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ELECRICAL-AND-THERMOELECTRIC PROPERTIES IN n(p)-TYPE DEGENERATE GaSb(1-x) P(x)-CRYSTALLINE ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW AND NEW ELECRICAL CONDUCTIVITY (III)

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ABSTRACT

In the $n^{+}(p^{+})-p(n)$ $\mathbf{GaSb_{1-x}P_x}$- crystalline alloy, $0 \le x \le 1$, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law given in Equations (1a, 1b) and new electrical conductivity in Eq. (14), and by our accurate Fermi energy given in Eq. (11), are now investigated, by basing on the same physical model and mathematical treatment method, as those used in our recent works (Van Cong, 2024, 2025). It should be noted here that, for x=0, these obtained numerical results may be reduced to those given in n(p)-type degenerate GaSb-crystal. Then, some remarkable results could be cited in the following. In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of the figure of merit ZT show a same maxi**mum**

$(\mathrm{ZT})_{\mathrm{max.}} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(\mathrm{ZT})_{\mathrm{Mott}}$, the Van-Cong coefficient VC, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(\mathrm{ZT})_{\mathrm{Mott}} = 1$. It seems that these same results could represent a new law in the thermoelectric properties, obtained in the degenerate case.

KEYWORDS: Electrical conductivity, Seebeck coefficient (S), Figure of merit (ZT), Van-Cong coefficient, (VC) Thomson coefficient (Ts), Peltier coefficient (Pt)

INTRODUCTION

In the $\mathbf{n^+(p^+) - p(n)\ X(x) \equiv GaSb_{1-x}P_x}$- crystalline alloy, $0 \le x \le 1$, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)},x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b) and new electrical conductivity, in Eq. (14), and also by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), are now investigated, by basing on the same physical model and mathematical treatment method, as those used in our recent works (Van Cong, 2024, 2025). It should be noted here that for x=0, these obtained numerical results may be reduced to those given in the n(p)-type degenerate GaSb-crystal (Van Cong, and Van Cong et al., 1980-2023; Hyun et al. 1998; Kim et al., 2015). Then, some remarkable results could be noted in the following.

(1) The generalized Mott criterium in the metal-insulator transition (**MIT**) is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N^{EBT}_{CDn(CDp)}$, obtained with a precision of the order of $\mathbf{2.92 \times 10^{-7}}$, as given in our recent work (2024), and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any density N^*.

(3) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} [8], and it is present in all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used for determining all the following electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density $\mathbf{N}$ and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given $\mathbf{T}$ and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of the figure of merit ZT show a same maxi**mum** $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the Van-Cong coefficient VC, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent a new law in the thermoelectric properties, obtained in the degenerate case.

(6) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used for determining the following electrical-and-thermoelectric coefficients.

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $\mathbf{n^+(p^+) - p(n)\,X(x)}$- crystalline alloy at T=0 K, we denote the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{Sb(Ga)}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, the unperturbed relative static dielectric constant by: $\varepsilon_o(x)$, the intrinsic band gap by: $E_{go}(x)$. Then, their values are reported in Table 1 in Appendix 1.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$E_{do(ao)}(x) = \frac{13600 \times [m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2}$ meV , and then, the isothermal bulk modulus, by: $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right)\times\left(r_{do(ao)}\right)^3}$.

Our Static Dielectric Constant Law

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) α, $\alpha_o = 0$. Further, the two important equations [4], used to determine the α -variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\alpha}{dV}$. giving: $\frac{d}{dV}(\frac{d\alpha}{dV}) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\alpha(r_{d(a)}, x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$$

Furthermore, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by: $\pm\,[\Delta\alpha(r_{d(a)}, x)]_{n(p)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +\,[\Delta\alpha(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -\,[\Delta\alpha(r_{d(a)}, x)]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new** **$\varepsilon(r_{d(a)}, x)$-law**,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (1a)$$

according to the increase in both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with increasing $r_{d(a)}$ and for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x) = \frac{\varepsilon_o(x)}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being a **new $\varepsilon(r_{d(a)},x)$-law**,

$$E_{gno(gpo)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (1b)$$

corresponding to the decrease in both $E_{gno(gpo)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with decreasing $r_{d(a)}$ and for a given x.

It should be noted that, in the following, all the electrical-and-thermoelectric properties strongly depend on this **new $\varepsilon(r_{d(a)},x)$-law.**

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)},x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)},x) \equiv \frac{\varepsilon(r_{d(a)},x)\times\hbar^2}{m_{c(v)}(x)\times m_o\times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)},x)}{m_{c(v)}(x)}. \quad (2)$$

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(NDp)}(r_{d(a)},x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as:

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = M_{n(p)},\ M_{n(p)} = 0.25, \quad (3)$$

depending thus on our **new $\varepsilon(r_{d(a)},x)$-law**.

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, by:

$$r_{sn(sp)}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)}, x)} = 1.1723 \times 10^{8} \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_{c(v)}(x) \times m_o}{\varepsilon(r_{d(a)}, x)}, \quad (4)$$

being equal to, in particular, at $N = N_{CDn(CDp)}(r_{d(a)}, x)$: $r_{sn(sp)}(N_{CDn(CDp)}(r_{d(a)}, x), r_{d(a)}, x) =$ **2.4813963**, for any $(r_{d(a)}, x)$-values. Then, from Eq. (4), one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} \times \frac{1}{2.4813963} = \mathbf{0.25} = (\mathbf{WS})_{n(p)} = \mathbf{M}_{n(p)}, \quad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = \mathbf{0.47137}$, as those given in our previous work (Van Cong, 2024), we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$**, with a precision of the order of $\mathbf{2.92 \times 10^{-7}}$.

It shoud be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $N^{EBT}_{CDn(CDp)}$ are found to be in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials:

$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) = N^*$, for a presentation simplicity. (6)

In summary, as observed in Table 4 of our previous paper (Van Cong, 2024), one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}(r_{d(a)}, x)$, $N_{CDn(NDp)}(r_{d(a)}, x)$ and $N^{EBT}_{CDn(CDp)}(r_{d(a)}, x)$ increase, affecting strongly all the physical properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+(p^+) - p(n)$ $\mathbf{X(x)}$- crystalline alloy, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{g_{c(v)}}\right)^{\frac{1}{3}}$, the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^*, is now defined by:

$$\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k^{-1}_{Fn(Fp)}}{a_{Bn(Bp)}} < 1,$$

being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{Fn(Fp)}^{-1}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}(r_{d(a)}, x)$ is determined in Eq. (2).

Then, **the ratio of the inverse effective screening length $k_{sn(sp)}$** to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + \left[R_{snTF(spTF)} - R_{snWS(spWS)}\right] e^{-r_{sn(sp)}} < 1, \quad (7)$$

being valid at any N^*.

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(snWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d\left[r_{sn(sp)}^2 \times E_{CE}(N^*)\right]}{dr_{sn(sp)}}\right), \quad (9)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \ \boldsymbol{\eta}_{n(p)}(\mathbf{N}^*) \equiv \frac{\sqrt{2\pi N^*}}{\varepsilon(r_{d(a)})} \times \mathbf{q}^2 \mathbf{k}_{sn(sp)}^{-1/2}, \quad (10)$$

which gives: $\mathbf{A}_{n(p)}(\mathbf{N}^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$.

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy and generalized Einstein relation

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$- crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$- crystalline alloy, according to the reduced Fermi energy, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N, r_{d(a)}, x, T)$, obtained in our previous paper (Van Cong, Debiais, and Doan Khanh, 1991- 1993), obtained with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)},\ A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T, x)}$,

$N_{c(v)}(T, x) = 2 g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_o \times k_B T}{2\pi \hbar^2}\right)^{\frac{3}{2}} (cm^{-3})$, $g_{c(v)} = 1$,

$F(u) = a u^{\frac{2}{3}} \left(1 + b u^{-\frac{4}{3}} + c u^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as $u \to 0$, **the limiting condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times a u^{\frac{2}{3}} \left(1 + b u^{-\frac{4}{3}} + c u^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_o}$ as $u \to \infty$, **the limiting condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, at T=0K, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_o}$, being proportional to $(N^*)^{2/3}$, and also equal to 0 at $N^* = 0$, according to the MIT.

In the following, it should be noted that all the electrical-and-thermoelectric properties strongly depend on such the accurate expression of $\xi_{n(p)}(N, r_{d(a)}, x, T)$.

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1+e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

So, the average of E^p, calculated using the FDDF-method, as developed in our previous work (Van Cong, 2018, 2025) is found to be given by:

$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE$, $-\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}$.

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p(E_{Fn(Fp)}) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E^{-\beta}_{Fn(Fp)} \times I_{\beta}$,

where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{(e^{\gamma/2}+e^{-\gamma/2})^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains:

$I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma$.

Now, using an identity $(1+e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n}-2) \times \pi^{2n} \times |B_{2n}|$. So, from Eq. (22), we get in the degenerate case the following ratio:

$$G_p(E_{Fn(Fp)}) \equiv \frac{\langle E^p \rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\ldots(p-2n+1)}{(2n)!} \times (2^{2n}-2) \times |B_{2n}| \times y^{2n} \equiv G_{p\geq 1}(y), \quad (12)$$

where $y \equiv \frac{\pi}{\xi_{n(p)}(N^*,T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*,T)}$.

Then, some usual results of $G_{p\geq 1}(y)$ are given **in Table 2 in Appendix 1, being needed to determine all the following electrical-and-thermoelectric properties.**

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by $\sigma(N, r_{d(a)}, x, T)$ expressed in $ohm^{-1} \times cm^{-1}$, the thermal conductivity by $\kappa(N, r_{d(a)}, x, T)$ in $\frac{W}{cm \times K}$, and the Lorenz number L defined by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8} \; (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\kappa(N, r_{d(a)}, x, T)}{\sigma(N, r_{d(a)}, x, T)} = L \times T. \quad (13)$$

We now determine the general form of σ as follows.

First of all, it is expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{Cn(Cp)} \times m_o}$, or the wave number k, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\eta_{n(p)}}\right)^{1/2},$$

which is thus proportional to ${E_k}^2$.

Then, for $\underline{E \geq 0}$, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E^2_{Fn(Fp)}$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N, r_{d(a)}, x, T)$ for a presentation simplicity. Therefore, one obtains (Van Cong, 2025):

$$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi \times \hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*)}\right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)}\right)^2\right] \left(\frac{1}{ohm \times cm}\right),$$

$$\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}\ ohm^{-1},\ A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)},\ R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}}, \quad (14)$$

which can be used to define the resistivity as: $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$, noting that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$. This $\sigma(N, r_{d(a)}, x, T)$-result is an essential one in this paper, being used to determine other electrical-and-thermoelectric properties.

In Eq. (14), one notes that at T= 0 K, $\sigma(N, r_{d(a)}, x, T=0K)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^*)^{\frac{4}{3}}$. Thus, $\sigma(N = N_{CDn(NDp)}, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by (Van Cong, 2025):

$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_{c(v)}(x)\times m_o}{q^2 \times N^*}$. Therefore, the mobility μ is given by:

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q\times\tau(N, r_{d(a)}, x, T)}{m_{c(v)}(x)\times m_o} = \frac{\sigma(N, r_{d(a)}, x, T)}{q\times N^*} \ \left(\frac{cm^2}{V\times s}\right). \quad (15)$$

Here, at T= 0K, $\mu(N^*, r_{d(a)}, T)$ is thus proportional to $(N^*)^{1/3}$, since $\sigma(N^*, r_{d(a)}, T = 0K)$ is proportional to $(N^*)^{4/3}$. Thus, $\mu(N^* = 0, r_{d(a)}, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Then, since τ and σ are both proportional to $E_{Fn(Fp)}(N^*, T)^2$, as given above, the Hall factor is defined by:

$r_H(N, r_{d(a)}, x, T) \equiv \frac{\langle\tau^2\rangle_{FDDF}}{[\langle\tau\rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}$, and therefore, the Hall mobility yields:

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T) \times r_H(N^*, T) \ \left(\frac{cm^2}{V\times s}\right), \quad (16)$$

noting that, at T=0K, since $r_H(N, r_{d(a)}, x, T) = 1$, one then gets: $\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T)$.

Finally, our **generalized Einstein relation** is found to be defined as:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3\times L}{\pi^2}} \times T \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right), \frac{k_B}{q} = \sqrt{\frac{3\times L}{\pi^2}} \quad (17)$$

where $D(N, r_{d(a)}, x, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility $\mu(N, r_{d(a)}, x, T)$ is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u, one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} = \frac{k_B \times T}{q} \times u\frac{V'(u)\times W(u) - V(u)\times W'(u)}{W^2(u)}$, where

$W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{8}{3}} e^{-du}(1 - du) + \frac{2}{3} Au^{B-1} F(u) \left[\left(1 + \frac{3B}{2}\right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}} \right]$.

One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and therefore: $\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}$, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2 u^{2B}$ and $u[V' \times W - V \times W'] \approx \frac{2}{3} a u^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, the **above generalized Einstein relation** is reduced to the **usual Einstein one**: $\frac{D(N, r_{d(a)}, x, T=0\,K)}{\mu(N, r_{d(a)}, x, T=0\,K)} \approx \frac{2}{3} E_{Fno(Fpo)}(N^*)/q$. In other words, **Eq. (17) verifies the correct limiting conditions**.

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (17) gives:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}\right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)} \right], \quad (18)$$

where $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{3}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{3}\right)^4$.

In Tables 3n(3p) given in Appendix 1, for given x, $N > N_{CDn}$ and T(=4.2 K and 77 K), and from Equations (14, 15, 16, 17), the numerical results of the coefficients: σ, μ, μ_H are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First off all, from Eq. (14), obtained for $\sigma(N, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S, is found to be given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \left.\frac{\partial \ln\sigma(E)}{\partial E}\right]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln\sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)}(N, r_{d(a)}, x, T) \gtrsim 1$, one gets, by putting

$$F_S(N, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3 \times G_2\left(y = \frac{\pi}{\xi_{n(p)}}\right)} \right],$$

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N^*, T)}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)} \left(\frac{V}{K}\right) < 0, \quad (19)$$

for the present degenerate case, giving here: (i) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, one thus gets: $S = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit, ZT, is found to be defined by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = (ZT)_{Mott} \times [2 \times F_S]^2,\ (ZT)_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}, \quad (20)$$

giving here: (i) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, one gets: $ZT = (ZT)_{Mott} = 1$, and (ii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $(ZT)_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Furthermore, from Eq. (19), one gets:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^{\ 2}}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^{\ 2}}{\pi^2}\right)^2}. \quad (21)$$

Finally, the Van-Cong coefficient, VC, is defined here by:

$VC(N, r_{d(a)}, x, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}$, being equal to 0 for

$$\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (22)$$

the Thomson coefficient, Ts, by:

$$Ts(N, r_{d(a)}, x, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

and the Peltier coefficient, Pt, as:

$$Pt(N, r_{d(a)}, x, T) \equiv T \times S\ (V). \quad (24)$$

One notes here that in next Tables 5n(p) and 6n(p) given I Appendix 1, obtained with such given physical conditions N(or T) for the decreasing $\xi_{n(p)}$, since $VC(N, r_{d(a)}, x, T)$ and $Ts(N, r_{d(a)}, x, T)$ are expressed in terms of $\frac{-dS}{dN^*}$ and $\frac{dS}{dT}$, one has: $[VC, Ts] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$, $[VC, Ts] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[VC, Ts] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating well that, at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, (i) S determined in Eq. (19) thus presents **a same minimum**

$(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$ and (ii) ZT determined in Eq. (20) thus presents **a same maximum:** $(ZT)_{max.}=1$, since $\frac{d(ZT)}{dN(or\, dT)}=2S\times\frac{dS}{dN(or\, dT)}$, $S<0$.

Furthermore, it is interesting to remark that the (VC)-coefficient is related to our generalized Einstein relation (17) by:

$$\frac{k_BT}{q}\times VC\left(N,r_{d(a)},x,T\right)\equiv-\frac{\partial S}{\partial\xi_{n(p)}}\times\frac{D\left(N,r_{d(a)},x,T\right)}{\mu\left(N,r_{d(a)},x,T\right)}\left(\frac{V^2}{K}\right),\ \frac{k_B}{q}=\sqrt{\frac{3\times L}{\pi^2}},\qquad(25)$$

according, in this work, to:

$$T\times VC\left(N,r_{d(a)},x,T\right)\equiv-\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)}\times 2\times\frac{\frac{3\times\xi_{n(p)}^2}{\pi^2}-1}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)^2}\ (V),\ \text{since}\ \frac{\partial S}{\partial\xi_{n(p)}}=\sqrt{\frac{3\times L}{\pi^2}}\times 2\times\frac{\frac{3\times\xi_{n(p)}^2}{\pi^2}-1}{\left(1+\frac{3\times\xi_{n(p)}^2}{\pi^2}\right)^2}.$$

Of course, our relation (25) is reduced to: $\frac{D}{\mu}$ and VC, being respectively determined by Equations (17, 22).

Now, in the lightly degenerate n(p)-type $X(x)-$ alloy, in which N=$5\times10^{17}\ cm^{-3}(3\times10^{19}\ cm^{-3})>N_{CDn(CDp)}$, and for T=3K (80K), the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1, noting that their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T, and in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘).

One notes here that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)}\simeq 1.8138$, while the numerical results of S present **a same minimum** $(S)_{min.}\left(\simeq -1.563\times10^{-4}\frac{V}{K}\right)$, those of ZT show **a same maximum** $(ZT)_{max.}=1$, (ii) for $\xi_{n(p)}=1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present **the same results as**: $-1.322\times10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times10^{-4}\frac{V}{K}$, and $1.657\times10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n\simeq 1.8138$, $(ZT)_{Mott}=1$. It seems that these results could represent a new law in the thermoelectric properties, obtained in the degenerate case.

CONCLUDING REMARKS

Finally, some concluding remarks are given as follows.

(1) The generalized Mott criterium in the MIT is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$, obtained with a precision of the order of $\mathbf{2.92 \times 10^{-7}}$, as given in our previous work (2024), and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any density N^*.

(3) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} (Van Cong and Debiais, 1993), and it is present in all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used for determining all the electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that with increasing T (or with decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of the figure of merit ZT show a same maxi**mum** $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the Van-Cong coefficient VC, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent a new law in the thermoelectric properties, obtained in the degenerate case.

(6) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used for determining all the electrical-and-thermoelectric coefficients.

(7) Our electrical-and-thermoelectric relation is given in Eq. (25) by:

$\frac{k_B T}{q} \times VC(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K}\right)$, $\frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}$, according, in this work, to:

$T \times VC(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2}\right)^2}$ (V), which should be new result.

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APPENDIX 1

Table 1: The values of energy-band-structure parameters are given in the following. BON

In $GaSb_{1-x}P_x$ **-crystalline alloy**, in which $r_{do(ao)}$ = $r_{Sb(Ga)}$ =0.136 nm (0.126 nm), we have: $g_{c(v)}(x) = 1 \times x + 1 \times (1-x)$, $m_{c(v)}(x)/m_o = 0.13\ (0.5) \times x + 0.047\ (0.3) \times (1-x)$, $\varepsilon_o(x) = 11.1 \times x + 15.69 \times (1-x)$, $E_{go}(x) = 1.796 \times x + 0.81 \times (1-x)$.

Table 2: Expressions for $G_{p\geq 1}(y \equiv \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}+\frac{7y^4}{640}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}-\frac{7y^4}{384}\right)$	$\left(1+y^2\right)$	$\left(1+\frac{35y^2}{24}+\frac{49y^4}{384}\right)$	$\left(1+2y^2+\frac{7y^4}{15}\right)$	$\left(1+\frac{21y^2}{8}+\frac{147y^4}{128}\right)$

Table 3n: Here, one notes that, for given x, $N > N_{CDn}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm \times cm}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^2 \times cm^2}{s}\right)$, decrease with increasing r_d.

Donor	P	As	Sb	Sn
r_d (nm) ↗	0.110	0.118	0.136	0.140
For x=**0**, the values of (σ, μ, μ_H, D) at $\mathbf{4.2K}$				
N (10^{18} cm^{-3})				
3	2.25, 4.694, 4.695, 5.05	1.98, 4.129, 4.130, 4.44	1.79, 3.724, 3.724, 4.00	1.78, 3.705, 3.705, 3.98
10	5.82, 3.632, 3.632, 8.72	5.07, 3.165, 3.165, 7.59	4.54, 2.835, 2.835, 6.80	4.52, 2.820, 2.820, 6.77
40	18.3, 2.864, 2.864, 17.3	15.8, 2.462, 2.462, 14.9	13.9, 2.181, 2.181, 13.2	13.9, 2.168, 2.168, 13.1
70	29.7, 2.648, 2.648, 23.2	25.4, 2.264, 2.264, 19.9	22.4, 1.998, 1.998, 17.5	22.3, 1.986, 1.986, 17.4
For x=**0.5**, the values of (σ, μ, μ_H, D) at $\mathbf{4.2K}$				
N (10^{18} cm^{-3})				
3	1.21, 2.540, 2.541, 1.44	1.07, 2.238, 2.239, 1.27	0.96, 2.018, 2.019, 1.14	0.95, 2.008, 2.009, 1.13
10	3.07, 1.291, 1.291, 2.44	2.69, 1.686, 1.686, 2.14	2.42, 1.518, 1.518, 1.93	2.41, 1.511, 1.511, 1.92
40	9.34, 1.458, 1.458, 4.68	8.09, 1.263, 1.263, 4.06	7.22, 1.127, 1.127, 3.62	7.18, 1.121, 1.121, 3.60
70	14.9, 1.328, 1.328, 6.19	12.8, 1.145, 1.145, 5.34	11.3, 1.016, 1.016, 4.74	11.3, 1.010, 1.010, 4.71
For x=**1**, the values of (σ, μ, μ_H, D) at $\mathbf{4.2K}$				
N (10^{18} cm^{-3})				
3	0.70, 1.516, 1.517, 0.58	0.61, 1.331, 1.332, 0.50	0.54, 1.196, 1.196, 0.44	0.53, 1.189, 1.190, 0.44

10	1.81, 1.139, 1.139, 0.98	1.59, 1.004, 1.004, 0.86	1.42, 0.907, 0.907, 0.77	1.42, 0.902, 0.902, 0.77
40	5.33, 0.834, 0.834, 1.82	4.65, 0.729, 0.729, 1.59	4.18, 0.655, 0.655, 1.43	4.16, 0.652, 0.652, 1.42
70	8.37, 0.747, 0.747, 2.37	7.27, 0.650, 0.650, 2.06	6.50, 0.581, 0.581, 1.84	6.47, 0.578, 0.578, 1.83

For **x=0**, the values of (σ, μ, μ_H, D) at **77 K**

N (10^{18} cm^{-3})				
3	2.28, 4.740, 4.844, 5.08	2.00, 4.170, 4.262, 4.47	1.80, 3.760, 3.842, 4.03	1.79, 3.741, 3.823, 4.01
10	5.83, 3.639, 3.655, 8.73	5.08, 3.171, 3.185, 7.61	4.55, 2.841, 2.853, 6.81	4.52, 2.826, 2.838, 6.78
40	18.3, 2.865, 2.867, 17.3	15.8, 2.462, 2.464, 14.9	14.0, 2.182, 2.183, 13.2	13.9, 2.169, 2.170, 13.1
70	29.7, 2.648, 2.649, 23.2	25.4, 2.265, 2.266, 19.9	22.4, 1.998, 1.999, 17.5	22.3, 1.986, 1.987, 17.4

For **x=0.5**, the values of (σ, μ, μ_H, D) at **77 K**

N (10^{18} cm^{-3})				
3	1.26, 2.629, 2.830, 1.48	1.10, 2.317, 2.495, 1.31	0.99, 2.089, 2.250, 1.18	0.99, 2.079, 2.239, 1.17
10	3.09, 1.934, 1.965, 2.46	2.71, 1.698, 1.724, 2.16	2.44, 1.529, 1.553, 1.94	2.43, 1.521, 1.545, 1.93
40	9.35, 1.459, 1.463, 4.68	8.10, 1.265, 1.268, 4.06	7.22, 1.128, 1.131, 3.62	7.18, 1.122, 1.125, 3.60
70	14.9, 1.329, 1.330, 6.20	12.8, 1.145, 1.147, 5.34	11.4, 1.017, 1.018, 4.74	11.3, 1.011, 1.012, 4.71

For **x=1**, the values of (σ, μ, μ_H, D) at **77 K**

N (10^{18} cm^{-3})				
3	0.76, 1.636, 1.903, 0.61	0.66, 1.438, 1.677, 0.53	0.58, 1.293, 1.511, 0.47	0.58, 1.286, 1.503, 0.47
10	1.83, 1.156, 1.196, 0.99	1.61, 1.020, 1.054, 0.87	1.45, 0.920, 0.955, 0.78	1.44, 0.916, 0.947, 0.78
40	5.35, 0.836, 0.841, 1.82	4.67, 0.731, 0.735, 1.59	4.19, 0.657, 0.660, 1.43	4.17, 0.653, 0.657, 1.42
70	8.38, 0.748, 0.750, 2.37	7.28, 0.651, 0.652, 2.06	6.51, 0.582, 0.583, 1.84	6.47, 0.579, 0.580, 1.83

Table 3p: Here, one notes that, for given x, $N > N_{CDp}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm \times cm}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{V \times s}, \frac{10 \times cm^2}{s}\right)$, decrease with increasing r_a.

Acceptor	Ga	Mg	In	Cd
r_a (nm) ↗	0.126	0.140	0.144	0.148

For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
3	5.72, 1.219, 1.219, 9.38	5.22, 1.119, 1.120, 8.59	4.94, 1.062, 1.062, 8.13	4.63, 0.999, 0.999, 7.62
5	8.94, 1.333, 1.333, 12.34	8.17, 1.037, 1.038, 11.3	7.72, 0.983, 0.983, 10.7	7.22, 0.922, 0.922, 9.99
8	13.5, 1.067, 1.067, 15.9	12.3, 0.975, 0.975, 14.5	11.7, 0.922, 0.922, 13.7	10.9, 0.863, 0.863, 12.9
10	16.5, 1.039, 1.039, 18.0	15.1, 0.948, 0.948, 16.4	14.2, 0.896, 0.896, 15.5	13.3, 0.838, 0.838, 14.5

For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
3	2.77, 0.636, 0.637, 3.50	2.53, 0.591, 0.591, 3.21	2.38, 0.565, 0.565, 3.04	2.22, 0.536, 0.536, 2.85
5	4.34, 0.574, 0.575, 4.56	3.97, 0.531, 0.531, 4.18	3.75, 0.506, 0.506, 3.96	3.51, 0.478, 0.478, 3.72
8	6.54, 0.529, 0.529, 5.83	5.98, 0.487, 0.487, 5.34	5.30, 0.436, 0.463, 4.74	5.30, 0.436, 0.436, 4.74
10	7.95, 0.510, 0.510, 6.55	7.26, 0.469, 0.469, 6.00	6.43, 0.419, 0.419, 5.33	6.43, 0.419, 0.419, 5.33

For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**

N (10^{19} cm^{-3})				
3	1.27, 0.389, 0.389, 1.41	1.11, 0.370, 0.370, 1.26	1.25, 0.336, 0.336, 1.33	0.88, 0.349, 0.349, 1.07
5	2.15, 0.333, 0.333, 1.90	1.93, 0.312, 0.312, 1.73	2.02, 0.291, 0.291, 1.74	1.65, 0.288, 0.288, 1.52
8	3.34, 0.296, 0.296, 2.45	3.03, 0.275, 0.275, 2.24	3.04, 0.259, 0.259, 2.21	2.64, 0.251, 0.251, 1.99
10	4.08, 0.281, 0.281, 2.75	3.71, 0.261, 0.261, 2.53	3.68, 0.246, 0.246, 2.46	3.26, 0.237, 0.237, 2.24

For x=**0**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{19} cm^{-3})				
3	5.82, 1.242, 1.295, 9.52	5.33, 1.141, 1.190, 8.71	5.04, 1.083, 1.129, 8.25	4.72, 1.018, 1.062, 7.74
5	9.03, 1.144, 1.168, 12.4	8.24, 1.047, 1.070, 11.4	7.79, 0.992, 1.013, 10.7	7.29, 0.930, 0.951, 10.1
8	13.6, 1.072, 1.085, 16.0	12.4, 0.979, 0.990, 14.6	11.7, 0.926, 0.937, 13.8	11.0, 0.867, 0.877, 12.9
10	16.6, 1.043, 1.052, 18.1	15.1, 0.951, 0.960, 16.5	14.3, 0.900, 0.907, 15.6	13.3, 0.842, 0.849, 14.6

For x=**0.5**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{19} cm^{-3})				
3	2.88, 0.660, 0.714, 3.60	2.62, 0.613, 0.665, 3.30	2.48, 0.587, 0.637, 3.13	2.31, 0.557, 0.606, 2.93
5	4.42, 0.585, 0.608, 4.62	4.04, 0.540, 0.562, 4.24	3.82, 0.515, 0.536, 4.02	3.57, 0.486, 0.507, 3.77
8	6.60, 0.534, 0.545, 5.87	6.04, 0.491, 0.502, 5.38	5.71, 0.467, 0.477, 5.10	5.35, 0.440, 0.449, 4.78
10	8.00, 0.514, 0.522, 6.59	7.31, 0.472, 0.479, 6.03	6.92, 0.448, 0.455, 5.71	6.48, 0.422, 0.428, 5.36

For x=**1**, the values of (σ, μ, μ_H, D) at **77K**

N (10^{19} cm^{-3})				
3	1.38, 0.423, 0.498, 1.50	1.21, 0.406, 0.487, 1.36	1.34, 0.360, 0.415, 1.40	0.99, 0.392, 0.487, 1.17
5	2.23, 0.344, 0.370, 1.96	2.00, 0.324, 0.350, 1.78	2.08, 0.300, 0.301, 1.78	1.72, 0.300, 0.326, 1.57
8	3.39, 0.301, 0.312, 2.48	3.08, 0.280, 0.291, 2.27	3.09, 0.263, 0.272, 2.23	2.69, 0.255, 0.266, 2.01
10	4.12, 0.285, 0.292, 2.78	3.76, 0.264, 0.272, 2.55	3.73, 0.249, 0.256, 2.49	3.30, 0.240, 0.246, 2.26

Table 4n: In the lightly degenerate n-type $X(x)$ – alloy, in which N=5×10^{17} cm^{-3}, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor	P	As	Sb	Sn

For x=0,

$\xi_{n(T=3K)}$	↘	188.604	188.461	188.309	188.301
$\xi_{n(T=80K)}$	↘	7.253	7.248	7.242	7.242
$\kappa_{(T=3K)}\left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	4.169	3.647	3.262	3.244
$\kappa_{(T=80K)}\left(\frac{10^{-3}\times W}{cm\times K}\right)$	↘	1.242	1.087	0.972	0.967
$-S_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	3.006	3.008	3.010	3.0108
$-S_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	7.357	7.362	7.366	7.367
$-VC_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	2.003	2.005	2.006	2.0067
$-VC_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	4.109	4.111	4.112	4.1125
$-Ts_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	3.005	3.007	3.009	3.010
$-Ts_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	6.164	6.166	6.168	6.1688
$-Pt_{(T=3K)}(10^{-6}\times V)$	↘	9.018	9.024	9.032	9.0325
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	5.886	5.889	5.893	5.8936
$ZT_{(T=3K)}(10^{-4})$	↗	3.699	3.704	3.710	3.7107
$ZT_{(T=80K)}(10^{-1})$	↗	2.217	2.218	2.221	2.2216

For x=0.5,

$\xi_{n(T=3K)}$	↘	98.0048	97.1798	93.3001	96.2510
$\xi_{n(T=80K)}$	↘	3.8258	3.7872	3.7456	3.7433
$\kappa_{(T=3K)}\left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	2.1436	1.8398	1.6164	1.6059
$\kappa_{(T=80K)}\left(\frac{10^{-3}\times W}{cm\times K}\right)$	↘	0.7584	0.6512	0.5722	0.5685
$-S_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	5.7834	5.8324	5.8857	5.8887
$-S_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	12.1004	12.1779	12.2620	12.2668
$-VC_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	3.8519	3.8846	3.9200	3.9219
$-VC_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	6.0897	6.1386	6.2146	6.2174
$-Ts_{(T=3K)}\left(\frac{10^{-6}\times V}{K}\right)$	↘	5.7779	5.8268	5.8799	5.8829
$-Ts_{(T=80K)}\left(\frac{10^{-5}\times V}{K}\right)$	↘	9.1345	9.2079	9.2838	9.2879
$-Pt_{(T=3K)}(10^{-6}\times V)$	↘	17.3501	17.4973	17.6570	17.6660
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	9.6803	9.7423	9.8096	9.8134
$ZT_{(T=3K)}(10^{-4})$	↗	13.6913	13.9246	14.1800	14.1945
$ZT_{(T=80K)}(10^{-1})$	↗	5.9935	6.0706	6.1547	6.1595

For x=1,					
$\xi_{n(T=3K)}$	↘	59.0286	56.6617	51.9721	51.7627
$\xi_{n(T=80K)}$	↘	1.8061	1.6340	1.4449	1.4341
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	1.0361	0.8269	0.6694	0.6619
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	0.3691	0.3013	0.2524	0.2502
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	9.5963	10.1756	10.8963	10.9403
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↗	15.6298	15.5452	15.2344	15.2086
$-VC_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	6.3809	6.7639	7.2398	7.2689
$VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	0.0742	1.8756	4.1980	4.3416
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	9.5714	10.1459	10.8598	10.9033
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↗	0.1113	-2.8134	-6.2970	-6.5124
$-Pt_{(T=3K)}(10^{-6}\times V)$	↘	28.7889	30.5268	32.6889	32.8207
$-Pt_{(T=80K)}(10^{-3}\times V)$	↗	12.5038	12.4361	12.1876	12.1669
$ZT_{(T=3K)}\ (10^{-4})$	↗	37.6958	42.3842	46.6005	48.9935
$ZT_{(T=80K)}\ (10^{-1})$	↘	9.9998	9.8918	9.5003	9.4680

Table 4p: In the lightly degenerate p-type $X(x)-$ alloy, in which N=$3\times 10^{19}\ cm^{-3}$, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Acceptor		Ga	Mg	In	Cd
For x=0,					
$\xi_{n(T=3K)}$	↘	446.416	445.062	444.099	442.816
$\xi_{n(T=80K)}$	↘	16.815	16.764	16.728	16.680
$\kappa_{(T=3K)}\ (\frac{10^{-4}\times W}{cm\times K})$	↘	4.190	3.830	3.622	3.390
$\kappa_{(T=80K)}\ (\frac{10^{-2}\times W}{cm\times K})$	↘	1.140	1.042	0.986	0.923
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	1.270	1.274	1.277	1.280
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	3.333	3.343	3.350	3.359
$-VC_{(T=3K)}\ (\frac{10^{-7}\times V}{K})$	↘	8.447	8.492	8.511	8.536
$-VC_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	2.152	2.157	2.162	2.167
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	1.270	1.274	1.277	1.280
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	3.227	3.236	3.243	3.251
$-Pt_{(T=3K)}(10^{-6}\times V)$	↘	3.810	3.822	3.830	3.841

$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	2.666		2.674		2.680		2.687
$ZT_{(T=3K)}\,(10^{-5})$	↗	6.603		6.643		6.672		6.711
$ZT_{(T=80K)}\,(10^{-2})$	↗	4.548		4.575		4.594		4.620
For x=0.5,								
$\xi_{n(T=3K)}$	↘	318.904		314.931		312.092		308.297
$\xi_{n(T=80K)}$	↘	12.063		11.916		11.810		11.669
$\kappa_{(T=3K)}\,(\frac{10^{-4}\times W}{cm\times K})$	↘	2.033		1.852		1.746		1.626
$\kappa_{(T=80K)}\,(\frac{10^{-2}\times W}{cm\times K})$	↘	0.564		0.514		0.485		0.452
$-S_{(T=3K)}\,(\frac{10^{-6}\times V}{K})$	↘	1.778		1.800		1.817		1.839
$-S_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$	↘	4.596		4.650		4.690		4.744
$-VC_{(T=3K)}\,(\frac{10^{-7}\times V}{K})$	↘	11.851		12.001		12.110		12.259
$-VC_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$	↘	2.877		2.906		2.928		2.956
$-Ts_{(T=3K)}\,(\frac{10^{-6}\times V}{K})$	↘	1.778		1.800		1.816		1.839
$-Ts_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$	↘	4.315		4.359		4.391		4.435
$-Pt_{(T=3K)}(10^{-6}\times V)$	↘	5.334		5.401		5.450		5.517
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	3.677		3.720		3.752		3.795
$ZT_{(T=3K)}\,(10^{-5})$	↗	12.939		13.267		13.510		13.844
$ZT_{(T=80K)}\,(10^{-2})$	↗	8.647		8.853		9.004		9.213
For x=1,								
$\xi_{n(T=3K)}$		210.626	↘	198.482	↗	229.974	↘	177.504
$\xi_{n(T=80K)}$		8.059	↘	7.614	↗	8.771	↘	6.848
$\kappa_{(T=3K)}\,(\frac{10^{-4}\times W}{cm\times K})$		0.932	↘	0.811	↗	0.918	↘	0.647
$\kappa_{(T=80K)}\,(\frac{10^{-2}\times W}{cm\times K})$		0.272	↘	0.239	↗	0.264	↘	0.195
$-S_{(T=3K)}\,(\frac{10^{-6}\times V}{K})$		2.692	↘	2.856	↗	2.465	↘	3.194
$-S_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$		6.696	↘	7.046	↗	6.199	↘	7.736
$-VC_{(T=3K)}\,(\frac{10^{-7}\times V}{K})$		17.941	↘	19.038	↗	16.433	↘	21.287
$-VC_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$		3.869	↘	4.001	↗	3.664	↘	4.230
$-Ts_{(T=3K)}\,(\frac{10^{-6}\times V}{K})$		2.691	↘	2.856	↗	2.465	↘	3.193
$-Ts_{(T=80K)}\,(\frac{10^{-5}\times V}{K})$		5.802	↘	6.001	↗	5.496	↘	6.345
$-Pt_{(T=3K)}(10^{-6}\times V)$		8.075	↘	8.569	↗	7.396	↘	9.582
$-Pt_{(T=80K)}(10^{-3}\times V)$		5.357	↘	5.637	↗	4.959	↘	6.189
$ZT_{(T=3K)}\,(10^{-5})$		29.658	↗	33.398	↘	24.879	↗	41.757

$ZT_{(T=80K)}(10^{-2})$	18.353	↗	20.325	↘	15.731	↗	24.500

Table 5n: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- $X(x)$ − alloy, for $N = 2\times N_{CDn}(r_P)$, one gets:

T(K)	↗	5		**5.6729**		6		**7.719467**		7.720
ξ_n	↘	2.225		**1.8138**		1.649		**1**		0.9998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.531	↘	**−1.563**	↗	−1.556	↗	**− 1.322**	↗	− 1.321
ZT		0.959	↗	**1**	↘	0.991	↘	**0.715**	↘	0.7149
$(ZT)_{Mott}$	↗	0.664		**1**		1.210		**3.290**		3.291
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.324	↗	**0**	↗	0.171	↗	**1.105**	↗	1.1052
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.487	↗	**0**	↗	0.256	↗	**1.657**	↗	1.6578
$Pt\ (10^{-3}V)$		−0.765	↘	−0.887	↘	−0.933	↘	−1.02028	↗	−1.02027

In the degenerate As- $X(x)$ − alloy, for $N = 2\times N_{CDn}(r_{As})$, one gets:

T(K)	↗	6.8		**6.90423**		7		**9.3950696**		9.4
ξ_n	↘	1.860		**1.8138**		1.772		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.951		**1**		1.047		**3.290**		3.298
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.043	↗	**0**	↗	0.040	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.065	↗	**0**	↗	0.061	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$		−1.062	↘	−1.079	↘	−1.084	↘	−1.242	↗	−1.241

In the degenerate Sb- $X(x)$ − alloy, for $N = 2\times N_{CDn}(r_{Sb})$, one gets:

T(K)	↗	7.95		**8.1009**		8.3		**11.023411**		11.05
ξ_n	↘	1.871		**1.8138**		1.741		**1**		0.994
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.318
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.939		**1**		1.085		**3.290**		3.327
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.054	↗	**0**	↗	0.072	↗	**1.105**	↗	1.114

$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.080	↗	0	↗	0.108	↗	**1.657**	↗	1.672
$Pt\ (10^{-3}V)$		−1.242	↘	−1.266	↘	−1.296	↘	−1.457	↗	−1.456

In the degenerate Sn- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sn})$, one gets:

T(K)	↗	7.95		**8.16501**		8.3		**11.110508**		11.12
ξ_n	↘	1.896		**1.8138**		1.764		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.561	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.998	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.915		**1**		1.056		**3.290**		3.303
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.075	↗	0	↗	0.048	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.113	↗	0	↗	0.072	↗	**1.657**	↗	1.662
$Pt\ (10^{-3}V)$		−1.241	↘	−1.276	↘	−1.297	↘	−1.4685	↗	−1.46816

For x=0.5,

In the degenerate P- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_P)$, one gets:

T(K)	↗	14.3		**14.6558**		15		**19.9431231**		19.95
ξ_n	↘	1.889		**1.8138**		1.744		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.321
ZT		0.998	↗	**1**	↘	0.998	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.922		**1**		1.081		**3.290**		3.295
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.070	↗	0	↗	0.069	↗	**1.105**	↗	1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.105	↗	0	↗	0.103	↗	**1.657**	↗	1.659
$Pt\ (10^{-3}V)$		−2.233	↘	−2.291	↘	−2.343	↘	−2.636	↗	−2.6356

In the degenerate As- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{As})$, one gets:

T(K)	↗	17.5		**17.8372**		18.2		**24.272017**		24.3
ξ_n	↘	1.872		**1.8138**		1.753		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.939		**1**		1.070		**3.290**		3.308
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.054	↗	0	↗	0.059	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.082	↗	0	↗	0.089	↗	**1.657**	↗	1.664
$Pt\ (10^{-3}V)$		−2.734	↘	−2.788	↘	−2.843	↘	−3.208	↗	−3.207

In the degenerate Sb- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sb})$, one gets:

T(K)	↗	20.5		**20.9285**		21.5		**28.478812**		28.5
ξ_n	↘	1.877		**1.8138**		1.733		**1**		0.998

$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.934		**1**		1.095		**3.290**		3.301
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.059	↗	0	↗	0.080	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.088	↗	0	↗	0.120	↗	**1.657**	↗	1.662
$Pt\ (10^{-3}V)$		−3.202	↘	−3.271	↘	−3.357	↘	−3.764	↗	−3.763

In the degenerate Sn- $X(x) -$ alloy, for $N = 2 \times N_{CDn}(r_{Sn})$, one gets:

T(K)	↗	20.5		**21.0938**		21.5		**28.703825**		28.7
ξ_n	↘	1.877		**1.8138**		1.733		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.321
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.71469
$(ZT)_{Mott}$	↗	0.934		**1**		1.095		**3.290**		3.293
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.059	↗	0	↗	0.080	↗	**1.105**	↗	1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.088	↗	0	↗	0.120	↗	**1.657**	↗	1.659
$Pt\ (10^{-3}V)$		−3.202	↘	−3.297	↘	−3.357	↘	−3.794	↗	−3.793

For x=1,

In the degenerate P- $X(x) -$ alloy, for $N = 2 \times N_{CDn}(r_P)$, one gets:

T(K)	↗	30.5		**31.351**		32		**42.661168**		42.7
ξ_n	↘	1.898		**1.8138**		1.752		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.561	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.998	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.913		**1**		1.071		**3.290**		3.304
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.078	↗	0	↗	0.060	↗	**1.105**	↘	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.117	↗	0	↗	0.091	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−4.762	↘	−4.900	↘	−4.999	↘	−5.638	↗	−5.637

In the degenerate As- $X(x) -$ alloy, for $N = 2 \times N_{CDn}(r_{As})$, one gets:

T(K)	↗	37.5		**38.156**		38.8		**51.921285**		52
ξ_n	↘	1.867		**1.8138**		1.763		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.319
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$	↗	0.944		**1**		1.058		**3.290**		3.313
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.049	↗	0	↗	0.049	↗	**1.105**	↗	1.111
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.074	↗	0	↗	0.074	↗	**1.657**	↗	1.666

$Pt\ (10^{-3}V)$		−5.859	↘	−5.964	↘	−6.062	↘	−6.862	↗	−6.860

In the degenerate Sb- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sb})$, one gets:

T(K)	↗	44		**44.7689**		45.5		**60.920214**		60.95
ξ_n	↘	1.867		**1.8138**		1.765		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.944		**1**		1.056		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.049	↗	**0**	↗	0.048	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.074	↗	**0**	↗	0.071	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$		−6.874	↘	−6.997	↘	−7.109	↘	−8.052	↗	−8.051

In the degenerate Sn- $X(x)$ − alloy, for $N = 2 \times N_{CDn}(r_{Sn})$, one gets:

T(K)	↗	44		**45.123**		45.5		**61.40155**		61.42
ξ_n	↘	1.867		**1.8138**		1.765		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.944		**1**		1.056		**3.290**		3.294
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.049	↗	**0**	↗	0.048	↗	**1.105**	↗	1.106
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.074	↗	**0**	↗	0.071	↗	**1.657**	↗	1.659
$Pt\ (10^{-3}V)$		−6.874	↘	−7.053	↘	−7.109	↘	−8.1155	↗	−8.1148

Table 5p: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate Ga- $X(x)$ − alloy, for $N = 2 \times N_{CDp}(r_{Ga})$, one gets:

T(K)	↗	50.5		**51.708**		53		**70.3622**		70.5
ξ_p	↘	1.886		**1.8138**		1.740		**1**		0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.318
ZT		0.998	↗	**1**	↘	0.998	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.925		**1**		1.086		**3.290**		3.320
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.067	↗	**0**	↗	0.073	↗	**1.105**	↗	1.113

$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.101	↗	0	↗	0.110	↗	**1.657**	↗	1.669
$Pt\,(10^{-3}V)$		−7.887	↘	−8.082	↘	−8.2768	↘	−9.2998	↗	−9.295

In the degenerate Mg- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets

T(K)	↗	56.5		**57.78394**		59		**78.629815**		78.7
ξ_p	↘	1.883		**1.8138**		1.751		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.928		**1**		1.072		**3.290**		3.304
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.064	↗	0	↗	0.061	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.096	↗	0	↗	0.092	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−8.825	↘	−9.032	↘	−9.216	↘	−10.3925	↗	−10.390

In the degenerate In- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K)	↗	60.5		**61.914**		63.3		**84.2501**		84.3
ξ_p	↘	1.884		**1.8138**		1.747		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT		0.998	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.926		**1**		1.077		**3.290**		3.300
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.066	↗	0	↗	0.065	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.098	↗	0	↗	0.098	↗	**1.657**	↗	1.661
$Pt\,(10^{-3}V)$		−9.449	↘	−9.677	↘	−9.887	↘	−11.1353	↗	−11.134

In the degenerate Cd- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K)	↗	66		**67.2**		68.7		**91.44384**		91.5
ξ_p	↘	1.869		**1.8138**		1.748		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.942		**1**		1.077		**3.290**		3.299
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.051	↗	0	↗	0.065	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.077	↗	0	↗	0.098	↗	**1.657**	↗	1.661
$Pt\,(10^{-3}V)$		−10.311	↘	−10.503	↘	−10.730	↘	−12.086	↗	−12.084

For x=0.5,

In the degenerate Ga- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Ga})$, one gets:

T(K)	↗	92.6		**94.592**		96.6		**128.717794**		128.9
ξ_p	↘	1.879		**1.8138**		1.751		**1**		0.997

$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.319
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$	↗	0.932		**1**		1.073		**3.290**		3.312
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.060	↗	0	↗	0.062	↗	**1.105**	↗	1.111
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.091	↗	0	↗	0.093	↗	**1.657**	↗	1.666
$Pt\ (10^{-3}V)$		−14.464	↘	−14.785	↘	−15.089	↘	−17.0127	↗	−17.006

In the degenerate Mg- $X(x)$ − alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets

T(K)	↗	103.5		**105.707**		108		**143.84224**		144
ξ_p	↘	1.878		**1.8138**		1.749		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.932		**1**		1.075		**3.290**		3.307
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.060	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.090	↗	0	↗	0.095	↗	**1.657**	↗	1.664
$Pt\ (10^{-3}V)$		−16.167	↘	−16.522	↘	−16.869	↘	−19.012	↗	−19.006

In the degenerate In- $X(x)$ − alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K)	↗	103.5		**113.263**		116		**154.12377**		155
ξ_p	↘	1.878		**1.8138**		1.742		**1**		0.987
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.312
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.705
$(ZT)_{Mott}$	↗	0.932		**1**		1.083		**3.290**		3.379
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.060	↗	0	↗	0.071	↗	**1.105**	↗	1.128
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.090	↗	0	↗	0.106	↗	**1.657**	↗	1.691
$Pt\ (10^{-3}V)$		−16.167	↘	−17.703	↘	−18.116	↘	−20.370	↗	−20.339

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In the degenerate Cd- $X(x)$ − alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K)	↗	120		**122.9335**		126		**167.2837**		168
ξ_p	↘	1.888		**1.8138**		1.740		**1**		0.990
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.314
ZT		0.998	↗	**1**	↘	0.998	↘	**0.715**	↘	0.707
$(ZT)_{Mott}$	↗	0.923		**1**		1.086		**3.290**		3.357
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.068	↗	0	↗	0.073	↗	**1.105**	↗	1.122
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.103	↗	0	↗	0.109	↗	**1.657**	↗	1.683
$Pt\ (10^{-3}V)$		−18.741	↘	−19.214	↘	−19.677	↘	−22.110	↗	−22.085

For x=1,

In the degenerate Ga- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Ga})$, one gets:

T(K) ↗	168.5		**172.189**		176		**234.30852**		134.5
ξ_p ↘	1.880		**1.8138**		1.748		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.931		**1**		1.076		**3.290**		3.302
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.062	↗	**0**	↗	0.065	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	**0**	↗	0.097	↗	**1.657**	↗	1.662
$Pt\ (10^{-3}V)$	−26.319	↘	−26.913	↘	−27.490	↘	−30.9686	↗	−30.962

In the degenerate Mg- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Mg})$, one gets:

T(K) ↗	188.3		**192.423**		197		**261.83996**		262
ξ_p ↘	1.880		**1.8138**		1.743		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$ ↗	0.931		**1**		1.082		**3.290**		3.299
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	**0**	↗	0.070	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	**0**	↗	0.104	↗	**1.657**	↗	1.661
$Pt\ (10^{-3}V)$	−29.412	↘	−30.076	↘	−30.767	↘	−34.607	↗	−34.602

In the degenerate In- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{In})$, one gets:

T(K) ↗	132.8		**135.779**		139		**184.76258**		185
ξ_p ↘	1.882		**1.8138**		1.744		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.319
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.929		**1**		1.082		**3.290**		3.310
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.063	↗	**0**	↗	0.069	↗	**1.105**	↗	1.110
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.095	↗	**0**	↗	0.104	↗	**1.657**	↗	1.665
$Pt\ (10^{-3}V)$	−20.742	↘	−21.222	↘	−21.709	↘	−24.420	↗	−24.412

In the degenerate Cd- $X(x) -$ alloy, for $N = 2 \times N_{CDp}(r_{Cd})$, one gets:

T(K) ↗	219		**223.78**		229		**304.5111**		305
ξ_p ↘	1.880		**1.8138**		1.745		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.319
ZT	0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.712

$(ZT)_{Mott}$	↗	0.931		**1**		1.080		**3.290**		3.315
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.068	↗	**1.105**	↗	1.111
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.102	↗	**1.657**	↗	1.667
$Pt\ (10^{-3}V)$		−34.208	↘	−34.977	↘	−35.766	↘	−40.247	↗	−40.231

Table 6n: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results: $-1.322\times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105\times 10^{-4}\frac{V}{K}$, and $1.657\times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- $X(x)$ – alloy, for T= **5.6729 K**, one gets:

$N(10^{15}cm^{-3})$	↘	3.4		**3.3064128**		3.25		**2.69469323**		2.69
ξ_n	↘	1.927		**1.8138**		1.745		**1**		0.993
$S\left(10^{-4}\frac{V}{K}\right)$		−1.560	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.317
ZT		0.996	↗	**1**	↘	0.998	↘	**0.715**	↘	0.710
$(ZT)_{Mott}$	↗	0.886		**1**		1.081		**3.290**		3.337
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.103	↗	0	↗	0.068	↗	**1.105**	↗	1.117
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.154	↗	0	↗	0.103	↗	**1.657**	↗	1.675
$Pt\ (10^{-3}V)$		−0.885	↘	−0.887	↗	−0.886	↗	−0.750	↗	−0.747

In the degenerate As- $X(x)$ – alloy, for T= **6.90423 K**, one gets:

$N(10^{15}cm^{-3})$	↘	4.5		**4.4394186**		4.38		**3.6180693**		3.615
ξ_n	↘	1.869		**1.8138**		1.759		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.319
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$	↗	0.942		**1**		1.062		**3.290**		3.313
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.051	↗	0	↗	0.053	↗	**1.105**	↗	1.111
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.077	↗	0	↗	0.080	↗	**1.657**	↗	1.666
$Pt\ (10^{-3}V)$		−1.0786	↘	−1.07913	↗	−1.0786	↗	−0.912	↗	−0.911

In the degenerate Sb- $X(x)$ – alloy, for T= **8.1009 K**, one gets:

$N(10^{15}cm^{-3})$	↘	5.75		**5.6422212**		5.55		**4.5983503**		4.595
ξ_n	↘	1.890		**1.8138**		1.747		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320

ZT		0.998	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.920		**1**		1.077		**3.290**		3.309
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.071	↗	**0**	↗	0.065	↗	**1.105**	↗	1.110
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.106	↗	**0**	↗	0.098	↗	**1.657**	↗	1.665
$Pt\,(10^{-3}V)$		−1.265	↘	−1.266	↗	−1.265	↗	−1.071	↗	−1.069

In the degenerate Sn- $X(x)$ − alloy, for T=**8.16501**, one gets:

$N(10^{15}cm^{-3})$	↘	5.78		**5.7092226**		5.6		**4.6529901**		4.65
ξ_n	↘	1.864		**1.8138**		1.736		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.947		**1**		1.091		**3.290**		3.307
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.047	↗	**0**	↗	0.077	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.070	↗	**0**	↗	0.116	↗	**1.657**	↗	1.664
$Pt\,(10^{-3}V)$		−1.2757	↘	−1.276	↗	−1.275	↗	−1.079	↗	−1.078

For x=0.5,

In the degenerate P- $X(x)$ − alloy, for T=**14.6558 K**, one gets:

$N(10^{16}cm^{-3})$	↘	3.6		**3.5475964**		3.5		**2.8912484**		2.88
ξ_n	↘	1.873		**1.8138**		1.759		**1**		0.984
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.310
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.703
$(ZT)_{Mott}$	↗	0.938		**1**		1.063		**3.290**		3.396
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.055	↗	**0**	↗	0.053	↗	**1.105**	↗	1.132
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.083	↗	**0**	↗	0.080	↗	**1.657**	↗	1.698
$Pt\,(10^{-3}V)$		−2.289	↘	−2.291	↗	−2.290	↗	−1.937	↗	−1.920

In the degenerate As- $X(x)$ − alloy, for T=**17.8372 K**, one gets:

$N(10^{16}cm^{-3})$	↘	4.9		**4.763246**		4.7		**3.8820133**		3.88
ξ_n	↘	1.929		**1.8138**		1.760		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.560	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.996	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.884		**1**		1.062		**3.290**		3.304
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.104	↗	**0**	↗	0.053	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.156	↗	**0**	↗	0.079	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−2.783	↘	−2.788	↗	−2.787	↗	−2.357	↗	−2.355

In the degenerate Sb- $X(x)$ – alloy, for T=**20.9285**, one gets:

$N(10^{16}cm^{-3})$	↘	4.9		**6.0537854**		4.7		**4.933764**		4.93
ξ_n	↘	1.929		**1.8138**		1.760		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.560	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.319
ZT		0.996	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.884		**1**		1.062		**3.290**		3.310
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.104	↗	**0**	↗	0.053	↗	**1.105**	↗	1.110
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.156	↗	**0**	↗	0.079	↗	**1.657**	↗	1.665
$Pt\ (10^{-3}V)$		−2.783	↘	−3.271	↗	−2.787	↗	−2.766	↗	−2.761

In the degenerate Sn- $X(x)$ – alloy, for T=**21.0938** one gets:

$N(10^{16}cm^{-3})$	↘	6.2		**6.125674**		6		**4.9923445**		4.99
ξ_n	↘	1.863		**1.8138**		1.730		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.948		**1**		1.098		**3.290**		3.302
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.046	↗	**0**	↗	0.083	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.068	↗	**0**	↗	0.124	↗	**1.657**	↗	1.662
$Pt\ (10^{-3}V)$		−3.296	↘	−3.297	↗	−3.293	↗	−2.788	↗	−2.785

For x=1,

In the degenerate P- $X(x)$ – alloy, for T=**31.351 K**, one gets:

$N(10^{17}cm^{-3})$	↘	1.999		**1.9760296**		1.95		**1.61044613**		1.61
ξ_n	↘	1.860		**1.8138**		1.760		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.950		**1**		1.061		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.044	↗	**0**	↗	0.052	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.066	↗	**0**	↗	0.079	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$		−4.898	↘	−4.900	↗	−4.898	↗	−4.144	↗	−4.141

In the degenerate As- $X(x)$ – alloy, for T=**38.156 K**, one gets:

$N(10^{17}cm^{-3})$	↘	2.69		**2.6531542**		2.62		**2.1622929**		2.16
ξ_n	↘	1.870		**1.8138**		1.763		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.319
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$	↗	0.941		**1**		1.058		**3.290**		3.318

$VC\left(10^{-4}\frac{V}{K}\right)$	−0.052	↗	0	↗	0.050	↗	**1.105**	↗	1.112
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.078	↗	0	↗	0.074	↗	**1.657**	↗	1.668
$Pt\ (10^{-3}V)$	−5.961	↘	−5.964	↗	−5.961	↗	−5.043	↗	−5.031

In the degenerate Sb- $X(x)-$ alloy, for T=**44.7689 K**, one gets:

$N(10^{17}cm^{-3})$	↘	3.43		**3.3719916**		3.32		**2.748129**	2.745
ξ_n	↘	1.883		**1.8138**		1.751		**1**	0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322** ↗	− 1.318
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715** ↘	0.711
$(ZT)_{Mott}$	↗	0.928		**1**		1.073		**3.290**	3.320
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.064	↗	0	↗	0.062	↗	**1.105** ↗	1.113
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.096	↗	0	↗	0.092	↗	**1.657** ↗	1.669
$Pt\ (10^{-3}V)$		−6.992	↘	−6.997	↗	−6.993	↗	−5.917 ↗	−5.902

In the degenerate Sn- $X(x)-$ alloy, for T=**45.123 K**, one gets:

$N(10^{17}cm^{-3})$	↘	3.46		**3.412034**		3.35		**2.7807765**	2.780
ξ_n	↘	1.870		**1.8138**		1.740		**1**	0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322** ↗	− 1.321
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715** ↘	0.714
$(ZT)_{Mott}$	↗	0.940		**1**		1.086		**3.290**	3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.053	↗	0	↗	0.073	↗	**1.105** ↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.079	↗	0	↗	0.110	↗	**1.657** ↗	1.660
$Pt\ (10^{-3}V)$		−7.049	↘	−7.053	↗	−7.047	↗	−5.964 ↗	−5.960

Table 6p: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S, ZT, $(ZT)_{Mott}$, VC, and T_s present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate Ga- $X(x)-$ alloy, for T=**51.708K**, one gets:

$N(10^{18}cm^{-3})$	↘	1.492		**1.4673047**		1.445		**1.19583942**	1.1952
ξ_p	↘	1.881		**1.8138**		1752		**1**	0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322** ↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715** ↘	0.713

$(ZT)_{Mott}$	↗	0.929		**1**		1.071		**3.290**		3.304
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.063	↗	0	↗	0.061	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.094	↗	0	↗	0.091	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−8.076	↘	−8.082	↗	−8.077	↗	−6.834	↗	−6.826

In the degenerate Mg- $X(x)$ − alloy, for T= **57.78394** K, one gets:

$N(10^{18}cm^{-3})$	↘	1.763		**1.7333732**		1.705		**1.4126857**		1.412
ξ_p	↘	1.882		**1.8138**		1.747		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.928		**1**		1.077		**3.290**		3.303
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.064	↗	0	↗	0.066	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.095	↗	0	↗	0.098	↗	**1.657**	↗	1.662
$Pt\ (10^{-3}V)$		−9.025	↘	−9.032	↗	−9.025	↗	−7.637	↗	−7.629

In the degenerate In- $X(x)$ − alloy, for T=**61.914 K**, one gets:

$N(10^{18}cm^{-3})$	↘	1.955		**1.9225022**		1.9		**1.56682126**		1.566
ξ_p	↘	1.882		**1.8138**		1.766		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.928		**1**		1.054		**3.290**		3.303
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.063	↗	0	↗	0.046	↗	**1.105**	↗	1.108
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.094	↗	0	↗	0.070	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−9.671	↘	−9.677	↗	−9.674	↗	−8.183	↗	−8.174

In the degenerate Cd- $X(x)$ − alloy, for T=**67.2 K**, one gets:

$N(10^{18}cm^{-3})$	↘	2.2		**2.1739166**		2.15		**1.77171312**		1.770
ξ_p	↘	1.862		**1.8138**		1.769		**1**		0.996
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.319
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.712
$(ZT)_{Mott}$	↗	0.949		**1**		1.051		**3.290**		3.316
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.045	↗	0	↗	0.043	↗	**1.105**	↗	1.112
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.068	↗	0	↗	0.065	↗	**1.657**	↗	1.667
$Pt\ (10^{-3}V)$		−10.500	↘	−10.503	↗	−10.500	↗	−8.882	↗	−8.863

For x=0.5,

In the degenerate Ga- $X(x)$ − alloy, for T=**94.592** K, one gets:

$N(10^{18} cm^{-3})$	↘	5.68		**5.5895544**		5.5		**4.5554186**		4.55
ξ_p	↘	1.879		**1.8138**		1.749		**1**		0.995
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.318
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.711
$(ZT)_{Mott}$	↗	0.932		**1**		1.076		**3.290**		3.322
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.060	↗	0	↗	0.064	↗	**1.105**	↗	1.113
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.091	↗	0	↗	0.096	↗	**1.657**	↗	1.670
$Pt\ (10^{-3}V)$		−14.775	↘	−14.785	↗	−14.775	↗	−12.502	↗	−12.470

In the degenerate Mg- $X(x)$ − alloy, for T=**105.707 K**, one gets:

$N(10^{18} cm^{-3})$	↘	6.68		**6.6031166**		6.5		**5.3814701**		5.38
ξ_p	↘	1.861		**1.8138**		1.750		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.950		**1**		1.074		**3.290**		3.297
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.044	↗	0	↗	0.062	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.066	↗	0	↗	0.094	↗	**1.657**	↗	1.660
$Pt\ (10^{-3}V)$		−16.517	↘	−16.522	↗	−16.511	↗	−13.971	↗	−13.963

In the degenerate In- $X(x)$ − alloy, for T=**113.263 K**, one gets:

$N(10^{18} cm^{-3})$	↘	7.4		**7.323585**		7.2		**5.9686534**		5.965
ξ_p	↘	1.855		**1.8138**		1.745		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.955		**1**		1.080		**3.290**		3.306
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.039	↗	0	↗	0.068	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.059	↗	0	↗	0.102	↗	**1.657**	↗	1.664
$Pt\ (10^{-3}V)$		−17.698	↘	−17.703	↗	−17.690	↗	−14.970	↗	−14.950

In the degenerate Cd- $X(x)$ − alloy, for T=**122.9335 K**, one gets:

$N(10^{18} cm^{-3})$	↘	8.39		**8.2813234**		8.15		**6.7491847**		6.747
ξ_p	↘	1.866		**1.8138**		1.749		**1**		0.999
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗	0.944		**1**		1.075		**3.290**		3.298
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.049	↗	0	↗	0.063	↗	**1.105**	↗	1.107

$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.074	↗	0	↗	0.095	↗	**1.657**	↗	1.661
$Pt\ (10^{-3}V)$	−19.206	↘	−19.214	↗	−19.202	↗	−16.248	↗	−16.237

For x=1,

In the degenerate Ga- X(x) − alloy, for T=**172.189** K, one gets:

$N(10^{19}cm^{-3})$	↘ 1.95		**1.9185205**		1.89		**1.56357393**		1.563
ξ_p	↘ 1.880		**1.8138**		1.753		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.321
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.714
$(ZT)_{Mott}$	↗ 0.931		**1**		1.070		**3.290**		3.300
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.059	↗	**1.105**	↗	1.107
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.089	↗	**1.657**	↗	1.661
$Pt\ (10^{-3}V)$	−26.896	↘	−26.913	↗	−26.898	↗	−22.758	↗	−22.740

In the degenerate Mg- X(x) − alloy, for T=**192.423** K, one gets:

$N(10^{19}cm^{-3})$	↘ 2.31		**2.2664086**		2.21		**1.84710825**		1.846
ξ_p	↘ 1.891		**1.8138**		1.712		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.560	↗	**−1.322**	↗	−1.320
ZT	0.998	↗	**1**	↘	0.997	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗ 0.920		**1**		1.122		**3.290**		3.306
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.071	↗	0	↗	0.102	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.107	↗	0	↗	0.153	↗	**1.657**	↗	1.664
$Pt\ (10^{-3}V)$	−30.049	↘	−30.076	↗	−30.026	↗	−25.433	↗	−25.399

In the degenerate In- X(x) − alloy, for T=**135.780** K, one gets:

$N(10^{19}cm^{-3})$	↘ 1.365		**1.3434**		1.314		**1.09486332**		1.094
ξ_p	↘ 1.878		**1.8138**		1.725		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.319
ZT	0.999	↗	**1**	↘	0.997	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗ 0.932		**1**		1.106		**3.290**		3.311
$VC\left(10^{-4}\frac{V}{K}\right)$	−0.060	↗	0	↗	0.089	↗	**1.105**	↗	1.110
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.090	↗	0	↗	0.133	↗	**1.657**	↗	1.666
$Pt\ (10^{-3}V)$	−21.209	↘	−21.222	↗	−21.196	↗	−17.946	↗	−17.915

In the degenerate Cd- X(x) − alloy, for T=**223.78** K, one gets:

$N(10^{19}cm^{-3})$	↘ 2.888		**2.842425**		2.78		**2.31654886**		2.315

ξ_ρ	↘	1.878		**1.8138**		1.724		**1**		0.997
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.561	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.997	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.932		**1**		1.106		**3.290**		3.308
$VC\left(10^{-4}\frac{V}{K}\right)$		−0.060	↗	0	↗	0.089	↗	**1.105**	↗	1.109
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.090	↗	0	↗	0.134	↗	**1.657**	↗	1.664
$Pt\ (10^{-3}V)$		−34.955	↘	−34.977	↗	−34.932	↗	−29.577	↗	−29.534