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AUTONOMOUS HUMAN BODY CONTROL, PART XIII: SERUM MAGNESIUM CONTROL USING I-FIRST ORDER AND I-SECOND ORDER COMPENSATORS COMPARED WITH PI CONTROLLER

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ABSTRACT

This paper is the thirteenth in a series of research papers presenting the autonomous control of the human body. It handles the control of serum magnesium concentration using I-first order and I-second order compensators from the second generation of control compensators. The proposed compensators are tuned using multiple approaches including zero/pole cancellation, specific performance characteristics fulfillment and trial and error. The step time response of the control system using the two proposed compensators is presented and compared with using a PI controller from the first generation of PID controllers to control the same patient dynamic characteristics of serum magnesium concentration and the time-based characteristics are compared. The comparison reveals the best controller among the three compensators/controller depending on a graphical and quantitative comparison study.

KEYWORDS: Autonomous human body control, serum magnesium concentration control, I-first order compensator, I-second order compensator, PI controller, compensators/controllers tuning.

INTRODUCTION

Magnesium is one of the human serum elements affecting too many functions in the human body and its increase or decrease causes health problems. Here, in the present research we concentrate on the decrease of magnesium concentration levels (hypomagnesemia) having many symptoms including: tremors, muscle spasms, muscle cramps, abnormal eye movements, fatigue, seizures, delirium and abnormal heart rhythms.[1] Usually, the problem of low or high magnesium concentration is solving during hemodialysis in an open-loop fashion. Here, we present a novel technique for closed-loop technique to control the magnesium concentration using two of the second-generation compensators presented by the author to the control engineering technologies since 2014. First of all, we will have a look into some of the international research regarding this important aspect since 1993.

Hutchison et al. (1993) examined the effect of a reduced calcium/magnesium dialysis flux on calcium/magnesium mass transfer in 1.36 % and 3.86 % glucose solutions. They concluded that this fluid formulation reduced hypercalcemia and hypermagnesemia in peritoneal dialysis patients.[2] Elsharkawy, Youssef and Zahoon (2006) outlined that magnesium plays a role of maintaining myocardial electrical stability in hemodialysis patients. They investigated the intradialytic changes and serum magnesium in chronic hemodialysis patients with different hemodialysis modalities. They found that magnesium level increased in the bicarbonate dialysate group from 2.73 at zero time to 5.73 mg/dL at 4 hr.[3]

Canningham, Rodriguez and Messa (2012) investigated the effect of various magnesium and calcium dialysate concentrations in hemodialysis and peritoneal dialysis in patients undergoing dialysis. They showed that dialysate magnesium of 0.75 mmol/L caused mild hypermagnesemia while 0.2 and 0.25 mmol/L resulted in serum magnesium levels mostly normal to hypomagnesaemia.[4] Harrith et al. (2016) found that there was a gradual decrease in serum magnesium levels over a two months period. They showed that patients had 3.29 mg/dL serum magnesium level at pre dialysis and 3.22 mg/dL post dialysis.[5] Li et al. (2019) investigated the clearance of magnesium in peritoneal dialysis patients and its influencing factors. They concluded that serum magnesium could be partly cleared by peritoneal dialysis and negatively correlated with the residual renal function and positively correlated with the nutritional status and daily peritoneal protein loss [6]. Garcia et al. (2020) investigated magnesium concentration in hemodialysis patients covering their predictive mortality rate and factors associated with hypomagnesemia and mortality in hemodialysis. They presented a graph for the variation of magnesium concentration with albumin and phosphorous.[7]

Correa et al. (2021) studied the data examining electrolyte changes during and immediately after hemodialysis and their relationships with dialysate prescriptions. They presented a magnesium concentration profile decreasing from 2.4 to 1.8 mg/dL from predialysis to 30 min after dialysis.[8] Kaneko, Ookawara and Morishita (2022) outlined that magnesium is an essential element associated with various physiological functions such as: maintenance of lood pressure, muscle contraction and nerve function. They presented a graphical change of serum potassium concentration with magnesium concentration in patients undergoing peritoneal dialysis and fitted a straight-line relationship for this correlation.[9] Shayanapour et al. (2023) investigated the impact of serum magnesium on incidence of dialysis complications. They outlined that decreasing serum magnesium levels throughout dialysis had a direct correlation with intradialytic hypertension. They concluded that intradialytic muscle cramps, nausea and vomiting had no significant relationship with serum magnesium levels before and after dialysis.[10] Yamada et al. (2024) outlined that conversion of dialysate magnesium from 1.0 to 1.20 mEq/L increased serum magnesium level leading to the increase of magnitude storage among patients undergoing hemodialysis. They presented serum magnesium concentration profile in mg/dL after dialysate conversion (three data points) over a period of 7 months.[11] Ravanshad et al. (2025) assessed magnesium disorders in patients with renal failure undergoing peritoneal dialysis. They outlined that more than 70 % of the patients had hypermagnesemia while only one (out of 91) had hypomagnesemia and diabetes patients had a lower magnesium level than non-diabetic patients.[12]

Serum Magnesium Concentration as a Process

Yamada et al. presented three data points for a hemodialysis patient following dialysate magnesium concentration change from 1.0 to 1.2 mEq/L (from 12.152 to 14.5824 mg/dL) at time of 0, 3 and 7 months (0, 2016 and 4704 hrs.).[11] The three data points was to investigate what is happening for dialysate concentration change and not for transfer function building since the three points will not lead to accurate dynamic transfer function model. To build a transfer function model for the magnesium concentration change, more data points are required which was not available in the published work. A new set of data of 6 points in the time span 0 to 5000 hrs. was designed after plotting the three data points of Yamada et al. using the MATLAB 'plot' command.[13] A 1/2 transfer function model for $G_p(s)$ was fitted to those assigned data using an ITAE performance index[14] and the optimization toolbox of MATLAB.[15] The transfer function gain K_p was assigned as the steady state magnesium concentration change over the dialysate magnesium change (0.23/2.4304 = 0.09463). The fitted model had the transfer function, $G_p(s)$ with an 0.9996 correlation coefficient is given by:

$$G_p(s) = K_p(T_z s + 1)/(T_p s + 1)^2 \quad (1)$$

Where:

$$K_p = 0.09463 \; ; \; T_z = 1750.678 \; h; \; T_p = 876.450 \; h \quad (2)$$

The step time response of this process for an step input of 2.4304 mg/dL using Eq.1 and the experimental modified data points is generated using the 'step' command of MATLAM[16] and shown in Fig.1.

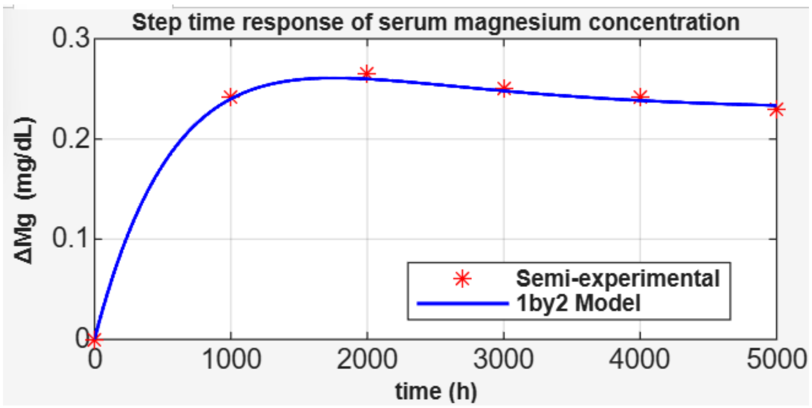

Figure 1: Step time response of the serum magnesium concentration.

The serum magnesium concentration of the hemodialysis patient under investigation as a process has the following time-based characteristics evaluated mostly by the '*stepinfo*' command of MATLAB.[17]

- Maximum percentage overshoot: 11.72 %
- Settling time within ± 2 % tolerance: 4722.10 h
- Rise time: 640.617 h
- Delay time: 246.6 h
- Steady-state error: 2.20 mg/dL

Controlling the Serum Magnesium Concentration using a Conventional PI Controller

For sake of comparison with other controllers/compensators we start the analysis by considering a conventional PI controller from the first generation of PID controllers. A PI controller has the transfer function, $G_{PI}(s)$ given by.

$$G_{PI}(s) = K_{pc1} + (K_{i1}/s) \quad (3)$$

Where: K_{pc1} = proportional gain.

K_{i1} = integral gain.

- The two parameters of the PI controllers are tuned through minimizing an ITAE performance index[14] using the MATLAB optimization toolbox.[15] The tuned PI controller parameters are given by:

$K_{pc1} = 0.0701$, $K_{i1} = 0.0161$ (4)

- Using the block diagram of the control system in a single-loop structure with controller coming after the error detector and cascaded by the controlled process, the transfer function is derived using Eqs.1 and 3 and the step time response of the control system using the step command of MATLAB[16] given in Fig.2 for:
- The normal range of the serum magnesium concentration is: 1.7 to 2.2 mg/dL.[18]
- We consider a desired serum magnesium concentration of 1.8 mg/dL (within the recommended limits). This will be the reference input of all the step time response values for the proposed control compensators.
- Only the lower level of the normal serum magnesium concentration is plotted in the step time responses.
- The left y-axis gives the changes in magnesium concentration and the right y-axis gives the absolute values.

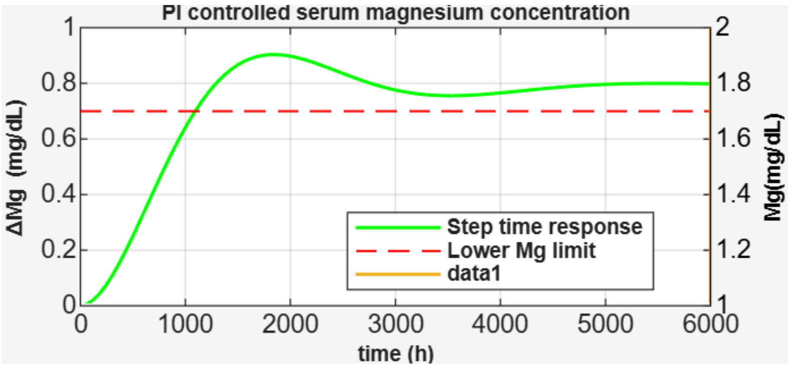

Figure 2: Step time response of the serum magnesium concentration using a PI controller.

COMMENTS

- Maximum percentage overshoot: zero (compared with 11.72 % without control)
- Settling time within ± 2 % tolerance: 4503.4 h (compared with 4722.10 h without control).
- Rise time: 875.6 h (compared with 640.617 h without control).

- Delay time: 420 h (compared with 246.6 h without control).
- Steady-state error: zero (compared with 2.20 mg/dL without control).

Controlling the Serum Magnesium Concentration using an I-first Order Compensator

The I- first order compensator is one of the second generation compensators presented by the author since 2014.[19]-[21] The I- first-order compensator has a transfer function $G_{I1st}(s)$ given by.[19]

$$G_{I1st}(s) = (K_{i2} / s)(T_{z2} s + 1) / (T_{p2} s + 1) \quad (5)$$

The I-first order compensator has three parameters to be tuned: K_{i2}, T_{z2} and T_{p2}. They are tuned as follows.

- The open-loop transfer function of the closed-loop control system of the magnesium concentration control $[G_{I1st}(s)G_p(s)]$ is obtained using Eqs.1 and 5.
- The zero/pole cancellation technique[22] is used to cancel the compensator zero with the process pole and the compensator pole with the process zero. This step reveals the values of the compensator zero and pole as follows.

$$T_{z2} = 876.45 \text{ h and } Tp_2 = 1750.687 \text{ h} \quad (6)$$

- The closed-loop transfer function, $M_2(s) = G_{I1st}(s)G_p(s)/[1+ G_{I1st}(s)G_p(s)]$ for a unit feedback control system. It gives $M_2(s)$ as.

$$M_2(s) = \omega_{n2}^2/(s^2+2\varsigma_2\omega_{n2}s+\omega_{n2}^2) \quad (7)$$

Where: $\omega_{n2}^2 = K_pK_{i2}/T_p \quad , \quad 2\varsigma_2\omega_{n2} = 1/T_p$ (8)

ω_{n2} is the natural frequency and ς is the damping ratio of a second order dynamic system.

- The control system defined by the transfer function in Eq.7 is a standard second-order dynamic system. Such a system has zero maximum overshoot if its damping ratio is unity (critically damped system). With this condition, Eq.8 gives the natural frequency of the system as.

  $\omega_{n2} = 1/T_p = 1/876.45 = 0.00114$ rad/h (9)

- Now, Eq.8 with ωn2 known gives the integral gain K*i2* as:

$K_{i2} = T_p\ \omega_{n2}^2/K_p = 0.012037$ (10)

The step time response of the control system incorporating the I-first order compensator and the magnesium concentration process is drawn using Eqs.7, 9 and 10 and the step command of MTLAB[16] as shown in Fig.3.

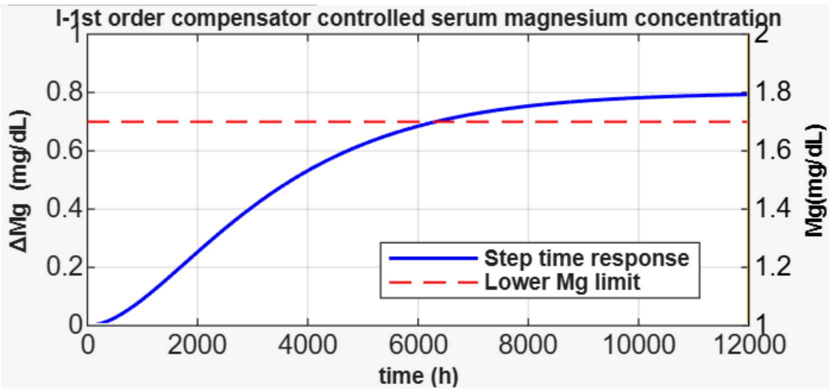

Figure 3: Step time response of the serum magnesium concentration using an I-first order compensator.

COMMENTS

- Maximum percentage overshoot: zero (compared with 11.72 % without control)
- Settling time within ± 2 % tolerance: 10227 h (compared with 4722.10 h without control).
- Rise time: 5886.8 h (compared with 640.617 h without control).
- Delay time: 2880 h (compared with 246.6 h without control).
- Steady-state error: zero (compared with 2.20 mg/dL without control).

Controlling the Serum Magnesium Concentration using an I-second Order Compensator

The I-second order compensator was introduced by the author as one of the second generation compensators introduced by the author since 2014.[23],[24] It is a feedforward compensator set in the forward path just after the error detector (summing point) in a single-loop control system structure. It has the transfer function $G_{I2nd}(s)$ given by.[20]

$$G_{I2nd}(s) = (K_{i3}/s)(T_{z3}s+1)^2/(s^2+b_{13}s+b_{23}) \quad (11)$$

This compensator has for gain parameters: K_{i3}, T_{z3}, b_{13} and b_{23} to be tuned to adjust the performance of the control system for the serum magnesium concentration control. The compensator parameters are tuned as follows.

- The open-loop transfer function of the control system, $G_{I2d}(s)G_p(s)$ is obtained using Eqs.1 and 11.
- The zero/pole cancellation technique[22] is applied to cancel the compensator zero with the process pole revealing.

$$T_{z3} = T_p = 876.45 \ h \quad (12)$$

- With few trials for b_{13}, b_{23} and K_{i3}, an excellent performance was obtained with:

$b_{13} = 0.6325 ; b_{23} = 0.10 ; K_{i3} = 0.0001$ (13)

- Using the closed-loop transfer function of the control system and the compensator parameter given by Eqs.12 and 13, the step time response of the control system is obtained using the MATLAB command '*step*'[16] as shown in Fig.4.

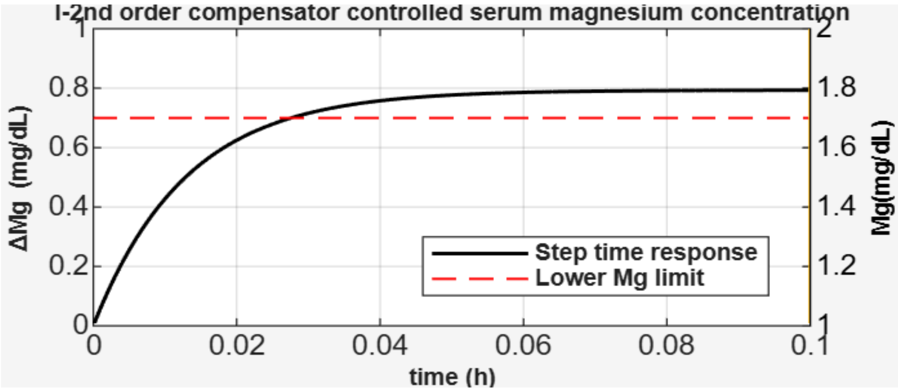

Figure 4: Step time response of the serum magnesium concentration using an I-second order compensator.

COMMENTS

- Maximum percentage overshoot: zero (compared with 11.72 % without control)
- Settling time within ± 2 % tolerance: 0.057 h (compared with 4722.10 h without control).
- Rise time: 0.0286 h (compared with 640.617 h without control).
- Delay time: 0.0095 h (compared with 246.6 h without control).
- Steady-state error: –zero (compared with 2.20 mg/dL without control).

Characteristics Comparison of the Two Compensators Compared with the PI Controllers

- The time-based characteristics of the control system for the serum magnesium concentration are graphically and quantitatively compared in Fig.5 and Table 1 for reference input tracking.

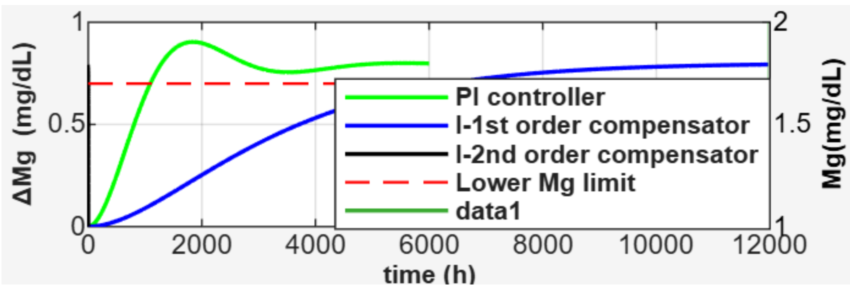

Figure 5: Compared step time response of the serum magnesium concentration.

Table 1: Time-based characteristics of the serum magnesium concentration using PI, I-first order and I-second order compensators.

Characteristics	Mg concentration without closed-loop control	PI controller	I-first order compensator	I-second order compensator
OS_{max} (%)	11.72	12.73	0	0
T_s (h)	4722.1	4303.4	10227	0.0570
T_d (h)	246.6	420	2880	0.0095
T_r (h)	640.617	875.6	5886.8	0.0286
e_{ss} (mg/dL)	2.20	0	0	0

OS_{max}: Maximum overshoot.

T_s: Settling time to 2% tolerance.

T_d: Delay time.

Tr: Rise time.

e_{ss}: steady-state error.

CONCLUSION

- The objective of the paper was to investigate the use and tuning of I-first order and I-second order compensators to control the serum magnesium concentration during hemodialysis with comparison with using a PI controller.
- The serum magnesium concentration process was a stable process having 11.72 % maximum overshoot and 4722.1 h settling time (2 % tolerance band).
- The proposed two compensators were from the second generation of control compensators introduced by the author since 2014 onward.
- The two controllers were tuned without any complex optimization procedures-based techniques but using different tuning techniques based on zero/pole cancellation, desired closed-loop characteristics of the control system and trial and error technique.
- The lower normal serum magnesium concentration level was imposed on the step time response for all the investigated controllers to help in selecting the best controller among the proposed ones and the PI controller.
- The proposed compensators succeeded to eliminate completely the maximum percentage overshoot compared with 12.73 % for the PI controllers.
- All the investigated controllers could eliminate completely the steady-state error.

- The proposed compensators could generate step time responses with settling time to ± 2 % tolerance of 10227 and 0.0570 h compared with 4503.4 h for the PI controller.
- The proposed compensators could generate step time responses with rise time of 2880 and 0.0095 h compared with 875.6 h for the PI controller.
- The I-second order compensator was chosen as the best compensator/controller for the control of `*serum magnesium concentration*' for its perfect time-based characteristics compared with the other compensator/controller.
- Extensive data collection during hemodialysis is required for best controller building in any control scheme for effective human body control.

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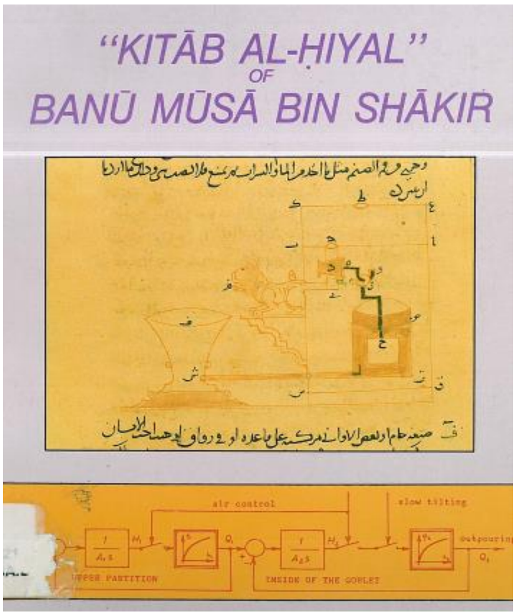

DEDICATION

BANU MUSA BIN SHAKER

- Three brothers born during the rein of the 7th Caliph Al-Ma'mun of the Abbasid Dynasty in Baghdad (813 – 833 AC).
- They were pioneers in mathematics, astronomy and applied mechanics.
- They wrote their '*Kitab al-hial (book of ingenious devices)*' comprising 100 Mechanical designs.
- They are the founders of feedback Control systems through the design and production of about 6 level control systems working with the feedback technology. Therefore, they were the fathers of automatic control in the 9th AC century.
- This is why I dedicate this research work on automatic control to the generous mechanical engineers, '*Banu Musa*'.