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SOLVING THE TRAIN SCHEDULING PROBLEM OF A SINGLE-TRACK LINE: A CASE STUDY ON THE MOMBASA-NAIROBI STANDARD GAUGE RAILWAY

Namu Eric Gitonga[1]*, Dr. Lingyun Meng[2]

Beijing Jiaotong University, China.

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***Corresponding Author**
Namu Eric Gitonga
Beijing Jiaotong University, China.

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ABSTRACT

Provision of punctual and reliable services is a major goal in railway operations and management. Due to the complexities of railroad operations planning, innovative operation management strategies must be developed in order to optimize the existing capacity, improve profitability and the general level of rail service. Train scheduling is an important stage in railway operations planning and is used as the basis for railroad organization. In this thesis, the train scheduling problem of a single-track line is formulated as a variable-based cumulative flow model so as to minimize the total completion time of trains traversing the network. Through the reformulation of the physical network infrastructure capacity on a time-space network, the model enables the decomposition of the initial complex train scheduling problem into a series of multiple single-train optimization sub-problems. The physical network of the entire railway line is constructed in NEXTA-Rail Network Editor. A train scheduling package Fast Train, which combines a time-dependent shortest path algorithm and a priority rule-based algorithm within a Lagrangian relaxation framework is used to solve the proposed model. The model is applied in a case study on Kenya's Mombasa-Nairobi Standard Gauge Railway. The construction of the physical network is first done for the current network consisting of 33 stations and then with 45 stations for the planned long-term network. The data for this line was obtained from the feasibility study reports. In the solution, a maximum of 10000 iterations was allowed after which the program terminates. Fast Train tends to converge to better solutions with increasing number of Lagrangian iterations and the optimality gap decreases with increasing computational time. The hardest problem comprised 36 trains and was solved in 43 min 18 s, with optimality gap of 16.1%, which is within acceptable time for a train scheduling problem. The impact of varying the traffic demand by increasing the number of scheduled trains as well as opening up the reserved passing stations on this network is discussed. The results obtained from this research can be used as a support tool to schedule trains on single-track railway networks as well as a planning tool to assess the implications of changes in traffic demand and railroad infrastructure.

KEYWORDS: Single-track; Train scheduling; Cumulative flow; Lagrangian Relaxation.

1 INTRODUCTION

This chapter presents the introduction of the topic under study. Section 1.1 presents the background to the problem and section 1.2 presents the motivation behind this topic. Section 1.3 presents the objective of the research, followed by a scope of the research in Section 1.4 and finally the rest of this thesis is outlined in Section 1.5.

1.1 BACKGROUND

Railway transportation is an energy efficient mode of transportation for people and cargo and plays an important role in the development of a country’s economy. In many countries, for instance China, railway transport has facilitated passenger transportation, large-scale freight movement and helped to alleviate highway congestion. Being a sustainable and environmentally friendly mode of transportation, many countries are improving and expanding their railway networks as an alternative mode of transportation.

In 2013, the government of Kenya embarked on constructing a new Standard Gauge Railway network, and hopes to extend it to more than 2985 km by 2040. The 472 km single-track section of the network from the Mombasa Port City to the capital Nairobi, which is in line with China’s Belt and Road Initiative and the first phase of East Africa Railway Network was completed and its operations began in May 2017. The Mombasa- Nairobi SGR is envisaged to decongest the Mombasa-Nairobi highway, realize faster and more efficient passenger and freight transportation as well as decongest the Mombasa port. Once completed, the planned network is expected to achieve efficient and economical rail transport that will spur economic growth not only in Kenya but also the East African region in general. To meet the projected traffic demand and realize a high- quality rail transport service, transportation and operation planners need to use modern decision-making support tools in the planning of railroad operations.

Provision of punctual and reliable services remains one of the major goals in railway operation and management. In the planning of railway operations, resources such as railroad infrastructure, rolling stock and staff must be allocated accordingly so as to meet the demand for rail transport at an appropriate cost. Since railway operations planning is a complex task comprising a large solution space, it is separated into several problems that are solved sequentially.[1]

There are different levels of railway planning process according to the planning horizon, i.e. strategic, tactical and operational planning as shown in Figure 1-1.

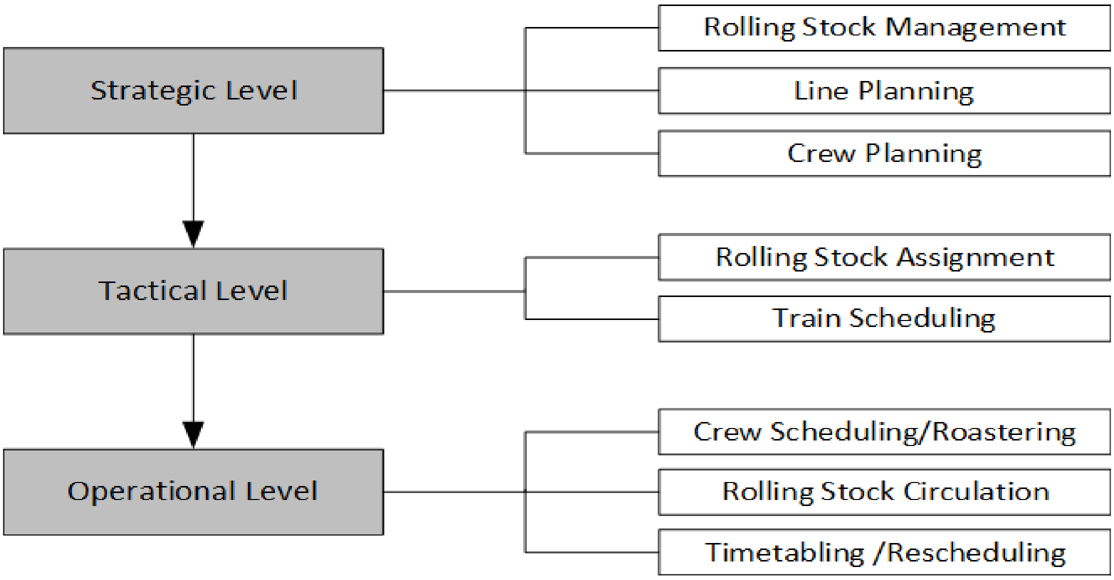


Figure 1-1: Levels of Railway Planning Process.

Strategic planning is long term and concerned with the construction or acquisition of sustainable resources that will remain active over a long period. While the tactical planning level is related to medium and short-term issues, and generally involves the specification of operational policies that are updated over a certain period of time, the operational level is concerned with the daily tasks that are performed, taking into consideration the particulars of the rail transportation system.[2]

Train scheduling is an essential tactical planning tool concerned with dispatching trains on the railway corridor under various constraints. The train scheduling problem (TSP) involves the development of a plan specifying a physical network route and detailed arrival/departure times of every train at each station, while aiming at optimizing certain goals such as to minimize the train travel time.[3] A critical constraint in train scheduling is to avoid any possible conflicts between trains which may cause delays, thus influencing the running times, dwell times and departing events of trains traversing the network. A train timetable must ensure that the expected transport demand is realized according to the requirements of passengers, shippers, train operation companies and infrastructure managers.

The train schedule is drawn on a time-distance graph called a train operation diagram and it is applied as the basis for railroad organization. If a train strays from its planned time-distance path, it might delay subsequent trains or conflict in meeting with or passing other trains scheduled on the same railway infrastructure. In such situations, the train timetable must be adjusted according to the real-time conditions to minimize the effects of such disruption, which is known as rescheduling. These conflicts are quite difficult to deal with for single-track railway networks, as trains run from opposite directions and are only allowed to cross or overtake each other at stations or sidings. For safe operation, if two trains are involved in a conflict, one must wait on the siding or station for the other to cross or pass. Therefore, the number of stations or sidings available along a single-track railway network has direct implications on the capacity of the line.

Several strategies have been adopted in the recent decades to obtain feasible timetables and assist transportation planners to fully optimize the capacity of railway lines. In this thesis, a mathematical programming model for the train scheduling problem is proposed to schedule trains on the Mombasa-Nairobi single-track line with an objective of minimizing total transit time of trains on this network. The model is also used for assessment of the impact of increasing traffic demand and adding more passing stations on this line.

1.2 Motivation

Rail transport is a capital-intensive means of transport and proper management and planning is essential to ensure the profitability of railway enterprises in highly competitive transportation markets. To increase the market share, it is important for rail service providers must offer reliable services and ensure the safety, commercial and environmental sustainability of the railway system. With the dynamically changing environment, technological advancement and increasing transport demands, railway companies must constantly upgrade the efficiency of their operations. Owing to the complications involved in these operations, limited possibilities of improving railroad infrastructure, increasing railroad traffic and customer demands, developing innovative operation management strategies while making use of the existing capacity is key in improving the level of rail service. Timetabling is an important planning stage in railway operation management and has a significant contribution to the attraction of travellers and shippers and the general level of rail service. With increase in traffic demand on a railway network, problems with safety, punctuality, reliability and service frequency begin to arise, hence the need for proper traffic management techniques.

Until recently, train scheduling process was done manually on the premise of the experience and expertise of timetable planners. However, as the rail networks become even more complex and real-time dispatching operations more complicated, scheduling based on manual calculation becomes so time-intensive and ineffective, affecting reliability, punctuality and overall service level. Due to the consequent need for improved techniques to solve complex scheduling problems, several computer-based methods have been studied and developed. Currently, several automated railroad scheduling systems are being used in practice, thanks to the recent advancements improvements in the computing power of computers and the available optimization techniques. Railway companies can achieve improved quality of the train operation diagram, improved service levels and reduced operational costs, while optimizing the utilization of the available infrastructure.

Most of the existing timetable optimization techniques are based on mathematical programming, simulation and heuristics among other approaches. However, some of these methods cannot achieve practically satisfactory results for real world timetabling problems, either because they need considerably large amount of running time and memory space to find optimal solutions or they cannot guarantee an adequate level of the solution quality. This is due to the reason that for real world applications, the design of a train timetable takes into account several constraints and the entire problem becomes really hard to solve. In this thesis, a Lagrangian relaxation method will be introduced to decompose the original complex train scheduling problem into several sub-problems so as to reduce the solution complexity of the proposed model, and to obtain feasible solutions of the original problem within reasonable computational time.

Due to high initial capital costs, a railway line must be designed as economically as possible and still have sufficient capacity to meet the forecast demand.[4] When the Mombasa-Nairobi SGR line was constructed, some of the designed passing stations were reserved for future construction because the line presently has low traffic demand and therefore train scheduling is not a major problem. However, passenger and freight volume on this section is predicted to increase every year and therefore it is essential to study the implications on train scheduling that might arise due to increased traffic demand and opening up of the reserved passing stations.

1.3 OBJECTIVE

The main goal of this research is to study the train scheduling problem of a single-track line for use as a support tool for decision-making by rail transportation planners and to help in railroad operations planning. The problem is formulated as a variable-based cumulative flow model with an objective to minimize total completion time of trains traversing the railway network, and is solved in an open-source train scheduling package, in which a train-based Lagrangian relaxation framework which provides an easy mechanism for decomposition of the problem is used to obtain feasible schedules. The study also evaluates the implications of the increase in traffic demand and changes in the infrastructure of a single-track railway network.

Specific Objectives

1. To describe the development of a variable-based cumulative flow optimization model for single-track railway scheduling problem.
2. To construct the physical rail network in rail network editor.
3. To use a suitable software to solve the model by simultaneously optimizing the routes and schedule of trains and output feasible solutions.
4. To investigate the impact of the increase in traffic demand and the addition of more passing stations on the average train travel time.

1.4 Scope of the Research

A train schedule is an important operation planning tool used by dispatchers and decision makers to organize trains running on a railway corridor. Train timetables are constructed in context of limited resources available on the railway network which must be efficiently utilized to meet traffic demands at minimum cost. Under limited resources and capacity of railway corridors, and increasing traffic demands, the development of suitable optimization techniques for cost efficient scheduling of trains become necessary. An effective timetable will have a great influence on transportation planning and management, satisfying customer demands, maximizing the company’s profits and creating a stable railway system with a high level of service.

In this thesis, a suitable model and solution approach for the train scheduling problem on Mombasa-Nairobi SGR line is presented. The feasible timetables for the planning years 2020 and 2025 obtained and the impact of increasing traffic demand and constructing additional passing stations evaluated. This can be used directly as a basis for action plan and operations planning on the railway line in the corresponding years. In addition, the solution approach to this problem can be used interactively in the scheduling of trains even in the entire network after completion.

1.5 Thesis Outline

The remainder of this thesis is organized as follows. Chapter 2 presents a review of the literature on the train scheduling problem. The chapter highlights some of the relevant work previously done by other researchers on this topic, including the modelling approaches and solution techniques. Chapter 3 details the model formulation for the single-track line scheduling problem adopted for this research. The solution approach adopted in solving the proposed model, including the algorithms is detailed in Chapter 4. In Chapter 5, the case study on the Mombasa-Nairobi Railway line is presented, including the solution for the train scheduling problem and the results and analysis of the solution. Finally, a conclusion of the research findings from the case study and recommendations are presented in Chapter 6.

2 Literature Review

The chapter presents an overview of information and some previous literature related to this study. Section 2.1 first introduces an overview of modeling approaches adopted by other researchers for the train scheduling problem, followed by a detailed description of the various solution techniques that have been used by other researchers. In Section 2.2, a summary and implications related to this problem is presented, including the framework proposed in this thesis.

2.1 Train Scheduling Problem Approaches

Train scheduling is an important issue in railway operations planning and thus it has attracted considerable attention. Many studies devoted towards solving railway traffic management problems have been done in the recent decades.

An overview of real-time rail traffic management models and algorithms is presented in Cacchiani et al.(2014)[5] and Corman and Meng (2014).[6] A recent study by Caimi et al. (2017)[7] provides an review of the railway timetable design approaches with a comparison of the different optimization models and solution methods proposed to solve the railway timetabling problem.

The train scheduling problem of the single-track railway is considered as a NP-hard problem[8] and in complex or large-scale and railway networks, optimal solutions are normally unattainable. To meet the computational requirements in real-world applications, efficient train scheduling techniques are necessary so as to generate feasible solutions in less time. While attempting to solve the TSP, a number of aspects has to be put into consideration. Traffic managers need to establish an objective, set preferences on various trains and consider several restrictions on railroad infrastructure and traffic parameters such as the position of other trains, safety headways, station capacity etc.

The existing literature has adopted various efficient methods such as heuristics, branch and bound approaches, LP relaxations and Lagrangian relaxation approach for solving this problem.

2.1.1 Mathematical programming

Mathematical programming continues to be relatively popular approach due to its higher effectiveness and computational efficiency. However, regarding large scale or real-world problems, mathematical programming methods take a long time to solve and their performance is not very satisfactory. These methods are applied in relatively smaller problems as it is hard or even impossible for them to find quick solutions in real world time timetabling problems.[9]

For the train timetabling problem of a single-track line, Szpigel (1973)[10] formulated a mixed-integer programming (MIP) model in order to determine the crossing and overtaking positions of trains with given and departure times routes and proposed a branch and bound solution algorithm for the model. Taking into account the need to minimize delays and yet meet traffic demands, Carey (1994a)[11] developed a train routing and scheduling mathematical model for complex rail networks with selection of lines, platforms and routes. This model was further extended in Carey (1994b)[12] for networks with one-way and two-way tracks.

To optimize train schedules for a single-track rail corridor, Higgins et al. (1996)[13] developed an optimization to minimize train tardiness and operation cost for trains with variable velocities. Depending on the estimate of the minimum train delay for the remaining conflicts in a schedule, train priority is determined and used within a branch and bound solution procedure to obtain optimal solutions. Further, in Higgins et al. (1997),[14] the solution is extended to incorporate heuristic decomposition techniques to solve the model in reasonable time and used to determine the number and optimal positioning of passing stations along a single-track corridor. Modifying the mathematical formulation presented in Higgins et al. (1996)[13] to accommodate a railway network, Karoonsoontawong and Taptana (2017)[15] considered the train scheduling problem in the application of planning for a single-track network. They proposed two branch and bound local-search heuristic algorithms based on the respective least lower bound branching rule[13] and least delay time branching rule[16] for the solution of the remaining conflicts.

For passenger train timetabling in single and multiple track corridors and different train capacities, Ghoseiri et al. (2004)[17] designed a multi-objective model to minimize total passenger time and fuel cost. To solve it, they used a two-step solution approach by initially using the ε-constraint method to obtain the Pareto frontier, and then adopting the distance-based method to seek a feasible solution.

In Zhou and Zhong (2007),[18] a resource-constrained formulation is proposed to minimize total train travel time. In their solution, a lower bound rule based on a Lagrangian relaxation is employed to dualize station and segment headway capacity constraints, followed by an exact lower bound rule for estimating least train delay to resolve meet-pass conflicts on a partial timetable, and finally a beam-search heuristic technique is used for constructing tight upper bounds. Using a similar model to Zhou and Zhong (2007)[18], Castillo et al. (2009)[19] and Castillo et al. (2011)[20] explored the optimization problem of the train schedule, in which user preferred departure times are used rather than the actual scheduled departure times. Castillo et al. (2009)[19] proposed a three-stage sequential optimization approach in which a combination of objective functions is introduced to minimizing relative train travel time, promptly allocate trains to circulate and minimize total train dwell times at stations. A bisection rule-based algorithm is utilized to ensure that an exact global optimum is attained. To decrease the computation time, Castillo et al. (2011)[20] uses a bisection rule based algorithm to obtain a sharp upper bound for the objective function, and a strategy to reduce the quantity of binary variables to be evaluated.

Investigating the robust periodic train scheduling of a single-track railway, Shafia et al. (2012)[21] proposed a PESP model by defining the problem as a fuzzy job shop scheduling problem. A two-stage heuristic algorithm solution approach based on Simulated Annealing (SA) is presented to solve large-scale scheduling problems. A hybrid job shop technique to generate more efficient and accurate train schedules is presented in Burdett and Kozan (2009).[22] Unique train scheduling characteristics are consolidated into the solution's disjunctive graph representation and then constructive algorithms that use this representation are developed. In Abid and Khan (2015a),[23] a formulation for the single- track line train scheduling problem is presented based on job-shop scheduling structure, to minimize total train travel time. A branch and bound technique with priority rules was used for solving the formulated problem in reasonable time. In the solution, an exact lower bound for estimating least train delay is used and a cut set dominance rule is applied to reduce the search space. Further in Abid and Khan (2015b),[24] this model was extended to minimize total delays and operational cost with regards to the position of the sidings. Considering trains running from opposite directions along a single-track line, Harbering et al. (2015)[25] proposed a formulation which was closely related to minimizing the make-span in a job shop scheduling problem consisting of two counter-routes and no pre-emption. A lower bound on the objective value was developed to provide a simple solution method.

While most of the existing literature consecutively determines the routes and thereafter schedules the trains, there is a definite tendency towards the development of more detailed mathematical models and effective solution algorithms that can simultaneously (re)route and (re)schedule trains.[3] To simultaneously route and schedule trains on a network, Caimi et al. (2010)[26] proposed an ILP formulation and employed the cutting-plane method to generate conflict-free train timetables for a microscopic model of the rail infrastructure. Based a stochastic, recourse-based programming framework, Meng and Zhou (2011)[27] incorporated various probabilistic scenarios into the rolling horizon decision making process for the rescheduling of trains following service interruption a single-track line. The model regularly optimizes timetables for a fairly long rolling horizon and selects and disseminates a rigid meet-pass plan for each rolling horizon. Pellegrini et al. (2014)[28] formulated a MILP model for the search for the best route and schedule of the train in the event of real-time rail traffic disruption.

An IP model reformulated with cumulative flow variables based on the network is presented in Meng and Zhou (2014)[3], to simultaneously reroute and reschedule trains on N-track networks. Using a similar method, Zhou and Teng (2016)[9] developed an ILP model which was restructured as a path-choice model based on a discrete time-space network to simultaneously route and schedule passenger trains on unidirectional and bidirectional railway networks.

2.1.2 Simulation and Heuristic approaches

Simulation and heuristic approaches have been extensively applied for the solution of real-world and large-scale and train scheduling problems in the recent years. This is due to the reason that they can usually obtain a satisfactory train timetable rather than the optimal one within an acceptable computing time.[9] A discrete event based rail traffic model was designed by Dorfman and Medanic (2004)[29] wherein a travel advance strategy (TAS) based on local feedback is developed to simulate train movement along the lines of a large-scale railway network and can rapidly manage disruptions in the schedule. Based on this travel advance strategy, Li et al. (2008)[30] further presented an improved simulation method. The ETAS (Effective Travel Advance Strategy) is achieved by the formulation of an algorithm which is based on global information of the train. Xu et al. (2014)[31] combined the improved TAS with a genetic algorithm so as to obtain an optimal balanced schedule, least train delay ratio and optimal velocities for trains on the railway line. Further, in Xu et al. (2018),[32] a heuristic method that is based on a simulation method for train movement is introduced to search for near-optimal train schedules within an acceptable computational time frame. Incorporating train status transition check and operation rules, a train scheduling method called the TSTA (Train Status Transition Approach) is then designed based on iterative discrete event simulation.

As far as heuristic algorithms are concerned, Carey and Crawford (2007)[33] developed an effective heuristic algorithm to help find and resolve conflicts in the draft train schedules in complex networks. Alternating a heuristic and a truncated branch and bound technique in a tabu-search scheme to compute train timetables in short computation times, Corman et al. (2010)[34] addressed the problem of detection and resolution of train conflicts. Mu and Dessouky (2011)[35] introduced two optimization-based heuristics based on an insertion procedure and a genetic algorithm to solve the train scheduling problem of freight trains on complex railway networks, taking into account flexible path of trains. An iterative heuristic for the solution the train scheduling problem in the situation of a profoundly congested railway node is proposed by Cacchiani et al. (2016).[36] The algorithm is able to provide good solutions for real world cases and can as well be used to assess capacity saturation of a railway node.

Simulation and heuristic approaches have their shortcomings. Simulation methods generally have low global optimization capacity since they mainly simulate train movement process with some given rules. Conversely, heuristic methods cannot at all times guarantee good solution qualities and may even trap in local optimum upon a limited number of iterations, although theoretically they have a global capacity with sufficient iterations.[9]

2.1.3 Lagrangian relaxation

In real-world applications, designing a train timetable takes into consideration several constraints and the entire train timetabling problem gets extremely hard to solve. In order to alleviate the complexity and difficulty of solving the train scheduling problems of large-scale and real-world cases, a good tactic is decomposing the original complex problem into several simple problems by either breaking down the larger railway network into multiple smaller ones or to decouple the set of interrelated trains into separate trains, then repeatedly coordinate the solving of multiple simple sub-problems so as to obtain the final solutions of the original problem.[9] However, in train scheduling models under generalized complex networks, capacity constraints are really hard to break down into different solution branches. There are several studies that have been dedicated to efficient decomposition mechanisms of reducing the train scheduling model complexity and heuristic algorithms to generate feasible solution in reasonable computational time, e.g. in the train-based decomposition by Lee and Chen (2009)[37] Zhou and Teng (2016),[9] and Liu en Dessouky (2017).[38] Some studies have employed classical Lagrangian relaxation of the conflicting constraints to break down the problem into shortest path problems on time discretized networks. For instance, in Brännlund et al. (1998)[39] a Lagrangian relaxation technique has been used to decompose the initial scheduling problem into train-based dynamic programs by relaxing the track capacity constraints and allotting usage prices for them.

A Lagrangian relaxation-based heuristic algorithm of track capacity constraints is presented in Cacchiani et al. (2010)[40] for a timetabling problem with mixed trains. The problem is modelled by a means of a time-space graph and using a generalization of the approach presented in Caprara et al. (2002)[8] and Caprara et al. (2006).[41] In Meng and Zhou (2014),[3] the complex problem of rerouting and rescheduling trains is simplified by decomposing it into single train-based optimization sub-problems using cumulative flow variable and later applying an effective time-dependent shortest path algorithm to resolve each sub-problem within the framework of a Lagrangian relaxation solution. Similarly, in Zhou and Teng (2016)[9] a train-based Lagrangian relaxation approach based on constructing a discretized time-space network is proposed to decompose the reformulated train path-choice model. By constructing weighted directed graphs, Jiang et al. (2017)[42] decomposed a multi-path searching model by introducing Lagrangian multipliers so as to relax the complicated constraints so as to solve a multi-periodic train scheduling problem to jointly optimize arrival and departure times as well trains operation periods on double- tracked railway network.

2.2 Summary and Implications

Even though extensive advancement has been made on both integer programming (IP) and linear programming (LP) formulations and solution algorithms for the train scheduling problem, the existing methods either rely on many heuristic rules to solve the formulated models, which could achieve practically satisfactory results but without an adequate assessment of the solution quality; or on commercial optimization solvers, which could take a considerable amount of running time and memory space to find solutions for real world problems to optimality. Many existing studies have shown that is difficult to find optimal solutions to the train scheduling problem using general optimization approaches such as branch and bound or cutting plane method within an acceptable computing time especially for a large real-world rail network.

Some existing literature e.g. Castillo et al. (2011)[20], Carey and Crawford (2007)[43] opted to determine the routes of trains and timetables sequentially so as to reduce the difficulty of solving the train timetabling problem. On the other hand, several studies consider these two components as closely related and prefer to optimize them together, which is a trend in the development of effective solution algorithms and CPU computing capabilities.

In this research, a variable-based cumulative flow model for routing and scheduling trains simultaneously on a single-track railway network is proposed. For reduction in the model solution complexity, a problem decomposition mechanism is adapted on the based on a time-dependent shortest path algorithm and a train-based Lagrangian relaxation framework, in which priority rules are incorporated to generate feasible solutions and guarantee solution quality.

3 Model Formulation

This chapter describes the formulation of a mathematical programming model for the single-track railway train scheduling problem adopted by this research to achieve the objectives stated in Section 1.3 of Chapter 1. The chapter introduces the conceptual illustration and notations, parameters and variables, followed by an objective function and the constraints considered in the formulation.

3.1 The Single-Track Train Scheduling Model Formulation

A mathematical formulation for the single-track line train scheduling problem is proposed based on the reformulation of the physical railway network into a time-space network structure. Cumulative flow variables are taken to model the trains temporal and spatial occupancy of on railway tracks as well as safety time headways, jointly optimizing the routes and passing time at every station along the selected route of each train. This way, the original complex TSP is hence decomposed into a sequence of multiple single-train optimization sub-problems, which are easier to solve.

The network variable-based cumulative flow model framework proposed by Meng and Zhou (2014)[3] is adapted. Whereas their model focuses on rerouting and rescheduling of trains during perturbations, the formulation presented in this study addresses the tactical scheduling problem, since real time traffic management is outside the scope of this research.

3.2 Conceptual Illustration

In this thesis, a railway network is represented by a set of nodes and links. Nodes represent the intersections of station tracks, switch lines or a point where tracks are merging or diverging in the physical railway network. A station is viewed as a sub-network comprising of a main track and a number of siding tracks corresponding to a set of links. In the proposed model, the track is modelled as a link and only one train is allowed on a link any given time. Each link connects two nodes and is presumed to be bidirectional, so that trains can traverse the track from both directions. The length of a train is taken to be zero for simplicity. Even though the model is flexible as regards the spatial granularity, it is proposed for a microscopic network view, and the granularity of time taken as one minute for this network.

A simple example of a rail network representation is shown in the Figure 3-1. The single- track rail network consists of two stations with 10 nodes connected by bidirectional links.

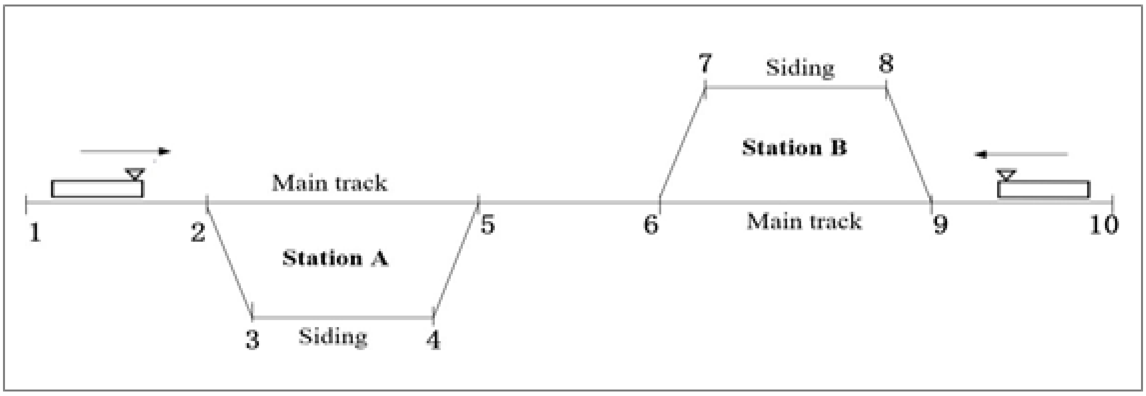


Figure 3-1 Bidirectional rail network with 10 nodes.

Nodes (2, 3, 4, 5) represent station A, and nodes (6, 7, 8, 9) represents station B. With the route being modelled as a series of nodes and the track being modelled as a link in the proposed model, the station minimum and maximum dwell times can be mapped as constraints on train traveling time in the corresponding link(s). For each link, input data such as free-flow running time, safety headways and dwell time requirements are given. The earliest departure time, origin and destination for each train is also given.

The train scheduling problem for the single-track railway network is defined as follows: Assuming a network of railway stations and segments, the problem requires to determine the arrival/departure times at every station for a set of trains, $f \in F$ from pre-specified origin stations to destination stations in a given planning horizon $t = 1,..., \mathrm{T}$, where; T represents the planning horizon. In order to capture the practical safety operational rules, the network is represented as a directed graph $G = (N, E)$, with a set of nodes N and a set of links E.

3.3 General Subscripts, Parameters and Variables

The general subscripts, parameters and decision variables of the proposed formulation are introduced in Tables 3-1, 3-2 and 3-3.

Table 3-1: General Subscripts.

Symbol	Description
i, j, k	Node index, $i, j, k \in N$, N is the set of nodes
e	Link index, (i, j), $e \in E$, E is the set of links
p	Route index, $p \subset P$, P is the set of all routes on a railway network
m	Link sequence number along a route p, $m \le np$, np is the number of links in route p
t	Scheduling time index, $t = 1,\dots T$, T is the planning horizon
f	Train index, $f \in F$, F is the set of trains

Table 3-2: Input Parameters.

Symbol	Description
E_p	Set of sequenced links of route p, $\|E_p\| = n_p$
P_f	Set of possible routes on which train f may run, $P_f \subset P$
E_f	Set of links train f may use, $E_f \subset E$
$FT_f (i, j)$	Free flow running time of train f to traverse link (i, j)
EST_f	Predetermined earliest start time of train f at its origin node
$w^{min}(i, j)\ f$	Minimum dwell time of train f on link (i, j), $(i, j) \in \Omega$
$w^{max}(i, j)\ f$	Maximum dwell time of train f on link (i, j), $(i, j) \in \Omega$
g	Safety time headway between the occupancy and the arrival of trains
h	Safety time headway between the departure and the release of trains
o_f	Origin node of train f
s_f	Destination node of train f
$E^{os}(i)$	Set of links starting from or ending at node i
$E^{o}(i)$	Set of links starting from node i
$E^{s}(i)$	Set of links ending at node i
Ω	Set of cells that allow dwell time, representing siding tracks in stations
$Cap(i, j,t)$	Flow capacity on link (i, j) at time t, $Cap(i, j,t) = 0$ due to maintenance on link (i, j) at time t, otherwise $Cap(i, j,t) = 1$.

Table 3-3: Decision Variables.

Symbol	Description
$TT_f (i, j)$	Running time of train f on link (i, j)
$x_f (i, j)$	Binary train routing variables, $x_f (i, j) = 1$ if train f selects link (i, j), otherwise,
$y_f (i, j,t)$	0-1 time-space network binary occupancy variables, $y_f (i, j,t) = 1$ = 1 if train f occupies link (i, j) at time t; otherwise, $y_f (i, j, t) = 0$.
$a_f (i, j,t)$	0-1 binary cumulative arrival flow variables, $a_f (i, j,t) = 1$ if train f has already arrived at link (i, j) by time t; otherwise $a_f (i, j, t) = 0$.
$d_f (i, j,t)$	0-1 binary cumulative departure flow variables, $d_f (i, j,t) = 1$, if train f has already departed from link (i, j) by time t; otherwise $d_f (i, j, t) = 0$.

3.4 Objective Function

The objective function in the model minimizes the total trip completion time of all the trains from the origin node to the destination node.

$$Z = \min \sum_f \left\{ \left| \sum_t t \times \sum_{i:(i,s_f) \in E^s(s_f) \cap E_f} \left[d_f(i,s_f,t) - d_f(i,s_f,t-1) \right] \right| \right\} \quad (1)$$

3.5 Constraints

3.5.1 Flow balance constraints

Constraints (2), (3) and (4) ensure flow balance on the network at the origin node, intermediate nodes, and destination node of train *f* respectively.

Flow balance constraints at origin node

$$\sum_{i,j:(i,j)\in E^o(o_f)\cap E_f} x_f(i,j)=1, \quad \forall f \tag{2}$$

Flow balance constraints at intermediate nodes

$$\sum_{i:(i,j)\in E^s(j)\cap E_f} x_f(i,j)=\sum_{k:(j,k)\in E^o(j)\cap E_f} x_f(j,k), \quad \forall f, j\in N-o_f-s_f \tag{3}$$

Flow balance constraints at the destination node

$$\sum_{i,j:(i,j)\in E^s(s_f)\cap E_f} x_f(i,j)=1, \quad \forall f \tag{4}$$

3.5.2 Time-space network constraints

Starting time constraints at origin node

Constraints (5) and (6) ensure that trains do not depart earlier than pre-determined earliest starting time at their origin nodes.

$$\sum_{j:(o_f,j)\in E_f} a_f(o_f,j,t)=0, \quad \forall f, t<EST_f \tag{5}$$

$$\sum_{j:(o_f,j)\in E_f} d_f(o_f,j,t)=0, \quad \forall f, t<EST_f \tag{6}$$

Within link transition constraints

These constraints represent the transition of train *f* within the link.

$$d_f(i,j,t+FT_f(i,j))\le a_f(i,j,t), \qquad \forall f,(i,j)\in E_f,t \tag{7}$$

Link-to-link transition constraints

Link-to-link transition constraints ensure link-to-link transition by guaranteeing that $a_f(j,k,t)=d_f(i,j,t)$ if the adjacent link (i,j) and link (j,k) are both used by train *f*.

$$\sum_{i,j:(i,j)\in E_f} d_f(i,j,t)=\sum_{j,k:(j,k)\in E_f} a_f(j,k,t), \quad \forall f, j\in N-o_f-s_f,t \tag{8}$$

Mapping constraints between the time-space network and the physical network

Mapping constraints are imposed to map the variables $a_f(i,j,t)$ in time-space network to variables $x_f\ (i, j)$ in the physical network, hence describing whether link (i, j) is selected by train f to traverse the network from its origin to destination.

$$x_f(i,j) = a_f(i,j,T), \qquad \forall f,\ (i,j) \in E_f \tag{9}$$

3.5.3 Running time and dwell time constraints

Running time constraints

The running time of train f on link (i, j) can be calculated by the following equation.

$$TT_f(i,j) = \sum_t \left\{ t \times \left[d_f(i,j,t) - d_f(i,j,t-1) \right] \right\} - \sum_t \left\{ t \times \left[a_f(i,j,t) - a_f(i,j,t-1) \right] \right\} \quad \forall f, (i,j) \in E_f \tag{10}$$

Minimum running time constraints

Minimum running constraints impose the required minimum train running time of train f.

$$TT_f(i,j) \geq FT_f(i,j) \qquad \forall f, (i,j) \in E_f \tag{11}$$

Minimum and maximum dwell time constraints

These constraints guarantee minimum and maximum station dwell times by the variables TT_f (i, j). In this study, $w(i, j)$ is taken as 1 hour.

$$w_f^{\min}(i,j) + FT_f(i,j) \leq TT_f(i,j) \leq w_f^{\max}(i,j) + FT_f(i,j), \quad \forall f, (i,j) \in E_f \Omega \tag{12}$$

3.5.4 Safety headway and capacity constraints

Link occupancy indication constraints

These constraints link time-space occupancy variables and cumulative arrival/departure variables of train f by mapping $y_f\ (i, j, t)$ with $a_f\ (i, j, t \,\square\, g)$ and $d_f\ (i, j, t \,\square\, g)$.

$$y_f(i,j,t) = a_f(i,j,t+g) - d_f(i,j,t-h), \qquad \forall f,\ (i,j) \in E_f, t \tag{13}$$

If train f has started occupying link (i, j) by time t, $y_f(i,j,t) = 1$, otherwise $y_f(i,j,t) = 0$. If train f has ended occupying link (i, j) by time t, $d_f(i,j,t) \geq d_f(i,j,t-1), \ \forall f, (i,j) \in E_f, t$ and otherwise, 0.

Link capacity constraints

Link capacity constraints enforce safety time headways by ensuring that the number of trains occupying link *(i, j)* is not more than the capacity of the respective link.

$$\sum_{f:(i,j)\in E_f} y_f(i,j,t) + \sum_{f':(i,j)\in E_{f'}} y_{f'}(j,i,t) \leq Cap(i,j,t), \qquad \forall i,j,t \tag{14}$$

3.5.5 Time-connectivity constraints

Constraints (15) and (16) represent time connectivity for cumulative flow variables.

$$a_f(i,j,t) \geq a_f(i,j,t-1), \quad \forall f,(i,j) \in E_f, t \tag{15}$$

$$d_f(i,j,t) \geq d_f(i,j,t-1), \quad \forall f,(i,j) \in E_f, t \tag{16}$$

If train f has arrived at or departed from link (i, j) by time t, then either $a_f(i,j,t)$ or $d_f(i,j,t)$ has to have a value of 1 in all later time periods, such that, $t^{'} \geq t$.

4 Solution Approach

This chapter describes the solution approach in FastTrain, an open-source software package adopted in the solution of the model proposed in this thesis. Section 4.1 describes the time-space representation of the physical network. Section 4.2 presents the transformation of physical network inflow variables into cumulative flow variables. Section 4.3 describes the use of cumulative flow variables to model the safety headways of trains. Section 4.4 describes the general Lagrangian relaxation framework applied in the solution approach. Section 4.5 presents the fundamental label-correcting algorithm used in solving the time-dependent shortest path problem and finally, Section 4.6 details the priority rule-based implementing algorithm for the transformation of the dual solutions into feasible solutions.

4.1 Time-Space Representation of Physical Rail Network

The train scheduling problem requires that the temporal and spatial train occupancy on the physical network infrastructure be precisely modelled with regard to various safety headway constraints. The solution approach proposed by Meng and Zhou (2014)[44] is adopted. In the train scheduling software package FastTrain, the input physical rail network is transformed into a time-space network according to discretized time units and constructed arcs. The network G is extended into a time-space network $TSG = (V, A)$ for each train f. Each node I in set N, is extended into a set of vertices (i, t) in the set of space-time network at each interval t in the planning horizon, $t = 1, 2, T$. Three types of arcs in the extended space-time network are defined so as to take into consideration the feasible transitions allowed in the network, i.e., link traveling arcs (some allow dwelling while others do not), link waiting arcs at the origin link and dummy arcs at the destination node. Through the different types of arcs, the state transition is restricted by setting an infinitely large cost for arcs which are invalid or infeasible so that the typical shortest path algorithm is adapted for train path choice.

The mapping constraints (9) between the physical network and the time-space network, and the flow balance constraints (2) - (4) on each link and the origin/destination nodes are taken into account by the network representation. The link occupancy capacity constraints (13) and (14) will be considered through the resource costs in the label-correcting algorithm discussed in section 4.5.

As illustrated in Figure 4-1, the physical network consisting of 4 nodes and 5 links (on the left) is transformed into the link-based time-space network on the right.

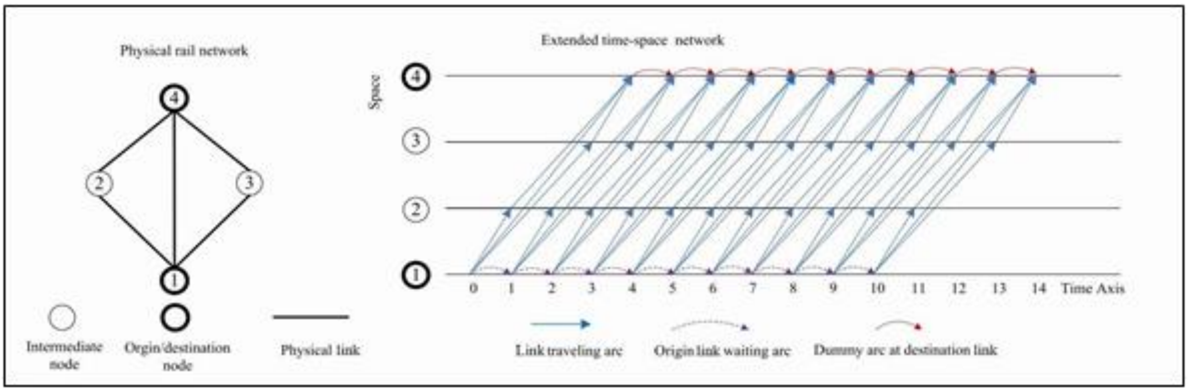


Figure 4-1: Extended time-space network representation of the physical network.[44]

A set of binary-based cumulative flow variables $a_f\ (i,\ j, t)$ and $d_f\ (i,\ j, t)$ is introduced to represent link occupancy in the extended space-time network, where; $a_f\ (i,\ j,t) = 1$ if train f has already arrived at link $(i,\ j)$ by time t, and otherwise $a_f\ (i, j,t) = 0$.
$d_f\ (i,\ j,t) = 1$ if train f has already departed from link $(i,\ j)$ by time t, and otherwise $d_f\ (i, j,t) = 0$.

4.2 Transformation of Network Inflow Variables into Cumulative Flow Variables

To model both the temporal and spatial train occupancy on the tracks as well as safety time headways between trains, inflow variables are linked to cumulative flow variables. The cumulative flow decision variables make it possible for a simultaneous solution approach, which enumerates all conceivable routes within the extended space-time network and then jointly optimizes both train routes and train arrival/departure times.

A set of network flow variables $u_f\ (i,\ j, t)$ and $u\ (i,\ j, t)$ are taken to represent the route selection and corresponding arrival/departure times of train *f*. These binary network inflow variables are linked to the cumulative flow variables by equations (17) and (18).

$$u_f(i,j,t) = a_f(i,j,t) - a_f(i,j,t-1) \quad (17)$$

$$u'_f(i,j,t) = d_f(i,j,t) - d_f(i,j,t-1) \quad (18)$$

where, $u_f(i,j,t) = 1$ represents train f arriving at the upstream node i of link (i,j) at time t, $u_f(i,j,t) = 0$, otherwise; $u'(i,j,t) = 1$ represents train f departing from the downstream node i of link (i,j) at time t', $u'(i,j,t) = 0$, otherwise. Figure 4-2 depicts an illustration on the usage of cumulative arrival/departure variables to describe selection of links and arrival/departure times of train f at link (1,2).

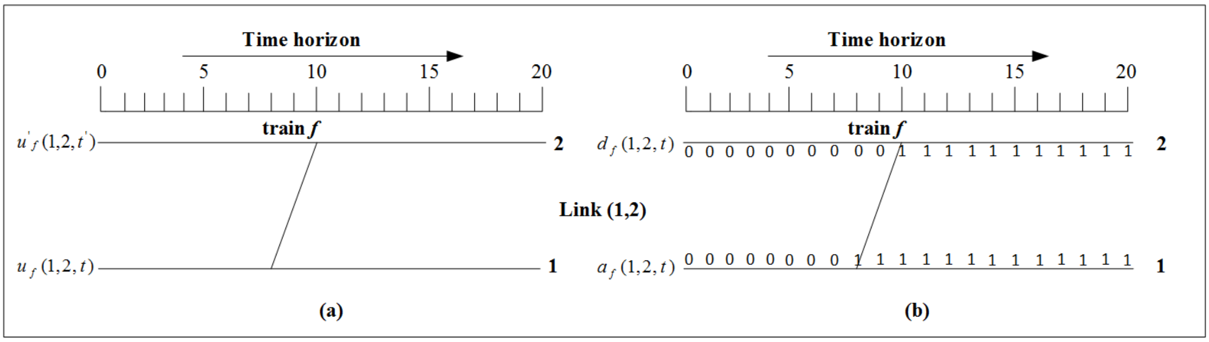


Figure 4-2: Transformation of inflow variables into cumulative flow variables.

In Figure 4-2 (a) above, train f arrives at link $(1, 2)$ at time $t = 8$ and departs at time $t = 10$ with $u_f(1,2,8) = 1$ and $u'(1,2,10) = 1$. With regard to the cumulative flow variables as shown in Figure 4-2 (b), $a_f(1,2,t) = 0$ for $t < 8$ and $a_f(1,2,t) = 1$, for $t \geq 8$; $d_f(1,2,t) = 0$ for $t < 10$ and $d_f(1,2,t) = 1$ for $t \geq 10$.

Moreover, $a_f(i,j,T) = 1$ demonstrates that the link (i, j) is used by train f to traverse the network, where T represents the planning horizon.

4.3 Modelling Safety Headway by Cumulative Flow Variables

In order to model the safety headways and spatial train occupancy, shifted cumulative flow variables $a_f(i,j,t+g)$ and $d_f(i,j,t-h)$ are introduced to denote if train f starts or ends occupying link (i, j) by time t, by considering the minimum safety headway times g and h. The spatial occupancy of train f is represented through the equation $y_f(i,j,t) = a_f(i,j,t+g) - d_f(i,j,t-h)$; where $y_f(i,j,t)$ represents a set of 0-1 binary occupancy variables. $y_f(i,j,t) = 1$, if train f occupies link (i, j) at time t, and otherwise, $y_f(i,j,t) = 0$.

The planning horizon is discretized and denoted by integer values from time index 1 to T. For instance, assuming that $g = h = 1$, the grey rectangular block in Figure 4-3 corresponds to $y_f(i, j, t) = 1$ for $t = 7 \ldots 10$, and $y_f(i, j, t) = 0$ otherwise; which implies that train f occupies link (i, j) from time 7 min to time 10 min.

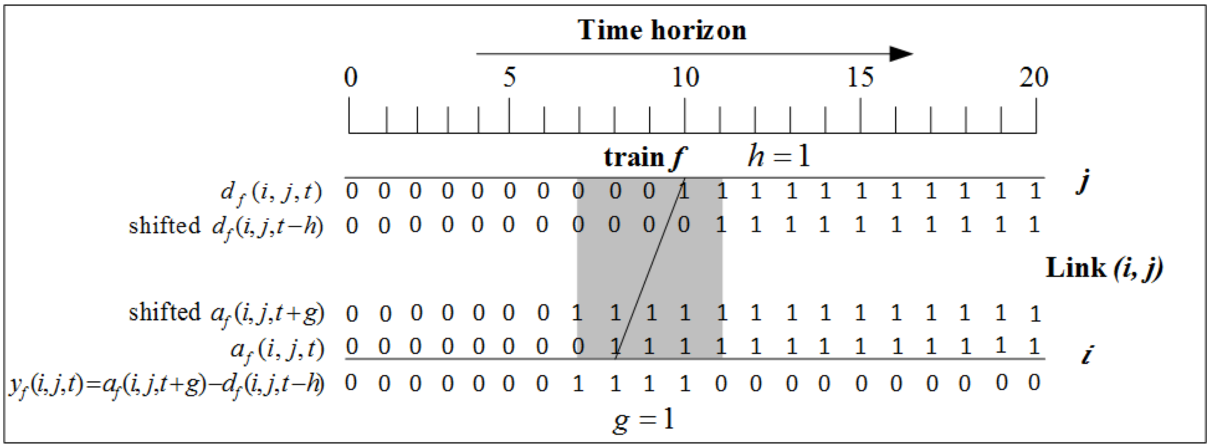


Figure 4-3: Spatial occupancy of link (*i*, *j*) by train *f*.

An illustration of a single-track case is depicted in Figure 4-4. A directed link e from station i to j and link e' from station j to i is introduced to let trains run on opposite directions. Considering train $Cap(i, j, t)$ using link e and train f' using link e'; since links e and e match the same segment, a constraint $y_f(i, j, t) + y_{f'}(i, j, t) \le 1$ can be utilized to model the safety headway requirement between the two trains.

Specifically, $y_f(i, j, t) + y_{f'}(i, j, t) = 1$ for t between 3 and 8, 10 and 16. Furthermore, $y_f(i, j, t) + y_{f'}(i, j, t) = 0$ for t between 0 and 3, 16 and 20, as well as 8 and 10, which indicates 2 time units buffer time.

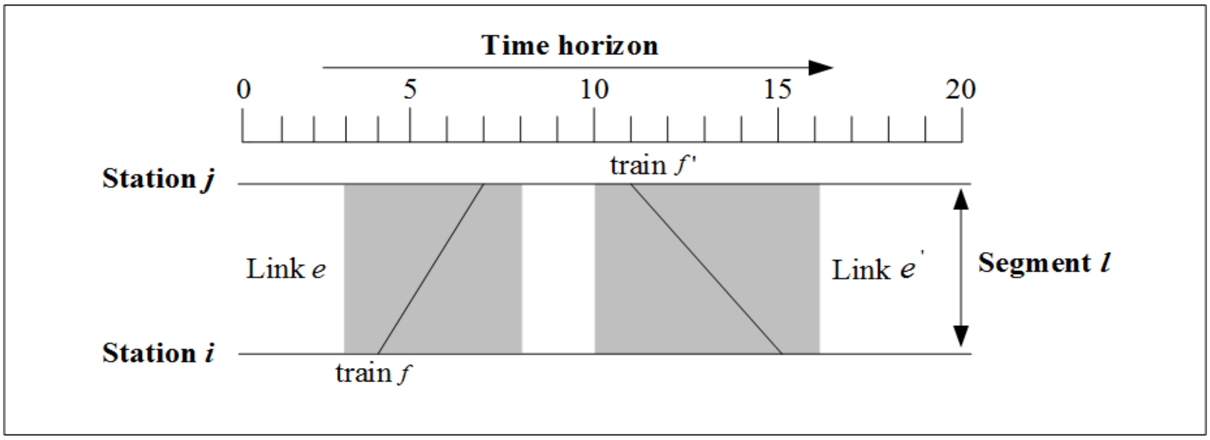


Figure 4-4: Two links corresponding to a single-track segment *l* from station *i* to *j*.

Based on the variables a_f (i, j,t), d_f (i, j,t) and y_f (i, j,t) safety headways g and h, and $Cap(i, j,t)$, the basic safety headway constraints are simply modeled by constraints (13) and (14). The additive structure of the capacity usage, which is the left part of constraints can break down the initial train scheduling problem into several train-specific sub- problems. This mechanism of decomposition is later used within the Lagrangian relaxation framework.

4.4 Lagrangian Relaxation Solution Framework

In the Lagrangian relaxation framework, capacity constraints of the network are relaxed and resource prices updated by Lagrangian multipliers. The original train scheduling problem is decoupled into a sequence of train-based sub-problems and a label-correcting based algorithm is employed to determine the least time-dependent shortest path and compute the lower bound for each train traversing the network. Lagrangian profits for each of the trains are computed and then a priority-based heuristic algorithm is applied in the construction of feasible solutions and calculate the optimality gap, upon which a termination condition is set. If the condition for termination is met, the algorithm then outputs the feasible solution and computational results at the current iteration. Otherwise, a sub-gradient method is invoked to update Lagrangian multipliers and then moves to the next iteration.

In the LR solution framework depicted in Figure 4-5 below, link capacity constraints are considered as hard constraints and relaxed as a penalty term in the objective function.

$$\min Z = \sum_f \left\{ \left| \sum_t t \times \sum_{i:(i,s_f)\in E^*(S_f)\cap E_f} \left[d_f(i,s_f,t) - d_f(i,s_f,t-1) \right] \right| \right\} + \sum_{i,j} \sum_t \rho_{i,j,t} \times \left\{ \sum_{f:(i,j)\in E_f} y_f(i,j,t) + \sum_{f':(j,i)\in E_{f'}} y_{f'}(j,i,t) - Cap(i,j,t) \right\} \quad (19)$$

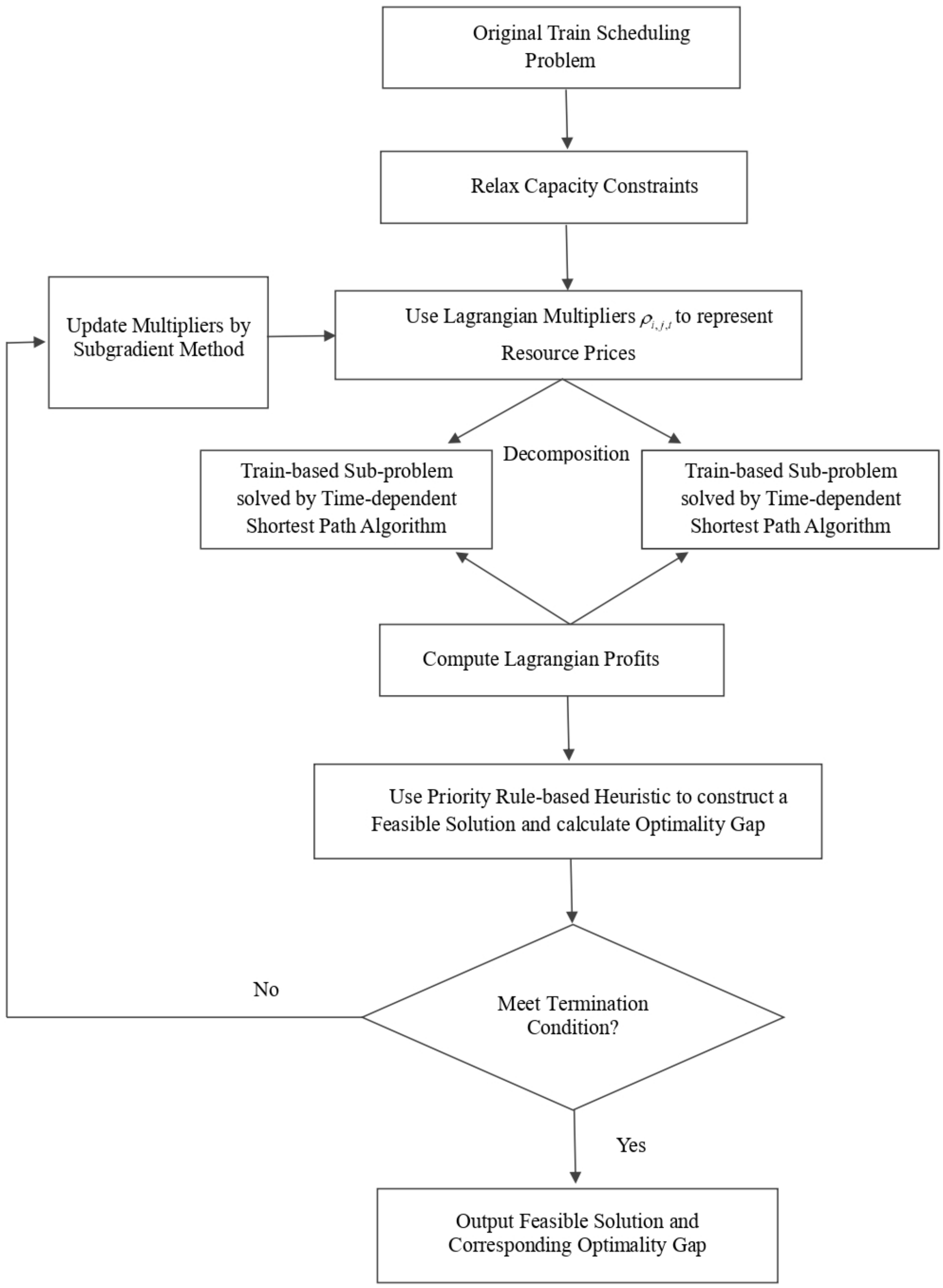


Figure 4-5: Lagrangian relaxation solution framework.

A set of non-negative Lagrangian multipliers $\rho_{i,j,t}$ is introduced, where $\rho_{i,j,t}$ can be construed as the cost incurred for utilizing a resource i.e. link (i, j) at time t; and ρ represents the iteration number. The original train scheduling problem is decomposed into train-based sub-problems LR_f as in equations (20) and (21).

$$\max_{\rho_{i,j,t}} LR = -\sum_{i,j}\sum_{t}\left[\rho_{i,j,t} \times Cap(i,j,t)\right] + \min \sum_{f} LR_f \qquad (20)$$

where,

$$LR_f = \left| \sum_t t \times \sum_{i:(i,s_f)E^a(s_f)\cap E_f} \left[d_f(i,e_f,t) - d_f(i,e_f,t-1) \right] \right| + \sum_{(i,j)\in E_f} \sum_t \left[\rho_{i,j,t} \times y_f(i,j,t) \right] \quad (21)$$

In a sub-problem with train *f*, the goal is to find the time-dependent least generalized cost path for the train, from its node of origin to destination node. The generalized cost comprises the schedule cost and the resource cost. For train *f* running on the network from the origin to the destination nodes, schedule cost denotes the total travel time of train *f*, while the resource cost i.e., the second portion of equation (21) is calculated by summing $\rho_{i,j,t}$ over all selected links within their associated time spans.

A label-correcting time-dependent shortest path algorithm is used to solve each sub- problem. Upon solving train-based sub-problems, Lagrangian profits for each train are computed and the trains are ranked according to decreasing Lagrangian profit values. The Lagrangian profit of each train is the ratio of the total free-flow travel time to the total travel time of the train in the dual solution. A priority rule-based heuristic algorithm is then used to convert dual solutions into feasible solutions. Train priority is determined by the corresponding Lagrangian profits.

The optimality gap at the current iteration is computed based on the dual solutions and feasible solutions, and the algorithm checks whether the condition for termination is met.

The criterion for termination is set as: if $q = Q_{max}$ (a pre-determined maximum number of iterations) then the algorithm ends.

If the condition for termination is met, the algorithm then outputs feasible solutions along with the corresponding quality measures, i.e. optimality gap. Otherwise, the subgradient method is invoked to update Lagrangian multipliers and then move to the next iteration.

The subgradient method iteratively adjusts resource prices by setting;

$$\rho_{i,j,t}^{q+1} = \max\left\{0, \rho_{i,j,t}^{q} + \alpha^q \times \left\{ \sum_{f:(i,j)\in E_f} y_f(i,j,t) + \sum_{f':(j,i)\in E_{f'}} y_{f'}(j,i,t) - Cap(i,j,t) \right\}\right\} \quad (22)$$

Where the superscript q denotes the index of iteration used within the dual updating process, while $\rho^{q}_{i,j,t}$ and α^{q} denote the link multiplier values and the step size at iteration q, respectively. In the optimum search process, the step size parameter is updated as $\alpha^{q} = 1/(q+1)$ and after a specific number of iterations, we stop reducing α^{q}.

4.5 Time-Dependent Shortest Path Algorithm

The framework for the label-correcting algorithm for the solution of the time-dependent least cost path problem is based on an extended time-space network.

In order to compute $\min \sum_{f} LR_f$ in equation (20), the shortest path problem has to be solved through a link-based network $\min G = (N, E)$.

All resource prices i.e., Lagrangian multipliers are presumed to be 0 and after the label correcting process in step 2, each vertex has its least cost label and preceding vertex. Table 4-1 introduces a list of symbols and the shortest path algorithm is then detailed.

Table 4-1: Notations for the time-dependent shortest path algorithm.

Symbol	Description
s	Origin node, corresponding to o_f
r	Destination node, corresponding to s_f
$\Theta(i, t)$	The corresponding node of vertex (i,t)
$\lambda_s(j,t)$	The least cost from vertex (s, EST_f) to vertex (j,t)
$\pi_s(j,t)$	The preceding least cost vertex (j, t'), denoted as time-space vertex (i,t)
$\sigma_{i,j}$	Free-flow running time of link (i, j), corresponding to $FT_f(i, j)$
$\Delta_{i,j}(t)$	Waiting time of link (i, j) at time t
$\vartheta_{i,j}(t, t+\sigma_{i,j}+\Delta_{i,j}(t))$	Resource cost of using link (i, j) from time t to $t+\sigma_{i,j}+\Delta_{i,j}(t)$, $\vartheta_{i,j}(t, t+\sigma_{i,j}+\Delta_{i,j}(t)) = \sum_{\xi=t}^{t+\sigma_{i,j}+\Delta_{i,j}(t)} \rho_{i,j,\xi}$
$\Gamma(i,t)$	Set of outgoing vertexes of vertex

Time-dependent shortest path algorithm
Input: Networks G and TSG origin node s (i.e., o_f), destination node r (i.e., s_f), starting time t (i.e., EST_f), and resource cost vector ρ at current iteration.
Output: The least cost path from s to r, at time t.
Step 1: Initialization
Create an empty SE list; Set $\lambda_s(j,t) = \infty, \forall j \in N/\{s\}, t = 1,...,T; \lambda_s(s,t) = 0, \forall t = 1,...,T;$ $\pi_s(s,t) = \varnothing, \forall t = 1,...,T;$ insert the source vertex (s,t) into the SE list.
Step 2: Label updating
While SE list is not empty **do**
Pop up the front vertex from the SE list, denoted by (i,t)

vertex $(j,e') \in \Gamma(i,t)$ **, do For** $t = ESTf$ **to T**
For $\Delta_{i,j}(t) = w_f^{\min}(i,j)$ **to** $w_f^{\max}(i,j)$
If $\Theta(j, t + \sigma_{i,j} + \Delta_{i,j}(t)) = r$ **Then**
Set candidate new cost label by $\lambda'_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t)) = \lambda_s(i,t) + \vartheta_{i,j}(t, t + \sigma_{i,j} + \Delta_{i,j}(t)) + t + \sigma_{i,j} + \Delta_{i,j}(t)$
Else
Set candidate new cost label by $\lambda'_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t)) = \lambda_s(i,t) + \vartheta_{i,j}(t, t + \sigma_{i,j} + \Delta_{i,j}(t))$
End
If $\lambda'_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t)) < \lambda_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t))$
Then
Set node cost label by $\lambda'_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t)) = \lambda_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t))$
Update preceding vertex by setting $\pi_s(j, t + \sigma_{i,j} + \Delta_{i,j}(t))$ to time-space vertex (i,t)
If vertex (j,t') **, i.e., vertex** $(j, t + \sigma_{i,j} + \Delta_{i,j}(t))$ **, has been in the SE list, Then**
Add vertex (j,t^*) to the front of SE list;
Else
Add vertex (j,t) to the back of SE list;
End
End // Updating node cost label End // for each link waiting time
End // for each possible starting time
End // for each vertex
Remove vertex (i,t) from the SE list.
End
Step 3: Fetch the time-dependent shortest path
Step 3.1: Find the vertex (j^*,t^*) corresponding to destination node r and with the least cost;
Set vertex (j^*,t^*)
as the current vertex (k,t) ;
Step 3.2: Backtrack from destination node r to node s;
While vertex (k,t) is not corresponding to the origin node s;
Find the preceding vertex (i,t') of the current vertex (k,t) ;
Update the preceding vertex (i,t') as the current (k,t) .
End
Step 3.3: Reverse the backward path and output the least cost path from s to r a t;
Step 3.4: Terminate the algorithm.

4.6 Priority Rule-Based Algorithm

At each Lagrangian iteration, a feasible solution based on priority rules is constructed to achieve a better upper bound estimate of the optimal solution. The priority rule implementing algorithm is detailed as below.

Priority rule-based implementing algorithm
Input: Network G, train set F, origin node of, destination node sf, earliest departure time $ESTf$ for each train f. **Output:** The routes and passing times at each station for each train f, and the updated upper bound.

Step 1: Train priority ranking
Rank the trains by decreasing values of Lagrangian profits. The Lagrangian profit of each train is the ratio of total free-flow travel time divided by total travel time in the dual solution.
Step 2: Schedule trains one by one
Step 2.1: For the train f^* with the highest priority, apply the shortest path algorithm introduced in Section 3.2.2 to find its route and passing times at each station;
Step 2.2: Fix the route and passing times at each station for train f^*; record the capacity usage of train f^* on network G;
Step 2.3: If all trains have been scheduled, move to Step 3, otherwise, loop back to Step 2.1.
Step 3: Update and output upper bound
Step 3.1: Compute the objective value of the heuristic solution obtained by step 2;
Step 3.2: Update the upper bound using the new objective value;
Step 3.3: Output the route, passing time at each station, and the new upper bound at the current Lagrangian iteration.

5. Case Study on the Mombasa-Nairobi Standard Gauge Railway

In this chapter, the basic information about the Mombasa-Nairobi SGR line is presented in Section 5.1. Section 5.2 presents an overview of stations on this line and information on the traffic demand. Transportation organization on this railway line is presented in Section 5.3. Section 5.4 describes the procedure for solving the train scheduling problem of this line, followed by results and analysis in Section 5.5.

5.1 Mombasa-Nairobi SGR

The Mombasa-Nairobi line is a single-track standard-gauge railway line connecting the Mombasa Port to Nairobi, the capital city of Kenya. The main line is 471.650km and the Mombasa Port Relief Line is 4.795km. This section is the Phase 1 of the proposed East African Railway Network, and its operations commenced in 2017. The second phase is currently under construction to connect Nairobi to Malaba, the border city with Uganda, and further extend to the rest of the countries in East Africa.

Figure 5-1: Mombasa-Nairobi Standard Gauge Railway.

Built to Chinese ‘Class 1’ standards, the line is designed with automatic inter-station blocking system, 25t axle loading, 4000t traction mass and internal combustion traction, with electrification conditions reserved. Passenger trains operate at up to 120 km/h while freight trains operate up to 80 km/h. For a summary of the technical data of the line, see Appendix A.

5.2 Station Overview

In the long-term design, a total of 45 stations are set on this line, of which there are 2 district stations, (Port Reitz and Nairobi Terminus Station), 8 intermediate stations, (Mombasa Terminus, Mariakani, Miasenyi, Voi, Mtito Andei, Kibwezi, Emali and Athi River) and 35 passing stations. However, since the traffic demand on the line is still low, the current short-term network comprises a total of 33 stations, and 12 passing stations (Samburu, Taru, Wangala, Manyani, Tsavo North, Kenani, Kibwezi South, Kiboko, Arroi Ranch, Konza South, Kapiti Plains and Marimbeti) have been reserved to be opened in future.

The stations on this line are transversely arranged. The short-term maximum distance between stations is 23.6km (Miasenyi ~ Maungu). The long-term average distance between stations on this line is 10.61km and the maximum distance between stations is 13.4km (Maungu ~ Ngutini). More details regarding the stations are provided in the Station List in Appendix B.

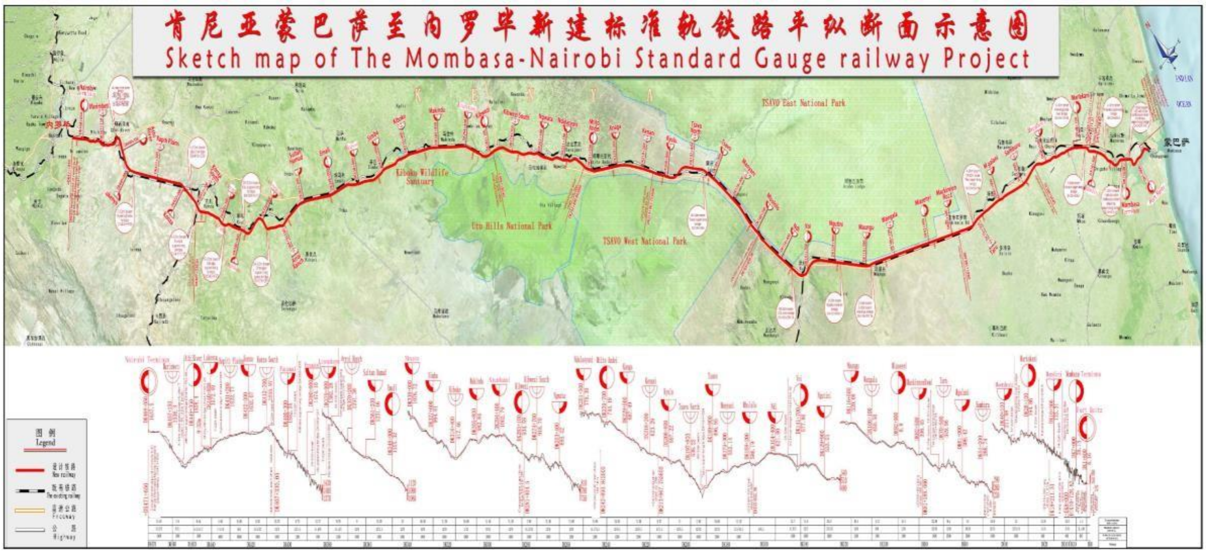


Figure 5-2: Stations along the Mombasa-Nairobi SGR.

Stations are distributed to balance the section passing capacity and to satisfy all kinds of technical operation requirements. The distribution of district and intermediate stations is considered so as to coordinate with the urban or regional planning. They are set around the cities and towns along the line with advanced economy and intensive population, and that which can form the local economic centers in accordance with urban and regional economic development planning. The intermediate stations undertake the passenger and cargo transportation business of this line.

Port Reitz Station is the originating station of this line and it undertakes the arrival/departure operation of section and non-stop trains of container, fuel oil and petroleum products in addition to other bulk supply tariff. A turnaround depot is set to undertake the servicing work of freight locomotive of this line and a train inspection and service point is set to undertake the train inspection operation of the freight rolling stock of this line.

Mombasa Terminus Station sets one locomotive waiting track to undertake the servicing work of passenger locomotive of this line and sets one train inspection and service point to undertake the train inspection operation of the passenger rolling stock of this line.

Nairobi Terminus Station is the terminal station of this line and is also the connection station of this line and the East Africa Railway. It undertakes the sending and receiving operations of non-stop trains of this line and East Africa Railway besides the departure, arrival, breaking-up and sorting operation of section trains of this line and the East Africa Railway. In addition, this station sets one locomotive depot to undertake the servicing work and overhaul operation of the locomotives, one train inspection and service point to undertake the train inspection operation of the freight rolling stock, one rolling stock depot to undertake the maintenance operation of rolling stock of this line and one freight yard to undertake the freight business of this station.

The distance between centers of tracks in station has been executed subject to the relevant provisions in the current Code for Design of Railway Stations and Terminals (GB50091-2006). The routes for receiving-departure tracks are designed as two-way routes, with effective length of 880m.

In terms of signaling, all signals adopt multi-lens color-light signaling. High post signal is used as starting signal of main line and dwarf signal without indicator is adopted as starting signal of receiving-departure track. A route for out-of-gauge freight train is set at all main lines in the station, and another receiving-departure track for out-of-gauge freight train has been set at intermediate stations besides the main line. In addition to the main lines of Tsavo, Makindu and Paranai passing stations, there is another line for out-of- gauge freight train. On Port Reitz and Nairobi Terminus district stations, there are another two arrival-departure tracks for out-of-gauge freight train in addition to the main lines.

5.3 Traffic Demand and Transportation Organization

This line provides freight and regional passenger service and uses the transportation organization pattern of passenger and cargo on the same line. To satisfy the transportation demand of this line, stations are set in main economic strongholds, regions with larger railway handling operation volume and regions handling passenger services, while passing stations are set to satisfy passing capacity needs and guarantee balanced transportation.

Freight flow of this line is mainly the freight flow of container, fuel oil and petroleum products, steel and other large amount of goods dispatched from Port Reitz to Nairobi and beyond, including a small amount of regional freight flow produced in the places along the line.

To meet the local passenger transport demand, two pairs of passenger trains are operated per day in between Mombasa and Nairobi, three pairs per day will be dispatched in 2025 and 4 pairs per day in 2035.

The pair number of passenger and freight trains on this line is shown in Table 5-1 in accordance with the predicted passenger and freight volume in the year of study.

Table 5-1: Pair Number of passenger and freight trains Unit: pairs/day.

Section	2020						2025						2035					
	Direct freight	Container	Section freight	District local train	Passenger train	Total	Direct freight	Container	Section freight	District local train	Passenger train	Total	Direct freight	Container	Section freight	District local train	Passenger train	Total
Port Reitz-Mombasa Terminus	2.5	4	1	0.5	0	8	4	7	3	0.5	0	14.5	7.5	9	5.5	0.5	0	22.5
Mombasa Terminus-Athi River	3.5	4	1	0.5	2	11	5	7	4	0.5	3	19.5	10.5	9	5.5	0.5	4	29.5
Athi River-Nairobi Terminus	2	4	1	0.5	2	9.5	3	7	4	0.5	3	17.5	7	9	5.5	0.5	4	26

The train plans for planning years 2020 and 2025 obtained from the Kenya Railways Corporation (KRC) feasibility study reports are used to capture the traffic demand on the line for the corresponding planning years and used in the determination of the feasible train schedules. The passenger and freight flow diagram is shown on Appendix B.

5.4 Solution of the Train Scheduling Problem

In this section, the formulated model and solution approach proposed is applied in solving the train scheduling problem of the Mombasa-Nairobi SGR line.

The physical network was first constructed in NEXTA-Rail Network Editor, with 33 nodes corresponding to the stations operational at the current period (short-term) and then with 45 nodes corresponding to the 45 stations in the long-term. The data for the nodes and links in the constructed networks is presented in Appendix B.

The model was then implemented in FastTrain on a 1.61GHz Intel(R) Core(TM) m3- 7Y30 CPU with 4 GB of RAM. The current and long-term networks were used and different number of trains were considered in each case (from 2 to 36) with a planning horizon, T = 1440 min. The input into the Solution Engine included the train information such as the origin and destination nodes, the speed multiplier to define the speed of different trains, intended train start time, cost per unit time the train is stopped, cost per unit time the train is running, weight of train, length of train and the type of goods.

The program was allowed to terminate after 10000 iterations, before which feasible solutions were found in all instances. The solution engine then output results with their corresponding quality measures.

5.5 RESULTS AND ANALYSIS

The output results provided values of the total travel time, total resource price, total trip time, computational time, upper bound (UB) and lower bound (LB) values with a corresponding optimality gap. The lower bound and upper bound tend to become better with increasing number of Lagrangian iterations. Considering the total travel time as the objective value, feasible results with the least optimality gap are considered. In the analysis, train schedules for the current (short-term) and long-term networks are obtained and compared.

Table 5-2 below shows the total travel times, upper bounds, lower bounds and the corresponding optimality gap for different number of trains when scheduled in both the current and the long-term networks.

Table 5-2: Results.

No. of Trains	Current network (33 stations)					Long-term network (45 stations)				
	Total travel time(min)	Lower Bound	Upper Bound	Op.Gap	Comp. Time(s)	Total Travel time (min)	Lower Bound	Upper Bound	Op.Gap	Comp. Time(s)
2	734	735.40	1019	0.278	1	729	731.00	1026	0.288	0
4	1616	1554.00	2432	0.361	0	1595	1570.00	2449	0.359	1
6	2384	2286.00	3325	0.312	0	2334	2308.00	3289	0.298	1
8	3306	3473.75	4546	0.236	126	3224	3395.50	4497	0.245	39
10	4260	4501.79	5617	0.195	116	4244	4409.14	5464	0.201	472
12	4956	5411.69	6545	0.173	114	5013	5263.38	6464	0.186	369
14	5821	6526.60	7546	0.147	442	5770	6252.32	7616	0.179	624
16	6674	7520.12	8914	0.156	797	6544	7135.62	8912	0.199	925
18	7548	8674.63	9963	0.129	334	7614	8202.42	10412	0.212	657
20	8790	9863.06	11277	0.125	664	8512	9297.23	11463	0.189	862
22	9617	11112.22	12630	0.12	953	9141	10387.87	12443	0.165	570
24	10608	12134.01	13902	0.165	781	10557	11515.52	13828	0.199	199
26	11123	13402.29	15375	0.126	1151	11597	12478.87	14490	0.139	587
28	-	-	-	-		12106	13604.46	16507	0.176	1062
30	-	-	-	-		12548	14710.81	16831	0.126	178
32	-	-	-	-		14434	15920.01	19394	0.179	1137
34	-	-	-	-		16336	17657.84	20552	0.141	1243
36	-	-	-	-		16230	18255.66	21766	0.161	2598

5.5.1 Model Performance and Solution Quality

The solution approach in FastTrain provides better results with the increasing number of Lagrangian iterations. The optimality gap, the lower bound and the upper bound values tend to converge with increasing computational time. The optimality gap is the relative difference between the upper bound value and the lower bound value of the solution, and it is given by the equation: Optimality Gap =(Upper bound - Lower bound)/Upper bound.

Given enough computational time i.e. number of iterations, the solution engine can converge towards more accurate solutions. However, in the solution of the problem in this case study, a maximum of 10000 iterations is allowed after which the program terminates. The hardest problem in the network current with 33 stations comprising 26 trains was solved in 19 min 11 s with an optimality gap of 12.6%; while the hardest problem in the long-term network with 45 stations comprised 36 trains and was solved in 43 min 18 s with an optimality gap of 16.1%, which is within acceptable time for a train scheduling problem.

The number of trains scheduled within a given time horizon has an effect on the solution quality. In the two networks, the program outputs feasible results after a few iterations and does not find better solutions even after 10000 iterations, and this explains why the optimality gap for the first 8 trains is above 20%. Figure 5-3 below shows how the optimality gap is affected by the traffic demand on both networks.

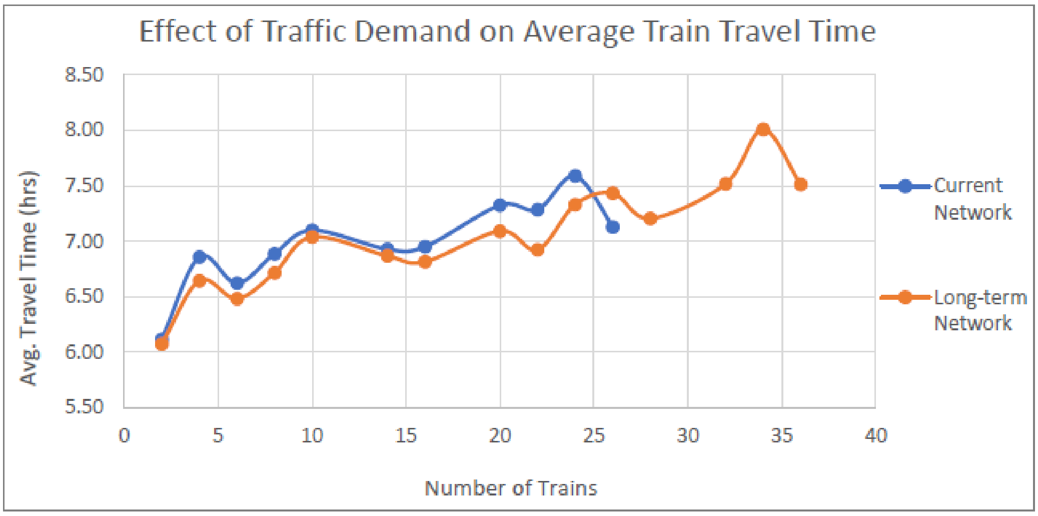


Figure 5-3: Effect of Travel Demand on Optimality Gap.

With an increase in the scheduled number of trains, the optimality gap decreases sharply at first and then with more trains, the change is not very significant. For any number of trains scheduled, the optimality gap for the long-term network appears to be higher than the one for the current network, and the difference tends to increase with an increase in number of trains. This is due to increasing number of constraints.

5.5.2 Impact of Traffic Demand and Number of Stations on the Average Train Travel Time

The average time taken by a train to traverse the network is affected by the number of trains scheduled in a given time horizon. In the case under consideration, for an increase in number of trains scheduled, there is a corresponding increase in the average train travel time. For instance, the average train travel time for 6 trains scheduled in the long-term network between Mombasa and Nairobi is 6 h 37 min as opposed to 8 h for 34 trains scheduled in the same network over the same time horizon.

Figure 5-4 below shows the impact of traffic demand and additional sidings on the average train travel time.

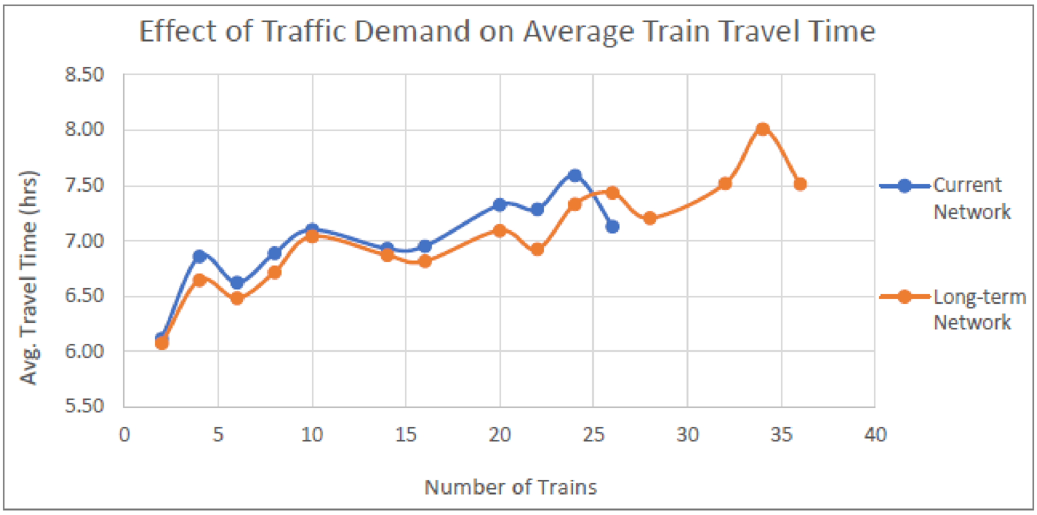


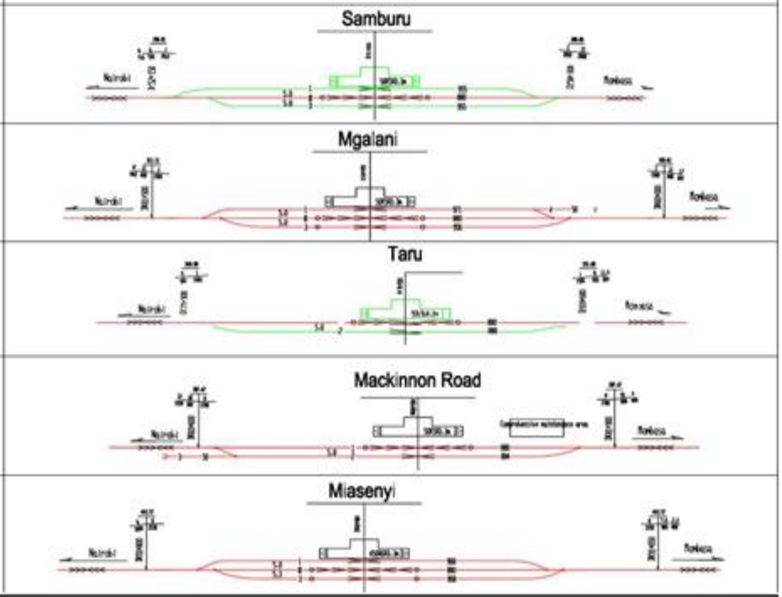
Figure 5-4: Effect of Traffic Demand on Average Train Travel Time.

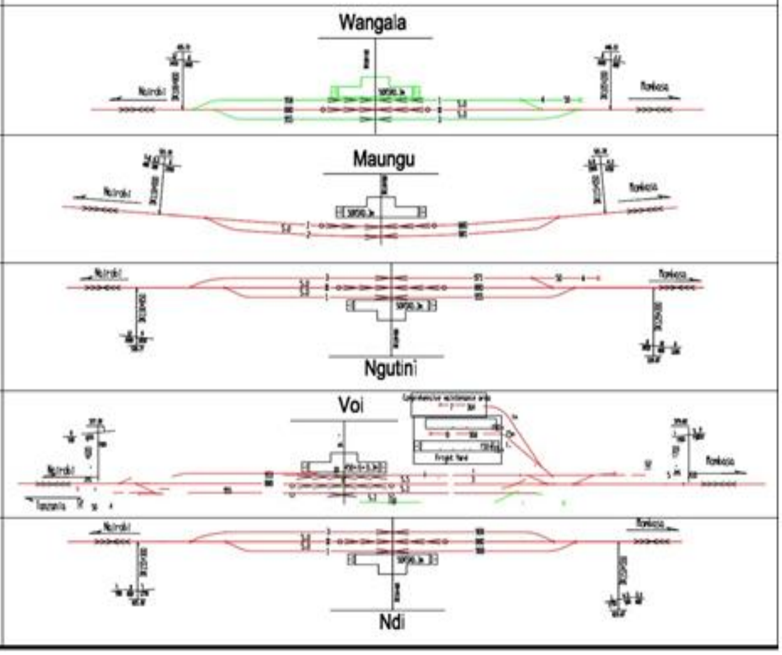
As can be noted from Figure 5-4 above, for any given number of trains, the corresponding average travel time on the current network is generally higher than corresponding average train travel time on the long-term network with more sidings. In addition, for the current network, feasible schedules could only be obtained for a maximum of 26 trains in the given time horizon, while more trains could be scheduled in the long-term network.

Appendix

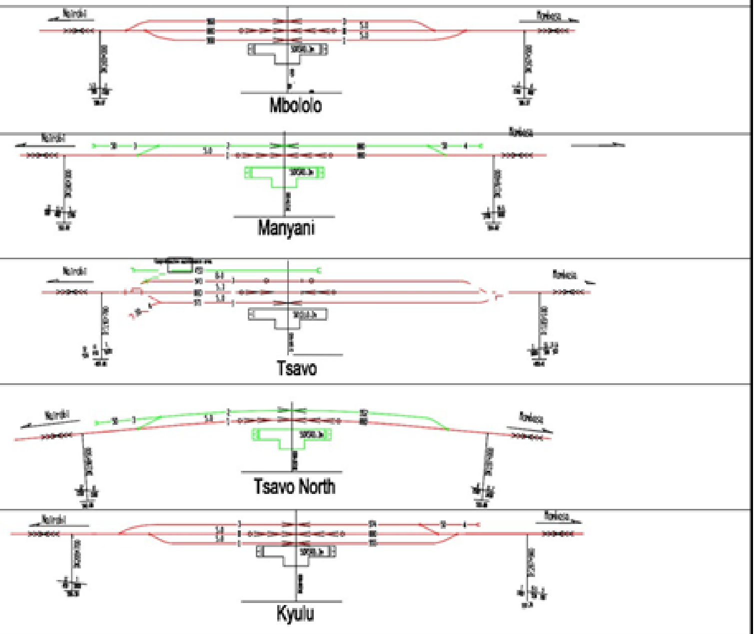
Appendix A: Station List

Destination Station Order	Station Name	Station Centre Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
6	Samburu	DK51+000	10.896	Reserved Passing Station		Samburu
7	Mgalani	DK60+850	9.85	Passing Station	DK60+250 ~ DK61+450	Mgalani
8	Taru	DK70+500	9.50	Reserved Passing Station		Taru
9	Mackinnon Road	DK82+500	12.289	Passing Station	DK81+900 ~ DK83+200	Mackinnon Road
10	Miaseny	DK92+900	10.40 13.20	Intermediate Station	DK92+200 ~ DK93+500	Miasenyi
11	Wangala	DK106+100	13.20	Reserved Passing Station		Wangala
12	Maungu	DK116+500	10.40	Passing Station	DK115+900 ~ DK117+100	Maungu
13	Ngutini	DK129+900	13.40	Passing Station	DK129+250 ~ DK130+550	Ngutini
14	Voi	DK141+700	11.80	Intermediate Station	DK140+700 ~ DK142+200	Voi
15	Ndi	DK154+400	12.70 13.359	Passing Station	DK153+700 ~ DK155+100	Ndi

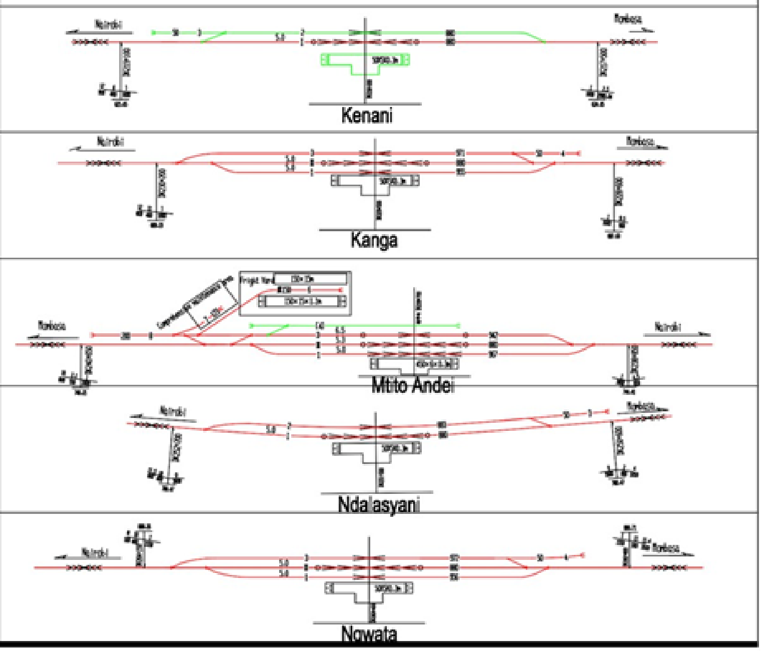


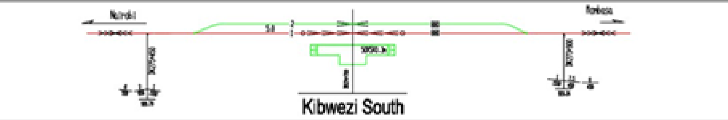
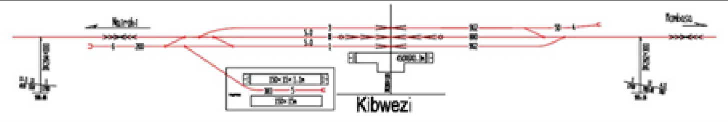
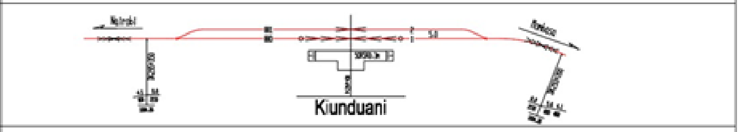
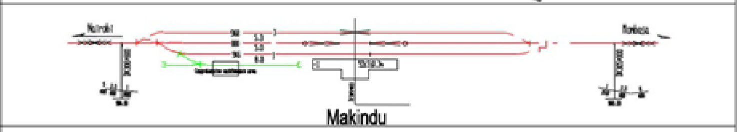
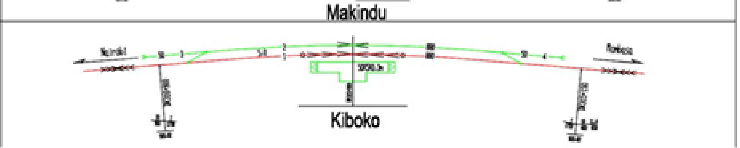


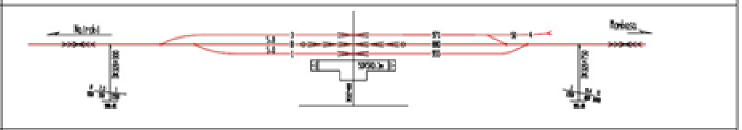
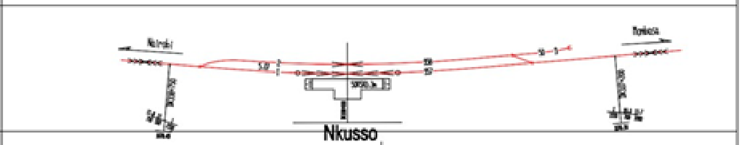
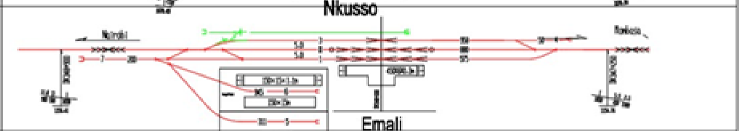
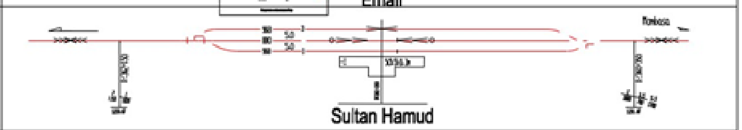
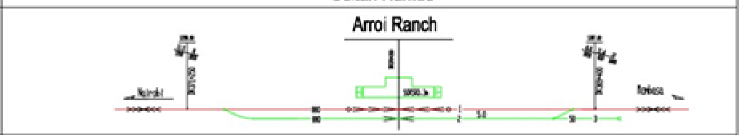
Destination Station Order	Station Name	Station Central Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations	Remarks
16	Mbololo	DK168+200	13.359	Passing Station	DK167+500 ~ DK168+800	Mbololo	
			11.10				
17	Manyani	DK179+300		Reserved Passing Station		Manyani	
			10.60				
18	Tsavo	DK189+900		Passing Station	DK189+350 ~ DK190+560	Tsavo	
			7.90				
19	Tsavo North	DK197+800		Reserved Passing Station		Tsavo North	
			11.00				
20	Kyulu	DK208+800	9.389	Passing Station	DK208+200 ~ DK209+450	Kyulu	

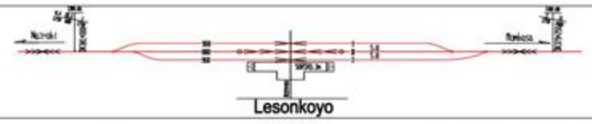
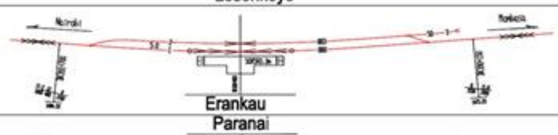
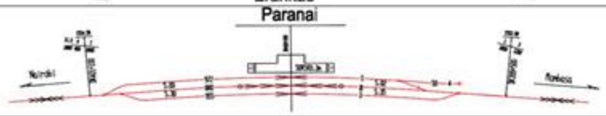
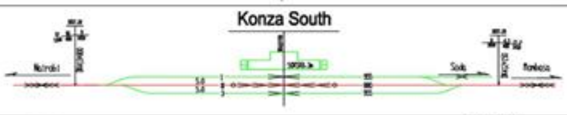
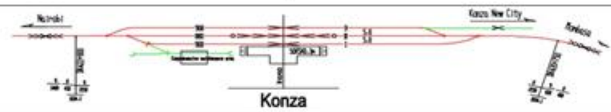
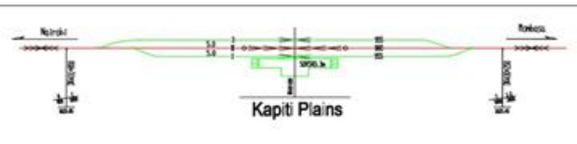
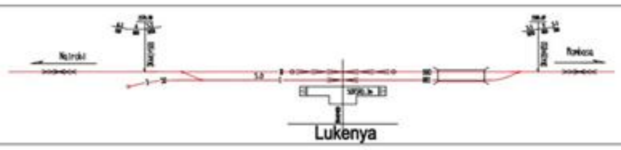
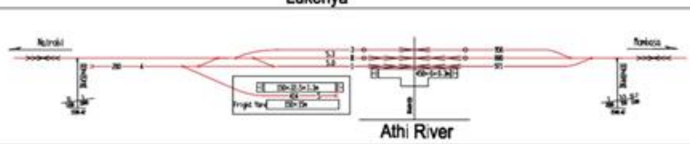
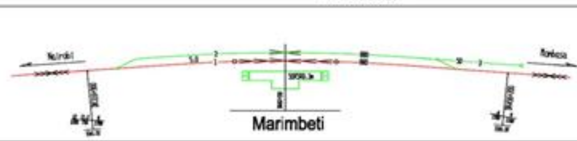


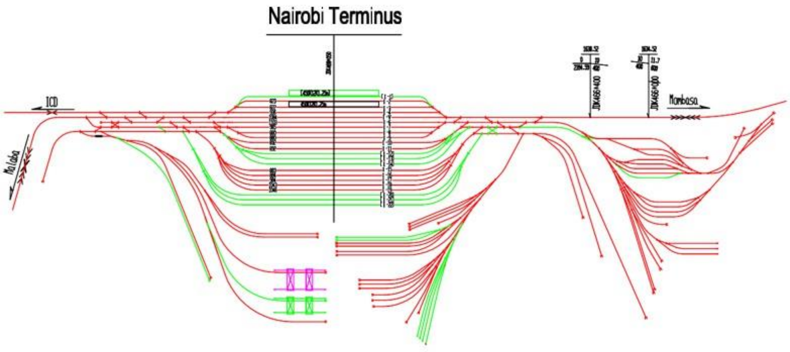
Destination Station Order	Station Name	Station Central Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
21	Kenani	DK218+200	9.389	Reserved Passing Station		Kenani
			11.30			
22	Kanga	DK229+500		Passing Station	DK228+850 ~ DK230+150	Kanga
			10.20			
23	Mtito Andei	DK239+700		Intermediate Station	DK239+100 ~ DK240+650	Mtito Andei
			11.703			
24	Ndalasyani	DK251+500		Passing Station	DK250+800 ~ DK251+950	Ndalasyani
			11.90			
25	Ngwata	DK263+400	11.30	Passing Station	DK262+850 ~ DK264+100	Ngwata



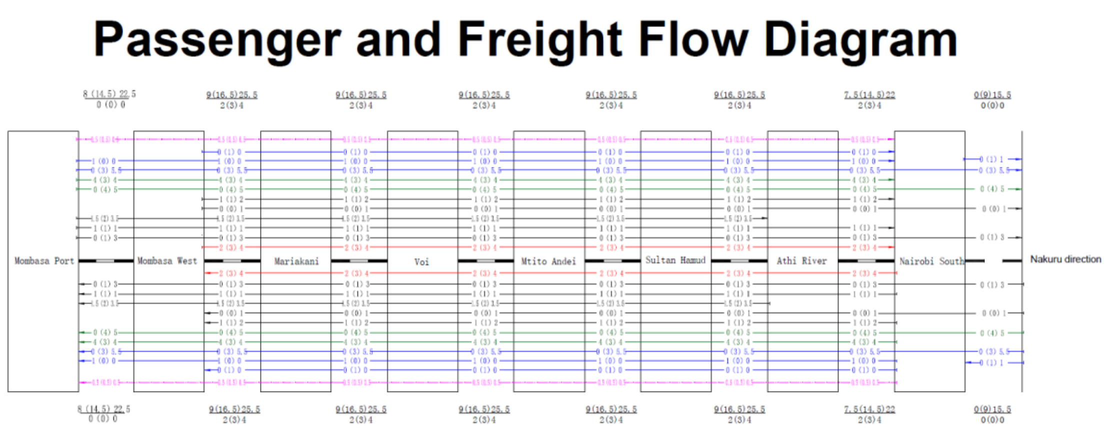
Destination Station Order	Station Name	Station Central Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
26	Kibwezi South	DK274+700	11.30	Reserved Passing Station		
27	Kibwezi	DK283+100	8.02	Intermediate Station	DK282+550 ~ DK284+050	
28	Kiunduani	DK294+400	11.30	Passing Station	DK293+800 ~ DK294+950	
29	Makindu	DK305+800	11.40	Passing Station	DK305+300 ~ DK306+600	
30	Kiboko	DK315+900	10.10 / 11.70	Reserved Passing Station		

Destination Station Order	Station Name	Station Central Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
31	Simba	DK327+600	11.70	Passing Station	DK327+000 ~ DK328+250	
32	Nkusso	DK338+000	10.40	Passing Station	DK337+500 ~ DK338+700	
33	Emali	DK348+000	10.00	Intermediate Station	DK347+450 ~ DK349+050	
34	Sultan Hamud	DK361+200	13.20	Passing Station	DK360+600 ~ DK361+850	
35	Arroi Ranch	DK370+200	9.00 / 9.70	Reserved Passing Station		

Destination Station Order	Station Name	Station Centre Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
36	Lesonkoyo	DK379+900	9.70 10.70	Passing Station	DK379+300 ~ DK380+550	
37	Erankau	DK390+600	9.70	Passing Station	DK390+200 ~ DK391+350	
38	Paranai	DK400+300	10.759	Passing Station	DK399+750 ~ DK401+000	
39	Konza South	DK412+700	9.30	Reserved Passing Station		
40	Konza	DK422+000	9.20	Passing Station	DK421+500 ~ DK422+800	
41	Kapiti Plains	DK431+200	9.20 9.60	Reserved Passing Station		
42	Lukenya	DK440+800	8.55	Passing Station	DK440+150 ~ DK441+300	
43	Athi River	DK449+350	7.80	Intermediate Station	DK448+750 ~ DK450+300	
44	Marimbeti	DK457+150	11.076	Reserved Passing Station		

Destination Station Order	Station Name	Station Central Mileage	Station Distance (km)	Station Nature	Boundary Mileage of Station Yard and section	Diagram of Stations
45	Nairobi Terminus	DK468+050	11.076	District Station	JDK465+450 = DK465+458.94 ~ JDK470+230	

Appendix B: Passenger and Freight Flow Diagram



Description:

1. "()" in the figure is the pair number of trains at the early stage, "()" is the pair number of trains in recent period, and "()" is the pair number of trains for long term;

2. Numerator is cargo train and denominator is passenger train;

3. Trailing Load is 4000t;

4. Non-stop cargo trains：48； Single deck container trains:57; Double deck container trains:41; Section cargo trains:50.

Legend
- Passenger train
- Non-stop cargo train
- Container cargo train
- Section cargo train
- Pick-up Trains
- Railway line
- Planned railway line

Appendix C: Node and Link Information for Current and Long- Term Networks

Node Information for Current Network

Station name	Node_id	x	y
Port Reitz	1	0.015404	-0.03081
Mombasa	2	2.468671	-0.00341
Mgadini	3	5.100082	-0.00341
Mariakani	4	7.529077	-0.74561
Mwembeni	5	9.958072	-0.2733
Mgalani	6	13.55658	0.941195
Mackinon Rd	7	17.46996	2.425581
Miasenyi	8	19.62907	3.572606
Maungu	9	23.60992	5.461824
Ngutini	10	25.90397	4.652159
Voi	11	28.33297	5.259408
Ndi	12	31.09932	6.001601
Mbololo	13	33.46085	5.056992
Tsavo	14	37.77906	1.75086
Kyulu	15	42.54709	2.448071
Kanga	16	47.3601	2.178183
Mtito Andei	17	50.37385	2.897885
Ndalasyani	18	53.07273	2.448071
Ngwata	19	55.68165	1.953276
Kibwezi	20	58.42552	1.413499
Kiunduani	21	61.39429	1.818332
Makindu	22	64.00321	2.448071
Simba	23	68.68127	3.257736
Nkusso	24	71.29019	3.662569
Emali	25	74.21398	4.202345
Sultan Hamud	26	77.18276	4.02242
Lesonkoyo	27	82.04075	4.292308
Enkarau	28	84.87457	4.922048
Paranai	29	87.43851	4.292308
Konza	30	92.0716	3.167773
Lukenya	31	95.98498	2.268146
Athi River	32	98.50393	1.683388
Nairobi Terminus	33	103.1595	0.806251

Link Information for Current Network

Link name	link_id	from_ node_id	to_ node_id	length (km)	speed_ limit	lane_capacity_ in_vhc_per_hour	link_type
1	1	1	2	5.2	100	1	1
2	2	2	3	12	100	1	1
3	3	3	4	11.8	100	1	1
4	4	4	5	12.1	100	1	1
5	5	5	6	20.7	100	1	1
6	6	6	7	21.8	100	1	1
7	7	7	8	10.4	100	1	1

8	8	8	9	23.6	100	1	1
9	9	9	10	13.4	100	1	1
10	10	10	11	11.8	100	1	1
11	11	11	12	12.7	100	1	1
12	12	12	13	13.4	100	1	1
13	13	13	14	21.7	100	1	1
14	14	14	15	18.9	100	1	1
15	15	15	16	20.7	100	1	1
16	16	16	17	10.2	100	1	1
17	17	17	18	11.7	100	1	1
18	18	18	19	11.9	100	1	1
19	19	19	20	19.3	100	1	1
20	20	20	21	11.3	100	1	1
21	21	21	22	11.4	100	1	1
22	22	22	23	21.8	100	1	1
23	23	23	24	10.4	100	1	1
24	24	24	25	10	100	1	1
25	25	25	26	13.2	100	1	1
26	26	26	27	18.7	100	1	1
27	27	27	28	10.7	100	1	1
28	28	28	29	9.7	100	1	1
29	29	29	30	20.1	100	1	1
30	30	30	31	18.8	100	1	1
31	31	31	32	8.6	100	1	1
32	32	32	33	18.7	100	1	1

Node Information for Long-Term Network

Station name	Node_id	x	y
Port Reitz	1	0.015404	-0.03081
Mombasa Terminus	2	2.468671	-0.00341
Mgadini	3	5.100082	-0.00341
Mariakani	4	7.529077	-0.74561
Mwembeni	5	9.958072	-0.2733
Samburu	6	12.51921	0.488428
Mgalani	7	14.84139	1.687677
Taru	8	17.22898	3.054094
Mackinnon Road	9	19.55116	4.173392
Miasenyi	10	22.23312	4.827528
Wangala	11	24.86057	5.372641
Maungu	12	27.67335	5.765122
Ngutini	13	30.61696	5.350836
Voi	14	33.25531	5.917754
Ndi	15	35.42486	4.783919
Mbololo	16	37.67072	3.606475
Manyani	17	39.54591	2.603467
Tsavo	18	41.59553	1.709482
Tsavo North	19	43.66696	0.902714
Kyulu	20	46.30531	1.469632

Kenani	21	49.20531	1.229782
Kanga	22	51.69103	1.426023
Mtitio Andei	23	54.26396	1.774895
Ndalasyani	24	56.35719	1.360609
Ngwata	25	58.62486	0.946323
Kibwezi South	26	61.29955	0.553842
Kibwezi	27	63.84705	0.924519
Kiunduani	28	67.05231	0.597451
Makindu	29	69.96322	0.597451
Kiboko	30	72.90683	0.989932
Simba	31	75.6869	1.644068
Nkusso	32	78.43427	2.23279
Emali	33	81.4433	2.527151
Sultan Hamud	34	84.1416	2.724755
Arroi Ranch	35	86.551	3.814981
Lesonkoyo	36	88.84048	3.61874
Enkarau	37	91.3589	3.71686
Paranai	38	93.35401	3.422499
Konza South	39	95.63259	2.572123
Konza	40	97.90026	1.917987
Kapiti Plains	41	100.1025	1.743551
Lukenya	42	103.0025	1.329265
Athi River	43	104.9649	0.838664
Marimbeti	44	106.96	0.249942
Nairobi Terminus	45	110.089	0.042799

Link Information for Long-Term Network

Link name	link_id	from_ node_id	to_ node_id	length (km)	speed_ limit	lane_capacity_ in_vhc_per_hour	link_type
1	1	1	2	5.2	10	1	1
2	2	2	3	12	10	1	1
3	3	3	4	11.	10	1	1
4	4	4	5	12.	10	1	1
5	5	5	6	10.	10	1	1
6	6	6	7	9.9	10	1	1
7	7	7	8	9.5	10	1	1
8	8	8	9	12.	10	1	1
9	9	9	10	10.	10	1	1
10	10	10	11	13.	10	1	1
11	11	11	12	10.	10	1	1
12	12	12	13	13.	10	1	1
13	13	13	14	11.	10	1	1
14	14	14	15	12.	10	1	1
15	15	15	16	13.	10	1	1
16	16	16	17	11.	10	1	1
17	17	17	18	10.	10	1	1
18	18	18	19	7.9	10	1	1
19	19	19	20	11	10	1	1

20	20	20	21	9.4	10	1	1
21	21	21	22	11.	10	1	1
22	22	22	23	10.	10	1	1
23	23	23	24	11.	10	1	1
24	24	24	25	11.	10	1	1
25	25	25	26	11.	10	1	1
26	26	26	27	8	10	1	1
27	27	27	28	11.	10	1	1
28	28	28	29	11.	10	1	1
29	29	29	30	10.	10	1	1
30	30	30	31	11.	10	1	1
31	31	31	32	10.	10	1	1
32	32	32	33	10	10	1	1
33	33	33	34	13.	10	1	1
34	34	34	35	9	10	1	1
35	35	35	36	9.7	10	1	1
36	36	36	37	10.	10	1	1
37	37	37	38	9.7	10	1	1
38	38	38	39	10.	10	1	1
39	39	39	40	9.3	10	1	1
40	40	40	41	9.2	10	1	1
41	41	41	42	9.6	10	1	1
42	42	42	43	8.6	10	1	1
43	43	43	44	7.8	10	1	1
44	44	44	45	10.	10	1	1

6 CONCLUSION

In this thesis, the train scheduling problem was formulated as a variable-based cumulative flow model for simultaneous routing and scheduling trains on a single-track railway network, and applied on the Mombasa-Nairobi Railway line as a case study. By the use of a vector of cumulative flow variables to reformulate the network infrastructure capacity, the model allowed the decomposition of the initial complex train scheduling problem into a sequence of multiple single-train sub-problems in order to optimize the routes and passing time of at every station along the selected route of each train.

The physical railway network of the section under study was first constructed in NEXTA-Rail Network Editor, with 33 nodes corresponding to the stations operational at the current period (short-term) and then with 45 nodes corresponding to the 45 stations in the long-term. An open-source software FastTrain, which combines a Lagrangian relaxation framework with an effective time-dependent shortest path algorithm and a priority rule-based implementing algorithm was used in the solution of the variable-based cumulative flow model, outputting feasible solutions with corresponding quality measures within reasonable time. The hardest problem comprising 36 trains was solved in 43 min 18 s, with optimality gap of 16.1%, which is within acceptable time range for a train scheduling problem.

The model presented in this study can be used as a reliable train scheduling tool for medium to large-scale networks as well as a support tool for railroad infrastructure and operations planning. It could also be used to assess the impact on the train schedule due to increase in traffic demand or constructing more passing stations. In the current network constructed with 33 stations, the average train travel time is generally higher than in the long-term network with 45 stations. The average train travel time also increases with increasing traffic demand in both networks.

In the current network, optimal schedules could only be obtained for a maximum of 26 trains while the long-term network with more passing stations could take 36 trains. The forecast demand for year 2025 is 33 trains per day, which implies that the current short- term network could not support the demand and hence some passing stations should be opened up before 2025. According to the feasibility study report, the traffic demand in the year 2035 is 51 trains per day, and therefore a capacity bottleneck is likely to occur in the long-term period.

The results obtained from this research can be used to inform decision-making on operations and infrastructure planning by transportation planners on this line. In future, a more detailed study on the train scheduling problem for the entire network upon completion can be carried out, as well as research on the ways to increase the capacity of this line.

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