

### EFFECT OF PARAMETER VARIATION ON CHARACTERISTICS OF LINEAR ANALOG VLSI CIRCUITS USING SIMPLE MATHEMATICAL EQUATION

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#### ABSTRACT

In this article we analyze the effect of various parameter variations on linear analog circuits. It is very necessary that the characteristics of circuit should be in stable form when there is some changes in value of any circuit parameter. Our studies focus on to get the knowledge how the system output depends upon the parameter of circuit. We also

model various circuits in this article to prove the effect of the parameter on output of system. In this article we perform analysis on three circuits. These are RLC circuit, Sallen Key circuit and Biquadratic filter circuit. All analysis and calculation done in this article are performed by using MATLAB software.

**KEYWORDS:** Parameter, Sensitivity, Variation, Circuits, Biquadratic.

#### INTRODUCTION

The output of any system depends upon the component associated with it. So it is very necessary to know the dependence of each component behaviour on characteristics of circuit. Some elements affect more and some affect less, but there is an effect on system output if there is a small change in its circuit parameter. Also the reliability and life span of the circuit depends upon the component associated with it. <sup>[1,2,3,4]</sup>

Before designing any circuit the engineer must know about the system behaviour that how does it vary with changes in value of components. If there is small change in parameter then the system output will be almost stable. But if there is large change in value then system output get unstable and it is very necessary to change that component.<sup>[5,6,7,8]</sup>

Here we use sensitivity analysis to analyze the effect of parameter variation on circuit characteristics.

Sensitivity analysis of analog circuit provide us the information about various component present in the circuit. With the help of sensitivity analysis we also know about the component characteristics variation in circuit and its effect on performance of system output.<sup>[9,10]</sup>

### Basic Principle of Analysis

A simple definition of sensitivity is how much specific system behavior/characteristic changes as a individual component value changes.<sup>[11,12]</sup> The general equation for sensitivity analysis is given below

$$S_x^y = \lim_{\Delta x \rightarrow 0} \frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{x}{y} \frac{\partial y}{\partial x} \quad (1)$$

Equation (1) is the general mathematical definition of circuit sensitivity: Where S represent sensitivity, X represent changing element/component and Y is the characteristic of circuit which one want to evaluate as component value is varied. The middle part of this equation shows that the percentage that the dependent variable  $\Delta y/y$  changes, relative to the percentage that the independent variable  $\Delta x/x$  changes. The sensitivity analysis done by using these formulae derived below. Let's take a transfer function H(s).

$$H(s) = \frac{N(s)}{D(s)} \quad (2)$$

Here N(s) represent the numerator part of transfer function and D(s) represent the denominator part of transfer function. From equation (1) and equation (2), we write a new equation which is same as equation (1). But its variable name are changed to make our calculation easy. In general, the AC-sensitivity is given by the following equation:

$$\text{Sens}(H(s), W) = \frac{W}{H(s)} \frac{\partial H(s)}{\partial W}$$

Substituting equation (1) into (2) and applying the chain rule have

$$\text{Sens}(H(s), W) = W \left( \frac{1}{N(s)} \frac{\partial N(s)}{\partial W} \right) - \left( \frac{1}{D(s)} \frac{\partial D(s)}{\partial W} \right) \quad (3)$$

Here  $W$  is the component which one want to vary w.r.t. circuit transfer function. By using above equation (3) we can calculate the sensitivity of circuit any circuit.

### Effect of Parameter Variation on Linear Analog VLSI

As we know that all circuit characteristics are function of their element values of circuit. Filter characteristics also dependent on elements values of circuit. Most filter very sensitive to their component values. Sensitivity analysis perform a major role to choose the component values according to their sensitivity.<sup>[13]</sup>

There are many was to calculate the sensitivity of filters with respect to their component value. One way is that analysis AC transfer behavior of filter with component variation. For simplicity here we write transfer function with  $Q$  (quality factor) and  $\omega_n$  (natural frequency).<sup>[14]</sup>

The transfer function of RLC filter circuit is given below

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} = \frac{N(s)}{D(s)} \quad (4)$$

Similarly

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{s}{\omega_n Q} + 1} = \frac{N(s)}{D(s)} \quad (5)$$

$$\omega_n = \frac{1}{\sqrt{L_1 C_1}} \quad (7)$$

$$Q = R_1 \sqrt{\frac{C_1}{L_1}} \quad (8)$$

Using nominal values of RLC circuit the  $Q=0.707$ .by using general equation of sensitivity the sensitivities of  $\omega_n$  and  $Q$  are given below.

Sensitivity using  $Q$  (quality factor)

$$S_{C_1}^Q = -S_{L_1}^Q = \frac{1}{2}$$

$$S_{R_1}^Q = 1$$

And sensitivity using  $\omega_n$  (natural frequency)

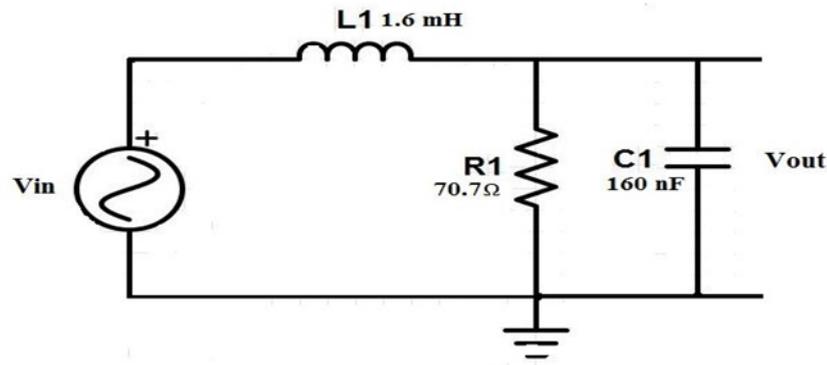


Figure 1: Circuit diagram of RLC filter circuit.

$$S_{C_1}^{\omega_n} = -S_{L_1}^{\omega_n} = -\frac{1}{2}$$

$$S_{R_1}^{\omega_n} = 0$$

As we saw that resistor value does not depend upon natural frequency in above equation.

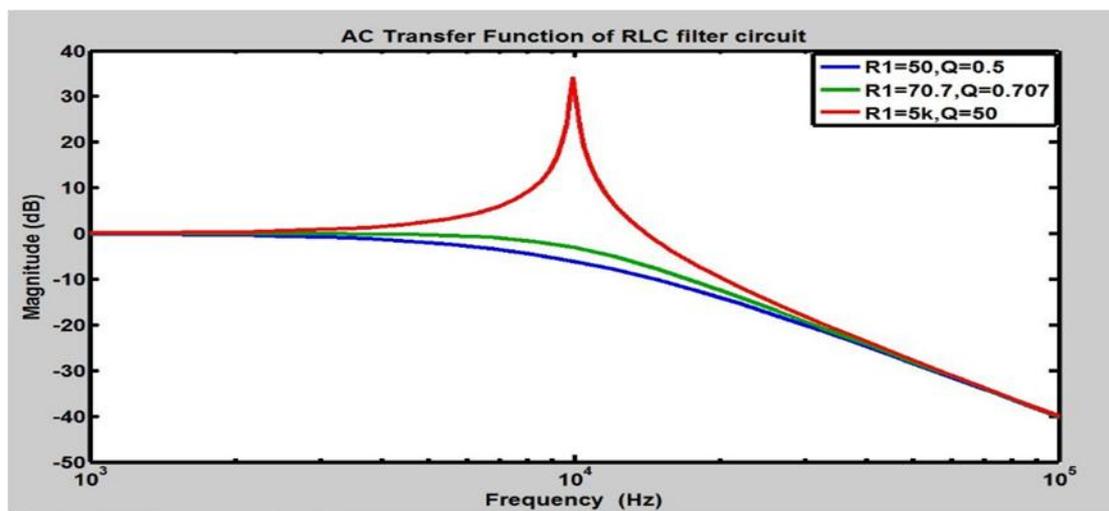


Figure 2: Variation of  $R_1$  w.r.t  $Q$ .

Figure 3 shows the variation of  $R_1$  w.r.t quality factor. Because it is most sensitive component in this circuit (in term of quality factor sensitivity). By using equation (3) and equation (6) we derive sensitivities of components which are given below.

Here  $W$  is the component which we want to vary. So  $W=L_1$

$$=L1 \left[ -\frac{1}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} \frac{\partial (s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1)}{\partial L_1} \right]$$

$$=L1 \left[ -\frac{(s^2 C_1 + \frac{s}{R_1})}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} \right]$$

$$S_{L_1}^{H(s)} = \left[ -\frac{(s^2 L_1 C_1 + s \frac{L_1}{R_1})}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} \right] \quad (9)$$

Similarly we calculate for  $R_1$  and  $C_1$ .

$$S_{R_1}^{H(s)} = \left[ -\frac{(s \frac{L_1}{R_1})}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} \right] \quad (10)$$

$$S_{C_1}^{H(s)} = \left[ -\frac{(s^2 L_1 C_1)}{s^2 L_1 C_1 + s \frac{L_1}{R_1} + 1} \right] \quad (11)$$

Here we plot sensitivity of passive elements with nominal values as well as some tolerance provided to nominal value and by using this we get the information related to sensitivity of elements in RLC circuit.

With the help of sensitivity analysis one also know about the tolerance specified at the time of design in particular element. Designer can provide cheap elements which does not affect circuit characteristics. By using equation (10), (11) and (12).

We analyzed the sensitivity of RLC filter circuit.

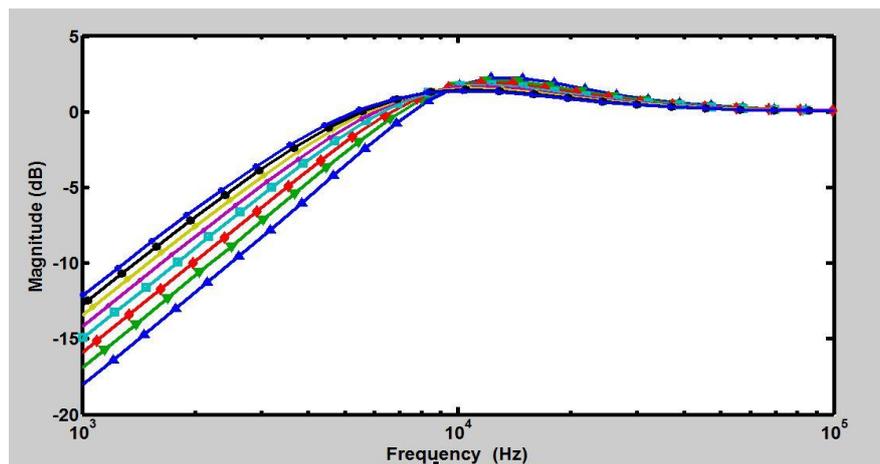
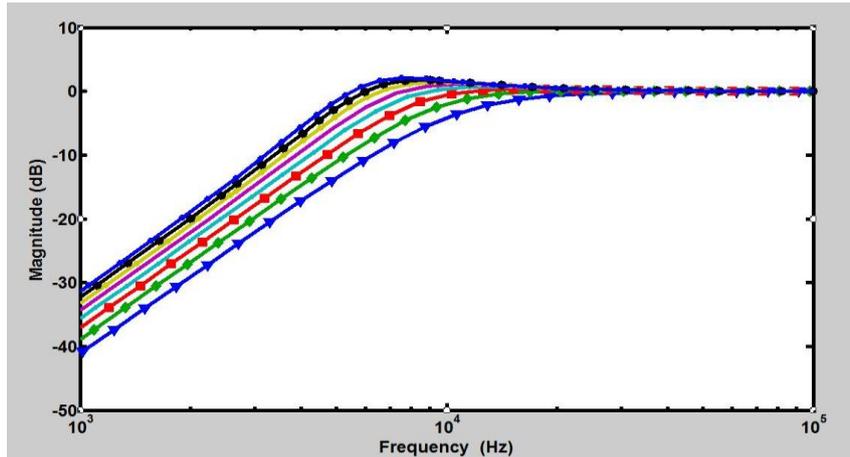


Figure 3: Sensitivity of inductor in RLC circuit.



**Figure 4: Sensitivity of capacitor in RLC circuit.**

In Figure 4, 5 and 6 we plot sensitivity analysis using nominal values of component and also provide some tolerance form their nominal value.

### Effect of Parameter Variation in Sallen-Key Filter

Sallen-key filter one of the most common filter.<sup>[14,16,17]</sup> The transfer function of this filter is

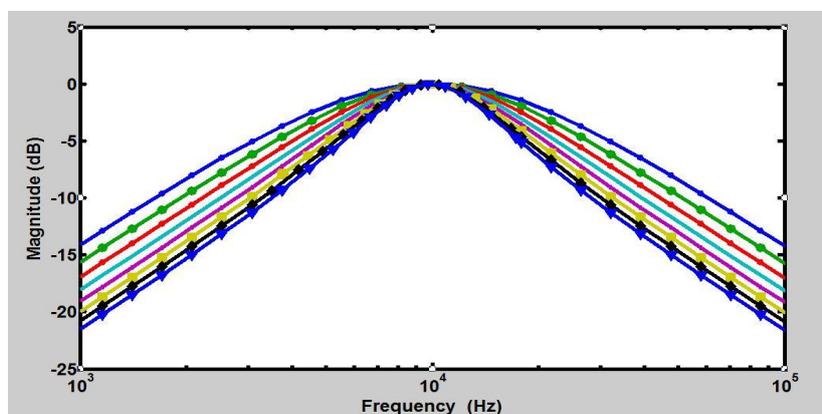
$$\frac{V_{out}}{V_{in}} = \frac{K}{s^2 (R_1 R_3 C_2 C_4) + s(R_1 C_2 + R_3 C_4 + R_1 C_4 - K R_1 C_2) + 1} \quad (12)$$

Where  $K = 1 + \frac{R_b}{R_n}$

$$\omega_n = \frac{1}{\sqrt{R_1 R_3 C_2 C_4}} \quad (13)$$

$$\frac{1}{Q} = \sqrt{\frac{R_3 C_4}{R_1 C_2}} + \sqrt{\frac{R_1 C_4}{R_3 C_2}} + (1 - K) \sqrt{\frac{R_1 C_2}{R_3 C_4}} \quad (14)$$

Sensitivity of K w.r.t Q is



**Figure 5: Sensitivity of resistor in RLC circuit.**

$$S_K^Q = QK \sqrt{\frac{R_1 C_2}{R_2 C_4}} \quad (15)$$

$$S_{R_a}^Q = -S_{R_b}^Q = -Q(K-1) \sqrt{\frac{R_1 C_2}{R_2 C_4}} = -Q \frac{R_b}{R_a} \sqrt{\frac{R_1 C_2}{R_2 C_4}} \quad (16)$$

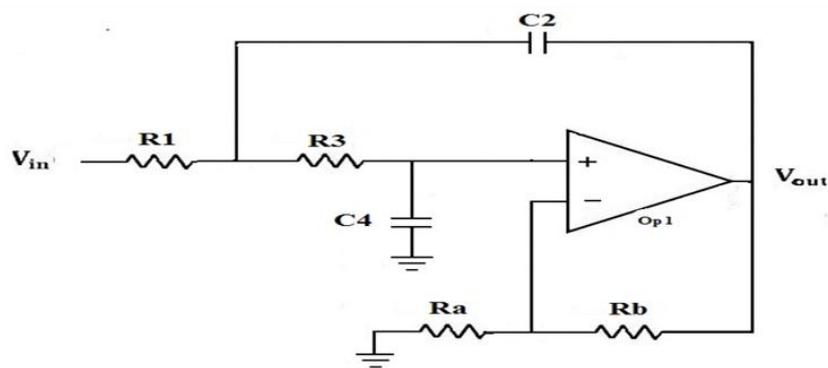
Sensitivity of active filter component w.r.t  $\omega_n$  is

$$S_{R_1}^{\omega_n} = S_{R_3}^{\omega_n} = S_{C_2}^{\omega_n} = S_{C_4}^{\omega_n} = -\frac{1}{2} \quad (17)$$

$$S_K^{\omega_n} = S_{R_a}^{\omega_n} = S_{R_b}^{\omega_n} = 0 \quad (18)$$

From above equation it is clear that natural frequency sensitivities of passive filter is  $\pm 0.5\%$  or Zero. But Q sensitivities are little complex.

The nominal values of component in Sallen key filter are  $R=10K\Omega$  and  $C=16nF$ .<sup>[10]</sup>



**Figure 6: Sallen-key filter circuit diagram.**

The Sallen key filter in figure is 1 kHz (low pass filter) with  $Q=1$ . In this filter we vary the value of  $R_b$  by keeping natural frequency constant. We vary the value of  $R_b$  from  $1K\Omega$  to  $19.9K\Omega$ . At  $R_b = 20K\Omega$  the Q goes to very high so we avoided it. As we know that  $R_b$  is main gain determine component in filter changing with Q in Figure10. Figure 11 shows the variation of different values for  $R_b$ .

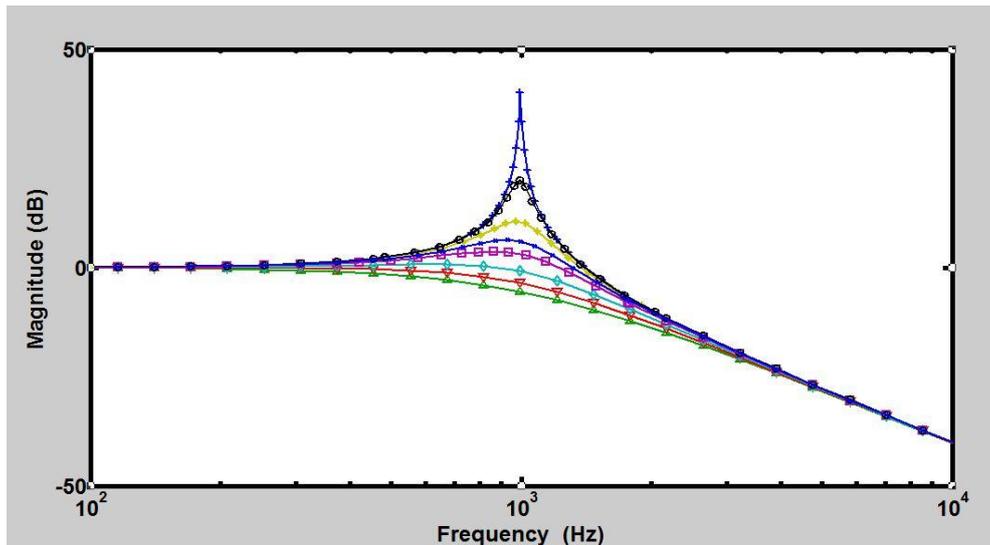


Figure 7: Response of Sallen-key filter by varying  $R_b$  from  $1K\Omega$  to  $19.9K\Omega$ .

### 5. Effect of Parameter Variation in Biquadratic Filter Circuit

Figure 12 shows the biquadratic filter circuit. Now we perform analysis on this circuit and analyze the effect of parameter variation on characteristics of circuit. First we calculate the transfer function of filter circuit and then apply sensitivity analysis as given in equation (1),<sup>[18,19,20]</sup>

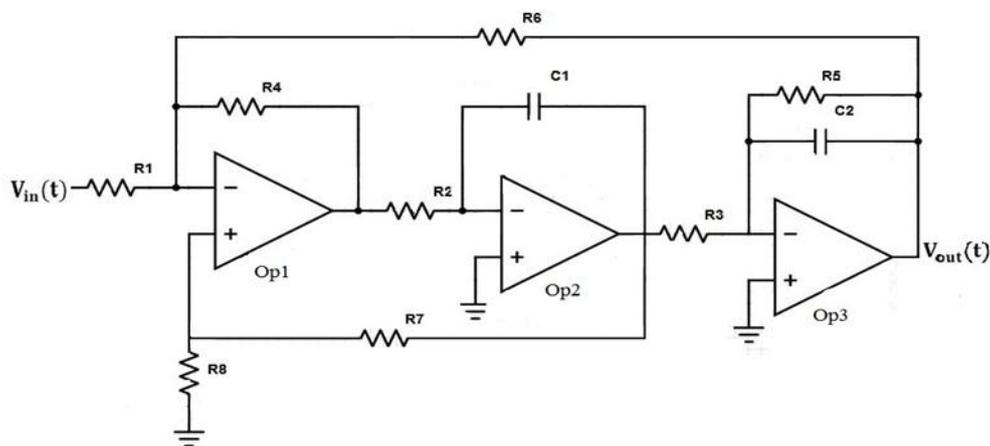


Figure 8: Circuit diagram of Biquadratic filter circuit.

Now we calculate the transfer function of biquad filter circuit from above Figure 12

$$V_2 = -\frac{R_4}{R_1}V_1 - \frac{R_4}{R_6}V_4 + \frac{R_8}{R_7+R_8} * \left(1 + \frac{R_4}{R_{11}R_6}\right)V_3 \quad (19)$$

$$V_2 = -\frac{R_4}{R_1}V_1 - \frac{R_4}{R_6}V_4 + \frac{R_8}{R_7+R_8} * \left(1 + \frac{(R_1+R_6)R_4}{R_1R_6}\right)V_3 \quad (20)$$

Now by calculation of voltages at different nodes

$$\frac{V_3}{V_2} = -\frac{1}{SR_2C_1} \quad (21)$$

$$\frac{V_4}{V_3} = -\frac{R_5}{R_3(SR_5C_2+1)} \quad (22)$$

By putting the value of equation (20) in equation (21) we get

$$V_2 = \frac{R_3SR_2C_1(SR_5C_2+1)}{R_5} V_4 \quad (23)$$

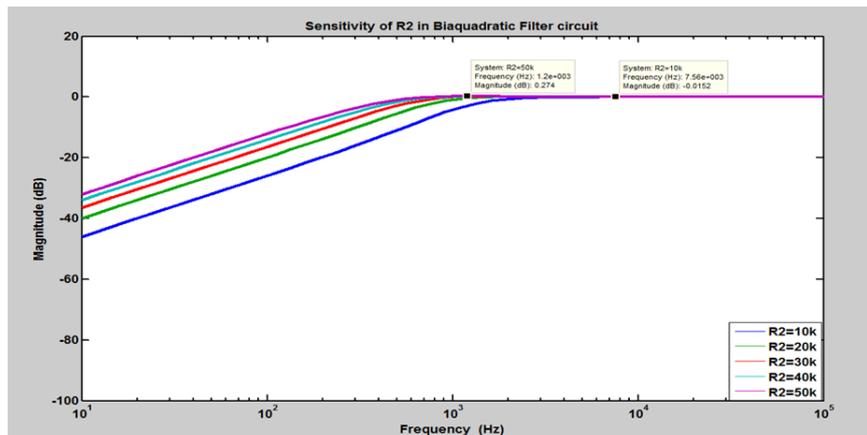
By putting the value of equation (20) and (21) in equation (22) we get

$$\begin{aligned} -\frac{R_4}{R_1} V_1 &= \frac{R_6}{R_1} V_4 + \frac{R_8}{R_7 + R_8} \left(1 + \frac{R_4}{R_{11}R_6}\right) \frac{R_3(SR_5C_2 + 1)}{R_5} V_4 + \frac{R_3SR_2C_1(SR_5C_2 + 1)}{R_5} V_4 \\ &= \frac{-\frac{R_4}{R_1}}{\frac{R_6}{R_1} + \frac{R_8}{R_7+R_8} \left(1 + \frac{R_4}{R_{11}R_6}\right) \frac{R_3(SR_5C_2+1)}{R_5} + \frac{R_3SR_2C_1(SR_5C_2+1)}{R_5}} \\ &= \frac{-R_4R_5}{R_5R_6 + \frac{R_8}{R_7+R_8} \left(1 + \frac{R_4}{R_{11}R_6}\right) R_3 + R_1R_2R_3C_1S + \frac{R_8}{R_7+R_8} \left(1 + \frac{R_4}{R_{11}R_6}\right) R_1R_3R_5C_2S + R_1R_2R_3R_5C_1C_2S^2} \end{aligned} \quad (24)$$

$$\text{Here } U = \left(\frac{R_8}{R_7+R_8}\right) \left(\frac{R_1R_6 + R_1R_4 + R_4R_6}{R_1R_6}\right) \quad (25)$$

$$H(s) = \frac{V_4}{V_1} = -\frac{R_4R_5}{(R_5R_6 + UR_1R_3) + (R_1R_2R_3C_1 + UR_1R_3R_5C_2)S + R_1R_2R_3R_5C_1C_2S^2} \quad (26)$$

Equation (26) represent the transfer function of biquadratic filter circuit.



**Figure 9: Sensitivity of  $R_2$  in Biquadratic filter circuit.**

As shown in Figure 13 if we increase the value of  $R_2$  the plot shift upward and constant at 3 kHz frequency. In first case to calculate deviation in  $R_2$  we can increase the value. And in second case we keep  $R_2$  value constant and vary the frequency, the frequency at which gain is constant, we take as test signal frequency for  $R_2$ .

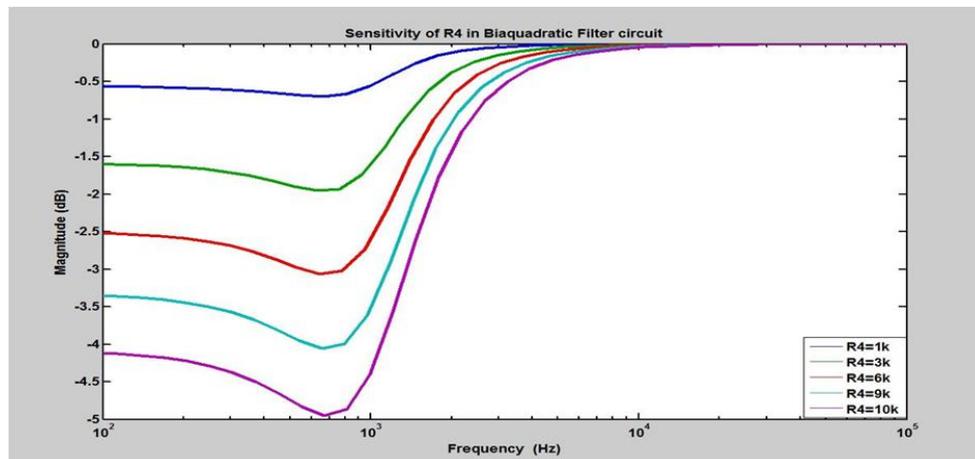
Here in Figure 5.9 value of  $R_2=50k\Omega$ . We analyse  $R_4$  sensitivity at nominal values. And we found that if we decrease  $R_4$  value then it is easy to detect fault. We derived  $R_4$  sensitivity of biquadratic filter circuit from equation (24). and after applying equation (1) we get this equation.

$$\text{Here } K = \frac{R_1 + R_6}{R_6(R_7 + R_8)}$$

$$U = \left( \frac{R_8}{R_7 + R_8} \right) \left( \frac{R_1 R_6 + R_1 R_4 + R_4 R_6}{R_1 R_6} \right)$$

$$S_{R_4}^{H(s)} = \frac{-R_3 R_4 R_5 R_8 C_2 K s + R_3 R_4 R_8 K}{(R_5 R_6 + U R_1 R_3) + (R_1 R_2 R_3 C_1 + U R_1 R_3 R_5 C_2) s + R_1 R_2 R_3 R_5 C_1 C_2 s^2} \quad (27)$$

Using above equation we can find the sensitivity of  $R_4$  in biquadratic filter circuit.



**Figure 10: Sensitivity of  $R_4$  in Biquadratic filter circuit.**

Using equation 26 we did sensitivity analysis for  $R_4$  and we found two results.

Using sensitivity analysis of  $R_4$  we know that if we decrease the  $R_4$  value then we can calculate tolerance very easily. In sensitivity analysis we keep all values nominal except  $R_4$  and also keep frequency constant.  $R_5$  Sensitivity analysis is done in same way as we did all other component. Form equation (26) and (1) we derived sensitivity equation for  $R_5$ .

$$\text{Here } U = \left( \frac{R_8}{R_7 + R_8} \right) \left( \frac{R_1 R_6 + R_1 R_4 + R_4 R_6}{R_1 R_6} \right)$$

$$H(s) = - \frac{R_4 R_5}{(R_5 R_6 + U R_1 R_3) + (R_1 R_2 R_3 C_1 + U R_1 R_3 R_5 C_2) s + R_1 R_2 R_3 R_5 C_1 C_2 s^2}$$

$$S_{R_5}^H(s) = \frac{R_1 R_2 R_3 R_5 C_1 C_2 s^2 + UR_3 R_5 C_2 s}{(R_5 R_6 + UR_1 R_3) + (R_1 R_2 R_3 C_1 + UR_1 R_3 R_5 C_2) s + R_1 R_2 R_3 R_5 C_1 C_2 s^2} \quad (28)$$

$R_6$  Sensitivity analysis is done using equation (26) and we generate sensitive value of component and test signal to detect fault.

$$\text{Here } K = \frac{R_1 + R_4}{R_6 (R_7 + R_8)}$$

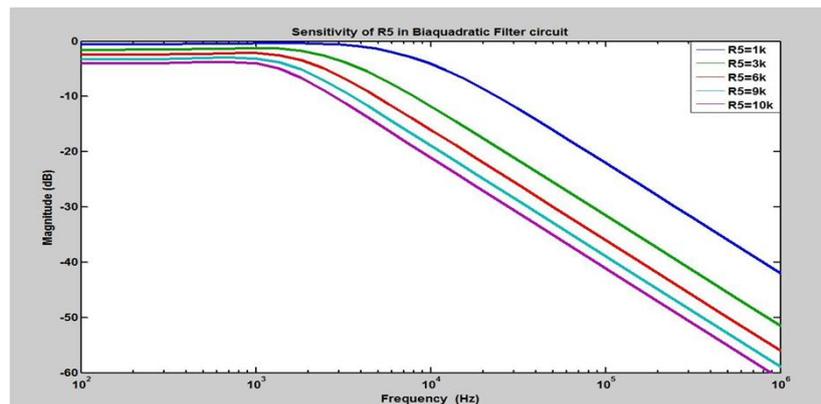


Figure 11: Sensitivity of  $R_5$  in Biquadratic filter circuit.

$$H(s) = - \frac{R_4 R_5}{(R_5 R_6 + UR_1 R_3) + (R_1 R_2 R_3 C_1 + UR_1 R_3 R_5 C_2) s + R_1 R_2 R_3 R_5 C_1 C_2 s^2}$$

$$S_{R_6}^H(s) = \frac{(R_3 R_5 R_6 R_8 C_2 K + UR_3 R_5 C_2) s - (R_5 R_6 + R_3 R_8 U)}{(R_5 R_6 + U R_1 R_3) + (R_1 R_2 R_3 C_1 + UR_1 R_3 R_5 C_2) s + R_1 R_2 R_3 R_5 C_1 C_2 s^2} \quad (29)$$

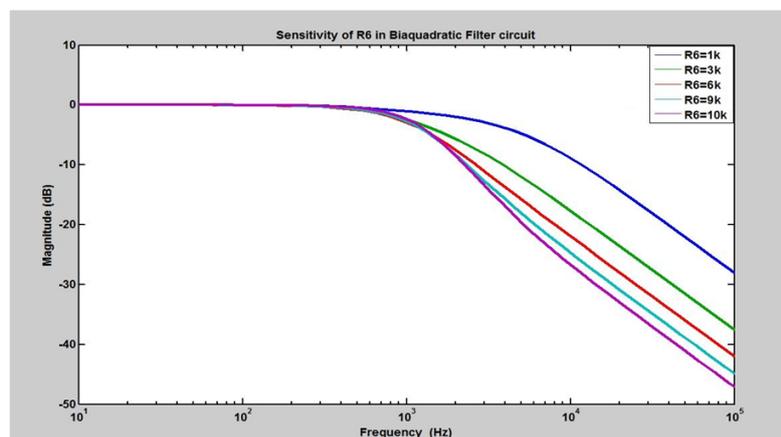


Figure 12: Sensitivity of  $R_6$  in Biquadratic filter circuit.

## CONCLUSION

In this article we perform parameter variation analysis using simple mathematical equation on linear analog VLSI circuits. This analysis help designer to choose proper value of component

so that the system output is remain stable for small changes in parameter. In this article we use sensitivity equation to perform analysis, which is done by using simple mathematical equation. The analysis is done on various linear analog circuitis including biquadratic filter circuit. All simulation are done with the help of MATLAB tool.

## REFERENCE

1. Y. Lu and R. Dandapani, Hard Fault Diagnosis in Analog Circuits Using Sensitivity Analysis, Proc. of the IEEE VLSI TestSymp., Apr., 1993; 225–229.
2. Van de Plassche, Rudy J. CMOS integrated analog-to-digital and digital-to-analog converters. Vol. 742. Springer Science & Business Media, 2013.
3. Thakur, Sandeep, and Baldev Raj. "Fault Modeling and Parametric Fault Detection in Analog VLSI Circuits using Discretization, 2016.
4. L. Milor and A.L. Sangiovanni-Vincentelli, "Minimizing Production Test Time to Detect Faults in Analog Circuits," IEEE Trans. on Computer-Aided Design, June 1994; 13: 796–813.
5. K.Maggard, P.Karunaratna, C.Stroud, "Built-In Self Test for analog circuits in Mixed Signal System," Proc. IEEE Southeast Regional Conf., 1999.
6. Cui, Jiang, and Youren Wang. "A novel approach of analog circuit fault diagnosis using support vector machines classifier." Measurement , 2011; 44(1): 281-289.
7. Osseiran, Adam, ed. Analog and mixed-signal boundary-scan: A guide to the IEEE 1149.4 test standard. Vol. 16. Springer Science & Business Media, 2013.
8. N.B. Hamida and B. Kaminska, "Multiple Fault Analog Circuit Testing by Sensitivity Analysis," J. Electronic Testing: Theory and Applications, 1993; 4: 331–343.
9. Thakur, Sandeep. A Comprehensive Approach for Modeling and Diagnosis of Various Faults in Analog VLSI Circuits, 2016.
10. Luo, Hui, Youren Wang, and Jiang Cui. "A SVDD approach of fuzzy classification for analog circuit fault diagnosis with FWT as preprocessor." Expert Systems with Applications, 2011; 38(8): 10554-10561.
11. N. Balabanian, T.A. Bickart, and S. Seshu, Electrical Network Theory, John Wiley & Sons, Inc, 1969.
12. Yuan, Lifen, Yigang He, Jiaoying Huang, and Yichuang Sun. "A new neural-network-based fault diagnosis approach for analog circuits by using kurtosis and entropy as a preprocessor." IEEE Transactions on Instrumentation and Measurement, 2010; 59(3): 586-595.

13. Thakur, Sandeep, and Baldev Raj. "Tolerance Analysis of Analog VLSI Circuits using Sensitivity, 2016.
14. Yang, Chenglin, Shulin Tian, Bing Long, and Fang Chen. "Methods of handling the tolerance and test-point selection problem for analog-circuit fault diagnosis." *IEEE Transactions on Instrumentation and Measurement*, 2011; 60(1): 176-185.
15. Lin, Tsung-Chih. "Analog circuit fault diagnosis under parameter variations based on type-2 fuzzy logic systems." *International Journal of Innovative Computing, Information and Control*, 2010; 6(5): 2137-2158.
16. Vineela, J., G. Praneetha, R. Harshad, K. Masrunnisa, M. Pruthvi Kumar, and T. Sandeep. "A Complete Analysis of Tolerance of Component in Analog VLSI Circuits Using Sensitivity." *International Journal of Hybrid Information Technology*, 2016; 9(7): 9-18.
17. Vasan, Arvind Sai Sarathi, Bing Long, and Michael Pecht. "Diagnostics and prognostics method for analog electronic circuits." *IEEE Transactions on Industrial Electronics*, 2013; 60(11): 5277-5291.
18. Adriana C. Sanabria-Borbon and Esteban Tlelo-Cuautle, Symbolic sensitivity analysis in the sizing of analog integrated circuits, *Computing Science and Automatic Control*, 2013.
19. Thakur, Sandeep, K. V. V. Satyanarayana, and K. Chinna Malla Reddy. "Diagnosis of parametric faults in linear analog VLSI circuits." In *Intelligent Systems and Control (ISCO)*, 10th International Conference on, 2016; 1-5.
20. Huelsman, L.P. and Allen, P.E., *Introduction to the Theory and Design of Active Filters*, McGraw-Hill, New York, 1980.