		3.381
$VC\left(10^{-4}\frac{V}{K}\right)$		0.057	↘	0	↘	−0.090	↘	**−1.105**	↘	−1.128
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.086	↗	0	↗	0.136	↗	**1.657**	↗	1.692
$Pt\,(10^{-2}V)$		−2.106	↘	−2.108	↗	−2.105	↗	−1.782	↗	−1.769

In the degenerate In- X(x) – alloy, for T=**135.779** K, one gets:

$N(10^{19}cm^{-3})$	↘	1.35		**1.3434**		1.3		**1.09485865**		1.09
ξ_p	↘	1.833		**1.8138**		1.682		**1**		0.982
$S\left(10^{-4}\frac{V}{K}\right)$		−1.5629	↘	**−1.563**	↗	−1.558	↗	**−1.322**	↗	−1.308
ZT		0.999	↗	**1**	↘	0.994	↘	**0.715**	↘	0.701
$(ZT)_{Mott}$	↗	0.978		**1**		1.163		**3.290**		3.412
$VC\left(10^{-4}\frac{V}{K}\right)$		0.019	↘	0	↘	−0.134	↘	**−1.105**	↘	−1.136
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.028	↗	0	↗	0.201	↗	**1.657**	↗	1.704
$Pt\,(10^{-2}V)$		−2.122	↘	−2.122	↗	−2.116	↗	−1.794	↗	−1.777

In the degenerate Cd- X(x) – alloy, for T=**136.735** K, one gets:

$N(10^{19}cm^{-3})$	↘	1.38		**1.3576148**		1.3		**1.10644304**		1.10
ξ_p	↘	1.880		**1.8138**		1.640		**1**		0.976
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.555	↗	**−1.322**	↗	−1.305
ZT		0.999	↗	**1**	↘	0.990	↘	**0.715**	↘	0.697
$(ZT)_{Mott}$	↗	0.931		**1**		1.223		**3.290**		3.451
$VC\left(10^{-4}\frac{V}{K}\right)$		0.061	↘	0	↘	−0.180	↘	**−1.105**	↘	−1.145
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.271	↗	**1.657**	↗	1.718
$Pt\,(10^{-2}V)$		−2.136	↘	−2.137	↗	−2.126	↗	−1.807	↗	−1.784