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S-I-S COMPARTMENTAL MODELING AND PREDICTIVE ANALYSIS OF EBOLA OUTBREAK IN CENTRAL AND WEST AFRICA.

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ABSTRACT

In this work, Ebola epidemic outbreak cases and deaths were modeled using compartmental model of epidemic mathematics and kinetics. Data were obtained from credible internet sources like Centers for Disease Control (CDC) and World Health Organization (WHO) which were used to validate our model, and the results show that it was almost a perfect model for some countries of West Africa but just good enough for Central Africa as the R^2 shows. The goodness of fit terms of coefficient of correlation (R^2) of the developed model on test data gives excellent validation. For Guinea, R^2 is 0.9997 for both cases and deaths test data. For Liberia, R^2 is 0.9977 for both cases and deaths test data. For Sierra Leone, R^2 is 0.9997 (for cases) and 0.9995 (for deaths data). For Nigeria, R^2 is 0.9914 (for cases data) and 0.9979 (for deaths data). For Central Africa, R^2 is 0.9343 (for cases data) and 0.9304 (for deaths data). The response plot of the cumulative cases and deaths as well as the monthly cases and deaths between the three worst-hit countries, namely (Liberia, Guinea. Sierra Leone) show very serious interaction since the EVD is spread by bats, gorillas, herbivorous animals that live in the bush and thick forest. We should reduce the rate of contact with the animals and stop eating them especially their carcasses found in the bushes and thickets. These findings can be used in studying future ebola epidemic and pandemic outbreaks by World Health Organization (WHO), Pan American Health Organization (Pan-AHO), Centers for Disease Control (CDC), etc.

KEYWORDS: Modeling, Predictive analysis, ebola virus outbreak, cases, deaths, fatality, West and Central Africa.

NOMENCLETURE

CACC	Central African cumulative cases
CACD	Central African cumulative deaths
CACF	Central African cumulative fatality
LCC	Liberia cumulative cases
LCD	Liberia cumulative deaths
LCF	Liberia cumulative fatality
SCC	Sierra-Leone cumulative cases
SCD	Sierra-Leone cumulative deaths
SCF	Sierra-Leone cumulative fatality
GCC	Guinea cumulative cases
GCD	Guinea cumulative deaths
GCF	Guinea cumulative fatality
NCC	Nigeria cumulative cases
NCD	Nigeria cumulative deaths
NCF	Nigeria cumulative fatality
EVD	Ebola Virus Diseases
SIS	susceptible Infected susceptible
EHF	Ebola hemorrhagic fever

1.0 INTRODUCTION

1.1 BACKGROUND INFORMATION

Infections with Ebola viruses originating from Africa cause a severe disease in humans, Ebola virus disease (EVD). Since the first documented EVD outbreak in Zaïre (now: the Democratic Republic of Congo) in 1976, five species of the genus Ebola virus (Filoviridae family) have been identified from samples collected from humans and non-human primates during outbreaks of the disease: Zaïre Ebola virus (EBOV), Sudan Ebola virus, Reston Ebola virus, Taï-Forest Ebola virus and Bundibugyo Ebola virus (Bustillo et al., 2014; CDC, 2014). Ebola viruses and Marburg virus, another member of the Filoviridae family, are classified as biosafety level 4 pathogens (BSL-4; risk group 4) and require special containment measures and barrier protection, in particular for healthcare workers. The map below (Fig 1) presents

the geographical distribution of Ebola outbreaks from 1976 to 2011 in Africa (CDC 2015). The advanced world was not taking the outbreak serious because they thought it's an African problem, an epidemic that span from 1986 to 2012 was not taken serious by the advanced world until the disease was carried by the animals in the oldest forest in the world (Congo). It did not stop there but began to spread to the advanced countries themselves. The virus is transmitted to people from wild animals and spreads in the human population through human-to-human transmission. Fruit bats of the Pteropodidae family are considered to be the natural host of the Ebola virus. Severely ill patients require intensive supportive care. No licensed specific treatment or vaccine is available for use in people or animals. Ebola first appeared in 1976 in 2 simultaneous outbreaks, in Nzara, Sudan, and in Yambuku, Democratic Republic of Congo. The latter was in a village situated near the Ebola River, from which the disease takes its name (WHO, 2014; Chowell et al., 2004).

Genus Ebolavirus is 1 of 3 members of the Filoviridae family (filovirus), along with genus Marburgvirus and genus Cuevavirus. Genus Ebolavirus comprises of 5 distinct species.

1. Bundibugyo ebolavirus (BDBV)
2. Zaire ebolavirus (EBOV)
3. Reston ebolavirus (RESTV)
4. Sudan ebolavirus (SUDV)
5. Taï Forest ebolavirus (TAFV).

BDBV, EBOV, and SUDV have been associated with large EVD outbreaks in Africa, whereas RESTV and TAFV have not. The RESTV species, found in Philippines and the People's Republic of China, can infect humans, but no illness or death in humans from this species has been reported to date (WHO, 2014; Chowell et al., 2004)..

1.2 TRANSMISSION

Ebola is introduced into the human population through close contact with the blood, secretions, organs or other bodily fluids of infected animals. In Africa, infection has been documented through the handling of infected chimpanzees, gorillas, fruit bats, monkeys, forest antelope and porcupines found ill or dead in the rainforest. Ebola then spread in the community through human-to-human transmission, with infection resulting from direct contact (through broken skin or mucous membranes) with the blood, secretions, organs or other bodily fluids of infected people, and indirect contact with environments contaminated with such fluids. Burial ceremonies in which mourners have direct contact with the body of

the deceased person can also play a role in the transmission of Ebola. Men who have recovered from the disease can still transmit the virus through their semen for up to 7 weeks after recovery from the illness (Nishira and Chowell, 2014; Althaus et al., 2014).

Health-care workers have frequently been infected while treating patients with suspected or confirmed EVD. This has occurred through close contact with patients when infection control precautions are not strictly practiced. Among workers in contact with monkeys or pigs infected with Reston Ebola virus, several infections have been documented in people who were clinically asymptomatic. Thus, RESTV appears less capable of causing disease in humans than other Ebola species (Althaus et al., 2014; Funk and Kumar, 2014). More studies of RESTV are needed before definitive conclusions can be drawn about the pathogenicity and virulence of this virus in humans.

EVD is a severe acute viral illness often characterized by the sudden onset of fever, intense weakness, muscle pain, headache and sore throat. This is followed by vomiting, diarrhoea, rash, impaired kidney and liver function, and in some cases, both internal and external bleeding. Laboratory findings include low white blood cell and platelet counts and elevated liver enzymes (Nishiura and Chowell, 2014). The incubation period, that is, the time interval from infection with the virus to onset of symptoms, is 2 to 21 days.

The problem is that these epidemic or rather pandemic outbreak cases and deaths trends has not been adequately modeled mathematically to really represent the real live situations. It is important to study the outbreak trend so as to mathematically model the process trend and so as to help Ebola nations predict the next outbreak, peak cases and deaths. The objective of this work is therefore to mathematically model the Ebola outbreak trend profile so as to study the characteristics of the peak cases and deaths for Central and West Africa. The scope of this work covers the mathematical modeling and prediction of the profile scenario of the disease outbreak in Central and West Africa, so that the Ebola countries can use this work findings as a guide, and does not include high and advanced mathematical models.

2.0 LITERETURE REVIEW

Ebola virus disease (EVD; also called Ebola hemorrhagic fever, or EHF), or simply Ebola, is a disease of humans and other primates caused by Ebola viruses. Signs and symptoms typically start between two days and three weeks after contracting the virus with a fever, sore throat, muscle pain, and headaches. Then, vomiting, diarrhea and rash usually follow along

with decreased function of the liver and kidneys. At this time some people begin to bleed both internally and externally (WHO and Pan-AHO, 2014). The disease has a high risk of death, killing between 25 and 90 percent of those infected with an average of about 50 percent. This is often due to low blood pressure from fluid loss, and typically follows six to sixteen days after symptoms appear. (Sunit, 2014).

The virus spreads by direct contact with body fluids, such as blood, of an infected human or other animals. This may also occur through contact with an item recently contaminated with bodily fluids. Spread of the disease through the air between primates, including humans, has not been documented in either laboratory or natural conditions (CDC, 2014). Semen or breast milk of a person after recovery from EVD may still carry the virus for several weeks to months. Fruit bats are believed to be the normal carrier in nature, able to spread the virus without being affected by it. Other diseases such as malaria, cholera, typhoid fever, meningitis and other viral hemorrhagic fevers may resemble EVD. Blood samples are tested for viral RNA, viral antibodies or for the virus itself to confirm the diagnosis. (CDC, 2014)
Ebola virus first broke out in Zaire and Sudan in 1976. Mortality rates were put at 88% in Zaire and 66% in Sudan, with 500 cases. In one small village in Zaire, 274 out of 300 people infected in an outbreak died. This disease resurfaced in 1995 according to the Zairian health ministry affecting medical professionals. 170 had died as of May 11th 1995 in response to people fleeing Kikwit and possibly spreading the virus. The government of Zaire attempted to cordon off the city to keep inhabitants from spreading the disease across the countryside and possibly even into the slums of Kinshasa, Zaire's capital. Kikwit was put under quarantine. It was unable, at that period, to contain the spread of the virus even with the support of international medical experts from Belgium, France, and South Africa. Poor health infrastructure, inadequate proactive measures can be attributed to the failure of the government during that time to contain the spread (Pourrut, 2005).

In recent times, the Ebola virus was reported first in Guinea in March 2014. Now the virus has spread to other West African nations like Liberia, Sierra Leone, Nigeria, and Senegal. The major scare of this deadly virus is the number of lives that have already been claimed ranging from 42%-66 %.(WHO, 2014; Salaam-Blyther, 2014). As of September 10th 2014, a total of 2,281 deaths had been recorded out of 4,614 suspected or confirmed cases. One immediate reaction by government authorities was to regulate movement through partial or total closure of borders (Morvan et al., 1999).

2.1 SIGNS AND SYMPTOMS OF EBOLA

The length of time between exposure to the virus and the development of symptoms (incubation period) is between 2 to 21 day. However, recent estimates based on mathematical models predict that around 5% of cases may take greater than 21 days to develop. Symptoms usually begin with a sudden influenza-like stage characterized by feeling tired, fever, weakness, decreased appetite, muscle pain, joint pain, headache, and sore throat. This is often followed by vomiting, diarrhea and abdominal pain (FART, 2015; Goeijenbier et al., 2014). In about half of the cases, the skin may develop a maculopapular rash, a flat red area covered with small bumps, 5 to 7 days after symptoms begin. In some cases, internal and external bleeding may occur, typically beginning in five to seven days after the first symptoms, all infected people showing some decreased blood clotting and bleeding from mucous membranes or from sites of needle punctures as reported in 40–50 percent of cases. This may cause vomiting or coughing out blood, or blood in stool. Bleeding into the skin may create petechiae, purpura, ecchymoses or hematomas (especially around needle injection sites). Heavy bleeding is uncommon; if it occurs, it is usually located within the gastrointestinal tract (Hoenen et al., 2012).

Recovery may begin between 7 and 14 days after first symptoms. Death, if it occurs, follows typically 6 to 16 days from first symptoms and is often due to low blood pressure from fluid loss. In general, bleeding often indicates a worse outcome, and blood loss may result in death. People are often in a coma near the end of life and those who survive often have ongoing muscle and joint pain, liver inflammation, decreased hearing, and may have constitutional symptoms such as feeling tired, continued weakness, decreased appetite, and difficulty returning to pre-illness weight. Additionally, they develop antibodies against Ebola that last at least 10 years, but it is unclear if they are immune to repeated infections. If someone recovers from Ebola, they can no longer transmit the disease. (CDC, 2014.)

2.2 CAUSE

EVD in humans is caused by four of five viruses of the genus Ebolavirus. The four are Bundibugyo virus (BDBV), Sudan virus (SUDV), Taï Forest virus (TAFV) and one simply called Ebola virus (EBOV, formerly Zaire Ebola virus), EBOV species, Zaire Ebola virus, is the most dangerous of the known EVD-causing viruses, and is responsible for the largest number of outbreaks. The fifth virus, Reston virus (RESTV), is not thought to cause disease

in humans, but has caused disease in other primates. All five viruses are closely related to Marburg viruses (Hoenen, 2012; Funk and Kumar, 2014).

2.3 VIROLOGY

Ebolaviruses contain single-stranded, non-infectious RNA genomes, which contain seven genes including 3'-UTR-NP-VP35-VP40-GP-VP30-VP24-L-5'-UTR. The genomes of the five different ebolaviruses (BDBV, EBOV, RESTV, SUDV and TAFV) differ in sequence, the number and location of gene overlaps. As with all filoviruses, ebolavirus virions are filamentous particles that may appear in the shape of a shepherd's crook, of a "U" or of a "6," and they may be coiled, toroid or branched. In general, ebolavirions are 80 nanometers (nm) in width and may be as long as 14,000 nm (Chipaux, 2014; Caihshort and Labiling, 1995).

Their life cycle is thought to begin with a virion attaching to specific cell-surface receptors such as C-type lectins, DC-SIGN, or integrins, which is followed by fusion of the viral envelope with cellular membranes. The virions taken up by the cell then travel to acidic endosomes and lysosomes where the viral envelope glycoprotein GP is cleaved. This processing appears to allow the virus to bind to cellular proteins enabling it to fuse with internal cellular membranes and release the viral nucleocapsid. The Ebolavirus structural glycoprotein (known as GP1,2) is responsible for the virus' ability to bind to and infect targeted cells.(Chippaux, 2014). The viral RNA polymerase, encoded by the L gene, partially uncoats the nucleocapsid and transcribes the genes into positive-strand mRNAs, which are then translated into structural and nonstructural proteins. The most abundant protein produced is the nucleoprotein, whose concentration in the host cell determines when L switches from gene transcription to genome replication. Replication of the viral genome results in full-length, positive-strand antigenomes that are, in turn, transcribed into genome copies of negative-strand virus progeny. Newly synthesized structural proteins and genomes self-assemble and accumulate near the inside of the cell membrane. Virions bud off from the cell, gaining their envelopes from the cellular membrane from which they bud from. The mature progeny particles then infect other cells to repeat the cycle. The genetics of the Ebola virus are difficult to study because of EBOV's virulent characteristics. (Feldmann, 2005; Goeijenbier et al., 2014)

2.4 TRANSMISSION (LIFE CYCLES OF THE EBOLAVIRUS)

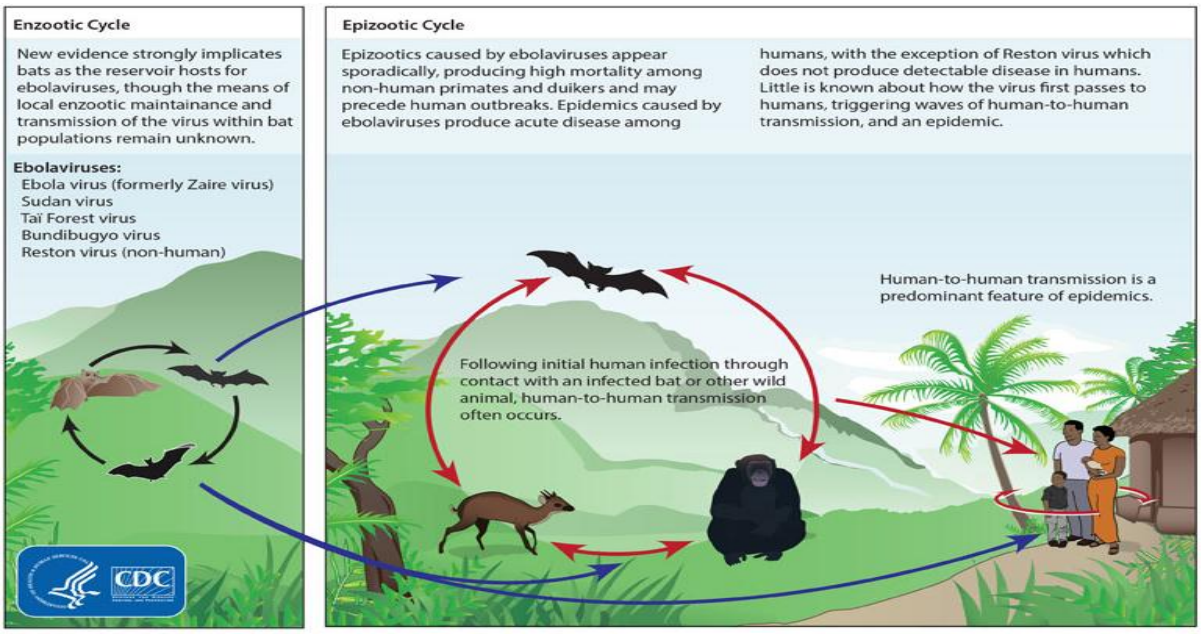


Figure 1: Life cycles of the *Ebolavirus* (CDC, 2015)

Between people, Ebola disease spreads only by direct contact with the blood or body fluids of a person who has developed symptoms of the disease. Body fluids that may contain ebola viruses include saliva, mucus, vomit, feces, sweat, tears, breast milk, urine and semen (Nishiura and Chowell, 2014). Entry points for the virus include the nose, mouth, eyes, open wounds, cuts and abrasions. Ebola may be spread through large droplets; however, this is believed to occur only when a person is very sick, that is if a person is splashed with droplets. Contact with surfaces or objects contaminated by the virus, particularly needles and syringes, may also transmit the infection. The virus is able to survive on objects for a few hours in a dried state, and can survive for a few days within body fluids.

The Ebola virus may be able to persist for up to 8 weeks in the semen after recovery, which could lead to infections via sexual intercourse. Ebola may also occur in the breast milk of women after recovery, and it is not known when it is safe to breastfeed again, otherwise, people who have recovered are not infectious. The potential for widespread infections in countries with medical systems capable of observing correct medical isolation procedures is considered low and usually when someone has symptoms of the disease, they are unable to travel without assistance. Dead bodies remain infectious; thus, people handling human remains, in practices, such as, traditional burial rituals or more modern processes, such as, embalming are at risk; 69% of the cases of Ebola infections in Guinea during the 2014 outbreak are believed to have been contracted via unprotected (or unsuitably protected)

contact with infected corpses during certain Guinean burial rituals (Althaus et al., 2014). Health-care workers treating people with Ebola are at greatest risk of infection as the risk increases when they do not have appropriate protective clothing such as masks, gowns, gloves and eye protection; do not wear it properly; or handle contaminated clothing incorrectly. There has been transmission in hospitals in some African countries that reuse hypodermic needles or health-care centers caring for people with the disease but do not have running water (Formenty et al., 2014). Human-to-human transmission of EBOV through the air has not been reported to occur during EVD outbreaks, and airborne transmission has only been demonstrated in very strict laboratory conditions, and, then, only from pigs to primates, but not from primates to primates has been effective (FART, 2015).

2.5 2013 TO 2015 WEST AFRICAN OUTBREAK

In March 2014, the World Health Organization (WHO) reported a major Ebola outbreak in Guinea, a western African nation as researchers traced the outbreak to a two-year-old child who died December 2013 and the disease then rapidly spread to the neighboring countries of Liberia and Sierra Leone. It is the largest Ebola outbreak ever documented, and the first recorded in the region, In a 26 September statement, the WHO said, "The Ebola epidemic ravaging parts of West Africa is the most severe acute public health emergency seen in modern times. Never before in recorded history has a biosafety level four pathogen infected so many people so quickly, over such a broad geographical area, for so long." As of 14 March 2015, 24,701 suspected cases and 10,194 deaths had been reported; however, the WHO has said that these numbers may be underestimated (WHO and Pan-AHO, 2014; CDC, 2015)

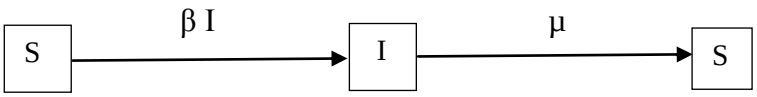
On 8 August 2014, the WHO declared the epidemic to be an international public health emergency and urging the world to offer aid to the affected regions, the Director-General said, "Countries affected to date simply do not have the capacity to manage an outbreak of this size and complexity on their own. I urge the international community to provide this support on the most urgent basis possible." By mid-August 2014, Doctors Without Borders reported the situation in Liberia's capital Monrovia as "catastrophic" and "deteriorating daily". They reported that fears of Ebola among staff members and patients had shut down much of the city’s health system, leaving many people without treatment for other conditions. By late August 2014, the disease had spread to Nigeria, and one case was reported in Senegal (WHO, 2014; Althaus, 2014). On 30 September 2014, the first confirmed case of Ebola in the United States was diagnosed, The patient died 8 days later.(Bustillo, 2014). On 29 December

2014 the first case was confirmed in the United Kingdom. Aside from the human cost, the outbreak has severely eroded the economies of the affected countries. A Financial Times report suggested that the economic impact of the outbreak could kill more people than the virus itself. As of 23 September, in the three hardest hit countries, Liberia, Sierra Leone and Guinea, only 893 treatment beds were available even though the then current need was 2122 beds. On 23 October, the Malian government confirmed its first case and, in response, UNMEER, in cooperation with the Logistics Cluster, air-lifted 1,050 kg of personal protective equipment (PPE) and body bags from Monrovia to Mali.(Salaam-Blyther, 2014).

2.6 2014 DRC CONGO OUTBREAK

An outbreak in Boende District in Equatorial Province was stopped effectively with flexible organization and funding, as well as social mobilization led by UNICEF advising action people could use. The DRC recent outbreak was from a local Ebola strain and not the one from West Africa (WHO and Pan-AHO, 2014).

3.0 DEVELOPMENT OF S-I-S COMPARTMENTAL MODEL



This is a case where a susceptible individual becoming infected is treaded, and then returns back to the susceptible population status as a compartment (Kamalu and Okolie, 2013)

$$\frac{dS}{dt} = -\beta SI + \mu I \quad (1)$$

$$\frac{dI}{dt} = \beta SI - \mu I \quad (2)$$

$$S + I = 1 \quad (3)$$

Putting $S = 1 - I$ into eqn (2)

$$\frac{dI}{dt} = [\beta(1 - I) - \mu]I = (\beta - \mu - \beta I)I$$

$$dt = \int \frac{dI}{(\beta-\mu-\beta I)I} \equiv \int\left(\frac{dI}{I} + \frac{dI}{\beta-\mu-\beta I}\right)$$

$$\frac{I}{(\beta - \mu - \beta I)I} \equiv \frac{A}{I} + \frac{B}{\beta - \mu - \beta I}$$

$$I \equiv A(\beta - \mu - \beta I) + BI$$

$$\text{If } I = 0: A = \frac{I}{\beta - \mu}$$

$$\text{If } B - \mu - BI = 0: \ I = B\frac{\beta - \mu}{\beta}, B = \frac{\beta}{\beta - \mu}$$

Integrating;

$$dt = \frac{I}{\beta-\mu}\int\frac{dI}{I} + \frac{I}{\beta-\mu}\int\frac{\beta dI}{\beta-\mu-\beta I}$$

$$= \frac{I}{\beta-\mu} lnI - \frac{I}{\beta-\mu}\ln(\beta - \mu - \beta I) = \frac{I}{\beta-\mu}\ln\frac{I}{\beta-\mu-\beta I}$$

$$t - to = \frac{I}{\beta-u}\left[ln\frac{I}{\beta-\mu-\beta I}\right]_{Io}^{I} = \frac{I}{\beta-\mu}\left[ln\frac{I}{\beta-\mu-\beta I} - ln\frac{Io}{\beta-\mu-\beta I}\right] = \frac{I}{\beta-\mu}\ln\frac{(\beta-\mu-\beta Io)}{(\beta-u-\beta I)I}$$

$$= \frac{I}{\theta}\ln\frac{(\theta-\beta Io)I}{(\theta-\beta I)Io} \ ; \ \theta = \beta - \mu$$

Then, $\theta(t-to) = ln\frac{(\theta-\beta Io)I}{(\theta-\beta I)Io}$

or $e^{\theta(t-to)} = \frac{(\theta-\beta Io)I}{(\theta-\beta I)Io}$; if $\psi = e^{\theta(t-to)} = e^{(\beta-\mu)(t-to)}$

Then, $\psi = \frac{(\theta-\beta Io)I}{(\theta-\beta I)Io} => \psi\theta Io - \psi\beta IIo = \theta I - \beta IoI$

or $\psi\theta Io = \theta I + \psi\beta IIo - \beta IIo = I(\theta + \psi\beta Io - \beta Io)$

$$I = \frac{\psi\theta Io}{\theta + \psi\beta Io - \beta Io} = \frac{\theta}{\frac{\theta}{Io\psi}+\beta-\frac{\beta}{\psi}} = \frac{\theta}{\beta+(\frac{\theta}{Io}-\beta)\frac{I}{\psi}} = \frac{\theta}{\beta+\left(\frac{\theta}{Io}-\beta\right)\psi-I}$$

Replacing θ; $I = \frac{\beta-\mu}{\beta+(\frac{\beta-\mu}{Io}-\beta)e^{-(\beta-\mu)(t-to)}}$; $Ro = \frac{\beta}{\mu}$

Still expanding the exponential term, we have;

$$I = \frac{\mu(Ro-I)}{\beta+(\frac{\mu(Ro-I)}{Io}-\beta)e^{-u(Ro-I)(t-to)}} = \frac{\mu(Ro-I)}{\beta+\underbrace{\left(\frac{\mu(Ro-I)}{Io}-\beta\right)e^{\mu(Ro-I)to}}_{a}e^{-\mu(Ro-I)t}}$$

(4) $$\boxed{I(t) = \frac{\mu(Ro-I)}{\beta + ae^{-\mu(Ro-I)t}}}$$

Taking the first derivative of equation (4) yields equation (5) thus;

$$DI(t) = \frac{[\beta+ae^{\mu(Ro-I)t}](o)-\mu(Ro-I)d(\beta+ae^{-\mu(Ro-I)t})}{[\beta+ae^{-\mu(Ro-I)t}]^2}$$

$$= \frac{-\mu(Ro-I)[-a\mu(Ro-I)e^{-\mu(Ro-I)t}]}{[\beta+ae^{-\mu(Ro-I)t}]^2}$$

$$\boxed{DI(t) = \frac{a\mu^2(Ro-I)^2e^{-\mu(Ro-I)t}}{[\beta+ae^{-\mu(Ro-I)t}]^2}} \quad (5)$$

Taking the second derivative of equation (4) or first derivative of equation (5) and equating to zero, yields equation (6) the peak time;

$D^2I(t) = 0$

$$= \frac{[\beta + ae^{-\mu(Ro-I)t}]^2[-a\mu^3(Ro-I)^3e^{-\mu(Ro-I)t} - a\mu^2(Ro-I)^2e^{-\mu(R0-I)t}.2[\beta+ae^{-\mu(Ro-I)t}]*[-a\mu(Ro-I)e^{-\mu(Ro-I)t}]}{[\beta + ae^{-\mu(Ro-I)t}]^4}$$

$$[\beta + ae^{-\mu(Ro-I)t}] = 2ae^{-\mu(Ro-I)t}$$

$$\beta = ae^{-\mu(Ro-I)t}$$

$$\boxed{t_{pk} = \frac{I}{\mu(Ro-I)}ln\frac{a}{\beta}} \quad (6)$$

3.1 COLLECTIONTION OF DATA

The data for the validation of the ebola models were obtained from credible internet sources, Althaus et al 2014, WHO (2014), CDC (2015) and Althaus (2014) as indicated in Tables 1a, 1b, 1c below.

Table 1a: Cases, Deaths of Ebola Fatality Epidemics In Guinea, Liberia, Sierraleone, CDC (2015), Althaus (2014)

Months	Feb-14	Mar-14	Apr-14	May-14	Jun-14	July-14	Aug-14	Sep-14	Oct-14	Nov-14	Dec-14	Jan-15	Feb-15	
Days	0	31	61	92	122	153	184	214	245	275	306	337	365	t_2
Gunea cases	0	122	196	240	300	410	475	720	1188	1750	2250	2820	3063	
Guinea death	0	80	100	125	188	300	340	475	730	1050	1375	1750	2000	
%	0	66%	51%	52%	63%	73%	72%	66%	61%	60%	61%	61%	65%	
Liberal Cases	0	0	0	0	107	125	350	1750	4000	6625	7750	7875	8875	
Liberal deaths	0	0	0	0	65	90	125	875	2000	2750	3250	3600	3875	
% L Fatality	0	0	0	0	61%	72%	36%	50	50	42	42	46%	44%	
S.Leone cases	0	0	0	50	150	250	500	1125	2625	5250	7750	10000	11125	
S.Leone deaths	0	0	0	6	69	100	250	400	625	1500	1750	3125	3625	
% S.L fatality	0	0	0	12%	46%	40%	50%	36%	24%	29%	23%	31%	33%	
W.A Cases	0	122	196	290	557	785	1325	3595	7813	13625	17750	20695	23063	
W.A death	0	80	100	131	322	490	715	1750	3355	5300	6375	8475	9500	
% Fatality	0	66	51	45	58	62	54	49	43	39	36%	41	41	
Gui cum. cases	0	122	318	558	858	1268	1743	2463	3651	5401	7651	10471	13534	GCC
Gui cum. Death	0	80	180	305	493	793	1133	1608	2338	2388	4763	6513	8513	GCD
	0	66	117	169	232	305	377	433	504	564	625	686	751	GCF
Lib.cum. cases	0	0	0	0	107	232	582	2332	6332	12957	20707	28582	37457	LCC
Lib.cum. deaths	0	0	0	0	65	155	280	1155	3155	5905	9155	12755	16630	LCD
Cum. Fatality	0	0	0	0	61	133	169	219	269	311	353	399	443	LCD
S.L. cum Cases	0	0	0	50	200	450	950	2075	4700	99500	17700	27700	38825	SCC
S.L.cum deaths	0	0	0	6	75	175	450	850	1450	2950	4700	7825	11450	SCD
Cum fatality	0	0	0	12	58	98	148	184	208	237	260	2991	324	SCF
W.A. cum. Cases	0	122	318	608	1165	1950	3275	6870	14683	28308	46058	66753	89816	WCC
W.A. cum. Deaths	0	80	180	311	633	1123	1838	3588	6943	12243	18618	27093	36593	WCD
Cum. fatality	0	66	117	162	220	282	336	385	428	467	503	544	585	WCF

Table 1b: Out Break Cases, Death Fatality and Previous Ebola Virus Epidemics in Central Africa, WHO (2014), Chowell et al (2004)

Country Virus	**1975**	**1976**	**1977**	**1978**	**1979**	**1980**	**1981**	**1982**	**1983**	**1984**	**1985**	**1986**	**1987**	**1988**	**1989**	**1990**	**1991**	**1992**	**1993**	**1994**	
Species	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	t_1
Congo, (Zaire Bundibugyo)		380 318	1 1																		
Gabon (Zaire)																				31 52	
Sudan (Sudan)		151 284			22 34																
Central Africa		431 602	1 1		22 34															31 52	
CACF	0	72	144	216	287	358	429	500	571	642	713	784	855	926	997	1068	1139	1210	1281	1352	
CACC CACD	0 0	431 602	432 603	432 603	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	454 637	485 689	
Country Virus	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012			
Species	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37			
Congo, (Zaire Bundibugyo)	254 315						44 59		157 178		10 12		187 264	14 32				29 57			
Gabon (Zaire)		67 92					53 65														
Ugander (Sudan Bundibugyo)						224 425							37 149				1 1	21 31			
Sudan (Sudan)										7 17											
Central Africa	254 315	67 92				224 425	97 124		157 178	7 17	10 12		224 413	14 32			1 1	50 88			
CACF	1426	1500	1574	1648	1722	1790	1859	1928	1998	2068	2138	2208	2275	2342	2409	2476	2543	2609		CACF	
	739 1004	806 1096	806 1096	806 1096	806 1096	1030 1521	1127 1645	1127 1645	1284 1823	1291 1840	1301 1852	1301 1852	1515 2265	1529 2297	1529 2297	1529 2297	1530 2298	1580 2386	66%	CACC CACD	

Table 1c: Cases, Death and Fatality of Ebola Epidemic In Nigeria (Althaus et al, 2014; Salaam-Blyther, 2014)

20 July	27 July	3rd Aug	10th Aug	17th Aug	24rd Aug	31st Aug	7 Sept	14th Sept	
$t_n 0$	7	14	21	28	35	42	49	56	t_3
NCC O No of cases	03	12	14	16	17	18	20	20	
NCD 0 No of deaths	0	2	4	5	6	7	8	8	
NCF	0	17	29	31	35	39	40	40	

3.2 CURVE FITTING

Cumulative data of the cases and deaths of Liberia, Guinea, Sierra Leone, Nigeria and Central African countries were used to produce scatter diagrams. The cumulative model equation (4) was superimposed on these cumulative scatter diagrams using matlab toolbox 7.9 to ascertain the goodness of fit of the data collected, which and all, give sigmoidal profiles.

Subsequently the first derivative of the developed model (5) were also used to plot the first derivative of the cumulative data produced by the computer as dumbbell profiles. In these way, the plots of different countries where made.

4.0 RESULTS PRESENTATION

The results of the curve-fitting done in the previous section are here presented in figs 2a to 6b and 7-10, and also the inherent tables 2a to 6b respectively.

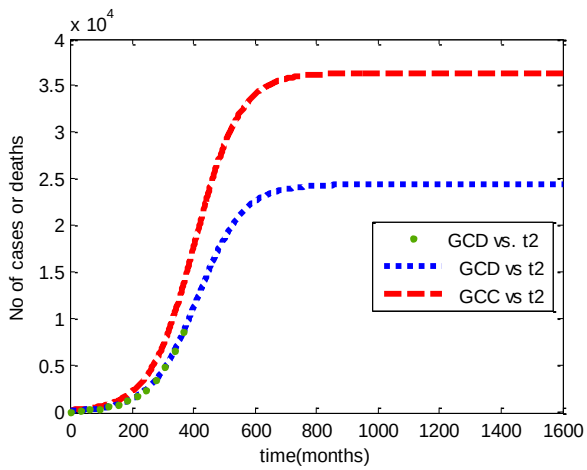


Fig 2a: No of Guinean cumulative cases or deaths vs time (Cases-R_2=0.9997,Deaths-R_2=0.9997)

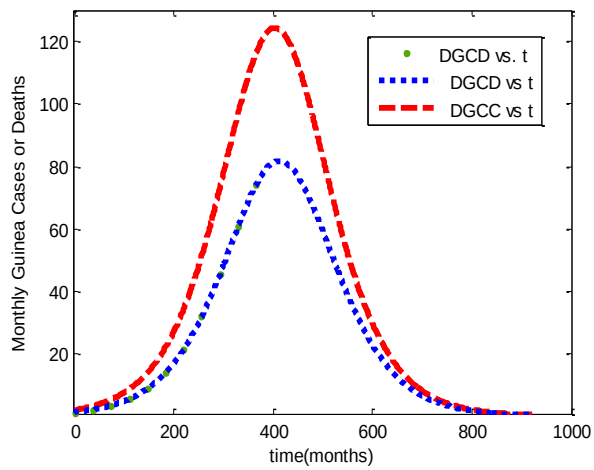


Fig 2b: Monthly Guinean Cases or Deaths vs time

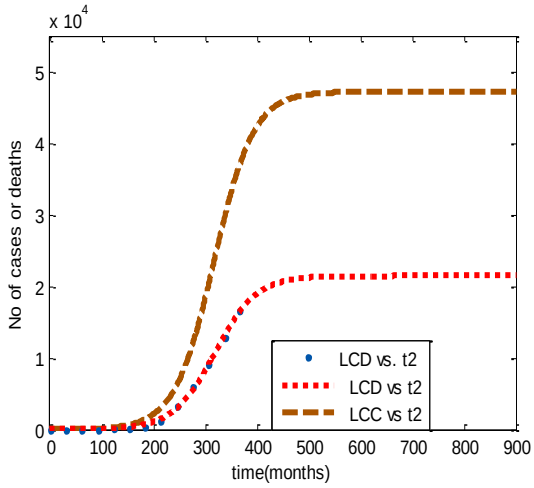


Fig 3a: No of Liberian cumulative cases or deaths vs time (Cases-R_2=0.9977,Deaths-R_2=0.9977)

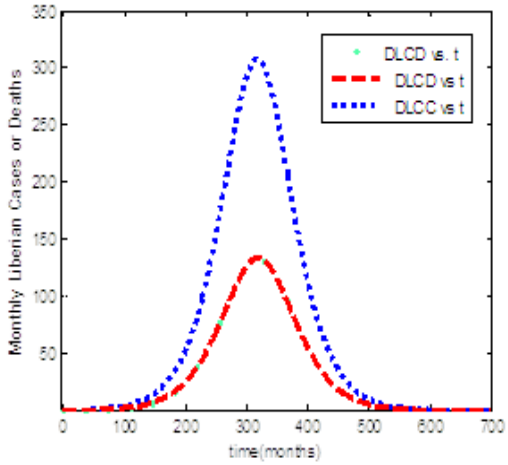


Fig 3b: Monthly Liberian Cases or Deaths vs time

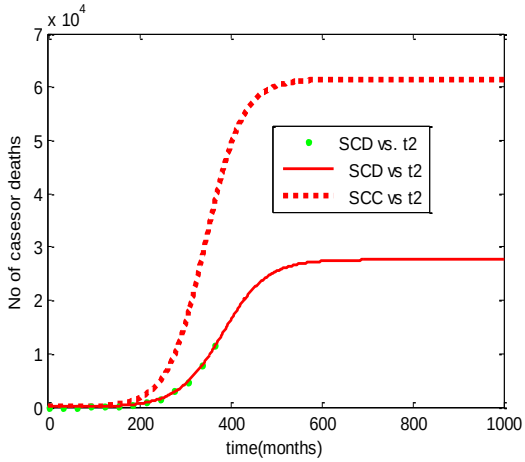


Fig 4a: No of S.Leone cumulative cases or deaths vs time (Cases-R_2=0.9996,Deaths-R_2=0.9995)

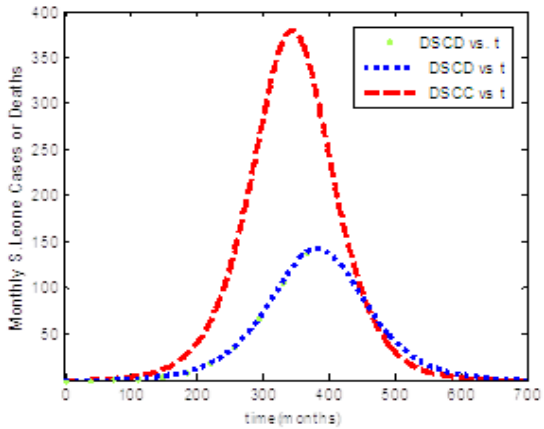


Fig 4b: Monthly S.Leone Cases or Deaths vs time

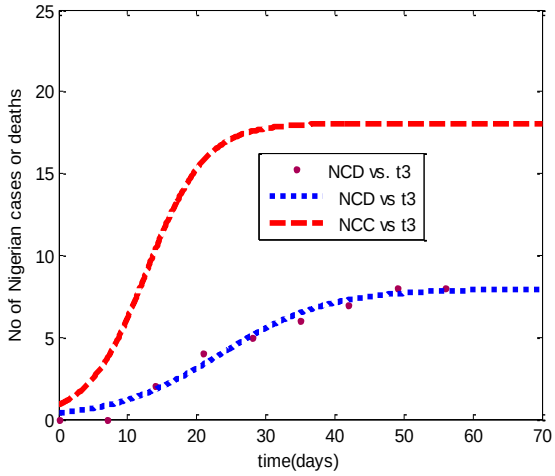


Fig 5a: No of Nigerian cumulative cases or deaths vs time (Cases-R_2=0.9566,Deaths-R_2=0.9914)

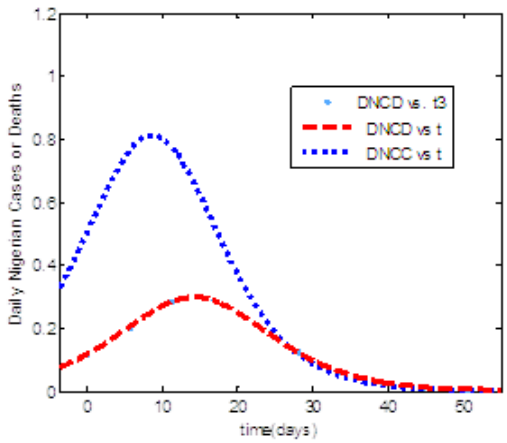


Fig 5b: Daily Nigerian Cases or Deaths vs time

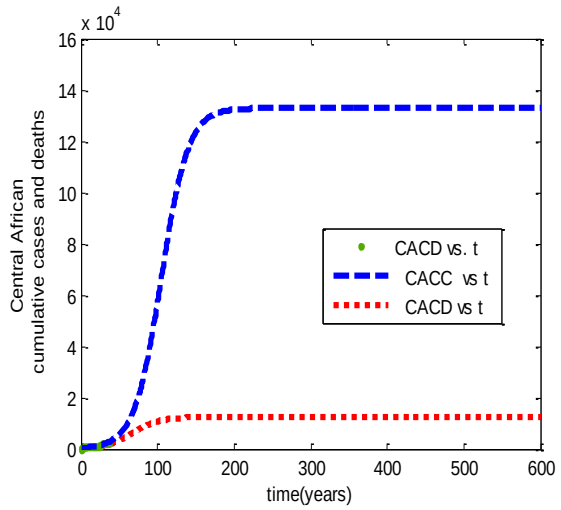


Fig 6a: Number of Central African Cummulative cases and deaths vs time

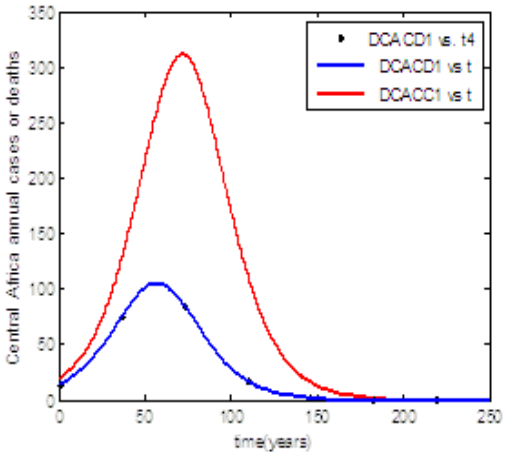


Fig 6b: Central Africa annual cases or deaths or deaths vs time

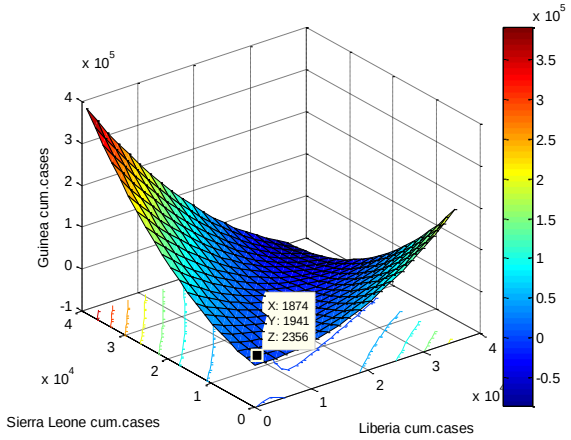


Fig 7: Guinean cumulative cases versus S.Leone and Liberian cumulative cases

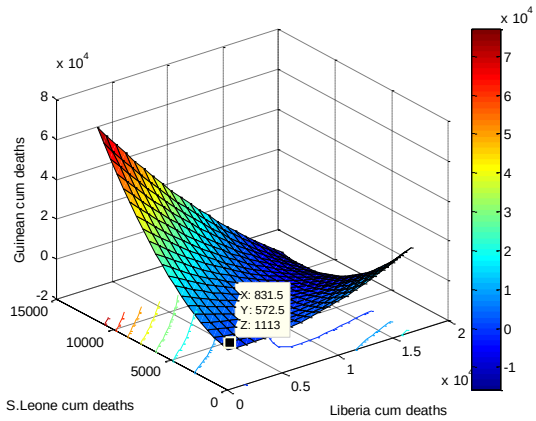


Fig 8: Guinean cumulative deaths versus S.Leone and Liberia cumulative deaths

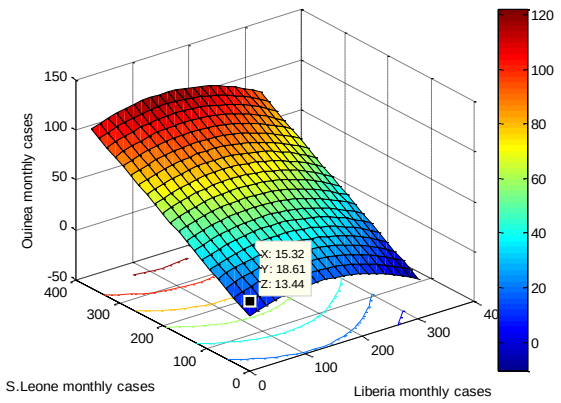


Fig 9: Guinean monthly cases versus S.Leone and Liberia monthly cases

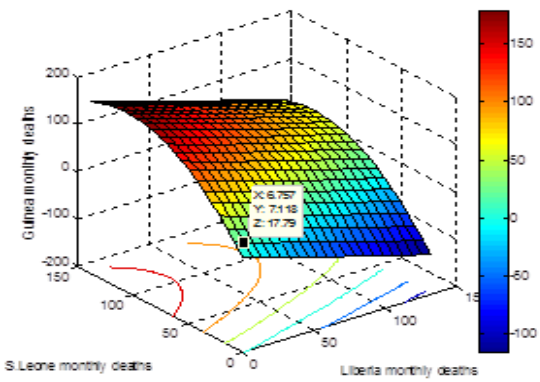


Fig 10: Guinean monthly deaths versus S.Leone and Liberia monthly deaths

Table 2a: Coefficients and Goodness of fits for Guinean ebola cumulative cases with 95 confidence bound

GCC	
B =0.00483 R0 =1.172 U = 1.283 x 10_4 a = 1.2 u = 0.07957 U: = F(1396)=36346.3 tpk = F(403=)124.329	SSE = 7.635 X 104 R2=0.9997 R adjusted =0.9995 RMSE =97.69

Table 2b: Coefficients and Goodness of fits for Guinean ebola cumulative deaths with 95 confidence bound

GCD	
B =0.0074 R0 = U = 1.78 a = 1.854 u = 0.07543 U: = F(1367)=24384.4 tpk = F(412)=81.8232	SSE = 2.23 X 104 R=0.9997 Adj R =0.9996 RMSE =5385f

Table 3a: Coefficients and Goodness of fits for Liberian ebola cumulative cases with 95 confidence bound

B =0.00748 R0 =1.33 U = 1.354 x 104 a = 28.87 u = 0.07909 U: = F(841)=47196 tpk = F(317)=307.947	SSE = 4.427 R=0.9977 R adjusted =0.9966

Table 3b: Coefficients and Goodness of fits for Liberian ebola cumulative deaths with 95 confidence bound

LCD	
B =0.01744 R0 =1.043 U = 1.502 x 10^4 a = 48.29 u = 0.5819 U: = F(774)=21462 tpk = F(319)133.71	SSE = 8.685 X 10^4 $R^{2=}$0.9977 R^2 Adjusted =0.9966 RMSE =329.5

Table 4a: Coefficients and Goodness of fits for Sierra Leonean ebola cumulative cases with 95 confidence bound

SCC	
B =0.00483 R_0 =1.172 U = 1.283 x 10^4 a = 1.2 u = 0.07957 U: = F(1396)=36346.3 tpk = F(403=)124.329	SSE = 7.635 X 10^4 R=0.9997 R adjusted =0.9995 RMSE =97.69

Table 4b: Coefficients and Goodness of fits for Sierra Leonean ebola cumulative deaths with 95 confidence bound

SCD	
B =0.01011 R^0 =1.142 U = 1.354 x 10^4 a = 26.63 u = 0.1456 U: = F(953628)=27 tpk = F(382)=142.496	SSE = 7.228 X 10^4 R=0.9995 R adjusted=0.9993 RMSE =95.05

Table 5a: Coefficients and Goodness of fits for Nigerian ebola cumulative cases with 95 confidence bound

NCC	
B =0.13 R0 =1.778 U = 15.28 a = 0.5497 u = 0.2144 U: = F(89.3)=19.5156 tpk = F(8.6)=0.813403	SSE = 2.227 R=0.9914 R adjusted =0.98 RMSE =0.8616

Table 5b: Coefficients and Goodness of fits for Nigerian ebola cumulative deaths with 95 confidence bound

NCD	
B =0.3353 R0 =1.098 U = 18.68 a = 2.706 u = 1.495 U: = F(98.4)=8.1468 tpk = F(14.5)=0.297542	SSE = 0.118 R=0.9979 R adjusted =0.9951 RMSE =0.1983

Table 6a: Coefficients and Goodness of fits for Central African ebola cumulative cases with 95 confidence bound.

CACC	
B =0.00731 R0 =2.191 U = 2739 a = 0.4553 u = 0.04848 U: = F(70)=311.617 tpk = F(281)=21631.3	SSE = 1.116 X 10^6 R=0.9343 R adjusted =0.9264 RMSE =183.9

Table 6b: Coefficients and Goodness of fits for Central African ebola cumulative deaths with 95 confidence bound

CACD	
B =8.564 R_0 =1.35 U = 1.053 x 10^6 a = 239.5 u = 0.1673 U: = F(297)=105.519 tpk = F(403=)124.329	SSE = 5.178 X 10^5 R=0.9304 R adjusted =0.9219 RMSE =125.3

4.1 RESULT DISCUSSION

Generally, it is seen from the plots that the cumulative number of cases or ultimate cases are higher than that of cumulative number or ultimate deaths as should be. Also, in the timely cases or deaths the peaks of the timely cases are higher than the peak of the timely death as it should be. In the case of Guinea, the ultimate number of cases and deaths are 36346 and 24384.

The peak of the monthly cases and deaths occur at 403 months with 124 cases and 412 months for 82 deaths respectively, so that the peak fatality will be $\frac{82}{124}$ or 65.8%. The reproductive number of cases is R_0 = 1.172.

The Liberia ultimate number of cases and deaths are 47196 and 21462 respectively. The peaks of the monthly cases and deaths occur at 317 months with 308 cases and 319 months with 134 deaths so that the peak percentage fatality will be $\frac{134}{308}$ or 43.4%. The reproductive number of cases and deaths are 1.33 and 1.043 respectively.

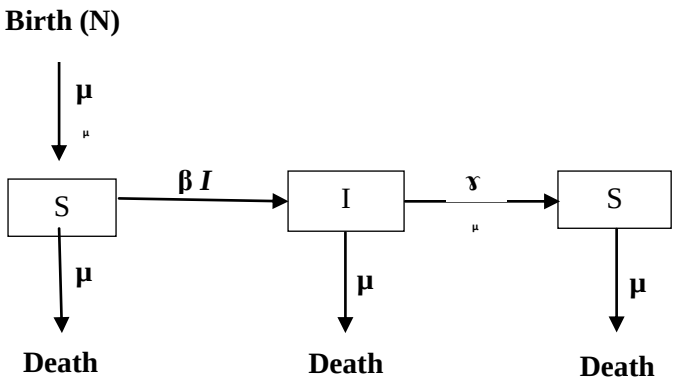
The Sierra Leone ultimate cases is 61467, and 27628 deaths. The peaks of the monthly cases and deaths will be 344 months at 379 cases and 382 months at 142 deaths, so that the monthly fatality will be $\frac{142}{379}$ or 37.6%. The reproductive number for the cases and deaths are 1.172 and 1.142 respectively.

In Nigeria, the ultimate cases was 20 and the number of deaths 8. The daily peak of the cases occur at 8.6 days as 0.8134 and that of the deaths occur at 14.5 days as 0.2975 so that the daily fatality will be 36.6%.

For Central Africa, the ultimate cases was 21631 while the ultimate deaths stood at 12433. The annual peaks occurred at 70 years as 312 cases and 57 years for 106 deaths, so that the annual percentage fatality is $\frac{106}{312}$ or 33.9%. The reproductive number for cases and deaths in Central Africa are 2.191 and 1.35 respectively.

Hence, amongst the worst hot West African countries, Guinea had the highest monthly peak fatality 66% followed by Liberia 43%, then Sierra Leone 38%. The numerical values from Central Africa, the 3-worst-hit countries of West Africa and Nigeria cannot be compared strictly because these regions have test data on different time-frames, ie in years, months and days respectively.

APPENDIX A (S-I-S ENDEMIC MODEL)



$$\frac{ds}{dt} = \mu N - \beta IS - \mu S + \gamma I - \mu S = \mu N - 2\mu S + (\gamma - \beta S)I$$

(A1)

$$\frac{dI}{dt} = \beta IS - \mu I - \gamma I = (\beta S - (\gamma + \mu))I$$

(A2)

$$N = S + I \qquad \text{(A3)}$$

Put (A3) into (A2)

$$\frac{dI}{dt} = \beta(N + I) - (\gamma + \mu)I = (a - \beta I)I$$

$$\boxed{a = \beta N - \gamma - \mu = (Ro - 1)\mu - \gamma\,;\; Ro = \frac{\beta N}{\mu}}$$

Integrating;

$$\int dt = \int \frac{dI}{(a-\beta I)I} = \int \frac{dI}{(a-\beta I)} = \frac{I}{a}\{\int \frac{dI}{I} + \int \frac{\beta dI}{a-\beta I}\}$$

$$t\int_{to}^{t} = \frac{I}{a}\ln\frac{I}{a-\beta I}\int_{Io}^{I} = \frac{I}{a}\ln\frac{I(a-\beta Io)}{Io(a-\beta I)} \qquad \textbf{or} \qquad a(t - to) = \ln\frac{I(a-\beta Io)}{Io(a-\beta I)}$$

Taking antilog; $e^{a(t-to)} = \theta = \frac{I(a-\beta Io)}{Io(a-\beta I)}$; $a\theta Io = (\theta\beta Io + a - \beta Io)I$

$$DI_{(t)} = \frac{(\beta+be^{-at})0 - a(-abe^{-at})}{(\beta+be^{-at})^2}$$

$$\boxed{I(t) = \frac{a}{\beta + a\left(\frac{I}{Io} - \frac{\beta}{a}\right)e^{-a(t-to)}} = \frac{\mu(Ro-1)-\gamma}{\beta + be^{-[\mu(Ro-I)-\gamma]t}}} \qquad \text{(A6)}$$

Differentiating (A4) twice or eqn (A6) once and equate to zero to yield eqn (A7) the peak time;

$$DI_{(t)} = 0: \frac{(\beta+be^{-at})^2.(-a^3be^{-at}) - a^2be^{-at}.2[\beta+be^{-at}](-abe^{-at})}{(\beta+be^{-at})^4} = 0$$

$$(\beta + be^{-at})(-a^3be^{-at}) = 2a^2be^{-at}(-abe^{-at})$$

$$\beta + be^{-at} = 2be^{-at}$$

$$\beta = be^{-at}$$

$$\boxed{t_{pk} = \frac{I}{a}ln\frac{b}{\beta} = \frac{I}{a}ln\frac{a}{\beta}\left(\frac{I}{Io} - \frac{\beta}{a}\right)}$$

e^{-ato}(A7)

where $a = \mu(Ro - 1) - \gamma$

and reproductive number $Ro = \frac{\beta N}{\mu}$

$$\boxed{DI_{(t)} = \frac{a^2be^{-at}}{(\beta+be^{-at})^2} = \frac{a^3\left(\frac{I}{Io} - \frac{\beta}{a}\right)e^{-a(t-to)}}{[\beta + a\left(\frac{I}{Io} - \frac{\beta}{a}\right)e^{-a(t-to)}]^2}}$$

5.0 CONCLUSION

In this work, Ebola epidemic outbreak cases and deaths were modeled using compartmental model of epidemic mathematics and kinetics. Data were obtained from credible internet sources like Centers for Disease Control (CDC) and World Health Organization (WHO) which were used to validate our model, and the results show that it was almost a perfect model for some countries of West Africa but just good enough for Central Africa as the R^2 shows. The goodness of fit terms of coefficient of correlation (R^2) of the developed model on test data gives excellent validation. For Guinea, R^2 is 0.9997 for both cases and deaths test data. For Liberia, R^2 is 0.9977 for both cases and deaths test data. For Sierra Leone, R^2 is 0.9997 (for cases) and 0.9995 (for deaths data). For Nigeria, R^2 is 0.9914 (for cases data) and 0.9979 (for deaths data). For Central Africa, R^2 is 0.9343 (for cases data) and 0.9304 (for deaths data)The response plot of the cumulative cases and deaths as well as the monthly cases and deaths between the three worst-hit countries, namely (Liberia, Guinea. Sierra Leone) show very serious interaction since the EVD is spread by bats, gorillas, herbivorous animals that live in the bush and thick forest. We should reduce the rate of contact with the animals and stop eating them especially their carcasses found in the bushes and thickets. These findings can be used in studying future ebola epidemic and pandemic outbreaks by World Health Organization (WHO), Pan American Health Organization (Pan-AHO), Centers for Disease Control (CDC), etc.

6.0 RECOMMENDATIONS

The developed world should not treat with levity an outbreak of this magnitude until it becomes almost uncontrollable; like they did with the Central African case.

And, if the world had taken serious study to stop the outbreak from Central Africa, perhaps, it could not have spread to West Africa and other countries as it did, since we know now that the world is a global village.

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