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REGULARIZATION BASED FUZZY GRAPH

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ABSTRACT

The aim of this paper is introduce some definitions of regular fuzzy graphs, total degree and total regular fuzzy graph. Further we introduce concept of regularization based fuzzy graph and to obtain some result related to regular fuzzy graph and total regular fuzzy graph. Some basic theorems related to the stated graphs have also been presented.

AMS Subject Classification: 05C72.

KEYWORDS: Strong fuzzy graph, Regular fuzzy graphs, Totally regular fuzzy graph, Complete fuzzy graph, Degree of a vertex in fuzzy graphs.

INTRODUCTION

In 1736, the concept of graph theory was first introduced by Euler. The perception of fuzzy set was discussed by L.A. Zadeh, in 1965. In 1973, Kaufmann gave the first definition of a fuzzy graph which was based on Zadeh's fuzzy relations. A.Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. Zadeh introduced the concept of fuzzy relations, in 1987. The theory of fuzzy graphs was developed J. N. Mordeson studied fuzzy line graphs and developed its basic properties, in 1993.

In this paper we discussed the concept of regular fuzzy graphs and totally regular fuzzy graphs. A comparative study between regular fuzzy graphs and totally regular fuzzy graph is made. Also some results on regular fuzzy graphs are studied and examined whether they hold for totally regular fuzzy graphs.

PRELIMINARIES

Definition 2.1: A graph G* = (V, E) is a pair of vertex set (V) and an edge set (E) where $E \subseteq V \times V$ i.e. E is a relation on V.

Definition 2.2: A fuzzy graph G = (σ, μ) is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ with $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$, ∀ x, y ∈ V, where V is a finite non- empty set and ∧ denote minimum. Where σ is a fuzzy subset of a non-empty set V and μ is a symmetric fuzzy relation on σ.

Definition 2.3: A fuzzy graph G = (σ, μ) is said to be a strong fuzzy graph if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$, ∀ (x, y) ∈ E.

Definition 2.4: A fuzzy graph G = (σ, μ) is said to be a complete fuzzy graph if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$, ∀ x, y ∈ V.

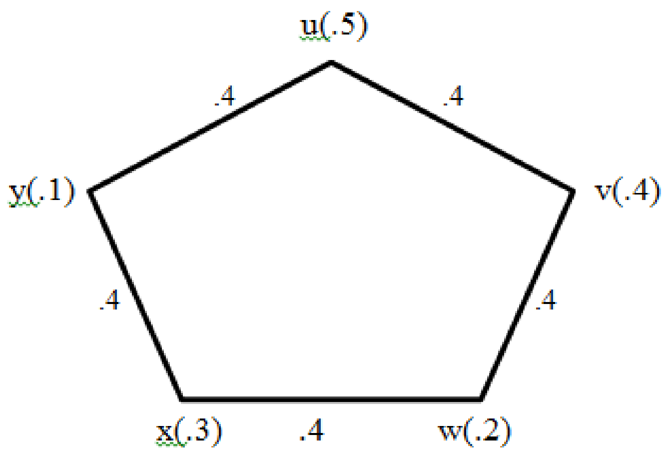
Definition 2.5: The degree of a vertex of G is denoted by

$$d_G(x) = \sum_{x \neq y} \mu(x, y)$$

REGULARIZATION ON FUZZY GRAPH

Definition 3.1: Let G = (σ, μ) be a fuzzy graph. If degree of each vertex is same say k, then G is called k-regular fuzzy graph.

Example 3.2: Let G be a fuzzy graph.



Here G is regular fuzzy graph, because degree of each vertex is .8, which is same.

Definition 3.3: Let G = (σ, μ) be a fuzzy graph. The total degree of a vertex of G is denoted by

$$Td_G(x) = \sum_{x \neq y} \mu(x, y) + \sigma(x) = d_G(x) + \sigma(x)$$

If total degree of each vertex is same say k, then G is called k- totally regular fuzzy graph.

Result 3.4

(I) Every regular fuzzy graph may or may not be a totally regular fuzzy graph.

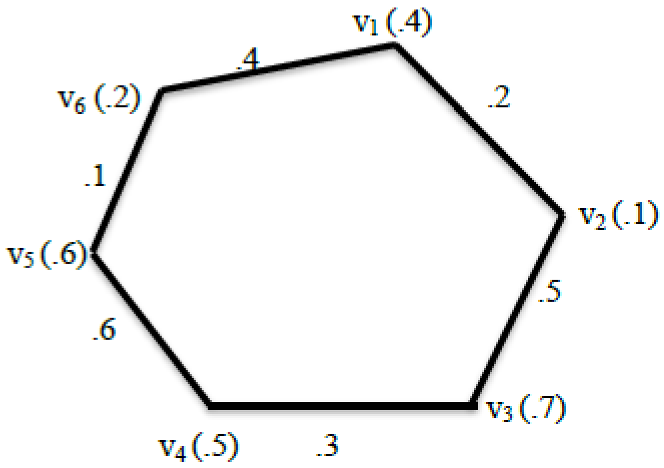
(II) If fuzzy graph is not regular fuzzy graph, it may or may not be a totally regular fuzzy graph.

Result 3.5: If fuzzy graph is not regular, it may or may not be a totally regular fuzzy graph.

(i) If G is not regular fuzzy graph, G is also not totally regular fuzzy graph.

Example 3.6: Consider G = (V,E) where V= {v1,v2,v3,v4,v5,v6} and E={v1 v2,v2v3,v3v4,v4v5,v5v6,v6v1}

Define fuzzy graph G as below.



Then d(v1) = .4+.2=.6, d(v2) =.2+.5= .7, similarly d(v3) = .8,d(v4) = .9, d(v5) =.7and d(v6) = .5 Since, d(v1) ≠ d(v2) ≠ d(v3) ≠ d(v4) ≠ d(v5) ≠ d(v6).

Therefore, G is not regular fuzzy graph.

But *Td*(v1) = .4+.4+.2=1.0 , *Td*(v2) = .2+.1+.5=.8, similarly *Td*(v3) =1.5, Td(v4) =1.4, *Td*(v5) =1.3, Td(v6) =.7

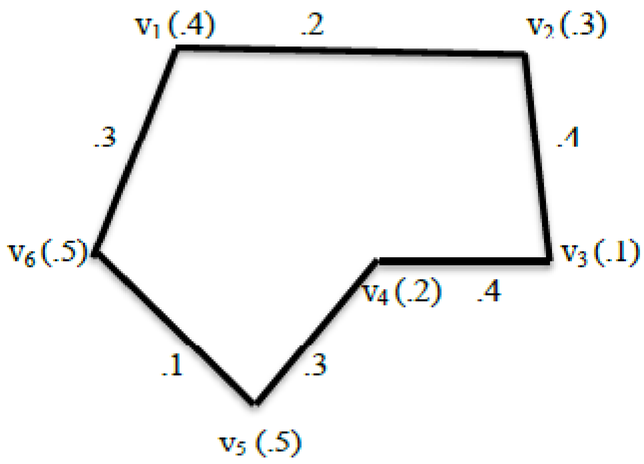
Again *Td*(v1) ≠ Td(v2) ≠ Td(v3) ≠ Td(v4) ≠ Td(v5) ≠ Td(v6).

Therefore, G is not totally regular fuzzy graph.

(ii) G is not regular fuzzy graph but G is totally regular fuzzy graph.

Example 3.7: Consider G = (V, E) where V= {v1, v2, v3, v4, v5, v6} and E = {v1 v2, v2v3, v3v4, v4 v5, v5v6, v6v1}.

Define fuzzy graph G as below.



Then d(v1) = .3+.2=.5, d(v2) =.2+.4= .6, similarly d(v3) = .8,d(v4) = .7, d(v5) =.4and d(v6) = .4 Since, d(v1) ≠ d(v2) ≠ d(v3) ≠ d(v4) ≠ d(v5) ≠ d(v6).

Therefore, G is not regular fuzzy graph. But Td(v1) = .9 for all i = 1,2,3,4,5,6.

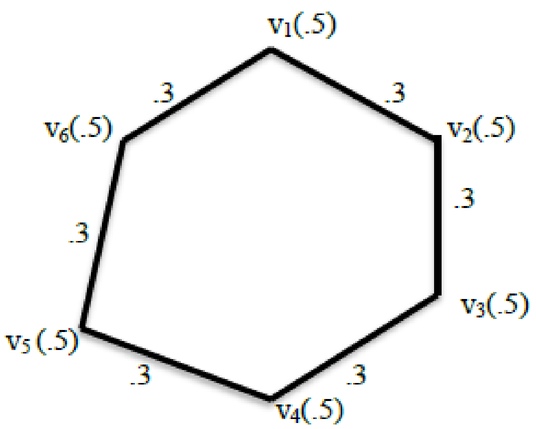
Therefore, G is a totally regular fuzzy graph.

Result: 3.8: Every regular fuzzy graph may or may not be a totally regular fuzzy graph.

(i) A fuzzy graph G is regular as well as totally regular fuzzy graph.

Example 3.9: Consider G = (V,E) where V= {v1,v2,v3,v4,v5,v6} and E={v1v2,v2v3,v3v4,v4v5,v5v6,v6v1}.

Define fuzzy graph G as below.



Here d(vi) = .6 ∀ i = 1,2,3,4,5,6

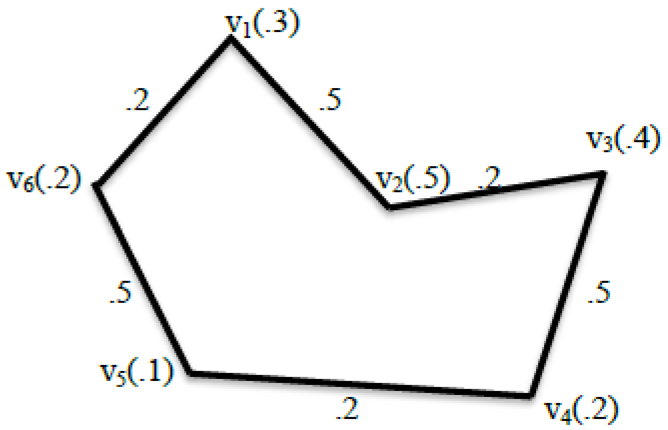
So G is regular. Also, Td(vi) =1.1 ∀ i = 1,2,3,4,5,6

∴ G is a totally regular fuzzy graph.

(ii) G is regular fuzzy graph, but it is not totally regular fuzzy graph.

Example 3.10: Consider G = (V,E) where V= {v1,v2,v3,v4,v5,v6} and E={v1 v2,v2v3,v3v4,v4v5,v5v6,v6v1}.

Define fuzzy graph G as below.



Then d(v1) = 0.7= d(v2) = d(v3) = d(v4) = d(v5) = d(v6)

So G is regular fuzzy graph.

But Td(v1) = .2+.3+.5=1.0 , Td(v2) = .5+.5+.2=1.2, similarly Td(v3) =1.1, Td(v4) =.9, Td(v5) =.8, Td(v6) =.9

So Td(v1) $\neq$ Td(v2) $\neq$ Td(v3) $\neq$ Td(v4) $\neq$ Td(v5) $\neq$ Td(v6).

$\therefore$ G is not totally regular fuzzy graph.

Theorem 3.11: Let σ be a constant function of fuzzy graph G. Prove that G is a regular fuzzy graph if G is a totally regular fuzzy graph.

Proof: Let σ be a constant function say c, i.e. σ (x) = c.

Let G be a k1 – totally regular fuzzy graph.

$\therefore$ Td(x) = k1, for all x $\in$ V.

Using definition of k1 – totally regular fuzzy graph.

$\Rightarrow$ d(x) + σ (x) = k1, for all x $\in$ V.

$\Rightarrow$ d(x) + c = k1, for all x $\in$ V.

$\Rightarrow$ d(x) = k1 - c, for all x $\in$ V.

$\therefore$ G is a regular fuzzy graph.

Thus G is a regular fuzzy graph if G is a totally regular fuzzy graph.

Theorem 3.12: Let σ be a constant function of fuzzy graph G. Prove that if G is a regular fuzzy graph, G is a totally regular fuzzy graph.

Proof: Let σ be a constant function say c, i.e. σ (x) = c.

Now, Let G be a k2 – regular fuzzy graph.

Then d(x) = k2, $\forall$ x $\in$ V.

Since we know that by definition of totally regular fuzzy graph.

$\therefore$ Td(x) = d(x) + σ (x), $\forall$ x $\in$ V.

$\Rightarrow$Td(x) = k2 + c, $\forall$ x $\in$ V.

Hence G is a totally regular fuzzy graph.

Thus if G is a regular fuzzy graph then G is a totally regular fuzzy graph.

Theorem 3.13: When a fuzzy graph G is regular as well as totally regular, then vertex set is a constant function.

Proof: Let σ be a vertex set and G be a k1–regular and k2– totally regular fuzzy graph.

So d(x) = k1 and Td(x) = k2, $\forall$ x $\in$ V.

Now Td(x) = k2, $\forall$ x $\in$ V.

Using definition of k2 – totally regular fuzzy graph.

$\Rightarrow$ $\therefore$ d(x) + σ (x) = k2 , $\forall$ x $\in$ V.

$\Rightarrow$ k1 + σ (x) = k2 , $\forall$ x $\in$ V.

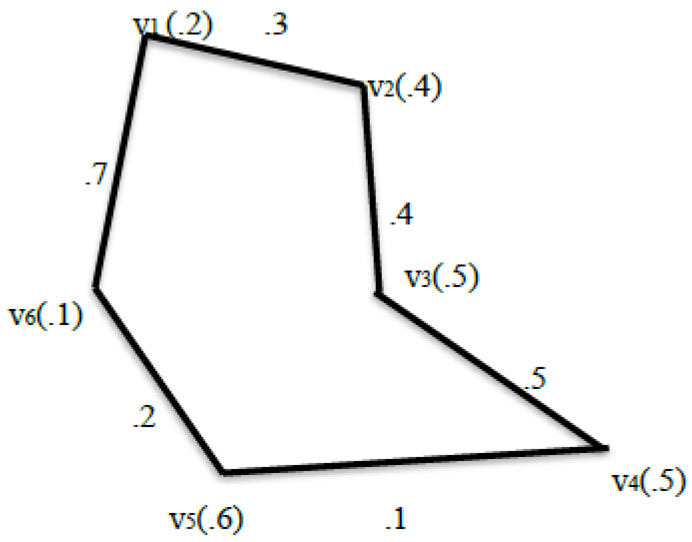
$\Rightarrow$ σ (x) = k2 - k1, $\forall$ x $\in$ V.

Hence σ is a constant function.

But converse of the above theorem 3.13 need not be true.

Example 3.15: Consider G=(V,E) where V= {v1,v2,v3,v4,v5,v6} and E={v1v2,v2v3,v3v4,v4v5,v5v6,v6v1}.

Define fuzzy graph G as below



Then d(v1) = .7+.3 = 1.0, d(v2) = .3+.4 = .7 similarly d(v3) = .9, d(v4) = .6, d(v5) =.3 and d(v6) = .9

Since, d(v1) ≠ d(v2) ≠ d(v3) ≠ d(v4) ≠ d(v5) ≠ d(v6).

Therefore, G is not regular fuzzy graph.

But, Td(v1) = .7+.2+.3=1.2 , Td(v2) = .3+.4+.4=1.1, similarly Td(v3) =1.4, Td(v4) =1.1, Td(v5) =.9, Td(v6) =1.0

Again Td(v1) ≠ Td(v2) ≠ Td(v3) ≠ Td(v4) ≠ Td(v5) ≠ Td(v6).

∴G is not totally regular fuzzy graph.

Whereas σ is a constant function, because σ(x) = Td(x) - d(x)

σ(v1) = Td(v1) - d(v1)

σ(v1) = 1.2 – 1.0

σ(v1) = 0.4 , which is constant.

SUMMARY AND CONCLUSION

Therefore, there is no relation between regular fuzzy graphs and totally regular fuzzy graphs. A regular fuzzy graph may or may not be a totally regular fuzzy graph and in addition if fuzzy graph is not regular, it does not provide assurance for it to be a totally regular fuzzy graph.

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