



**VARIOUS ELECTRICAL-AND-THERMOELECTRIC LAWS,
RELATIONS AND COEFFICIENTS IN NEW n(p)-TYPE
DEGENERATE “COMPENSATED” GaAs(1-x)Te(x)-CRYSTALLINE
ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT
LAW, ACCURATE FERMI ENERGY, AND ELECTRICAL
CONDUCTIVITY MODEL (IX)**

Prof. Dr. Huynh Van Cong*

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS),
EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

Article Received on 03/02/2025

Article Revised on 23/02/2025

Article Accepted on 15/03/2025



***Corresponding Author**

**Prof. Dr. Huynh Van
Cong**

Université de Perpignan Via
Domitia, Laboratoire de
Mathématiques et Physique
(LAMPS), EA 4217,
Département de Physique,
52, Avenue Paul Alduy, F-
66 860 Perpignan, France.

ABSTRACT

In the $n^+(p^+) - p(n) \text{X}(x) \equiv \text{GaAs}_{1-x}\text{Te}_x$ - crystalline alloy, $0 \leq x \leq 1$, various electrical-and-thermoelectric laws, relations and coefficients, enhanced by our static dielectric constant law given in Equations (1a, 1b), being due to the effects of the size of donor (acceptor) $d(a)$ -radius $r_{d(a)}$ and the x -concentration, by our accurate Fermi energy given in Eq. (11), and finally by our electrical conductivity model given in Eq. (14), are now investigated, basing on the same physical model and mathematical treatment method, as those used in our recent works (Van Cong, 2024, 2025). It should be noted here that, for $x=0$, these obtained numerical results may be reduced to those given in the n(p)-type degenerate GaAs-crystal (Van Cong,

2023). Then, some remarkable results could be cited in the following. In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T , and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that, for any given x , $r_{d(a)}$ and N (or T), with increasing T (or decreasing N) one obtains: (i) for

$\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{\min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K} \right)$, those of the figure of merit ZT show a same maximum $(ZT)_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S , ZT , the Mott figure of merit $(ZT)_{\text{Mott}}$, the first Van-Cong coefficient $VC1$, and the Thomson coefficient Ts , present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$. It seems that these same results could represent a new law in the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$).

KEYWORDS: Electrical conductivity, Seebeck coefficient (S), Figure of merit (ZT), First Van-Cong coefficient ($VC1$), Second Van-Cong coefficient ($VC2$), Thomson coefficient (Ts), Peltier coefficient (Pt)

INTRODUCTION

In the $n^+(p^+) - p(n) X(x) \equiv GaAs_{1-x}Te_x$ - crystalline alloy, $0 \leq x \leq 1$, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) $d(a)$ -radius, given in Equations (1a, 1b) and our electrical conductivity model, in Eq. (14), and also by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), are now investigated, by basing on the same physical model and mathematical treatment method, as those used in our recent works (Van Cong, 2024, 2025). It should be noted here that for $x=0$, these obtained numerical results may be reduced to those given in the $n(p)$ -type degenerate GaAs-crystal (Van Cong, 2023; Hyun et al. 1998; Kim et al., 2015). Then, some remarkable results could be noted in the following.

(1) The generalized Mott criterium in the metal-insulator transition (**MIT**) is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{CDn(CDp)}^{EBT}$, obtained with a precision of the order of 2.89×10^{-7} , as given in our recent work (Van Cong, 2024), and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^* .

(3) The Fermi energy for any N and T , $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} (Van Cong and Debais, 1993), and it is present in all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ , and for the Seebeck coefficient, S , determined respectively in Equations (14, 19) are the basic expressions, used to determine all the following electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T , and further in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: $\nearrow$, decrease: $\searrow$). Furthermore, one notes in these Tables that, for any given x , $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{\min.} (\simeq -1.563 \times 10^{-4} \frac{V}{K})$, those of the figure of merit ZT show a same maximum $(ZT)_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S , ZT , the Mott figure of merit $(ZT)_{\text{Mott}}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts , present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$. It seems that these same results could represent **a new law in the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $n^+(p^+) - p(n) X(x)$ - crystalline alloy at $T=0$ K, we denote the donor (acceptor) $d(a)$ -radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{As(Ga)}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, the unperturbed relative static dielectric constant by: $\epsilon_o(x)$, and the intrinsic band gap by: $E_{go}(x)$. Then, their values are reported in Table 1 in Appendix 1.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$$E_{do(ao)}(x) = \frac{13600 \times [m_c(v)(x)/m_o]}{[\varepsilon_o(x)]^2} \text{ meV}, \text{ and then, the isothermal bulk modulus, by :}$$

$$B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right) \times (r_{do(ao)})^3}.$$

Our Static Dielectric Constant Law

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p , $p_o = 0$, and for the deformation potential energy (or the strain energy) α , $\alpha_o = 0$. Further, the two important equations, used to determine the α -variation, $\Delta \alpha \equiv \alpha - \alpha_o = \alpha$, are defined by:

$\frac{dp}{dv} = -\frac{B}{V}$ and $p = -\frac{d\alpha}{dv}$, giving rise to: $\frac{d}{dv}\left(\frac{d\alpha}{dv}\right) = \frac{B}{V}$. Then, by an integration, one gets:

$$\left[\Delta \alpha(r_{d(a)}, x)\right]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by: $\pm [\Delta \alpha(r_{d(a)}, x)]_{n(p)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = + [\Delta \alpha(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = - [\Delta \alpha(r_{d(a)}, x)]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new $\varepsilon(r_{d(a)}, x)$ -law**,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \geq 0, \quad (1a)$$

according to the increase in both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with increasing $r_{d(a)}$ and for a given x , and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a

condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 < 1$, being a **new $\varepsilon(r_{d(a)}, x)$ -law**,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 - 1 \right] \times \ln \left(\frac{r_{d(a)}}{r_{do(ao)}} \right)^3 \leq 0, \quad (1b)$$

corresponding to the decrease in both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with decreasing $r_{d(a)}$ and for a given x .

It should be noted that, in the following, all the electrical-and-thermoelectric properties strongly depend on this **new $\varepsilon(r_{d(a)}, x)$ -law**.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)}, x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}, x) \equiv \frac{\varepsilon(r_{d(a)}, x) \times \hbar^2}{m_{c(v)}(x) \times m_o \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)}, x)}{m_{c(v)}(x)}. \quad (2)$$

Generalized Mott Criterion in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at $T=0$ K, $N_{CDn(NDp)}(r_{d(a)}, x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as:

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = M_{n(p)}, \quad M_{n(p)} = 0.25, \quad (3)$$

depending thus on our **new** $\varepsilon(r_{d(a)}, x)$ -law.

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp)}$, characteristic of interactions, by:

$$r_{sn(sp)}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)}, x)} = 1.1723 \times 10^8 \times \left(\frac{1}{N}\right)^{1/3} \times \frac{m_c(v)(x) \times m_0}{\varepsilon(r_{d(a)}, x)}, \quad (4)$$

being equal to, in particular, at $N=N_{CDn(CDp)}(r_{d(a)}, x)$: $r_{sn(sp)}(N_{CDn(CDp)}(r_{d(a)}, x), r_{d(a)}, x) = 2.4813963$, for any $(r_{d(a)}, x)$ -values. Then, from Eq. (4), one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)}, x) = \left(\frac{3}{4\pi}\right)^{1/3} \times \frac{1}{2.4813963} = 0.25 = (WS)_{n(p)} = M_{n(p)}, \quad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$, as those given in our previous work (Van Cong, 2024), we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail, $N_{CDn(CDp)}^{EBT}$** , with a precision of the order of 2.89×10^{-7} .

It should be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $N_{CDn(CDp)}^{EBT}$ are found to be in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials:

$$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) = N^*, \quad \text{for a presentation simplicity.} \quad (6)$$

In summary, as observed in Table 4 of our previous paper (Van Cong, 2024), one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}(r_{d(a)}, x)$, $N_{CDn(NDp)}(r_{d(a)}, x)$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)}, x)$ increase, affecting strongly all electrical-and-thermoelectric properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+(p^+) - p(n)$ $X(x)$ - crystalline alloy, if denoting the Fermi wave number by:

$$k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{8c(v)} \right)^{\frac{1}{3}}, \text{ the reduced effective Wigner-Seitz (WS) radius } r_{sn(sp)},$$

characteristic of interactions, being given in Eq. (4), in which N is replaced by N^* , is now defined by:

$$\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < 1,$$

being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{Fn(Fp)}^{-1}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}(r_{d(a)}, x)$ is determined in Eq. (2).

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} = R_{snWS(spWS)} + [R_{snTF(spTF)} - R_{snWS(spWS)}]e^{-r_{sn(sp)}} < 1, \quad (7)$$

being valid at any N^* .

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k_{Fn(Fp)}^{-1}}{k_{snTF(spTF)}^{-1}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(spWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{s}{2\pi} - \gamma \frac{d[r_{sn(sp)}^2 \times E_{CE}(N^*)]}{dr_{sn(sp)}} \right), \quad (9)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908 + r_{sn(sp)}} + \left(\frac{2[1 - \ln(2)]}{\pi^2} \right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67878876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \eta_{n(p)}(N^*) \equiv \frac{\sqrt{2\pi N^*}}{s(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (10)$$

which gives: $A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$.

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$ - crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$ - crystalline alloy, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N, r_{d(a)}, x, T)$, obtained in our previous paper (Van Cong, Debiais, and Doan Khanh, 1991- 1993), obtained with a precision of the order of 2.11×10^{-4} , is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, \quad A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T, x)}$,

$$N_{c(v)}(T, x) = 2g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_0 \times k_B T}{2\pi \hbar^2} \right)^{\frac{3}{2}} (\text{cm}^{-3}), \quad g_{c(v)} = 1, \quad F(u) = au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}},$$

$$a = [3\sqrt{\pi}/4]^{2/3}, \quad b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2, \quad c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4, \quad \text{and } G(u) \simeq \text{Ln}(u) + 2^{-\frac{3}{2}} \times u \times e^{-du};$$

$$d = 2^{3/2} \left[\frac{1}{\sqrt{27}} - \frac{3}{16} \right] > 0.$$

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times \text{Ln}(u)$ as $u \rightarrow 0$, the limiting non-degenerate condition, and in the very degenerate case ($u \gg 1$),

one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_0}$

as $u \rightarrow \infty$, the limiting degenerate condition. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, at $T=0K$, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{c(v)}(x) \times m_0}$, being proportional to $(N^*)^{2/3}$, and also equal to 0 at $N^* = 0$, according to the MIT.

In the following, it should be noted that all the electrical-and-thermoelectric properties strongly depend on such the accurate expression of $\xi_{n(p)}(N, r_{d(a)}, x, T)$.

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^Y)^{-1}$, $Y \equiv (E - E_{Fn(Fp)})/(k_B T)$.

So, the average of E^p , calculated using the FDDF-method, as developed in our previous work (Van Cong, 2018, 2025) is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E_{Fn(Fp)}^p \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E} \right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^Y}{(1+e^Y)^2}.$$

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fn(Fp)})$, $\delta(E - E_{Fn(Fp)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fn(Fp)}) = 1$.

Then, at low T, by a variable change $Y \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$$G_p(E_{Fn(Fp)}) \equiv 1 + E_{Fn(Fp)}^{-p} \times \int_{-\infty}^{\infty} \frac{e^Y}{(1+e^Y)^2} \times (k_B T Y + E_{Fn(Fp)})^p dY = 1 + \sum_{\mu=1,2,\dots}^p C_p^\beta \times (k_B T)^\beta \times E_{Fn(Fp)}^{-\beta} \times I_\beta,$$

where $C_p^\beta \equiv p(p-1) \dots (p-\beta+1)/\beta!$ and the integral I_β is given by:

$I_\beta = \int_{-\infty}^{\infty} \frac{Y^\beta \times e^Y}{(1+e^Y)^2} dY = \int_{-\infty}^{\infty} \frac{Y^\beta}{(e^{Y/2} + e^{-Y/2})^2} dY$, vanishing for odd values of β . Then, for even values of $\beta = 2n$, with $n=1, 2, \dots$, one obtains.

$$I_{2n} = 2 \int_0^{\infty} \frac{Y^{2n} \times e^Y}{(1+e^Y)^2} dY.$$

Now, using an identity $(1 + e^Y)^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{Y(s-1)}$, a variable change: $sY = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1} \pi^{2n} |B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. So, from above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case the following ratio:

$$G_p(E_{Fn(Fp)}) \equiv \frac{\langle E^p \rangle_{FDDF}}{E_{Fn(Fp)}^p} = 1 + \sum_{n=1}^p \frac{p(p-1)\dots(p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n} \equiv G_{p \geq 1}(y), \quad (12)$$

$$\text{where } y \equiv \frac{\pi}{\xi_{n(p)}(N^*, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, T)}.$$

Then, some usual results of $G_{p \geq 1}(y)$ are given in Table 2 in Appendix 1, being needed to determine all the following electrical-and-thermoelectric properties.

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by $\sigma(N, r_{d(a)}, x, T)$ expressed in $\text{ohm}^{-1} \times \text{cm}^{-1}$, the thermal conductivity by $\kappa(N, r_{d(a)}, x, T)$ in $\frac{W}{\text{cm} \times K}$, and the

Lorenz number L defined by:

$L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times \text{ohm}}{K^2}\right) = 2.4429637 \times 10^{-8} (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature $T(K)$, as:

$$\frac{\kappa(N, r_{d(a)}, x, T)}{\sigma(N, r_{d(a)}, x, T)} = L \times T. \quad (13)$$

We now determine the general form of σ in the following.

First of all, it is expressed in terms of the kinetic energy of the electron (hole),

$$E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{Cn(Cp)} \times m_0}, \text{ or the wave number } k, \text{ as:}$$

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{\eta_{n(p)}}\right)^{1/2},$$

which is thus proportional to E_k^2 .

Then, for $E \geq 0$, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E_{Fn(Fp)}^2$, and

$G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N, r_{d(a)}, x, T)$ for a presentation simplicity. Therefore, one obtains (Van Cong, 2025):

$$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi \times \hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*)} \right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)} \right)^2 \right] \left(\frac{1}{\text{ohm} \times \text{cm}} \right),$$

$$\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5} \text{ ohm}^{-1}, A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}, R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}}, \quad (14)$$

which can be used to define the resistivity as: $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$. This $\sigma(N, r_{d(a)}, x, T)$ -result is an essential one in this paper, being used to determine other electrical-and-thermoelectric properties.

In Eq. (14), one notes that at $T = 0 \text{ K}$, $\sigma(N, r_{d(a)}, x, T = 0 \text{ K})$ is proportional to $E_{Fno(Fpo)}^2$, or to $(N^*)^{\frac{4}{3}}$. Thus, $\sigma(N = N_{CDn(NDp)}, r_{d(a)}, x, T = 0 \text{ K}) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by (Van Cong, 2025):

$$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_{c(v)}(x) \times m_0}{q^2 \times N^*}. \text{ Therefore, the mobility } \mu \text{ is given by:}$$

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N, r_{d(a)}, x, T)}{m_{c(v)}(x) \times m_0} = \frac{\sigma(N, r_{d(a)}, x, T)}{q \times N^*} \left(\frac{\text{cm}^2}{\text{V} \times \text{s}} \right). \quad (15)$$

Here, at $T = 0 \text{ K}$, $\mu(N^*, r_{d(a)}, T)$ is thus proportional to $(N^*)^{1/3}$, since $\sigma(N^*, r_{d(a)}, T = 0 \text{ K})$ is proportional to $(N^*)^{4/3}$. Thus, $\mu(N^* = 0, r_{d(a)}, T = 0 \text{ K}) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Then, since τ and σ are both proportional to $E_{Fn(Fp)}(N^*, T)^2$, as given above, the Hall factor is defined by:

$$r_H(N, r_{d(a)}, x, T) \equiv \frac{\langle \tau^2 \rangle_{FDDF}}{[\langle \tau \rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}, y \equiv \frac{\pi}{\xi_{n(p)}(N, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}, \text{ and therefore,}$$

$$\text{the Hall mobility yields:}$$

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T) \times r_H(N^*, T) \left(\frac{\text{cm}^2}{\text{V} \times \text{s}} \right), \quad (16)$$

noting that, at $T = 0 \text{ K}$, since $r_H(N, r_{d(a)}, x, T) = 1$, one then gets:

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T).$$

Our generalized Einstein relation

Our generalized Einstein relation is found to be defined as:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn}(F_p)}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}$$

where $D(N, r_{d(a)}, x, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility $\mu(N, r_{d(a)}, x, T)$ is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u , one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} = \frac{k_B \times T}{q} \times u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)},$$

where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}} e^{-du}(1 - du) + \frac{2}{3} Au^{B-1} F(u) \left[\left(1 + \frac{3B}{2} \right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}} \right]$.

One remarks that: (i) as $u \rightarrow 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and

therefore: $\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}$, and (ii) as $u \rightarrow \infty$, one has: $W^2 \approx A^2 u^{2B}$ and

$u[V' \times W - V \times W'] \approx \frac{2}{3} au^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at

$T=0K$, the **above generalized Einstein relation** is reduced to the **usual Einstein one**:

$\frac{D(N, r_{d(a)}, x, T=0 K)}{\mu(N, r_{d(a)}, x, T=0 K)} \approx \frac{2}{3} E_{Fno}(F_{po}) (N^*)/q$. In other words, **Eq. (17) verifies the correct limiting conditions**.

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (17) gives:

$$\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \simeq \frac{2}{3} \times \frac{E_{Fno}(F_{po})(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}} \right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)} \right], \quad (18)$$

where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2$ and $c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4$.

In Tables 3n(3p) given in Appendix 1, for given x , $N > N_{CDn}$ and $T(=4.2 K \text{ and } 77 K)$, and from Equations (14, 15, 16, 17), the numerical results of the coefficients: σ , μ , μ_H and D are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First of all, from Eq. (14), obtained for $\sigma(N, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S , is found to be given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q > 0} \times k_B T \times \left. \frac{\partial \ln \sigma(E)}{\partial E} \right|_{E=E_{Fn}(Fp)} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln \sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$F_S(N, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3 \times G_2 \left(y = \frac{\pi}{\xi_{n(p)}} \right)} \right],$$

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N^*, T)}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{s \times \xi_{n(p)}^2}{\pi^2} \right)} =$$

$$-2\sqrt{L} \times \frac{\sqrt{(ZT)_{Mott}}}{1 + (ZT)_{Mott}} \left(\frac{V}{K} \right) < 0, \quad (ZT)_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}, \quad (19)$$

according to:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{s \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{s \times \xi_{n(p)}^2}{\pi^2} \right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{(ZT)_{Mott} \times [1 - (ZT)_{Mott}]}{[1 + (ZT)_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \rightarrow +\infty$ or $\xi_{n(p)} \rightarrow +0$, one has a same limiting value of S:

$S \rightarrow -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum

$(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K} \right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains:

$S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K} \right)$.

Further, the figure of merit, ZT, is found to be defined by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{s^2 \times \sigma \times T}{\kappa} = \frac{s^2}{L} = \frac{4 \times (ZT)_{Mott}}{[1 + (ZT)_{Mott}]^2}. \quad (20)$$

Here, one notes that: (i) $\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{s}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since

$\frac{\partial (ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{max.} = 1$, and $(ZT)_{Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one

obtains: $ZT \simeq 0.715$ and $(ZT)_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient, VC1, can be defined by:

$$VC1(N, r_{d(a)}, x, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K} \right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (21)$$

and the second Van-Cong coefficient, VC2, as:

$$VC2(N, r_{d(a)}, x, T) \equiv T \times VC1(V), \quad (22)$$

the Thomson coefficient, Ts, by:

$$Ts(N, r_{d(a)}, x, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K} \right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

and the Peltier coefficient, Pt, as:

$$Pt(N, r_{d(a)}, x, T) \equiv T \times S(V). \quad (24)$$

One notes here that in next Tables 5n(p) and 6n(p) given in Appendix 1, obtained with such given physical conditions N(or T) for the decreasing $\xi_{n(p)}$, since $VC1(N, r_{d(a)}, x, T)$ and

$Ts(N, r_{d(a)}, x, T)$ are expressed in terms of $\frac{-dS}{dN^*}$ and $\frac{dS}{dT}$, one has: $[VC1, Ts] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$,

$[VC1, Ts] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[VC1, Ts] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating also that for

$$\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}:$$

(i) S, determined in Eq. (19), thus presents a same minimum

$$(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K} \right),$$

(ii) ZT, determined in Eq. (20), therefore presents a same maximum: $(ZT)_{\max.} = 1$, since the variations of ZT are expressed in terms of $[VC1, Ts] \times S$, $S < 0$.

Furthermore, it is interesting to remark that the (VC2)-coefficient is related to our generalized Einstein relation (17) by:

$$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \quad (25)$$

according, in this work, with the use of our Eq. (21), to:

$$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{\text{Mott}} \times [1 - (ZT)_{\text{Mott}}]}{[1 + (ZT)_{\text{Mott}}]^2} (V).$$

Of course, our relation (25) is reduced to: $\frac{D}{\mu}$, VC1 and VC2, being determined respectively by Equations (17, 21, 22).

Now, in the degenerate n(p)-type $X(x)$ – alloy, and for $N > N_{CDn(CDp)}$, and for $T=3K$ (80K), the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1, noting that their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T , and in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and various thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘).

CONCLUDING REMARKS

Here, some concluding remarks can be given as follows.

(1) In the $n^+(p^+) - p(n) X(x) \equiv GaAs_{1-x}Te_x$ - crystalline alloy, $0 \leq x \leq 1$, the electrical-and-thermoelectric laws, relations, and various coefficients are found to be enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, being, for a given x , decreased with increasing $r_{d(a)}$, as given in Equations (1a, 1b) and also in Table 2 of our recent work (2024), by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), and in particular by our electrical conductivity model given in Eq. (14).

(2) The generalized Mott criterium in the MIT is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail, $N_{CDn(CDp)}^{EBT}$, obtained with a precision of the order of 2.89×10^{-7} , as given in our previous work (2024), and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, as that observed in the compensated crystals.

(3) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid for any density N^* .

(4) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T , and then in Tables 6n(6p) given Appendix 1, for a given T and with

decreasing N , the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). One remarks in these Tables that, for any given x , $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a **same minimum** $(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, those of the figure of merit ZT show a **same maximum** $(ZT)_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S , ZT , the Mott figure of merit $(ZT)_{\text{Mott}}$, the Van-Cong coefficient $VC1$, and the Thomson coefficient Ts , present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$. It seems that these same results could represent a **new law given for the thermoelectric properties, obtained in the degenerate case**.

(5) Finally, our electrical-and-thermoelectric relation is given in Eq. (25) by:

$$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \left(\frac{V^2}{K}\right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}}, \quad \text{according, in this}$$

work, to:

$$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N, r_{d(a)}, x, T)}{\mu(N, r_{d(a)}, x, T)} \times 2 \times \frac{(ZT)_{\text{Mott}} \times [1 - (ZT)_{\text{Mott}}]}{[1 + (ZT)_{\text{Mott}}]^2} (V), \quad \text{being reduced to: } \frac{D}{\mu},$$

$VC1$ and $VC2$, determined respectively in Equations (17, 21, 22). This should be a **new result**.

REFERENCES

1. Van Cong, H. Electrical-and-Thermoelectric Laws, Relations, and Coefficients in n(p)-type Degenerate GaP(1-x)As(x)-Crystalline Alloy, Enhanced by Our Static Dielectric Constant Law and Electrical Conductivity (VI). WJERT, 2025; 11(3): 184-215.
2. Van Cong, H. New Critical Impurity Density in Metal-Insulator Transition, obtained in various n(p)-Type Degenerate Crystalline Alloys, being just That of Carriers Localized in Exponential Band Tails. WJERT, 2024; 10(4): 05-23.
3. Van Cong, H. New critical impurity density in Metal-Insulator Transition, obtained in n(p)-type Degenerate $\text{InP}_{1-x}\text{As}_x(\text{Sb}_x)$, $\text{GaAs}_{1-x}\text{Te}_x(\text{Sb}_x, \text{P}_x)$, $\text{CdS}_{1-x}\text{Te}_x(\text{Se}_x)$ – crystalline alloys (II). EJASET, 2024; 2(2): 186-211.

4. Van Cong, H. Effects of donor size and heavy doping on optical, electrical and thermoelectric properties of various degenerate donor-silicon systems at low temperatures. American Journal of Modern Physics, 2018; 7(4): 136-165.
5. Van Cong, H. et al. Size effect on different impurity levels in semiconductors. Solid State Communications, 1984; 49: 697-699.
6. Van Cong, H. Diffusion coefficient in degenerate semiconductors. Phys. Stat. Sol. (b), 1984; 101: K27.
7. Van Cong, H. Same Maximal Figure of Merit $ZT(=1)$, Due to the Effect of Impurity Size, Obtained in the n(p)-Type Degenerate GaAs-Crystal ($\xi_{n(p)} \geq 1$), at Same Reduced Fermi Energy $\xi_{n(p)} (= 1.8138)$ and Same Minimum Seebeck Coefficient $S \left(= -1.563 \times 10^{-4} \frac{V}{K} \right)$, at which Same $(ZT)_{\text{Mott}} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1 \right)$. SCIREA Journal of Physics, 2023; 8(2): 133-156.
8. Kim, H. S. et al. Characterization of Lorenz number with Seebeck coefficient measurement. APL Materials, 2015; 3(4): 041506.
9. Hyun, B. D. et al. Electrical-and-Thermoelectric Properties of $90\%Bi_2Te_3 - 5\%Sb_2Te_3 - 5\%Sb_2Se_3$ Single Crystals Doped with SbI_3 . Scripta Materialia, 1998; 40(1): 49-56.
10. Van Cong, H. and Debais, G. A simple accurate expression of the reduced Fermi energy for any reduced carrier density. J. Appl. Phys., 1993; 73: 1545-1546.
11. Van Cong, H. and Doan Khanh, B. Simple accurate general expression of the Fermi-Dirac integral $F_j(a)$ and for $j > -1$. Solid-State Electron., 1992; 35(7): 949-951.
12. Van Cong, H. New series representation of Fermi-Dirac integral $F_j(-\infty < a < \infty)$ for arbitrary $j > -1$, and its effect on $F_j(a \geq 0_+)$ for integer $j \geq 0$. Solid-State Electron., 1991; 34(5): 489-492.

APPENDIX 1: Tables

Table 1: The values of energy-band-structure parameters are given in the following.

In the $\mathbf{X(x) \equiv GaAs_{1-x}Te_x}$ -crystalline alloy, in which $r_{do(ao)} = r_{As(Ga)} = 0.118$ nm (0.126 nm), we have:
 $\epsilon_{c(v)}(x) = 1 \times x + 1 \times (1 - x)$, $m_{c(v)}(x)/m_0 = 0.209$ (0.4) $\times x + 0.066$ (0.291) $\times (1 - x)$, $\epsilon_0(x) = 12.3 \times x + 13.13 \times (1 - x)$,
 $E_{go}(x) = 1.796 \times x + 1.52 \times (1 - x)$.

Table 2: Expressions for $G_{p \geq 1}(y \equiv \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$(1 + \frac{y^2}{8} + \frac{7y^4}{640})$	$(1 + \frac{y^2}{3})$	$(1 + \frac{5y^2}{8} - \frac{7y^4}{384})$	$(1 + y^2)$	$(1 + \frac{35y^2}{24} + \frac{49y^4}{384})$	$(1 + 2y^2 + \frac{7y^4}{15})$	$(1 + \frac{21y^2}{8} + \frac{147y^4}{128})$

Table 3n: Here, one notes that, for given x , $N > N_{CDn}$ and $T(=4.2$ K and 77 K), the functions: σ, μ, μ_H, D , expressed respectively in $(\frac{10^3}{ohm \times cm}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^2 \times cm^2}{s})$, decrease with increasing r_d .

Donor	P	As	Sb	Sn
r_d (nm)	0.110	0.118	0.136	0.140

For $x=0$, the values of (σ, μ, μ_H, D) at **4.2K**

$N (10^{18} cm^{-3})$					
3	1.23, 2.566, 2.566, 1.96	1.19, 2.499, 2.499, 1.91	1.05, 2.189, 2.190, 1.67	0.98, 2.062, 2.063, 1.57	
10	3.10, 1.936, 1.936, 3.30	3.01, 1.884, 1.884, 3.22	2.64, 1.648, 1.648, 2.81	2.48, 1.553, 1.553, 2.65	
40	9.34, 1.458, 1.458, 6.28	9.07, 1.415, 1.415, 6.09	7.83, 1.223, 1.223, 5.27	7.34, 1.147, 1.147, 4.94	
70	14.8, 1.324, 1.324, 8.28	14.4, 1.284, 1.284, 8.03	12.4, 1.103, 1.013, 6.90	11.6, 1.031, 1.031, 6.45	

For $x=0.5$, the values of (σ, μ, μ_H, D) at **4.2K**

$N (10^{18} cm^{-3})$					
3	0.72, 1.565, 1.566, 0.56	0.70, 1.525, 1.526, 0.54	0.60, 1.336, 1.337, 0.47	0.56, 1.258, 1.259, 0.44	
10	1.85, 1.171, 1.172, 0.95	1.80, 1.141, 1.141, 0.93	1.57, 1.001, 1.001, 0.81	1.48, 0.944, 0.945, 0.76	
40	5.52, 0.865, 0.865, 1.78	5.37, 0.840, 0.840, 1.73	4.65, 0.730, 0.730, 1.50	4.37, 0.686, 0.686, 1.41	
70	8.71, 0.778, 0.778, 2.33	8.46, 0.756, 0.756, 2.26	7.30, 0.652, 0.652, 1.95	6.83, 0.611, 0.611, 1.83	

For $x=1$, the values of (σ, μ, μ_H, D) at **4.2K**

$N (10^{18} cm^{-3})$					
3	0.46, 1.145, 1.147, 0.25	0.44, 1.117, 1.119, 0.24	0.36, 0.984, 0.986, 0.20	0.33, 0.930, 0.932, 0.18	
10	1.29, 0.845, 0.845, 0.44	1.25, 0.823, 0.824, 0.43	1.08, 0.725, 0.725, 0.37	1.01, 0.685, 0.685, 0.35	
40	3.87, 0.611, 0.611, 0.82	3.76, 0.594, 0.594, 0.80	3.26, 0.519, 0.519, 0.70	3.07, 0.488, 0.488, 0.65	
70	6.07, 0.545, 0.545, 1.07	5.90, 0.530, 0.530, 1.04	5.10, 0.460, 0.460, 0.90	4.78, 0.432, 0.432, 0.84	

For $x=0$, the values of (σ, μ, μ_H, D) at **77 K**

$N (10^{18} cm^{-3})$					
3	1.25, 2.615, 2.728, 1.99	1.22, 2.547, 2.657, 1.94	1.07, 2.232, 2.329, 1.69	1.00, 2.102, 2.193, 1.60	
10	3.11, 1.943, 1.960, 3.31	3.03, 1.891, 1.908, 3.23	2.65, 1.655, 1.669, 2.82	2.49, 1.559, 1.573, 2.66	
40	9.35, 1.459, 1.461, 6.28	9.07, 1.416, 1.418, 6.10	7.84, 1.224, 1.225, 5.27	7.35, 1.147, 1.149, 4.94	
70	14.8, 1.324, 1.325, 8.28	14.4, 1.284, 1.285, 8.03	12.4, 1.103, 1.014, 6.90	11.6, 1.031, 1.032, 6.45	

For $x=0.5$, the values of (σ, μ, μ_H, D) at **77 K**

$N (10^{18} cm^{-3})$					
3	0.78, 1.705, 2.018, 0.60	0.76, 1.662, 1.967, 0.58	0.66, 1.459, 1.732, 0.50	0.61, 1.375, 1.636, 0.47	
10	1.88, 1.191, 1.237, 0.96	1.83, 1.161, 1.205, 0.94	1.60, 1.018, 1.058, 0.82	1.51, 0.961, 0.998, 0.77	
40	5.54, 0.867, 0.872, 1.79	5.38, 0.843, 0.848, 1.74	4.67, 0.732, 0.736, 1.51	4.38, 0.687, 0.692, 1.41	
70	8.73, 0.779, 0.782, 2.34	8.47, 0.756, 0.759, 2.27	7.30, 0.653, 0.655, 1.96	6.84, 0.612, 0.614, 1.83	

For $x=1$, the values of (σ, μ, μ_H, D) at 77 K

$N (10^{19} \text{ cm}^{-3})$				
3	0.57, 1.422, 2.045, 0.30	0.55, 1.390, 2.007, 0.29	0.46, 1.245, 1.847, 0.25	0.42, 1.188, 1.791, 0.24
10	1.34, 0.880, 0.959, 0.45	1.30, 0.858, 0.935, 0.44	1.13, 0.756, 0.826, 0.38	1.05, 0.715, 0.782, 0.36
40	3.89, 0.615, 0.624, 0.83	3.78, 0.598, 0.607, 0.80	3.29, 0.522, 0.529, 0.70	3.08, 0.491, 0.498, 0.66
70	6.09, 0.547, 0.551, 1.07	5.92, 0.531, 0.535, 1.04	5.12, 0.461, 0.464, 0.90	4.80, 0.433, 0.436, 0.85

Table 3p: Here, one notes that, for given x , $N > N_{CDP}$ and $T(=4.2 \text{ K and } 77 \text{ K})$, the functions: σ, μ, μ_H, D , expressed respectively in $\left(\frac{10^3}{\text{ohm}\times\text{cm}}, \frac{10^2 \times \text{cm}^2}{V \times s}, \frac{10^2 \times \text{cm}^2}{V \times s}, \frac{10 \times \text{cm}^2}{s}\right)$, decrease with increasing r_a .

Acceptor	Ga	Mg	In	Cd
r_a (nm)	0.126	0.140	0.144	0.148

For $x=0$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	4.50, 9.743, 9.744, 7.65	4.12, 8.973, 8.974, 7.02	3.89, 8.531, 8.532, 6.65	3.65, 8.040, 8.040, 6.24
5	7.03, 8.982, 8.982, 10.0	6.43, 8.244, 8.245, 9.18	6.08, 7.821, 7.821, 8.69	5.69, 7.351, 7.351, 8.14
8	10.6, 8.404, 8.404, 12.9	9.69, 7.693, 7.693, 11.8	9.16, 7.285, 7.286, 11.1	8.57, 6.833, 6.833, 10.4
10	12.9, 8.162, 8.162, 14.6	11.8, 7.464, 7.464, 13.3	11.1, 7.063, 7.063, 12.6	10.4, 6.618, 6.618, 11.8

For $x=0.5$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	3.24, 7.250, 7.251, 4.69	2.96, 6.711, 6.712, 4.30	2.79, 6.402, 6.403, 4.08	2.61, 6.058, 6.059, 3.82
5	5.06, 6.594, 6.595, 6.12	4.63, 6.078, 6.078, 5.61	4.37, 5.781, 5.782, 5.31	4.09, 5.451, 5.452, 4.98
8	7.62, 6.105, 6.105, 7.83	6.96, 5.609, 5.609, 7.17	6.59, 5.324, 5.324, 6.79	6.17, 5.007, 5.007, 6.37
10	9.26, 5.903, 5.903, 8.82	8.46, 5.416, 5.416, 8.07	8.00, 5.136, 5.136, 7.64	7.49, 4.825, 4.825, 7.16

For $x=1$, the values of (σ, μ, μ_H, D) at 4.2K

$N (10^{19} \text{ cm}^{-3})$				
3	2.40, 5.682, 5.684, 3.06	2.18, 5.291, 5.292, 2.80	2.05, 5.067, 5.069, 2.65	1.90, 4.820, 4.821, 2.48
5	3.78, 5.090, 5.091, 3.99	3.45, 4.713, 4.713, 3.66	3.26, 4.496, 4.497, 3.47	3.04, 4.256, 4.257, 3.25
8	5.70, 4.659, 4.660, 5.10	5.21, 4.297, 4.297, 4.67	4.93, 4.089, 4.089, 4.43	4.61, 3.857, 3.857, 4.15
10	6.92, 4.484, 4.484, 5.73	6.33, 4.128, 4.029, 5.24	5.99, 3.924, 3.924, 4.97	5.61, 3.697, 3.697, 4.66

For $x=0$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	4.59, 9.921, 10.33, 7.76	4.19, 9.139, 9.516, 7.12	3.97, 8.689, 9.050, 6.74	3.71, 8.191, 8.534, 6.33
5	7.09, 9.063, 9.248, 10.1	6.48, 8.319, 8.490, 9.24	6.13, 7.892, 8.055, 8.75	5.74, 7.418, 7.572, 8.20
8	10.7, 8.444, 8.535, 12.9	9.74, 7.730, 7.814, 11.8	9.21, 7.320, 7.400, 11.2	8.61, 6.866, 6.941, 10.5
10	13.0, 8.191, 8.257, 14.6	11.8, 7.490, 7.550, 13.3	11.2, 7.088, 7.145, 12.6	10.5, 6.642, 6.695, 11.8

For $x=0.5$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	3.33, 7.446, 7.890, 4.79	3.04, 6.895, 7.315, 4.39	2.87, 6.580, 6.985, 4.16	2.68, 6.230, 6.620, 3.91
5	5.13, 6.680, 6.877, 6.18	4.69, 6.158, 6.341, 5.67	4.43, 5.858, 6.034, 5.37	4.15, 5.525, 5.692, 5.04
8	7.67, 6.147, 6.242, 7.87	7.01, 5.647, 5.736, 7.21	6.63, 5.360, 5.445, 6.83	6.21, 5.042, 5.121, 6.40
10	9.30, 5.933, 6.001, 8.85	8.50, 5.443, 5.506, 8.10	8.04, 5.162, 5.222, 7.67	7.53, 4.850, 4.906, 7.19

For $x=1$, the values of (σ, μ, μ_H, D) at 77K

$N (10^{19} \text{ cm}^{-3})$				
3	2.50, 5.904, 6.406, 3.15	2.27, 5.505, 5.988, 2.89	2.14, 5.277, 5.781, 2.73	1.98, 5.026, 5.492, 2.56
5	3.85, 5.183, 5.396, 4.05	3.52, 4.801, 5.001, 3.71	3.32, 4.581, 4.775, 3.52	3.10, 4.338, 4.525, 3.30
8	5.76, 4.703, 4.803, 5.13	5.26, 4.338, 4.431, 4.71	4.98, 4.128, 4.217, 4.46	4.66, 3.895, 3.980, 4.19
10	6.97, 4.515, 4.586, 5.76	6.38, 4.157, 4.223, 5.28	6.03, 3.952, 4.014, 5.00	5.64, 3.723, 3.783, 4.69

Table 4n: In the lightly degenerate n-type $X(x)$ – alloy, and for $T=3K$ and $80K$, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor	P	As	Sb	Sn
For $x=0$ and $N=5 \times 10^{17} \text{ cm}^{-3}$,				
$\xi_n(T=3K)$ ↘	132.351	132.207	131.342	130.858
$\xi_n(T=80K)$ ↘	5.206	5.200	5.168	5.151
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$ ↘	2.165	2.098	1.791	1.665
$\kappa_{(T=80K)} \left(\frac{10^{-4} \times W}{\text{cm} \times K} \right)$ ↘	7.120	6.905	5.906	5.496
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.283	4.288	4.316	4.332
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	9.713	9.720	9.767	9.794
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	2.854	2.857	2.876	2.886
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.748	4.750	4.765	4.773
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	8.562	8.571	8.628	8.659
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right)$ ↘	3.798	3.800	3.812	3.818
$-Ts_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	4.281	4.286	4.314	4.330
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	7.122	7.126	7.147	7.159
$-Pt_{(T=3K)} (10^{-5} \times V)$ ↘	1.285	1.286	1.295	1.299
$-Pt_{(T=80K)} (10^{-3} \times V)$ ↘	7.770	7.776	7.814	7.835
$ZT_{(T=3K)} (10^{-4})$ ↗	7.510	7.526	7.625	7.682
$ZT_{(T=80K)} (10^{-3})$ ↗	3.861	3.868	3.905	3.926
For $x=0.5$ and $N=2 \times 10^{18} \text{ cm}^{-3}$,				
$\xi_n(T=3K)$ ↘	155.964	155.526	152.879	151.390
$\xi_n(T=80K)$ ↘	6.066	6.050	5.954	5.900
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$ ↘	3.790	3.675	3.129	2.900
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{\text{cm} \times K} \right)$ ↘	1.184	1.149	0.983	0.914
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	3.635	3.645	3.708	3.745
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	8.580	8.599	8.714	8.780
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	2.422	2.429	2.471	2.495
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.460	4.464	4.493	4.509
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	7.267	7.287	7.414	7.486
$-VC2_{(T=80K)} \left(\frac{10^{-3} \times V}{K} \right)$ ↘	3.568	3.571	3.594	3.607
$-Ts_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	3.633	3.644	3.707	3.743
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	6.689	6.696	6.739	6.763
$-Pt_{(T=3K)} (10^{-5} \times V)$ ↘	1.090	1.093	1.112	1.123
$-Pt_{(T=80K)} (10^{-3} \times V)$ ↘	6.864	6.879	6.971	7.024
$ZT_{(T=3K)} (10^{-4})$ ↗	5.408	5.439	5.629	5.740
$ZT_{(T=80K)} (10^{-3})$ ↗	3.014	3.027	3.109	3.156
For $x=1$ and $N=7 \times 10^{18} \text{ cm}^{-3}$,				
$\xi_n(T=3K)$ ↘	235.387	234.650	230.186	227.673
$\xi_n(T=80K)$ ↘	8.970	8.943	8.779	8.686

$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	7.068	6.859	5.873	5.458
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	2.026	1.967	1.689	1.572
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.409	2.416	2.463	2.490
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	6.072	6.089	6.194	6.255
$-VC1_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	1.605	1.610	1.642	1.660
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	3.608	3.616	3.662	3.688
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	4.816	4.831	4.925	4.979
$-VC2_{(T=80K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	2.886	2.892	2.929	2.950
$-Ts_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.408	2.416	2.463	2.490
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	5.412	5.424	5.492	5.531
$-Pt_{(T=3K)} (10^{-6} \times V)$ ↘	7.226	7.248	7.389	7.471
$-Pt_{(T=80K)} (10^{-6} \times V)$ ↘	4.858	4.871	4.955	5.004
$ZT_{(T=3K)} (10^{-4})$ ↗	2.375	2.390	2.483	2.538
$ZT_{(T=80K)} (10^{-4})$ ↗	1.509	1.518	1.570	1.601

Table 4p: In the lightly degenerate p-type $X(x)$ – alloy, in which $N=2 \times 10^{19} \text{ cm}^{-3}$, and for $T=3K$ and $80K$, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Acceptor	Ga	Mg	In	Cd
For x=0,				
$\xi_n(T=3K)$ ↘	343.331	340.811	339.015	336.619
$\xi_n(T=80K)$ ↘	12.972	12.878	12.811	12.722
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	23.190	21.191	20.030	18.726
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	6.400	5.851	5.533	5.175
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	1.651	1.664	1.672	1.684
$-S_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.287	4.317	4.339	4.368
$-VC1_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	1.101	1.109	1.115	1.223
$-VC1_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.707	2.723	2.735	2.752
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	3.302	3.327	3.344	3.368
$-VC2_{(T=80K)} \left(\frac{10^{-6} \times V}{K} \right)$ ↘	2.165	2.179	2.188	2.201
$-Ts_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	1.651	1.663	1.672	1.684
$-Ts_{(T=80K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	4.060	4.085	4.103	4.128
$-Pt_{(T=3K)} (10^{-5} \times V)$ ↘	0.495	0.499	0.502	0.505
$-Pt_{(T=80K)} (10^{-5} \times V)$ ↘	3.430	3.454	3.471	3.494
$ZT_{(T=3K)} (10^{-4})$ ↗	1.116	1.133	1.145	1.161
$ZT_{(T=80K)} (10^{-4})$ ↗	0.752	0.763	0.770	0.781
For x=0.5,				
$\xi_n(T=3K)$ ↘	279.243	275.256	272.405	268.591
$\xi_n(T=80K)$ ↘	10.592	10.444	10.338	10.197
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	16.589	15.092	14.214	13.218
$\kappa_{(T=80K)} \left(\frac{10^{-5} \times W}{cm \times K} \right)$ ↘	4.658	4.244	4.002	3.727
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right)$ ↘	2.030	2.060	2.081	2.111

$-S_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	5.201	5.270	5.320	5.390
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	1.353	1.373	1.387	1.407
$-VC1_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	3.194	3.229	3.254	3.288
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	4.060	4.119	4.162	4.221
$-VC2_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	2.555	2.583	2.603	2.631
$-T_s_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	2.030	2.059	2.081	2.111
$-T_s_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	4.791	4.843	4.881	4.932
$-Pt_{(T=3K)} (10^{-5} \times V) \searrow$	0.609	0.618	0.624	0.633
$-Pt_{(T=30K)} (10^{-3} \times V) \searrow$	4.160	4.216	4.256	4.312
$ZT_{(T=3K)} (10^{-4}) \nearrow$	1.687	1.737	1.773	1.824
$ZT_{(T=30K)} (10^{-1}) \nearrow$	1.107	1.137	1.159	1.189
For x=1,				
$\xi_n(T=3K) \searrow$	227.487	221.389	217.004	211.105
$\xi_n(T=30K) \searrow$	8.679	8.455	8.294	8.077
$\kappa_{(T=3K)} \left(\frac{10^{-5} \times W}{cm \times K} \right) \searrow$	12.095	10.879	10.154	9.320
$\kappa_{(T=30K)} \left(\frac{10^{-5} \times W}{cm \times K} \right) \searrow$	3.484	3.147	2.947	2.717
$-S_{(T=3K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	2.492	2.561	2.613	2.686
$-S_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	6.259	6.411	6.524	6.683
$-VC1_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	1.661	1.707	1.741	1.790
$-VC1_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	3.690	3.754	3.800	3.864
$-VC2_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	4.984	5.121	5.224	5.370
$-VC2_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	2.952	3.003	3.040	3.091
$-T_s_{(T=3K)} \left(\frac{10^{-6} \times V}{K} \right) \searrow$	2.492	2.560	2.612	2.685
$-T_s_{(T=30K)} \left(\frac{10^{-5} \times V}{K} \right) \searrow$	5.534	5.630	5.700	5.795
$-Pt_{(T=3K)} (10^{-5} \times V) \searrow$	0.748	0.768	0.784	0.806
$-Pt_{(T=30K)} (10^{-3} \times V) \searrow$	5.007	5.129	5.219	5.346
$ZT_{(T=3K)} (10^{-4}) \nearrow$	2.542	2.684	2.794	2.952
$ZT_{(T=30K)} (10^{-1}) \nearrow$	1.604	1.682	1.742	1.828

Table 5n: Here, for a given N and with increasing T , the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: $\nearrow$, decrease: $\searrow$). One notes here that with increasing T : (i) for $\xi_n \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{\min.} (\approx -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $(ZT)_{\max.} = 1$, (ii) for $\xi_n = 1$, those of S , ZT , $(ZT)_{\text{Mott}}$, $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_n \approx 1.8138$, $(ZT)_{\text{Mott}} = 1$.

For x=0,

In the degenerate P- $X(x)$ - alloy, for $N = 2 \times N_{\text{Cn}}(r_p) = 2.507675 \times 10^{18} \text{ cm}^{-3}$, one gets:

$T(K) \nearrow$	15.263	15.5943916	15.929	21.2202591	21.242
$\xi_n \searrow$	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K} \right) \searrow$	-1.562	-1.563	-1.562	-1.322	-1.320
$ZT \nearrow$	0.999	1	0.999	0.715	0.713
$(ZT)_{\text{Mott}} \nearrow$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K} \right) \nearrow$	-0.061	0	0.063	1.105	1.109

$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.933 ↗	0 ↗	0.999 ↗	23.447 ↗	23.558
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-2.384 ↘	-2.437 ↘	-2.488 ↘	-2.8047 ↗	-2.8039

In the degenerate As- $X(x)$ – alloy, for $N = 2 \times N_{CDn} (r_{As}) = 2.6660176 \times 10^{18} \text{ cm}^{-2}$, one gets:

$T(K)$ ↗	15.8977	16.244123	16.593	22.1043894	22.127
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.975 ↗	0 ↗	1.042 ↗	24.424 ↗	24.540
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-2.483 ↘	-2.539 ↘	-2.592 ↘	-2.9215 ↗	-2.9207

In the degenerate Sb- $X(x)$ – alloy, for $N = 2 \times N_{CDn} (r_{Sb}) = 3.619757 \times 10^{18} \text{ cm}^{-2}$, one gets:

$T(K)$ ↗	19.493	19.917762	20.346	27.103339	27.13
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.195 ↗	0 ↗	1.279 ↗	29.948 ↗	30.084
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-3.045 ↘	-3.113 ↘	-3.178 ↘	-3.5822 ↗	-3.5813

In the degenerate Sn- $X(x)$ – alloy, for $N = 2 \times N_{CDn} (r_{Sn}) = 4.152416 \times 10^{18} \text{ cm}^{-2}$, one gets:

$T(K)$ ↗	21.3612	21.8267041	22.296	29.700956	29.731
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.310 ↗	0 ↗	1.402 ↗	32.818 ↗	32.971
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
$Pt (10^{-3}V)$	-3.337 ↘	-3.411 ↘	-3.483 ↘	-3.9256 ↗	-3.9246

For $x=0.5$,

In the degenerate P- $X(x)$ – alloy, for $N = 2 \times N_{CDn} (r_p) = 2.4968508 \times 10^{17} \text{ cm}^{-2}$, one gets:

$T(K)$ ↗	33.905	34.6436699	35.389	47.1418	47.19
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$ ↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.079 ↗	0 ↗	2.226 ↗	52.089 ↗	52.336

$T_s \left(10^{-4} \frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)	−5.296	↘	−5.415	↘	−5.528	↘	−6.2307	↗	−6.2291

In the degenerate As- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 2.65451 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K)	↗	35.318	36.08708		36.86		49.105937		49.156
ξ_n	↘	1.880	1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$		−1.562	↘ −1.563	↗	−1.562	↗	− 1.322	↗	−1.320
ZT		0.999	↗ 1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	↗	0.931	1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$		−0.061	↗ 0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$		−2.165	↗ 0	↗	2.308	↗	54.259	↗	54.515
$T_s \left(10^{-4} \frac{V}{K}\right)$		−0.092	↗ 0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)		−5.517	↘ −5.640	↘	−5.757	↘	−6.4903	↗	−6.4886

In the degenerate Sb- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 3.6041328 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K)	↗	43.3045	44.248242		45.2001		60.21134		60.273
ξ_n	↘	1.880	1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$		−1.562	↘ −1.563	↗	−1.562	↗	− 1.322	↗	−1.320
ZT		0.999	↗ 1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	↗	0.931	1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$		−0.061	↗ 0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$		−2.656	↗ 0	↗	2.843	↗	66.530	↗	66.845
$T_s \left(10^{-4} \frac{V}{K}\right)$		−0.092	↗ 0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)		−6.764	↘ −6.916	↘	−7.060	↘	−7.9581	↗	−7.9560

In the degenerate Sn- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 4.1344928 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K)	↗	47.455	48.489048		49.53		65.982067		66.049
ξ_n	↘	1.880	1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$		−1.562	↘ −1.563	↗	−1.562	↗	− 1.322	↗	−1.320
ZT		0.999	↗ 1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	↗	0.931	1		1.074		3.290		3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$		−0.061	↗ 0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$		−2.910	↗ 0	↗	3.109	↗	72.906	↗	73.249
$T_s \left(10^{-4} \frac{V}{K}\right)$		−0.092	↗ 0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)		−7.412	↘ −7.579	↘	−7.737	↘	−8.7209	↗	−8.7186

For $x=1$,

In the degenerate P- $X(x)$ – alloy, for $N = 2 \times N_{CDn}(r_p) = 9.6862824 \times 10^{17} \text{ cm}^{-2}$, one gets:

T(K)	↗	55.072	56.271695		57.482		76.5724		76.65
ξ_n	↘	1.880	1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$		−1.562	↘ −1.563	↗	−1.562	↗	− 1.322	↗	−1.320
ZT		0.999	↗ 1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	↗	0.931	1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$		−0.061	↗ 0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$		−3.377	↗ 0	↗	3.615	↗	84.608	↗	85.005
$T_s \left(10^{-4} \frac{V}{K}\right)$		−0.092	↗ 0	↗	0.094	↗	1.657	↗	1.663

Pt (10^{-3}V) -8.602 ↘ -8.795 ↘ -8.979 ↘ -10.1206 ↗ -10.1180

In the degenerate As- $\text{X}(\text{x})$ – alloy, for $\text{N} = 2 \times \text{N}_{\text{CDn}}(\text{r}_{\text{As}}) = 1.0297905 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	57.37	58.616224	59.877	79.762746	79.844
ξ_{p}	↘	1.880	1.8138	1.750	1	0.998
$\text{S} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT		0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.306
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$\text{VC2} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-3.508	↗ 0 ↗	3.766	↗ 88.133 ↗	88.549
$\text{T}_s \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt (10^{-3}V)		-8.961	↘ -9.162 ↘	-9.353	↘ -10.5423 ↗	-10.5395

In the degenerate Sb- $\text{X}(\text{x})$ – alloy, for $\text{N} = 2 \times \text{N}_{\text{CDn}}(\text{r}_{\text{Sb}}) = 1.3981871 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	70.34	71.872394	73.418	97.801241	97.901
ξ_{p}	↘	1.880	1.8138	1.750	1	0.998
$\text{S} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT		0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.306
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$\text{VC2} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-4.313	↗ 0 ↗	4.617	↗ 108.065 ↗	108.575
$\text{T}_s \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt (10^{-3}V)		-10.987	↘ -11.234 ↘	-11.468	↘ -12.9264 ↗	-12.9230

In the degenerate Sn- $\text{X}(\text{x})$ – alloy, for $\text{N} = 2 \times \text{N}_{\text{CDn}}(\text{r}_{\text{Sn}}) = 1.603935 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	77.081	78.760733	80.45	107.174635	107.284
ξ_{p}	↘	1.880	1.8138	1.750	1	0.998
$\text{S} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320
ZT		0.999	↗ 1 ↘	0.999	↘ 0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.306
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.061	↗ 0 ↗	0.063	↗ 1.105 ↗	1.109
$\text{VC2} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-4.728	↗ 0 ↗	5.046	↗ 118.422 ↗	118.981
$\text{T}_s \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-0.092	↗ 0 ↗	0.094	↗ 1.657 ↗	1.663
Pt (10^{-3}V)		-12.040	↘ -12.310 ↘	-12.566	↘ -14.1653 ↗	-14.1616

Table 5p: Here, for a given N and with increasing T , the reduced Fermi-energy ξ_{p} decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_{\text{p}} \approx 1.8138$, while the numerical results of S present a same minimum $(\text{S})_{\text{min.}} (\approx -1.563 \times 10^{-4} \frac{\text{V}}{\text{K}})$, those of ZT show a same maximum $(\text{ZT})_{\text{max.}} = 1$, (ii) for $\xi_{\text{p}} = 1$, those of S , ZT , $(\text{ZT})_{\text{Mott}}$, VC1 , and T_s present the same results: $-1.322 \times 10^{-4} \frac{\text{V}}{\text{K}}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{\text{V}}{\text{K}}$, and $1.657 \times 10^{-4} \frac{\text{V}}{\text{K}}$, respectively, and (iii) for $\xi_{\text{p}} \approx 1.8138$, $(\text{ZT})_{\text{Mott}} = 1$.

For $\text{x}=0$,

In the degenerate Ga- $\text{X}(\text{x})$ – alloy, for $\text{N} = 2 \times \text{N}_{\text{CDn}}(\text{r}_{\text{Ga}}) = 2.285126 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	70.095	71.621813	73.16	97.46026	97.56
ξ_{p}	↘	1.880	1.8138	1.750	1	0.998
$\text{S} \left(10^{-4} \frac{\text{V}}{\text{K}} \right)$		-1.562	↘ -1.563 ↗	-1.562	↗ -1.322 ↗	-1.320

ZT	0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931 ↗	1	1.074 ↘	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.043 ↗	0 ↗	0.046 ↗	1.077 ↗	1.082
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.095 ↘	-1.119 ↘	-1.143 ↘	-1.2881 ↗	-1.2878

In the degenerate Mg- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Mg}) = 2.6994914 \times 10^{12} \text{ cm}^{-2}$, one gets

T(K)	↗	78.331	80.037437	81.759	108.911924	109.023
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$		0.931 ↗	1	1.074	3.290	3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		-0.048 ↗	0 ↗	0.051 ↗	1.203 ↗	1.209
$T_s\left(10^{-4}\frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.223 ↘	-1.251 ↘	-1.277 ↘	-1.4395 ↗	-1.4391

In the degenerate In- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{In}) = 2.9940338 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	83.93	85.758335	87.6	116.696705	116.81
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$		0.931 ↗	1	1.074 ↘	3.290	3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		-0.051 ↗	0 ↗	0.055 ↗	1.289 ↗	1.295
$T_s\left(10^{-4}\frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.311 ↘	-1.340 ↘	-1.368 ↘	-1.5424 ↗	-1.5420

In the degenerate Cd- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Cd}) = 3.3855772 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	91.1	93.080853	95.08	126.660914	126.79
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	↗	0.931	1	1.074	3.290	3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		-0.056 ↗	0 ↗	0.060 ↗	1.399 ↗	1.406
$T_s\left(10^{-4}\frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.423 ↘	-1.455 ↘	-1.485 ↘	-1.6741 ↗	-1.6736

For $x=0.5$,

In the degenerate Ga- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Ga}) = 4.2113412 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	88.743	90.67699	92.628	123.38983	123.51
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.998 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$		0.931 ↗	1	1.074 ↘	3.290	3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109

$VC2 \left(10^{-2} \frac{V}{K}\right)$	-0.054 ↗	0 ↗	0.058 ↗	1.363 ↗	1.369
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)	-1.386 ↘	-1.417 ↘	-1.447 ↘	-1.6308 ↗	-1.6304

In the degenerate Mg- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Mg}) = 4.97499 \times 10^{12} \text{ cm}^{-2}$, one gets

T(K)	↗	99.171	101.331612	103.51	137.888235	138.028
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.065 ↗	1.523 ↗	1.531
$T_s \left(10^{-4} \frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.549 ↘	-1.584 ↘	-1.617 ↘	-1.8225 ↗	-1.8220

In the degenerate In- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{In}) = 5.5178128 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	106.259	108.57457	110.91	147.74418	147.895
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	↗	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$		-0.065 ↗	0 ↗	0.070 ↗	1.632 ↗	1.640
$T_s \left(10^{-4} \frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.660 ↘	-1.697 ↘	-1.732 ↘	-1.9527 ↗	-1.9522

In the degenerate Cd- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Cd}) = 6.2394024 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	115.332	117.845266	120.38	160.35939	160.52
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$		-0.071 ↗	0 ↗	0.076 ↗	1.772 ↗	1.780
$T_s \left(10^{-4} \frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-2}V$)		-1.801 ↘	-1.842 ↘	-1.880 ↘	-2.1195 ↗	-2.1189

For $x=1$,

In the degenerate Ga- $X(x)$ – alloy, for $N = 2 \times N_{CDP}(r_{Ga}) = 7.2192156 \times 10^{12} \text{ cm}^{-2}$, one gets:

T(K)	↗	109.792	112.184161	114.597	152.655973	152.81
ξ_p	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	↗	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-2} \frac{V}{K}\right)$		-0.067 ↗	0 ↗	0.072 ↗	1.687 ↗	1.695
$T_s \left(10^{-4} \frac{V}{K}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663

Pt (10^{-2}V) -1.715 ↘ -1.753 ↘ -1.790 ↘ -2.0176 ↗ -2.0171

In the degenerate Mg- $\text{X}(\text{x})$ – alloy, for $N = 2 \times N_{\text{CDP}}(r_{\text{Mg}}) = 8.5282866 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	122.692	125.365892	128.063	170.59318	170.76
ξ_{F}	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.305
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$\text{VC2} \left(10^{-2} \frac{\text{V}}{\text{K}}\right)$		-0.075 ↗	0 ↗	0.080 ↗	1.885 ↗	1.893
$T_{\text{S}} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2}V)		-1.916 ↘	-1.959 ↘	-2.000 ↘	-2.2547 ↗	-2.2542

In the degenerate In- $\text{X}(\text{x})$ – alloy, for $N = 2 \times N_{\text{CDP}}(r_{\text{In}}) = 9.458811 \times 10^{18} \text{ cm}^{-2}$, one gets:

T(K)	↗	131.462	134.32677	137.216	182.7868	182.97
ξ_{F}	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.305
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$\text{VC2} \left(10^{-2} \frac{\text{V}}{\text{K}}\right)$		-0.081 ↗	0 ↗	0.086 ↗	2.020 ↗	2.029
$T_{\text{S}} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2}V)		-2.053 ↘	-2.099 ↘	-2.143 ↘	-2.4159 ↗	-2.4153

In the degenerate Cd- $\text{X}(\text{x})$ – alloy, for $N = 2 \times N_{\text{CDP}}(r_{\text{Cd}}) = 1.0695783 \times 10^{19} \text{ cm}^{-2}$, one gets:

T(K)	↗	142.687	145.796326	148.933	198.39414	198.59
ξ_{F}	↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT		0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(\text{ZT})_{\text{Mott}}$	↗	0.931	1	1.074	3.290	3.305
$\text{VC1} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$\text{VC2} \left(10^{-2} \frac{\text{V}}{\text{K}}\right)$		-0.087 ↗	0 ↗	0.094 ↗	2.192 ↗	2.202
$T_{\text{S}} \left(10^{-4} \frac{\text{V}}{\text{K}}\right)$		-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt (10^{-2}V)		-2.229 ↘	-2.279 ↘	-2.326 ↘	-2.6222 ↗	-2.6215

Table 6n: Here, for a given T and with decreasing N , the reduced Fermi-energy ξ_{F} decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_{\text{F}} \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{\text{min.}} (\approx -1.563 \times 10^{-4} \frac{\text{V}}{\text{K}})$, those of ZT show a same maximum $(ZT)_{\text{max.}} = 1$, (ii) for $\xi_{\text{F}} = 1$, those of S , ZT , $(ZT)_{\text{Mott}}$, VC1 , and T_{S} present the same results: $-1.322 \times 10^{-4} \frac{\text{V}}{\text{K}}$, 0.715 , 3.290 , $-1.105 \times 10^{-4} \frac{\text{V}}{\text{K}}$, and $1.657 \times 10^{-4} \frac{\text{V}}{\text{K}}$, respectively, and (iii) for $\xi_{\text{F}} \approx 1.8138$, $(ZT)_{\text{Mott}} = 1$.

For $x=0$,

In the degenerate P- $\text{X}(\text{x})$ – alloy, for $T = 15.5943916 \text{ K}$, one gets:

$N(10^{18} \text{ cm}^{-2})$	↘	2.5488	2.507675	2.4683	2.0437298	2.04253
ξ_{F}	↘	1.880	1.8138	1.750	1	0.998

$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	0.931		1		1.074		3.290		3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.955	↗	0	↗	0.980	↗	17.231	↗	17.294
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)	-2.436	↘	-2.437	↗	-2.436	↗	-2.0611	↗	-2.0585

In the degenerate As- $X(x)$ – alloy, for $T=16.244123$ K, one gets:

$N(10^{25}cm^{-3})$	↘ 2.7098		2.6660176		2.6242		2.1727774		2.1715
ξ_n	↘ 1.880		1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	0.931		1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-0.996	↗	0	↗	1.020	↗	17.949	↗	18.015
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)	-2.537	↘	-2.539	↗	-2.537	↗	-2.1470	↗	-2.1442

In the degenerate Sb- $X(x)$ – alloy, for $T=19.917762$ K, one gets:

$N(10^{25}cm^{-3})$	↘ 3.6792		3.619757		3.563		2.95006533		2.94832
ξ_n	↘ 1.880		1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	0.931		1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.221	↗	0	↗	1.250	↗	22.008	↗	22.089
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)	-3.111	↘	-3.113	↗	-3.111	↗	-2.6325	↗	-2.6291

In the degenerate Sn- $X(x)$ – alloy, for $T=21.8267041$ K, one gets:

$N(10^{25}cm^{-3})$	↘ 4.2206		4.152416		4.0873		3.384177		3.38217
ξ_n	↘ 1.880		1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mett}$	0.931		1		1.074		3.290		3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-1.338	↗	0	↗	1.370	↗	24.117	↗	24.207
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
Pt ($10^{-3}V$)	-3.409	↘	-3.411	↗	-3.409	↗	-2.8848	↗	-2.8811

For $x=0.5$,

In the degenerate P- $X(x)$ – alloy, for $T=34.6436699$ K, one gets:

$N(10^{27}cm^{-3})$	↘ 2.5378		2.4968508		2.45762		2.03490814		2.0337
ξ_n	↘ 1.880		1.8138		1.750		1		0.997
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320

ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.121 ↗	0 ↗	2.179 ↗	38.279 ↗	38.422
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-5.411 ↘	-5.415 ↗	-5.411 ↗	-4.5789 ↗	-4.5730

In the degenerate As- $X(x)$ – alloy, for T= **36.08708 K**, one gets:

$N(10^{27}cm^{-3})$ ↘	2.69813	2.65451	2.6128	2.1633988	2.16212
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.213 ↗	0 ↗	2.270 ↗	39.874 ↗	40.022
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-5.637 ↘	-5.640 ↗	-5.637 ↗	-4.7696 ↗	-4.7635

In the degenerate Sb- $X(x)$ – alloy, for T=**44.248242 K**, one gets:

$N(10^{27}cm^{-3})$ ↘	3.6633	3.6041328	3.5475	2.9373318	2.9356
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.711 ↗	0 ↗	2.784 ↗	48.892 ↗	49.072
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-6.911 ↘	-6.916 ↗	-6.911 ↗	-5.8483 ↗	-5.8408

In the degenerate Sn- $X(x)$ – alloy, for T=**48.489048 K** one gets:

$N(10^{27}cm^{-3})$ ↘	4.2024	4.1344928	4.0696	3.36956978	3.3676
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713
$(ZT)_{Mett}$	0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061 ↗	0 ↗	0.063 ↗	1.105 ↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-2.973 ↗	0 ↗	3.047 ↗	53.578 ↗	53.774
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092 ↗	0 ↗	0.094 ↗	1.657 ↗	1.663
Pt ($10^{-3}V$)	-7.574 ↘	-7.579 ↗	-7.574 ↗	-6.4088 ↗	-6.4007

For x=1,

In the degenerate P- $X(x)$ – alloy, for T=**56.271695 K**, one gets:

$N(10^{27}cm^{-3})$ ↘	9.845	9.6862824	9.5341	7.8942223	7.8896
ξ_n ↘	1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562 ↘	-1.563 ↗	-1.562 ↗	-1.322 ↗	-1.320
ZT	0.999 ↗	1 ↘	0.999 ↘	0.715 ↘	0.713

$(ZT)_{Mott}$	0.931	↗	1	↘	1.074	↗	3.290	↘	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.442	↗	0	↗	3.540	↗	62.177	↗	62.405
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt (10^{-3}V)$	-8.790	↘	-8.795	↗	-8.790	↗	-7.4374	↗	-7.4280

In the degenerate As- $X(x)$ – alloy, for $T=58.616224$ K, one gets:

$N(10^{22}cm^{-2})$	↘ 1.0467		1.0297905		1.0136		0.83926887		0.83877
ξ_n	↘ 1.880		1.8138		1.750		1		0.997
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mott}$	0.931	↗	1	↘	1.074	↗	3.290	↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-3.593	↗	0	↗	3.690	↗	64.768	↗	65.009
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt (10^{-3}V)$	-9.156	↘	-9.162	↗	-9.156	↗	-7.7473	↗	-7.7373

In the degenerate Sb- $X(x)$ – alloy, for $T=71.872394$ K, one gets:

$N(10^{22}cm^{-2})$	↘ 1.42116		1.3981871		1.37622		1.13950837		1.13884
ξ_n	↘ 1.880		1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mott}$	0.931	↗	1	↘	1.074	↗	3.290	↘	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-4.408	↗	0	↗	4.521	↗	79.415	↗	79.707
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt (10^{-3}V)$	-11.226	↘	-11.234	↗	-11.226	↗	-9.4994	↗	-9.4873

In the degenerate Sn- $X(x)$ – alloy, for $T=78.760733$ K, one gets:

$N(10^{22}cm^{-2})$	↘ 1.6302		1.603935		1.5788		1.3071908		1.30642
ξ_n	↘ 1.880		1.8138		1.750		1		0.998
$S \left(10^{-4} \frac{V}{K}\right)$	-1.562	↘	-1.563	↗	-1.562	↗	-1.322	↗	-1.320
ZT	0.999	↗	1	↘	0.999	↘	0.715	↘	0.713
$(ZT)_{Mott}$	0.931	↗	1	↘	1.074	↗	3.290	↘	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	-0.061	↗	0	↗	0.063	↗	1.105	↗	1.109
$VC2 \left(10^{-4} \frac{V}{K}\right)$	-4.815	↗	0	↗	4.942	↗	87.026	↗	87.347
$T_s \left(10^{-4} \frac{V}{K}\right)$	-0.092	↗	0	↗	0.094	↗	1.657	↗	1.663
$Pt (10^{-3}V)$	-12.302	↘	-12.310	↗	-12.302	↗	-10.4098	↗	-10.3965

Table 6p: Here, for a given T and with decreasing N , the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T : (i) for $\xi_p \approx 1.8138$, while the numerical results of S present a same minimum $(S)_{min.} (\approx -1.563 \times 10^{-4} \frac{V}{K})$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S , ZT , $(ZT)_{Mott}$, $VC1$, and T_s present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_p \approx 1.8138$, $(ZT)_{Mott} = 1$.

For $x=0$,

In the degenerate Ga- $X(x)$ – alloy, for $T=71.621813$ K, one gets:

$N(10^{22}\text{cm}^{-3})$	↘ 2.3226	2.285126	2.2492	1.8623546	1.8613
ξ_p	↘ 1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K})$	-1.562	↘ -1.563	↗ -1.562	↗ -1.322	↗ -1.320
ZT	0.999	↗ 1	↘ 0.999	↘ 0.715	↘ 0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1(10^{-4}\frac{V}{K})$	-0.061	↗ 0	↗ 0.063	↗ 1.105	↗ 1.109
$VC2(10^{-2}V)$	-0.044	↗ 0	↗ 0.045	↗ 0.791	↗ 0.794
$T_s(10^{-4}\frac{V}{K})$	-0.092	↗ 0	↗ 0.094	↗ 1.657	↗ 1.663
$Pt(10^{-2}V)$	-1.1187	↘ -1.1194	↗ -1.1187	↗ -0.9466	↗ -0.9455

In the degenerate Mg- $X(x)$ – alloy, for $T=80.037437$ K, one gets:

$N(10^{22}\text{cm}^{-3})$	↘ 2.7438	2.6994914	2.6571	2.2000582	2.1988
ξ_p	↘ 1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K})$	-1.562	↘ -1.563	↗ -1.562	↗ -1.322	↗ -1.320
ZT	0.999	↗ 1	↘ 0.999	↘ 0.715	↘ 0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1(10^{-4}\frac{V}{K})$	-0.061	↗ 0	↗ 0.063	↗ 1.105	↗ 1.109
$VC2(10^{-2}V)$	-0.049	↗ 0	↗ 0.050	↗ 0.884	↗ 0.887
$T_s(10^{-4}\frac{V}{K})$	-0.092	↗ 0	↗ 0.094	↗ 1.657	↗ 1.663
$Pt(10^{-2}V)$	-1.25019	↘ -1.25098	↗ -1.25019	↗ -1.0578	↗ -1.0565

In the degenerate In- $X(x)$ – alloy, for $T=85.758335$ K, one gets:

$N(10^{22}\text{cm}^{-3})$	↘ 3.0432	2.9940338	2.947	2.4401073	2.4387
ξ_p	↘ 1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K})$	-1.562	↘ -1.563	↗ -1.562	↗ -1.322	↗ -1.320
ZT	0.999	↗ 1	↘ 0.999	↘ 0.715	↘ 0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1(10^{-4}\frac{V}{K})$	-0.061	↗ 0	↗ 0.063	↗ 1.105	↗ 1.109
$VC2(10^{-2}V)$	-0.052	↗ 0	↗ 0.054	↗ 0.947	↗ 0.951
$T_s(10^{-4}\frac{V}{K})$	-0.092	↗ 0	↗ 0.094	↗ 1.657	↗ 1.663
$Pt(10^{-2}V)$	-1.339	↘ -1.340	↗ -1.339	↗ -1.1335	↗ -1.1320

In the degenerate Cd- $X(x)$ – alloy, for $T=93.080853$ K, one gets:

$N(10^{22}\text{cm}^{-3})$	↘ 3.441	3.3855772	3.3325	2.7592112	2.7576
ξ_p	↘ 1.880	1.8138	1.750	1	0.998
$S(10^{-4}\frac{V}{K})$	-1.562	↘ -1.563	↗ -1.562	↗ -1.322	↗ -1.320
ZT	0.999	↗ 1	↘ 0.999	↘ 0.715	↘ 0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1(10^{-4}\frac{V}{K})$	-0.061	↗ 0	↗ 0.063	↗ 1.105	↗ 1.109
$VC2(10^{-2}V)$	-0.057	↗ 0	↗ 0.058	↗ 1.028	↗ 1.032
$T_s(10^{-4}\frac{V}{K})$	-0.092	↗ 0	↗ 0.094	↗ 1.657	↗ 1.663
$Pt(10^{-2}V)$	-1.454	↘ -1.455	↗ -1.454	↗ -1.2302	↗ -1.2287

For $x=0.5$,

In the degenerate Ga- $X(x)$ – alloy, for $T=90.67699$ K, one gets:

$N(10^{22}\text{cm}^{-3})$	↘ 4.2805	4.2113412	4.1452	3.4322005	3.4302
----------------------------	----------	------------------	--------	------------------	--------

ξ_p	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.055	0	0.057	1.002	1.005
$T_s\left(10^{-4}\frac{V}{K}\right)$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.416	-1.417	-1.416	-1.1985	-1.1970

In the degenerate Mg- $X(x)$ – alloy, for $T=101.331612$ K, one gets:

$N(10^{23}cm^{-3})$	5.0567	4.97499	4.8969	4.0545665	4.0522
ξ_p	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.062	0	0.064	1.120	1.124
$T_s\left(10^{-4}\frac{V}{K}\right)$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.583	-1.584	-1.583	-1.3393	-1.3376

In the degenerate In- $X(x)$ – alloy, for $T=108.57457$ K, one gets:

$N(10^{23}cm^{-3})$	5.6084	5.5178128	5.4321	4.4969616	4.4943
ξ_p	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.066	0	0.068	1.200	1.204
$T_s\left(10^{-4}\frac{V}{K}\right)$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.696	-1.697	-1.696	-1.4350	-1.4332

In the degenerate Cd- $X(x)$ – alloy, for $T=117.845266$ K, one gets:

$N(10^{23}cm^{-3})$	6.3419	6.2394024	6.1414	5.0850498	5.0821
ξ_p	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713
$(ZT)_{Mott}$	0.931	1	1.074	3.290	3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$	-0.061	0	0.063	1.105	1.109
$VC2(10^{-2}V)$	-0.072	0	0.074	1.302	1.307
$T_s\left(10^{-4}\frac{V}{K}\right)$	-0.092	0	0.094	1.657	1.663
$Pt(10^{-2}V)$	-1.841	-1.842	-1.841	-1.5576	-1.5556

For $x=1$

In the degenerate Ga- $X(x)$ – alloy, for $T=112.184161$ K, one gets:

$N(10^{23}cm^{-3})$	7.3378	7.2192156	7.1058	5.8835877	5.8801
ξ_p	1.880	1.8138	1.750	1	0.998
$S\left(10^{-4}\frac{V}{K}\right)$	-1.562	-1.563	-1.562	-1.322	-1.320
ZT	0.999	1	0.999	0.715	0.713

$(ZT)_{Mott}$	$\nearrow$ 0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	$-0.061 \nearrow$	0	$\nearrow$ 0.063	$\nearrow$ 1.105	$\nearrow$ 1.109
$VC2(10^{-2}V)$	$-0.069 \nearrow$	0	$\nearrow$ 0.070	$\nearrow$ 1.239	$\nearrow$ 1.244
$T_s \left(10^{-4} \frac{V}{K}\right)$	$-0.092 \nearrow$	0	$\nearrow$ 0.094	$\nearrow$ 1.657	$\nearrow$ 1.663
$Pt(10^{-2}V)$	$-1.752 \searrow$	$-1.753 \nearrow$	$-1.752 \nearrow$	$-1.4827 \nearrow$	-1.4808

In the degenerate Mg- $X(x)$ – alloy, for $T=125.365892$ K, one gets:

$N(10^{12}cm^{-2})$	$\searrow$ 8.668	8.5282866	8.3943	6.9504673	6.9464
ξ_p	$\searrow$ 1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	$-1.562 \searrow$	$-1.563 \nearrow$	$-1.562 \nearrow$	$-1.322 \nearrow$	-1.320
ZT	0.999 $\nearrow$	1 $\searrow$	0.999 $\searrow$	0.715 $\searrow$	0.713
$(ZT)_{Mott}$	$\nearrow$ 0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	$-0.061 \nearrow$	0	$\nearrow$ 0.063	$\nearrow$ 1.105	$\nearrow$ 1.109
$VC2(10^{-2}V)$	$-0.077 \nearrow$	0	$\nearrow$ 0.079	$\nearrow$ 1.385	$\nearrow$ 1.390
$T_s \left(10^{-4} \frac{V}{K}\right)$	$-0.092 \nearrow$	0	$\nearrow$ 0.094	$\nearrow$ 1.657	$\nearrow$ 1.663
$Pt(10^{-2}V)$	$-1.958 \searrow$	$-1.959 \nearrow$	$-1.958 \nearrow$	$-1.6570 \nearrow$	-1.6549

In the degenerate In- $X(x)$ – alloy, for $T=134.32677$ K, one gets:

$N(10^{12}cm^{-2})$	$\searrow$ 9.614	9.458811	9.3105	7.7088353	7.7043
ξ_p	$\searrow$ 1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	$-1.562 \searrow$	$-1.563 \nearrow$	$-1.562 \nearrow$	$-1.322 \nearrow$	-1.320
ZT	0.999 $\nearrow$	1 $\searrow$	0.999 $\searrow$	0.715 $\searrow$	0.713
$(ZT)_{Mott}$	$\nearrow$ 0.931	1	1.074	3.290	3.306
$VC1 \left(10^{-4} \frac{V}{K}\right)$	$-0.061 \nearrow$	0	$\nearrow$ 0.063	$\nearrow$ 1.105	$\nearrow$ 1.109
$VC2(10^{-2}V)$	$-0.082 \nearrow$	0	$\nearrow$ 0.084	$\nearrow$ 1.484	$\nearrow$ 1.490
$T_s \left(10^{-4} \frac{V}{K}\right)$	$-0.092 \nearrow$	0	$\nearrow$ 0.094	$\nearrow$ 1.657	$\nearrow$ 1.663
$Pt(10^{-2}V)$	$-2.098 \searrow$	$-2.099 \nearrow$	$-2.098 \nearrow$	$-1.7754 \nearrow$	-1.7731

In the degenerate Cd- $X(x)$ – alloy, for $T=145.796326$ K, one gets:

$N(10^{12}cm^{-2})$	$\searrow$ 1.0871	1.0695783	1.0528	0.87169546	0.8712
ξ_p	$\searrow$ 1.880	1.8138	1.750	1	0.998
$S \left(10^{-4} \frac{V}{K}\right)$	$-1.562 \searrow$	$-1.563 \nearrow$	$-1.562 \nearrow$	$-1.322 \nearrow$	-1.320
ZT	0.999 $\nearrow$	1 $\searrow$	0.999 $\searrow$	0.715 $\searrow$	0.713
$(ZT)_{Mott}$	$\nearrow$ 0.931	1	1.074	3.290	3.305
$VC1 \left(10^{-4} \frac{V}{K}\right)$	$-0.061 \nearrow$	0	$\nearrow$ 0.063	$\nearrow$ 1.105	$\nearrow$ 1.109
$VC2(10^{-2}V)$	$-0.089 \nearrow$	0	$\nearrow$ 0.091	$\nearrow$ 1.611	$\nearrow$ 1.617
$T_s \left(10^{-4} \frac{V}{K}\right)$	$-0.092 \nearrow$	0	$\nearrow$ 0.094	$\nearrow$ 1.657	$\nearrow$ 1.663
$Pt(10^{-2}V)$	$-2.277 \searrow$	$-2.279 \nearrow$	$-2.277 \nearrow$	$-1.9270 \nearrow$	-1.9246