wjert, 2018, Vol. 4, Issue 4, 458-474. *Original Article* ISSN 2454-695X

World Journal of Engineering Research and Technology
WJERT
www.wjert.org SJIF Impact Factor: 5.218

RELIABILITY AND FUZZY ANALYSIS OF A SYSTEM HAVING THREE UNITS

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Article Received on 23/05/2018 **Article Revised on 13/06/2018** **Article Accepted on 04/07/2018**

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ABSTRACT

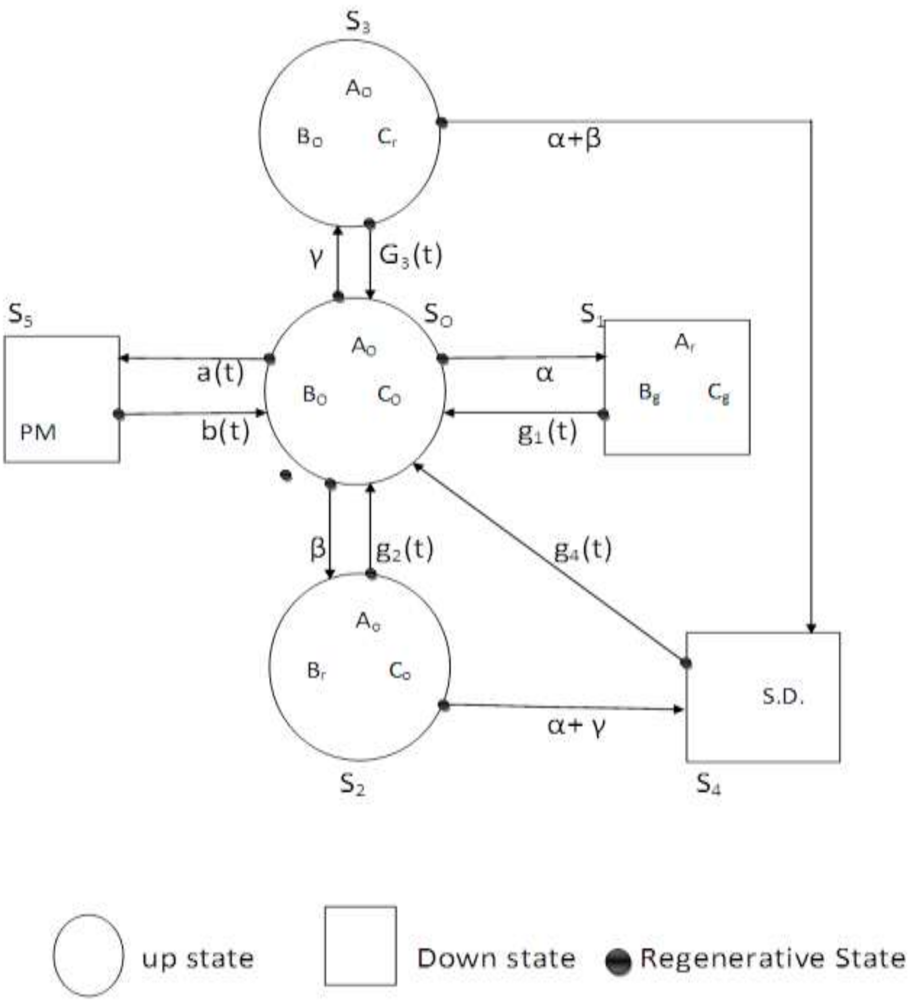
This paper presents a reliability analysis of a system having three units. The first step in the proposed methodology is assumption of model. Second step involves calculating Transition Probabilities, Mean Time to system failure, Availability analysis, Busy Period Analysis. Third step is to take particular Cases using fuzzy logic and Erlangian distributions. Fourth step is to calculate profit analysis and defuzzify the fuzzy profit analysis by using signed distance ranking method for defuzzification. It helps to allocate reliability of model before the actual system is built. It also helps to estimate the exact value of profit.

KEYWORDS: Triangular Fuzzy Number, MTSF, Availability, Busy Period, Signed Distance Ranking Method.

1. INTRODUCTION

In the present generation functional reliability of machines is top priority for system managers and they are in dire need for such system which is fault free. Reliability analysis provides an opportunity for keeping the system fault tolerant and it is vital for proper utilization and maintenance of any system. Reliability analysis basically involves having standby redundant system and timely maintenance of whole system. This technique helps for increasing system effectiveness through reducing failure and cost minimization. Standby redundant systems represent one approach to improving system reliability. Spare parts and back systems are examples of standby redundancy. Barlow and Prochan,[1] carried our pioneer work in the field of Reliability and Life Testing of Probabilistic models. Then on, redundant system with parallel configuration was discussed by the likes of Osaki.[2] Chandrasekhar et al[5] performed the same study taking the Eglangian distribution in repair time. In recent years various reliability models have been formulated for predicting, estimating or optimizing the probability of survival, the mean life or more generally the life distribution of components or systems. Industries are trying to develop more and more automation in their industrial processes in order to meet the ever increasing demands of society. The complexities of industrial systems as well as their products are increasing day by day. A parallel work in this field was done by Rander[3] by performing analysis of cold standby system with preventive maintenance and replacement of standby unit. Singh et al[6] studied reliability characteristics of an integrated steel plant. This kind of analysis is of immense help to the owners of small scale industry. Also the involvement of preventive maintenance and replacement of standby unit in the models increase the reliability of the functioning units to great extent. At last fuzzy technique is used to assess the cost of the system.

Figure 1: state transition diagram



2. Definitions and Preliminaries

A ***fuzzy set*** *A* is defined by a membership function $\mu_A(x)$ which maps each and every element of *X* to [0 1] i.e.,

$$\mu_A(x) \to [0\ 1] \qquad [2.1]$$

where, *X* is the underlying ground set. In simple, a fuzzy set is a set whose boundary is not clear. On the other hand, a fuzzy set is a set whose elements are characterized by a membership function as above.

The ***α- cut of*** a fuzzy set *A* is the crisp subset of the ground set *X*, that contains all the elements whose membership grade is greater than or equal to α. It is denoted by A_α and is defined by

$$A_{\alpha} = \{x \mid \mu_A(x) \geq \alpha\ ,\ x \in X\} \qquad [2.2]$$

A ***triangular fuzzy number*** is a fuzzy set. It is denoted by *A = (a,b,c)* and defined by the following membership function

$$\mu_A(x) = \begin{cases} 0; a \leq x \\ \dfrac{x-a}{b-a}; a \leq x \leq b \\ \dfrac{c-x}{c-b}; b \leq x \leq c \\ 0; x \geq c \end{cases} \qquad [2.3]$$

where, $a, b, c \in$ R, $A \in F_N$ where F_N is the set of triangular fuzzy numbers and is represented graphically as

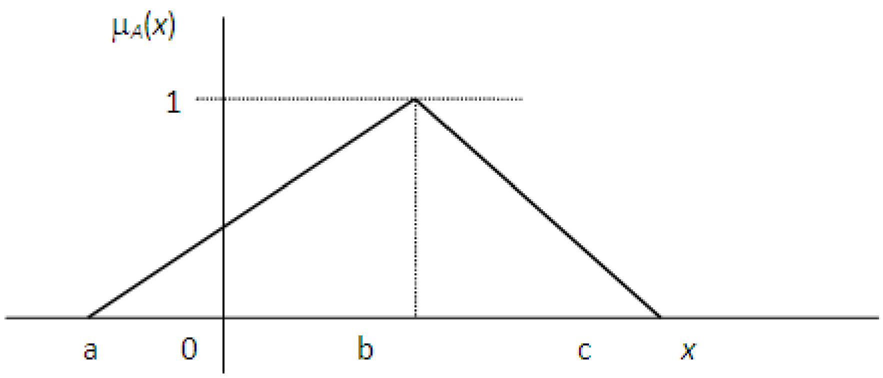


Fig. 2: Variation of membership degree w.r.t. x.

Properties of triangular fuzzy numbers

If A & B are two fuzzy numbers then their sum is also a fuzzy number. Suppose A = (a, b, c) & $B = (u, v, w)$ then,

$$A \oplus B = (a, b, c) \oplus (u, v, w)$$

$$= (a+u, b+v, c+w)$$

$$A - B = (a, b, c) - (u, v, w)$$

$$= (a, b, c) \oplus (-w, -v, -w)$$

$$= (a-w, b-v, c-u)$$

Example: Let $A = (-2, 2, 3)$ & $B = (-1, 0, 4)$

$$A \oplus B = (-1, 2, 3) + (-1, 0, 4)$$

$$= (-2-1\ , 2+0\ , 3+4)$$

$$= (-3, 2, 7)$$

$$A - B = (-2, 2, 3) - (-1, 0, 4)$$

$$= (-2, 2, 3) \oplus (-4, 0, 1)$$

$$= (-6, 2, 4)$$

Def 1: - Let d*(c, 0) = c; c, 0 $\in$ R

Geometrically, c>0 means that c lies to the right of the origin O and the distance between c and O is denoted by c = d*(c, 0). Similarly, c<0 means that c lies to the left of the origin O and the corresponding distance between c & o is denoted by –c = d*(c, 0). Therefore, d*(c, O) denotes the signed distance of c which is measured form O.

Def 2:- Signed-distance for $A = (a, b, c) \in F_N$, a triangular fuzzy number, the signed-distance of A measured from $\tilde{O}_1$ is defined by

$$d(A, \tilde{O}_1) = \frac{1}{4}(a + 2b + c)$$

Def 3:- Let $A=(a, b, c)$ & $B = (u, v, w) \in F_N$. Then the ranking of fuzzy numbers on F_N is defined by

$$A \prec B \text{ iff } d(A, \tilde{O}_1) < d(B, \tilde{O}_1)$$

& $$A \approx B \text{ iff } d(A, \tilde{O}_1) = d(B, \tilde{O}_1)$$

Fuzzy number are fuzzy subsets of set on real number satisfying some additional condition. Fuzzy number allow us to model non-probabilistic uncertainties in an easy way. Triangular and Trapezoidal fuzzy numbers are commonly used. Therefore here I am discuss about these two numbers only. Triangular and Trapezoidal fuzzy numbers can represented by (a, b, c) and (a, b, c, d) respectively. Triangular fuzzy numbers are special case of trapezoidal fuzzy numbers when b equal c.

Let A and B be two triangular fuzzy numbers, parameterized by (a1,a2,a3) and (b1,b2,b3). Their arithmetic can be described following

A+B=(a1+b1, a2+b2, a3+b3); A-B=(a1-b3, a2-b2, a3-b1);

A*B=(a1*b1, a2*b2, a3*b3); A/B=(a1/b3, a2/b2, a3/b1)

Let A and B be two triangular fuzzy numbers, parameterized by (a1,a2,a3,a4) and (b1,b2,b3,b4). Their arithmetic can be described following

A+B=(a1+b1, a2+b2, a3+b3, a4+b4); A-B=(a1-b4, a2-b3, a3-b2, a4-b1)

A*B=(a1*b1, a2*b2, a3*b3, a4*b4); A/B=(a1/b4, a2/b3, a3/b2, a4/b1)

These are the operations performed on fuzzy numbers. However, these values need to be mapped to real values for the calculation. Process of converting fuzzy numbers into crisp numbers is called defuzzification. Formula for performing defuzzification operations on triangular and trapezoidal fuzzy numbers. These formulas are given below. Let A(a1, a2, a3) and B(b1,b2,b3) are two triangular fuzzy numbers. Their defuzzification formula is given as

$$G(t) = \frac{1}{4}(a + 2b + c)$$

3. System Description of the model

The system consists of three units namely one main unit A and two associate units B & C. Here the associate unit B and C are dependent upon main unit A when the system is in operation. The system functions when the main unit and at least one of the associate units are working. The system is taken in down mode when the main unit is not functioning. As soon as a job arrives, all the units work with load. It is further assumed that only one job is taken for processing at a time. There is a single repairman who repairs the failed units on first come first served basis. Using regenerative point technique several system characteristics such as transition probabilities, mean sojourn times, availability and busy period of the repairman are evaluated. Chapman Kolmogorov equations are used to develop the recursive relations. In the end, profit analysis is done with the help of values of MTSF, Availability and Busy Periods and fuzzy technique is used to calculate the actual profit.

4. Assumptions used in the model

a. The system consists of one main unit A and two associate units B and C.
b. The main unit A works with the help of associate units B and C.
c. There is a single repairman which repairs the failed units on priority basis.
d. After a random period of time the whole system goes for preventive maintenance.
e. All units work as new after repair.
f. The failure rates of all the units are taken to be exponential whereas the repair time distributions are arbitrary.
g. Switching devices are perfect and instantaneous.

5. Symbols and Notations

p_{ij} = Transition probabilities from S_i to S_j

μ_i = Mean sojourn time at time t

E_0 =State of the system at epoch t=0

E=set of regenerative states $S_0 - S_5$

$q_{ij}(t)$ = Probability density function of transition time from S_i to S_j

$Q_{ij}(t)$ = Cumulative distribution function of transition time from S_i to S_j

$\pi_i(t)$ = Cdf of time to system failure when starting from state $E_0 = S_i \in E$

$\mu_i(t)$ = Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t)$ = Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3 / r_4$ =Constant repair rate of Main unit A /Unit B/Unit C/ Shut down state

$\alpha / \beta / \gamma$ =Failure rate of Main unit A /Unit B/Unit C

η = Repair rate from P.M.

$g_1(t) / g_2(t) / g_3(t)$ =Probability density function of repair time of Main unit A/Unit B/Unit C

$\overline{G}_1(t) / \overline{G}_2(t) / \overline{G}_3(t)$ =Cumulative distribution function of repair time of Main unit A/Unit B/Unit C

$g_4(t) / \overline{G}_4(t)$ = Pdf / Cdf of repair time of Shut down state.

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

$\overline{A}(t)$ = Cumulative distribution functions of preventive maintenance.

$\overline{B}(t)$ = Cumulative distribution functions of preventive maintenance completion time.

$\boxed{s}$ Symbol for Laplace -Stieltjes transforms.

$\boxed{c}$ Symbol for Laplace-convolution.

6. Symbols used for states of the system

$A_0 / A_g / A_r$ -- Main unit 'A' under operation/good and non-operative mode/ repair mode

$B_0 / B_r / B_g$ -- Associate Unit 'B' under operation/repair/ good and non-operative mode

$C_0 / C_r / C_g$ -- Associate Unit 'C' under operation/repair/good and non-operative mode

P.M. -- System under preventive maintenance.

S.D. – System under shut down mode.

Up states: $S_0 = (A_0, B_0, C_0); S_2 = (A_0, B_r, C_0); S_3 = (A_0, B_0, C_r)$

Down States: $S_1 = (A_r, B_o, C_o); S_4 = (S.D.); S_5 = (P.M.)$

7. Transition Probabilities

Simple probabilistic considerations yield the following non-zero transition probabilities:

1. $p_{01} = \int_0^\infty \alpha e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\alpha}{x_1}[1-a^*(x_1)],$
2. $p_{02} = \int_0^\infty \beta e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\beta}{x_1}[1-a^*(x_1)]$
3. $p_{03} = \int_0^\infty \gamma e^{-(\alpha+\beta+\gamma)t} \overline{A}(t)dt = \frac{\gamma}{x_1}[1-a^*(x_1)],$
4. $p_{05} = \int_0^\infty a(t) e^{-(\alpha+\beta+\gamma)t} dt = a^*(x_1)$
5. $p_{20} = \int_0^\infty e^{-(\alpha+\gamma)t} g_2(t)dt = g_2^*(\alpha+\gamma),$
6. $p_{24} = \int_0^\infty (\alpha+\gamma) e^{-(\alpha+\gamma)t} \overline{G}_2(t)dt = 1 - g_2^*(\alpha+\gamma)$
7. $p_{30}(t) = \int_0^\infty e^{-(\alpha+\beta)t} g_3(t)dt = g_3^*(\alpha+\beta)$
8. $p_{34}(t) = \int_0^\infty (\alpha+\beta) e^{-(\alpha+\beta)t} \overline{G}_3(t)dt = 1 - g_3^*(\alpha+\beta)$
9. $p_{10} = p_{40} = p_{50} = 1$ Where $x_1 = \alpha+\beta+\gamma$ [7.1-7.9]

And mean sojourn time are given by

10. $\mu_0 = \dfrac{1}{x_1}[1 - a^*(x_1)]$,

11. $\mu_1 = \int_0^\infty \overline{G}_1(t)dt$,

12. $\mu_2 = \dfrac{1}{\alpha + \gamma}[1 - g_2^{\,*}(\alpha + \gamma)]$,

13. $\mu_3 = \dfrac{1}{\alpha + \beta}[1 - g_3^{\,*}(\alpha + \beta)]$

14. $\mu_4 = \int_0^\infty \overline{G}_4(t)dt$

15. $\mu_5 = \int_0^\infty \overline{B}(t)dt$

[7.10-7.15]

8. Mean Time to System Failure

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regard the down state as absorbing. Using the argument as for the regenerative process, we obtain the following recursive relations.

$$\pi_0(t) = Q_{01}(t) + Q_{02}(t)\ \boxed{s}\ \pi_2(t) + Q_{03}(t)\ \boxed{s}\ \pi_3(t) + Q_{05}(t)$$
$$\pi_2(t) = Q_{20}(t)\ \boxed{s}\ \pi_0(t) + Q_{24}(t)$$
$$\pi_3(t) = Q_{30}(t)\ \boxed{s}\ \pi_0(t) + Q_{34}(t) \qquad [8.1\text{-}8.3]$$

Taking Laplace - Stieltjes transform of above equations and writing in matrix form, We get

$$\begin{bmatrix} 1 & -\tilde{Q}_{02} & -\tilde{Q}_{03} \\ -\tilde{Q}_{20} & 1 & 0 \\ -\tilde{Q}_{30} & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_2 \\ \pi_3 \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{01} + \tilde{Q}_{05} \\ \tilde{Q}_{24} \\ \tilde{Q}_{34} \end{bmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -\tilde{Q}_{02} & -\tilde{Q}_{03} \\ -\tilde{Q}_{20} & 1 & 0 \\ -\tilde{Q}_{30} & 0 & 1 \end{vmatrix} = 1 - \tilde{Q}_{20}\tilde{Q}_{02} - \tilde{Q}_{03}\tilde{Q}_{30} \text{ and}$$

$$N_1(s) = \begin{vmatrix} \tilde{Q}_{01} + \tilde{Q}_{05} & -\tilde{Q}_{02} & -\tilde{Q}_{03} \\ \tilde{Q}_{24} & 1 & 0 \\ \tilde{Q}_{34} & 0 & 1 \end{vmatrix} = (\tilde{Q}_{01} + \tilde{Q}_{05} + \tilde{Q}_{02}\tilde{Q}_{24} + \tilde{Q}_{03}\tilde{Q}_{34}) \qquad [8.4\text{-}8.5]$$

Now letting $s \to 0$ and noting that $\underset{t\to\infty}{Lim} Q_{ij}(t) = p_{ij}$, we get,

$D_1(0) = 1 - p_{02}p_{20} - p_{03}p_{30}$ and $N_1(0) = p_{01} + p_{05} + p_{02}p_{24} + p_{03}p_{34}$

The mean time to system failure when the system starts from the state S_0 is given by

$$\text{MTSF}= E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)} \qquad [8.6]$$

To obtain the numerator of the above equation, we collect the coefficients of relevant of $m_{i\,j}$ in $D_1^{/}(0) - N_1^{/}(0)$.

Coeff. of ($m_{01} = m_{02} = m_{03} = m_{05}$) =1

Coeff. of (m_{20}) = p_{02}

Coeff. of ($m_{30} = m_{34}$) = p_{03} From equation [8.6]

$$\text{MTSF}= E(T) = -[\frac{d}{ds}\tilde{\pi}_0(s)]_{s=0} = \frac{D_1^{/}(0) - N_1^{/}(0)}{D_1(0)} \qquad = \frac{\mu_0 + \mu_2 p_{02} + \mu_3 p_{03}}{1 - p_{02}p_{20} - p_{03}p_{30}} \qquad [8.7]$$

9. Availability Analysis

Let $M_i(t)(i = 0,1,2)$ denote the probability that system is initially in regenerative state $S_i \in E$ is up at time t without passing through any other regenerative state or returning to itself through one or more non regenerative states .i.e. either it continues to remain in regenerative S_i or a non regenerative state including itself . By probabilistic arguments, we have the following recursive relations

$$M_0(t) = e^{-(\alpha+\beta+\gamma)t}\overline{A}(t), \qquad M_2(t) = e^{-(\alpha+\beta)t}\overline{G}_2(t), \qquad M_3(t) = e^{-(\alpha+\gamma)t}\overline{G}_3(t), \quad [9.1\text{-}9.3]$$

Recursive relations giving point wise availability $A_i(t)$ given as follows:

$$A_0(t) = M_0(t) + \sum_{i=1,2,3,5} q_{0i}(t) \boxed{c} A_i(t)\ ; \qquad A_1(t) = q_{10}(t) \boxed{c} A_0(t);$$

$$A_2(t) = M_2(t) + \sum_{i=0,4} q_{2i}(t) \boxed{c} A_i(t)\ ; \qquad A_3(t) = M_3(t) + \sum_{i=0,4} q_{3i}(t) \boxed{c} A_i(t);$$

$$A_4(t) = q_{40}(t) \boxed{c} A_0(t)\ ; \qquad A_5(t) = q_{50}(t) \boxed{c} A_0(t); \qquad [9.4\text{-}9.9]$$

Taking Laplace transformation of above equations; and writing in matrix form, we get

$$q_{6X6}[A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*]^{/} = [M_0^*, 0, M_2^*, M_3^*, 0, 0]^{/}$$

Where $q_{6X6} = \begin{bmatrix} 1 & -q_{01}^{\ *} & -q_{02} & -q_{03}^{\ *} & 0 & -q_{05}^{\ *} \\ -q_{10}^{\ *} & 1 & 0 & 0 & 0 & 0 \\ -q_{20} & 0 & 1 & 0 & -q_{24}^{\ *} & 0 \\ -q_{30}^{\ *} & 0 & 0 & 1 & -q_{34}^{\ *} & 0 \\ -q_{40} & 0 & 0 & 0 & 1 & 0 \\ -q_{50}^{\ *} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$ [9.9a]

Therefore $D_2(s) = \begin{bmatrix} 1 & -q_{01}^{\ *} & -q_{02} & -q_{03}^{\ *} & 0 & -q_{05}^{\ *} \\ -q_{10}^{\ *} & 1 & 0 & 0 & 0 & 0 \\ -q_{20} & 0 & 1 & 0 & -q_{24}^{\ *} & 0 \\ -q_{30}^{\ *} & 0 & 0 & 1 & -q_{34}^{\ *} & 0 \\ -q_{40} & 0 & 0 & 0 & 1 & 0 \\ -q_{50}^{\ *} & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$

$$= 1 - q_{01}^{\ *} q_{10}^{\ *} - q_{02}^{\ *}(q_{20}^{\ *} + q_{24}^{\ *} q_{40}^{\ *}) - q_{03}^{\ *}(q_{30}^{\ *} + q_{34}^{\ *} q_{40}^{\ *}) - q_{05}^{\ *} q_{50}^{\ *} \qquad [9.10]$$

If $s \to 0$ we get $D_2(0) = 0$ which is true

Now $N_2(s) = \begin{bmatrix} M_0^{\ *} & -q_{01}^{\ *} & -q_{02} & -q_{03}^{\ *} & 0 & -q_{05}^{\ *} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ M_2^{\ *} & 0 & 1 & 0 & -q_{24}^{\ *} & 0 \\ M_3^{\ *} & 0 & 0 & 1 & -q_{34}^{\ *} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$

Solving this Determinant, we get

$$N_2(s) = M_0^{\ *} + M_2^{\ *} q_{02}^{\ *} + M_3^{\ *} q_{03}^{\ *} \qquad [9.11]$$

If $s \to 0$ we get

$$N_2(0) = \mu_0 + \mu_2 p_{02} + \mu_3 p_{03} \qquad [9.12]$$

To find the value of $D_2^{\ /}(0)$, we collect the relevant coefficient m_{ij} in $D_2(s)$ we get

Coeff. of $(m_{01} = m_{02} = m_{03} = m_{05}) = 1 = L_0$

Coeff. of $(m_{10}) = p_{01} = L_1$ Coeff. of $(m_{20} = m_{24}) = p_{02} = L_2$

Coeff.of $(m_{30} = m_{34}) = p_{03} = L_3$ Coeff.f $m_{40} = p_{03}p_{34} + p_{03}p_{34} = L_4$

Coeff. of $m_{50} = p_{05} = L_5$ [9.13-9.18]

Thus the solution for the steady-state availability is given by

$$A_0^*(\infty) = \underset{t\to\infty}{Lim} A_0^*(t) = \underset{S\to 0}{Lim}\, sA_0^*(s) = \frac{N_2(0)}{D_2^{/}(0)} = \frac{\mu_0 L_0 + \mu_2 L_2 + \mu_3 L_3}{\sum\limits_{i=0,1,2,3,4,5} \mu_i L_i} \qquad [9.19]$$

10. BUSY PERIOD ANALYSIS

(a) Busy period of the Repairman in performing Normal repair in time (0, t]

Let $W_i(t)$ $(i = 1, 2, 3)$ denote the probability that the repairman is busy initially with normal repair in regenerative state S_i and remain busy at epoch t without transiting to any other state or returning to itself through one or more regenerative states.

By probabilistic arguments we have

$$W_1(t) = \overline{G}_1(t), W_2(t) = \overline{G}_2(t), W_3(t) = \overline{G}_3(t) \qquad [10.1\text{-}10.3]$$

Developing similar recursive relations as in availability, we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t) \boxed{c}\; B_i(t) \quad ; \quad B_1(t) = W_1(t) + q_{10}(t) \boxed{c}\; B_0(t) \;;$$

$$B_2(t) = W_2(t) + \sum_{i=0,4} q_{2i}(t) \boxed{c}\; B_i(t) \quad ; \quad B_3(t) = W_3(t) + \sum_{i=0,4} q_{3i}(t) \boxed{c}\; B_i(t);$$

$$B_4(t) = q_{40}(t) \boxed{c}\; B_0(t) \quad ; \quad B_5(t) = q_{50}(t) \boxed{c}\; B_0(t); \qquad [10.4\text{-}10.9]$$

Taking Laplace transformation of above equations; and writing in matrix form, we get

$$q_{6X6}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*]^{/} = [0, W_1^*, W_2^*, W_3^*, 0, 0]^{/} \qquad [10.10]$$

Where q_{6x6} is denoted by [9.9a] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

Now $$N_3(s) = \begin{bmatrix} 0 & -q_{01}^* & -q_{02} & -q_{03}^* & 0 & -q_{05}^* \\ W_1^* & 1 & 0 & 0 & 0 & 0 \\ W_2^* & 0 & 1 & 0 & -q_{24}^* & 0 \\ W_3^* & 0 & 0 & 1 & -q_{34}^* & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$$

So, we get the value of this determinant after putting $s \to 0$ is

$$N_3(0) = (\mu_1 p_{01} + \mu_2 p_{02} + \mu_3 p_{03})$$

$$= \mu_1 L_1 + \mu_2 L_2 + \mu_3 L_3 = \sum_{i=1,2,3} \mu_i L_i \qquad [10.11]$$

Thus, in the long run, the fraction of time for which the repairman is busy with normal repair of the failed unit is given by:

$$B_0^{1^*}(\infty) = \lim_{t\to\infty} B_0^{1^*}(t) = \lim_{s\to 0} sB_0^{1^*}(s) = \frac{N_3(0)}{D_2^{/}(0)} = \frac{\sum_{i=1,2,3}\mu_i L_i}{\sum_{i=0,1,2,3,4,5}\mu_i L_i} \qquad [10.12]$$

(b) Busy period of the Repairman in preventive maintenance in time (0, t]

By probabilistic arguments we have

$$W_5(t) = \overline{B}(t) \qquad [10.13]$$

Developing similar recursive relations as in 10(a), we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t)\ \boxed{c}\ B_i(t)\ ; \qquad B_1(t) = q_{10}(t)\ \boxed{c}\ B_0(t)\ ;$$

$$B_2(t) = \sum_{i=0,4} q_{2i}(t)\ \boxed{c}\ B_i(t)\ ; \qquad B_3(t) = \sum_{i=0,4} q_{3i}(t)\ \boxed{c}\ B_i(t)\ ;$$

$$B_4(t) = q_{40}(t)\ \boxed{c}\ B_0(t)\ ; \qquad B_5(t) = W_5(t) + q_{50}(t)\ \boxed{c}\ B_0(t) \qquad [10.14\text{-}10.19]$$

Taking Laplace transformation of above equations; and writing in matrix form, we get

$$q_{6X6}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*]^{/} = [0,0,0,0,0,W_5^*]^{/}$$

Where q_{6x6} is denoted by [9.9a] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

$$\text{Now } N_4(s) = \begin{bmatrix} 0 & -q_{01}^* & -q_{02} & -q_{03}^* & 0 & -q_{05}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 \\ 0^* & 0 & 0 & 1 & -q_{34}^* & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ W_5^* & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$$

Solving this Determinant, In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_4(0) = \mu_5 p_{05} = \mu_5 L_5 \qquad [10.20]$$

Thus, in the long run, the fraction of time for which the system is under preventive maintenance is given by:

$$B_0^{2^*}(\infty) = \underset{t\to\infty}{Lim}\, B_0^{2^*}(t) = \underset{s\to 0}{Lim}\, sB_0^{2^*}(s) = \frac{N_4(0)}{D_2^{/}(0)} = \frac{\mu_5 L_5}{\sum\limits_{i=0,1,2,3,4,5} \mu_i L_i} \qquad [10.21]$$

(c) Busy period of the Repairman in Shut Down repair in time (0, t]

By probabilistic arguments we have

$$W_4(t) = \overline{G}_4(t) \qquad [10.22]$$

Developing similar recursive relations as in 10(b), we have

$$B_0(t) = \sum_{i=1,2,3,5} q_{0i}(t)\ \boxed{c}\ B_i(t)\ ; \qquad B_1(t) = q_{10}(t)\ \boxed{c}\ B_0(t);$$

$$B_2(t) = \sum_{i=0,4} q_{2i}(t)\ \boxed{c}\ B_i(t)\ ; \qquad B_3(t) = \sum_{i=0,4} q_{3i}(t)\ \boxed{c}\ B_i(t);$$

$$B_4(t) = W_4(t) + q_{40}(t)\ \boxed{c}\ B_0(t)\ ;\ B_5(t) = q_{50}(t)\ \boxed{c}\ B_0(t)\ ; \qquad [10.23\text{-}10.28]$$

Taking Laplace transformation of above equations; and writing in matrix form, we get

$$q_{6X6}[B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*]^{/} = [0,0,0,0,W_4^*,0]^{/}$$

Where q_{6x6} is denoted by [9.9a] and therefore $D_2^{/}(s)$ is obtained as in the expression of availability.

$$\text{Now } N_5(s) = \begin{bmatrix} 0 & -q_{01}^* & -q_{02} & -q_{03}^* & 0 & -q_{05}^* \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 \\ 0^* & 0 & 0 & 1 & -q_{34}^* & 0 \\ W_5^* & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6X6}$$

In the long run, we get the value of this determinant after putting $s \to 0$ is

$$N_5(0) = \mu_4(p_{02}p_{24} + p_{03}p_{30}) = \mu_4 L_4 \qquad [10.29]$$

Thus the fraction of time for which the system is under shut down is given by:

$$B_0^{3^*}(\infty) = \underset{t\to\infty}{Lim}\, B_0^{3^*}(t) = \underset{s\to 0}{Lim}\, sB_0^{3^*}(s) = \frac{N_5(0)}{D_2^{/}(0)} = \frac{\mu_4 L_4}{\sum\limits_{i=0,1,2,3,4,5} \mu_i L_i} \qquad [10.30]$$

11. Particular cases

When all repair time distributions are n-phase Erlangian distributions i.e.

Density function $g_i(t) = \frac{nr_i (nr_i t)^{n-1} e^{-nr_i t}}{n-1!}$ and

Survival function $\overline{G}_j(t) = \sum_{j=0}^{n-1} \frac{(nr_i t)^j e^{-nr_i t}}{j!}$ [11.1-11.2] And

other distributions are negative exponential

$a(t) = \theta e^{-\theta t}, b(t) = \eta e^{-\eta t}, \overline{A}(t) = e^{-\theta t}, \overline{B}(t) = e^{-\eta t}$ [11.3-11.6]

For n=1 $g_i(t) = r_i e^{-r_i t}$, $\overline{G}_i(t) = e^{-r_i t}$ If i=1, 2, 3, 4

$g_1(t) = r_1 e^{-r_1 t}, g_2(t) = r_2 e^{-r_2 t}, g_3(t) = r_3 e^{-r_3 t}, g_4(t) = r_4 e^{-r_4 t}$

$\overline{G}_1(t) = e^{-r_1 t}$,$\overline{G}_2(t) = e^{-r_2 t}, \overline{G}_3(t) = e^{-r_3 t}$, $\overline{G}_4(t) = e^{-r_4 t}$ [11.7-11.14]

Also $p_{01} = \frac{\alpha}{x_1 + \theta}, p_{02} = \frac{\beta}{x_1 + \theta}, p_{03} = \frac{\gamma}{x_1 + \theta}, p_{05} = \frac{\theta}{x_1 + \theta}$

$p_{20} = \frac{r_2}{\alpha + \gamma + r_2}, p_{24} = \frac{\alpha + \gamma}{\alpha + \gamma + r_2}$

$p_{30} = \frac{r_3}{\alpha + \beta + r_3}, p_{34} = \frac{\alpha + \beta}{\alpha + \beta + r_3}, \mu_0 = \frac{1}{x_1 + \theta}, \mu_1 = \frac{1}{r_1},$

$\mu_2 = \frac{1}{\alpha + \beta + r_{2\,4}}$ $, \mu_3 = \frac{1}{\alpha + \gamma + r_3} =, \mu_4 = \frac{1}{r_4}, \mu_5 = \frac{1}{\eta}$ [11.15-11.30]

MTSF= $\frac{\mu_0 + \mu_2 p_{02} + \mu_3 p_{03}}{1 - p_{02} p_{20} - p_{03} p_{30}}$ $, A_0(\infty) = \frac{\mu_0 L_0 + \mu_2 L_2 + \mu_3 L_3}{\sum_{i=0,1,2,3,4,5} \mu_i L_i},$

$B_0^{1^*}(\infty) = \frac{\sum_{i=1,2,3} \mu_i L_i}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}$ $, B_0^{2^*}(\infty) = \frac{\mu_5 L_5}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}, B_0^{3^*}(\infty) = \frac{\mu_4 L_4}{\sum_{i=0,1,2,3,4,5} \mu_i L_i}$ [11.31-11.35]

Where $L_0 = 1; L_1 = p_{01}; L_2 = p_{02}$; $L_3 = p_{03}; L_4 = p_{02} p_{24} + p_{03} p_{34}$; $L_5 = p_{05}$;

[11.36-11.41]

12. Profit Analysis

The profit analysis of the system can be carried out by considering the expected busy period of the repairman in repair of the unit in (0,t].

Therefore, G(t) = Expected total revenue earned by the system in (0,t] -Expected repair cost of the failed units

Expected repair cost of the repairman in preventive maintenance -Expected repair cost of the Repairman in shut down

$$= C_1\mu_{up}(t) - C_2\mu_{b1}(t) - C_3\mu_{b2}(t) - C_4\mu_{b3}(t) \qquad [12.1]$$

Where $\mu_{up}(t) = \int_0^t A_0(t)\,dt$; $\mu_{b1}(t) = \int_0^t B_0^1(t)\,dt$; $\mu_{b2}(t) = \int_0^t B_0^2(t)\,dt$; $\mu_{b3}(t) = \int_0^t B_0{}^3(t)dt$ [12.2-12.5]

C_1 is the revenue per unit time and C_2, C_3, C_4 are the cost per unit time for which the system is under simple repair, preventive maintenance and shut down repair respectively.

Apply fuzzy concept in [12.1]

$$\tilde{G}(t) = \tilde{C}_1\mu_{up}(t) - \tilde{C}_2\mu_{b1}(t) - \tilde{C}_3\mu_{b2}(t) - \tilde{C}_4\mu_{b3}(t) \qquad [12.6]$$

Taking triangle fuzzy number

$\tilde{C}_1 = (24,5,8)$ $\tilde{C}_2 = (4,2,1)$ $\tilde{C}_3 = (5,3,1)$ $\tilde{C}_4 = (12,6,3)$

$\mu_{up} = 0.819$ $\mu_{b1} = 0.174$ $\mu_{b2} = 0.022$ $\mu_{b3} = 1.151$

$$\tilde{G}(t) = (24,5,8)*0.819 - (4,2,1)*0.174 - (5,3,1)*0.022 - (12,6,3)*1.151$$

$$= (19.656, 4.095, 6.552) - (0.696, 0.348, 0.174) - (0.11, 0.066, 0.022) - (13.813, 6.906, 3.453)$$

$$= (19.656, 4.095, 6.552) - (14.619, 7.32, 3.649)$$

$$= (19.656, 4.095, 6.552) + (-3.649, -7.32, -14.619)$$

$$= (16.047, -3.225, -8.067) \qquad [12.7]$$

Applying defuzzification using signed distance ranking method in eq. [12.7], we get

$$G(t) = \frac{1}{4}(16.047 - 2*3.225 - 8.067)$$

$$= \frac{1}{4}(7.98 - 6.45)$$

$$= \frac{1}{4}(1.53)$$

$$= 0.3825 \text{ (rounding off)}$$

13. DISCUSSION AND RESULT

It is seen from the table 1.1 that value of MTSF decreases with increase in the failure rate of main unit. The same can be predicted in the case of Availability. It is also seen that with the application of preventive maintenance technique Availability increases to some extent. The use of fuzzy theory in profit analysis removes the uncertainty in the cost of various parameters and gives the exact value of profit of any system.

Table 1.1: Variations in MTSF vis-à-vis Failure Rate of Main Unit.

α	**β**	**γ , θ**	**λ**	**r3,r4**	**r1, r2**	**MTSF**
0.1	0.01	0.01	0.1	0.01	0.01	51.84
0.2	0.02	0.01	0.1	0.01	0.02	46.02
0.3	0.03	0.01	0.1	0.01	0.03	30.81
0.4	0.04	0.01	0.1	0.01	0.04	25.42

Table 1.2: Variations in Availability vis-à-vis Failure Rate of Main Unit.

α	**B**	**γ , θ**	**λ**	**r3,r4**	**r1, r2**	**Availability**
0.1	0.01	0.01	0.1	0.01	0.01	125.22
0.2	0.02	0.01	0.1	0.01	0.02	97.57
0.3	0.03	0.01	0.1	0.01	0.03	46.66
0.4	0.04	0.01	0.1	0.01	0.04	35.89

Table 1.3: Variations in Profit vis-à-vis increase Failure Rate of Main Unit.

α	**B**	**γ , θ**	**λ**	**r3,r4**	**Profit**
0.1	0.01	0.01	0.1	0.01	62.592
0.2	0.02	0.01	0.1	0.01	41.222
0.3	0.03	0.01	0.1	0.01	22.129
0.4	0.04	0.01	0.1	0.01	11.027

Table 1.4: Variations in Profit vis-à-vis increase Repair Rate of Main Unit.

r3,r4	**η**	**r1, r2**	**γ , θ**	**λ**	**Profit**
0.01	0.01	0.01	0.01	0.1	0.981
0.01	0.01	0.02	0.01	0.1	18.273
0.01	0.01	0.03	0.01	0.1	31.752
0.01	0.01	0.04	0.01	0.1	42.826

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