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GENERALIZATION OF OPTIMIZATION TECHNIQUE WITH ANALYTICAL AND COMPUTATIONAL METHODS FOR SOLVING LINEAR PROGRAMMING PROBLEM

Ankitapuja Pradhan* and Dr. P. K. Mishra

Department of Mathematics, Odisha University of Technology and Research, Bhubaneswar, Odisha.

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***Corresponding Author**
Ankitapuja Pradhan
Department of Mathematics, Odisha University of Technology and Research, Bhubaneswar, Odisha.

ABSTRACT

In this work, we have studied the establishment of traditional simplex methods and the modified simplex methods for solving linear programming problem (LPP) by replacing one basic variable by one non-basic variable at each simplex iteration. After the generalization of traditional simplex methods for solving linear programming problem (LPP) by replacing more than one basic variables by non-basic variables at each simplex iteration and compare the methods between themselves. To apply these methods on large scale real life linear programming problem, we developed computer program based on MATHEMATICA language. This method is applicable for LPP based on garment industry and textile mill scheduling problem. **Introduction:-**Operations research is a science which deals with problem, formulation, solutions and appropriate decision making. It is most often used to analyze complex real-life problems with the goal of improving or optimizing performance. It is a quantitative approach for decision making based on the scientific method of solving linear programming problem (LPP) by graphical method, simplex method etc. Graphical method is only applicable for problems with two variables whereas simplex method is to solve the problems with any number of variables and constraints, using an iterative approach.

There are some important terms related to Linear Programming Problem.

Decision variable

Decision variables are the physical quantities used to be calculated and controlled by the decision-maker and represented by mathematical symbols. These variables are the physical quantities and the decision-maker has control over them. Such variables are usually denoted by x_1, x_2, …x_n where n is a finite positive integer i.e. $x_1 \geq 0, x_2 \geq 0,\ \ . .,x_n \geq 0$. These decision variables are usually interrelated in terms of the consumption of resources, and they require simultaneous solutions. These are unknown quantities and are expected to be estimated as an output of the LP problem.

Objective function

If c_1, c_2 , …., c_n are constants and x_1, x_2 , …., x_n are variables then the function $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ which is to be maximized or minimized is called objective function.

Constraints

The equations or inequations in variables of a LPP which describes the condition in which the optimization is done are called constraints.

Non-negative restrictions

These are the constraints which describe that the variables involved in LPP are non-negative.

Slack variable

The positive variables which are added to the lefthand side of the constraints in order to make them into equalities are called slack variable.

Example: $MaxZ = 2x + 3y + 4z$ subject to $2x + 8y - z \leq 10$

$5x - 6y + 3z \leq 20$

$7x + 8y + 9z \leq 30$

$x, y, z \geq 0$

$MaxZ = 2x + 3y + 4z + 0s_1 + 0s_2 + 0s_3$ subject to $2x + 8y - z + s_1 = 10$

$5x - 6y + 3z + s_2 = 20$

$7x + 8y + 9z + s_3 = 30$

$x, y, z \geq 0$

Surplus variable

The positive variables which are subtracted from the lefthand side of the constraints in order to make them into equalities are called surplus variable.

Example: $MaxZ = 2x + 3y + 4z$ subject to $2x + 8y - z \geq 10$

$5x - 6y + 3z \geq 20$

$7x + 8y + 9z \geq 30$

$x, y, z \geq 0$

$MaxZ = 2x + 3y + 4z + 0s_1 + 0s_2 + 0s_3$ subject to $2x + 8y - z - s_1 = 10$

$5x - 6y + 3z - s_2 = 20$

$7x + 8y + 9z - s_3 = 30$

$x, y, z \geq 0$

Artificial variable

In a LPP some constraints may have greater than or equal to all the elements of column B are positive. In this case we don't find a unit matrix. To solve this type of problem by simplex method, we need another variable called artificial variable.

Artificial variable is denoted as A.

Coefficient of artificial variable in the objective function is either -1 or -M. If coefficient of A is -1 then it is called as two phase method and if coefficient of artificial variable in the objective function is -M then Big M method.

Literature Review

Optimization techniques are fundamental in various fields such as engineering, economics, machine learning, and operations research. The generalization of optimization techniques is an ongoing area of research aimed at improving the applicability, efficiency, and robustness of optimization methods across diverse problem domains. This literature review explores the evolution of optimization techniques, highlighting key advancements and generalizations that enhance their practical usage.

Need of the study:- Operations Research (OR) offers numerous advantages, including improved decision-making through quantitative analysis, enhanced efficiency and productivity by optimizing resource allocation, and better control over operations by providing data-driven insights. The distinctive approach is to develop a scientific model of the system, incorporating measurements of factors such as change and risk, with which to predict and compare the outcomes of alternative decisions, strategies or controls. The purpose is to help management determine its policy and actions scientifically. The reason all good managers study operations management is to effectively accomplish the next layer of basic functions in the management process. Those are planning, organizing, staffing, leading, and controlling. The primary objective of operations management is to efficiently and effectively manage the production and distribution of goods and services.

Operations research can be used to optimize supply chain management by identifying ways to reduce inventory levels, improve the accuracy of demand forecasting, and optimize transportation networks on 27 May 2024.

Scope of the study:- Importance of Operations Research is a problem-solving and decision-making analytical technique. Many firms use this quantitative approach to problem-solving. When decision-making becomes complex owing to unclear situations or when specified objectives contradict, this strategy is applied. There are three phases of operations research that is formulation, analysis, and interpretation. In the formulation phase, the problem is defined, assessment criteria are determined, and alternatives are developed. The analysis phase involves using mathematical models and techniques to analyze the problem. Finally, the interpretation phase focuses on understanding the results, making recommendations, and implementing the solution.

Objective of the study:- In Operations Research, the Simplex method is a powerful algorithm used to solve linear programming problems, finding the optimal solution by iteratively improving a feasible solution until the objective function is maximized or minimized, while adhering to constraints.

Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem. Simplex tableau is used to perform row operations on the linear programming model as well as for checking optimality. In optimization, the objective function defines the relationship between the decision variables and the desired outcome. For example: Maximization: When the goal is to increase a value, such as maximizing profits, efficiency, or output. simplex method, standard technique in linear programming for solving an optimization problem, typically one involving a function and several constraints expressed as inequalities.

What are the steps in the Simplex method?

1. Set up the problem.
2. Convert the inequalities into equations.
3. Construct the initial simplex tableau.
4. The most negative entry in the bottom row identifies the pivot column.
5. Calculate the quotients.
6. Perform pivoting to make all other entries in this column zero.

Advantages of simplex method

The Simplex method offers several advantages for solving linear programming problems, including its efficiency in handling large-scale problems, its adaptability to both maximization and minimization problems, and its systematic approach to finding optimal solutions. Here is a more detailed breakdown of the advantages:

1. Number of variable is three or more
2. Better accuracy
3. Time management
4. Different software Programme available
5. Efficiency for Large Problems: The simplex method is particularly well-suited for solving linear programming problems with many variables and constraints, making it suitable for real-world applications.
6. Systematic Approach: It provides a structured, step-by-step process for finding the optimal solution, ensuring a reliable and efficient solution process.

Research Methodology

The Simplex method is an iterative algorithm used to find the optimal solution (maximum or minimum) for a linear programming problem by systematically moving from one feasible corner point to another until the optimal solution is reached. Here is a breakdown of the methodology:

1. Problem formulation

Standard Form: Convert the linear programming problem into standard form, which involves: Maximizing (or minimizing) a linear objective function. Expressing constraints as equations (by introducing slack, surplus, or artificial variables). Ensuring all variables are non-negative.

Initial Feasible Solution: Find an initial basic feasible solution, which is a corner point of the feasible region.

2. Simplex tableau

(a) Tableau structure: Organize the problem's data into a tableau, with columns representing variables (including slack/surplus/artificial variables) and rows representing constraints and the objective function.

(b) Initial tableau: Populate the tableau with the coefficients of the objective function and constraints, as well as the right-hand side values.

3. Iterative process

(i) Optimality condition:

Check if the current solution is optimal. In a maximization problem, this means that all coefficients in the objective function row (excluding the basic variables) are non-negative.

(ii) Entering variable: If the solution is not optimal, identify the entering variable (the variable with the most negative coefficient in the objective function row for a maximization problem).

(iii) Leaving variable: Determine the leaving variable by calculating the ratios of the right-hand side values to the corresponding coefficients in the entering variable's column. The smallest positive ratio identifies the leaving variable.

(iv) Pivot operation:

i. Perform a pivot operation, which involves:

ii. Making the pivot element (the intersection of the entering and leaving variable's columns) equal to 1.

iii. Making all other elements in the pivot column equal to 0.

v. Repeat steps 3-5 until optimal condition is achieved.

Optimal solution: Once the optimality condition is met, the values of the basic variables in the final tableau represent the optimal solution.

Objective function value: The value of the objective function at the optimal solution is found in the bottom right corner of the tableau.

Example

Maximize $Z = 3x_1 + 5x_2 + 4x_3$ subject to the constraint

$2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

Maximize $Z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$ subject to

$2x_1 + 3x_2 + s_1 = 8$

$2x_2 + 5x_3 + s_2 = 10$

$3x_1 + 2x_2 + 4x_3 + s_3 = 15$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

	C	B	x_1		x_2	S_1 x_3	S_2	S_3	Min Ratio
S_1	0	8	2	3	0	1	0	0	$\frac{8}{3}$
S_2	0	10	0	2	5	0	1	0	5
S_3	0	15	3	2	4	0	0	1	$\frac{15}{2}$
$\Delta_j = c_j - cx_j$	3	5	4	0	0	0			

	C	B	x_1		x_2	S_1 x_3	S_2	S_3	Min Ratio
x_2	5	$\frac{8}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	0	∞
S_2	0	$\frac{14}{3}$	$\frac{-4}{3}$	0	5	$\frac{-2}{3}$	1	0	$\frac{14}{15}$
S_3	0	$\frac{29}{3}$	$\frac{5}{3}$	0	4	$\frac{-2}{3}$	0	1	$\frac{29}{12}$
$\Delta_j = c_j - cx_j$	$\frac{-1}{3}$	0	4	$\frac{-5}{3}$	0	0			

	C	B	x_1		x_2	S_1 x_3	S_2	S_3	Min Ratio
x_2	5	$\frac{8}{3}$	$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	0	4
x_3	4	$\frac{14}{15}$	$\frac{-4}{15}$	0	1	$\frac{-2}{15}$	$\frac{1}{5}$	0	-ve
S_3	0	$\frac{89}{15}$	$\frac{41}{15}$	0	0	$\frac{-2}{15}$	$\frac{-4}{5}$	1	$\frac{89}{41}$
$\Delta_j = c_j - cx_j$	$\frac{11}{15}$	0	0	$\frac{-17}{15}$	$\frac{-4}{5}$	0			

	C	B	x_1		x_2	S_1	S_2	S_3	Min Ratio
x_2	5	$\frac{50}{41}$	0	1	0	$\frac{15}{41}$	$\frac{8}{41}$	$\frac{-10}{41}$	4
x_3	4	$\frac{62}{41}$	0	0	1	$\frac{-6}{41}$	$\frac{5}{41}$	$\frac{4}{41}$	-ve
x_1	3	$\frac{89}{41}$	1	0	0	$\frac{-2}{41}$	$\frac{-12}{41}$	$\frac{15}{41}$	$\frac{89}{41}$
$\Delta_j = c_j - cx_j$	0	0	0	$\frac{-45}{41}$	$\frac{-24}{41}$	$\frac{-11}{41}$			

Here all $\Delta_j \leq 0$, therefore this is the optimum solution.

$$x_1 = \frac{89}{41}, x_2 = \frac{50}{41}, x_3 = \frac{62}{41}$$

$$Z = 3x_1 + 5x_2 + 4x_3 = 18.65$$

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