

### FLOOD STUDIES IN CALABAR METROPOLIS: THE MARKOV CHAIN APPROACH

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**ABSTRACT**

The Markovian principle was applied to study the Calabar Metropolis catchment so as to determine the influence of some hydraulic and hydrologic parameters on the rainfall-runoff processes in the Catchment. The model framework relied significantly on literature

from Wang & Maritz (1990) and Chartfield (2004). The factor analysis approach was employed to discretize the chosen variables. The rotated component matrix showed velocity, drainage area and the degree of imperviousness to give the highest loading of 0.72, 0.95 and 0.82 to components 1,2 and 3 respectively. There was a very strong correlation between the cross sectional area of drain and discharge ( $r = 0.93, p < 0.05$ ) as well as the basin area ( $r = 0.76, p < 0.05$ ). Incorrect sizing and spread of drains as well as the existing slopes employed in the generation of the drains' invert during construction have been seen as some of the key factors that foster flooding in the Metropolis. Misalignment of the drains with the existing outlet does not help the expected discharge of storm runoff to receiving bodies. Sequel to these, revisiting the Calabar Master Plan of 1972 with the original design for six drainage outlets is recommended.

**KEYWORDS:** Markov chain, misalignment, discretize, imperviousness, catchment, metropolis, master plan, loading.

## 1.0 INTRODUCTION

Hydrological model is a simplified representation of a natural system. It can be said that “a model” is a collection of symbols, which represents the system in a concise form that works as a representation of natural system or some aspects of it.

The rainfall- runoff model is one of the most frequently used events in hydrology. It determines the runoff signal which leaves the watershed from the rainfall signal received by the basin.

Rainfall- runoff modeling plays a pivotal supportive decision role in resolving practical water resource management and planning issues in any given watershed. In Calabar Metropolis for instance, after every storm event, some streets look clean -, while others look dirty. In both cases, there are problems. Unpredicted storms with its resultant runoff rushing over paved surfaces picks up wastes and pollutants from the clean surfaces to the dirty ones and then flows either directly or via storm systems, to the various water bodies in the Metropolis.

In Calabar Metropolis storm drains (and especially the main channels) may have been designed without the basic data, and may have relied on empirically – derived criteria as pointed out by Adeleye (1978). Yet studies of the channels as indicated by Effiong-Fuller (1998) and Ekeng (1998) have all shown that the channels only helped to alter the points of incidence of floods, while solving the problem only in a few areas. Rainfall- runoff modeling plays a pivotal supportive decision role in resolving practical water resource management and planning issues in any given watershed.

Calabar Metropolis has witnessed a very rapid urbanization over the same period. One of the many complex problems resulting from increased urbanization globally is related to management of storm water from developed areas. Proper water management has been a perennial problem in the Metropolis, as it is in many parts of the globe. This, of course has led to the incessant flooding of the streets and reduction of the quality of water in rivers and receiving water bodies. The objectives of this research are to:

1. Identify the pertinent factors or variables of the rainfall-runoff processes of the Calabar Metropolis catchment.
2. Analyse the catchment’s rainfall-runoff processes.
3. Apply the Markovian principles in the classification of flood events in the catchment.

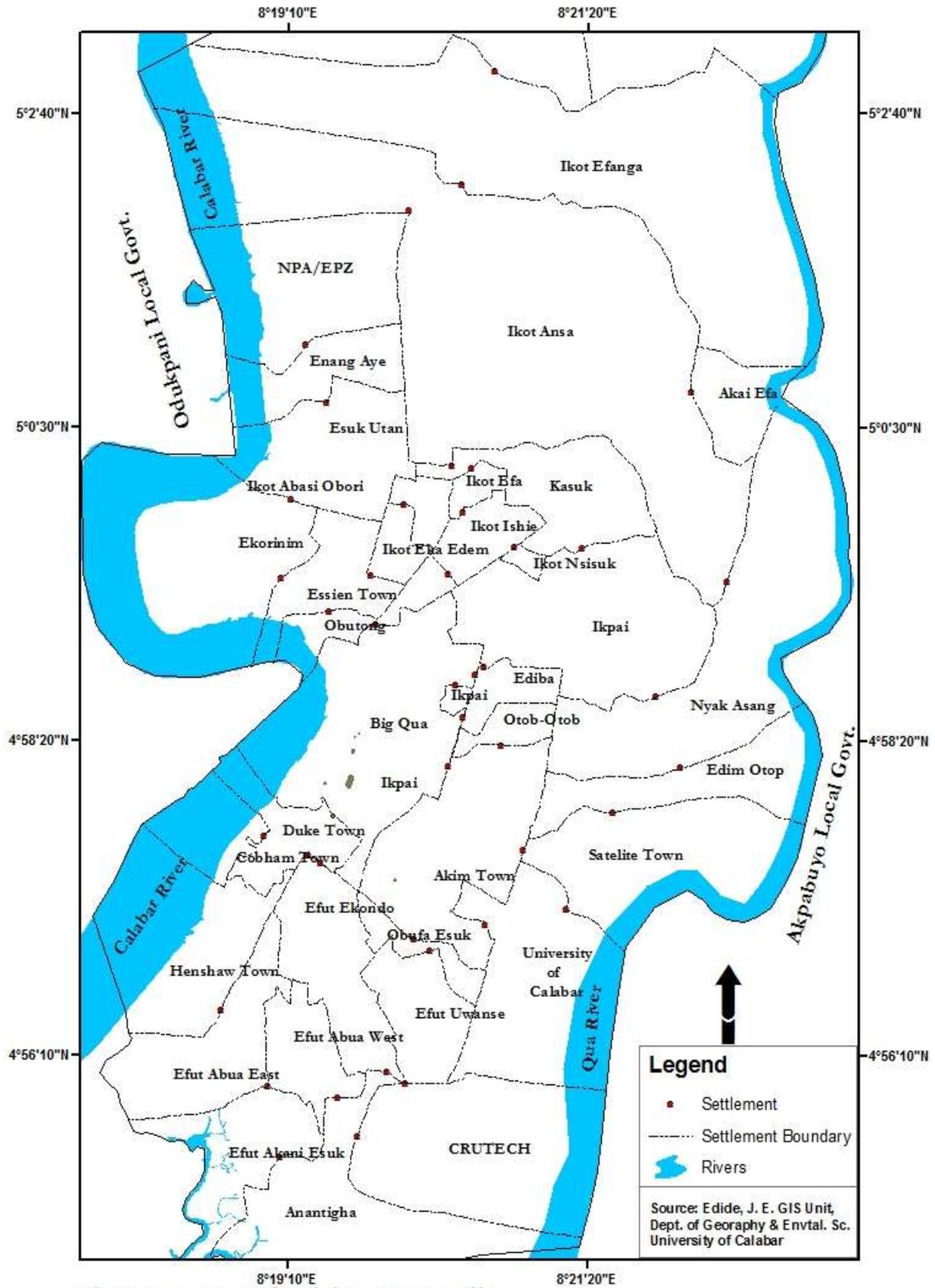


Fig. 3.1: Layout of Calabar Metropolis

## 2.0 MATERIALS AND METHODS

### 2.1 Description of Area of Study

Calabar Metropolis lies between latitudes  $04^{\circ} 45' 30''$  North and  $05^{\circ} 08'30''$  North of the Equator and longitudes  $8^{\circ} 11' 21''$  and  $8^{\circ}27'00''$  East of the Meridian. The town is flanked on its eastern and western borders by two large perennial streams viz: the Great Kwa River and the Calabar River respectively. These are aside from the numerous ephemeral channels which receive water after storm events to drain the area of study (Fig.1).

It occupies an area of about  $223.325 \text{ km}^2$  with major clans being Efut Uwanse, Obufa – Esuk, Old Calabar, Mbukpa, Anantigha, Archibong Town, Cobham Town, Henshaw Town, Old Town, Essien Town, Ikot Ansa, Ikot Effanga, Ikot Omin, Ikot Nkebre, Akim Qua Town, Big Qua Town, Kasuk, Satellite Town, Nyakasang etc.

The urban structure can best be explained in terms of the Hoytes (1939) as captured in Ugbong (2000) sectoral model. Population and settlements are concentrated in zones inhabited by the three ethnic groups-, the Efuts to the south, the Efiks to the west and the Quas to the east.

With a population of 202,585 in 1991, it now has a population of over 400,000, (C.R.S Ministry of Land and Housing, 2008). This shows a growth or an increase in population of 49.4% or an average annual population increase of 2.9%.

As a coastal town in Nigeria, Calabar Metropolis has a high relative humidity, usually between 80% and 100%. All the year round, temperature rarely falls below  $19^{\circ}\text{C}$  and average  $27^{\circ}\text{C}$ . The average daily maximum is above  $24^{\circ}\text{C}$  with a range of  $6^{\circ}\text{C}$ , and a seasonal variation of the same amount, between the hottest month (March) and the coolest month (August). Expectedly therefore, evaporation will be high (Antigha *et al*, 2014). The relative humidity however drops with the rise in temperature to about 70% in the afternoon during the dry season. Vapour pressure in the air averages 29 millibars throughout the year (CRBDA Report, 1995).

The Calabar River is about 7.58 metres deep (Tesko-Kutz, 1973). The city lies in a peninsular between the two rivers, 56km up the Calabar River away from the sea. Calabar has been described as an inter-fluvial settlement (Ugbong, 2000).

The present conditions as seen in terms of road network and settlements are as follows: The Calabar Road cum Murtala Muhammed Highway form the main artery of the city's roads network, running from north to south, linking all other major lines. Other major routes are the Ndidem Usang Iso Road, which runs parallel to the Highway, and MCC Road which runs perpendicular to both the Highway and Usang Iso Roads. Other streets spread like branches of a tree throughout the city.

## 2.2 Rainfall Data

Rainfall and evapotranspiration measurements for the data were done at the Nigerian Meteorological Centre (NIMET) of the Margaret Ekpo International Airport, Calabar, Cross River State. Two sets of rainfall data were obtained for the study. The first was a 28 year daily/hourly rainfall data, while the second was a 43 year yearly/monthly rainfall data. The daily rainfall readings which were personally obtained for 23 months from January, 2008 to November, 2009, out of the 333 months rain fell were subjected to closer scrutiny. (This aligns with Wilson's (2006) 510 rainfall data from 17 different catchments in the U.K). This was because the runoff readings also personally obtained from the area for the study were obtained during some of these storms' event. A total number of 346 storm events were recorded. 5587 millimeters of rain was recorded for the period monitored. Total hours that rain fell were 1084.67hours. This gave an average rainfall of 242.9mm per month of rain, and an average intensity of 5.15mm per hour of rain. Both the cylindrical and self-recording rain gauges were used for the rainfall readings.

## 2.3 Runoff Data

Flow measurements are critical to monitoring storm water best management practices (BMPs). Accurate flow measurements are necessary for accurate computing of samples used to characterize storm runoff and for the estimation of volumes. A total of 10 drainage outlets were selected as points for storm runoff readings. The choice of these 10 locations was informed by their vulnerability to flooding, being major flood outlets in the catchment. 20 storm events were monitored at each recording point and 80 runoff readings were taken from each reading point with the propeller- type current metre type F<sub>4</sub> (A.Ott –Kempton, Bavaria). The metre had a reading range of  $n < 4.67, v = 0.0560n + 0.040$ ;  $n \geq 4.67, v = 0.0545n + 0.047$  for propeller one and  $n < 1.28, v = 0.0905n + 0.040$ ;  $n \geq 1.2, v = 0.1030n + 0.024$  for propeller two, where  $v$  is in m/s and  $n$  is the number of revolutions. The readings were taken during the months selected as the wettest part of the year (May to October) 24 months and for

storms with duration of not less than 120 minutes (Dayaratne, 1996, 2000). These gave a total of 800 runoff readings.

The readings were taken at the five minutes, 10 minutes, 15 minutes up to the 120 minutes rainfall intervals. These were recorded as  $I_5 I_{10} I_{15}$  to  $I_{120}$  respectively (Drainage Criteria Manual, 2007).

#### **2.4 Land Use Data**

Land use data were obtained from the most recent topo map (2014) of the study area obtained from the State Ministry of Lands and Survey, Calabar. The Land Development software (Bentley Systems, Inc. USA). was employed with the application of the polyline approach in discretizing both the total basin area and the area that has become built-up. The degree of imperviousness was obtained by determining from the total area as calculated with the software, the relative percentages of the built-up areas with reference to the total area as scaled, (Ugbong, 2000, Okon, 2012).

#### **2.5 Gradient Data**

The gradient data for the sub- catchments were generated from the topo map with the aid of the Land Development software (Bentley Systems, Inc. USA). The profiling approach was equally employed for confirmation. A global positioning system (G.P.S) instrument was used. Mapping was done by marking off 25m interval on ground. This was made to run to cover the whole length of the overland flow of the sub-drainage basins drained by the channels network. The inlet elevation was subtracted from the outlet elevation. The plot of the inlet and outlet elevations was obtained with reference to the overland flow length. The tangent of the angle so formed gave the value of the slope ( Uyanah, 2006, Okon, 2012).

#### **2.6 Model Development**

To develop a model, it is important to define what purpose or purposes a model should have. Mulligan and Wainwright (2004) have identified three purposes to which a general model is usually put. They included amongst others, an aid to research, a tool for simulation and prediction as well as a research product.

The Markov Chain principles were applied to generate a model to attempt a solution to the flooding problems in the Metropolis. The Markov chain is a natural extension of sequences of independent random variables. A Markov process usually refers to a first order process of

autoregressive processes. The future development of this process is completely determined by the present state and is independent of the way in which the present state has developed, which explains the “first order” (Chatfield, 2004). A Markov chain is a discrete state random process and it is a simple linear model (Meyn & Tweedie 2005).

A Markov chain is a special case of a Markov process, which itself is a special case of a random or stochastic process. In the most general terms, a random process is a family, or ordered set of related random variables  $X(\epsilon)$ , where  $\epsilon$  is a time parameter. There are many kinds of random processes. Two of the most distinguishing characteristics of a random process are:

- (i) Its state space, or the set of values that the random variables of the process can have, and
- (ii) The nature of the indexing parameter.

It is possible to classify random process along each of these dimensions (Wang and Maritz, 1990, Chartfield, 2004):

**State Space:** Continuous state  $X(t)$  can take on any value or a finite or infinite continuous interval or set of such intervals.

**Discrete State:**  $X(t)$  has only a countable number of possible values  $[S_0, S_1, S_2 \dots S_k]$ . A discrete state random process is also often called a chain.

**Index Parameter (Time):** Discrete-time: Permitted time at which changes in value may occur are finite or countable; continuous state may occur anywhere within a finite or infinite interval or set of such intervals. In a first order Markov chain,

$X_{n+1}$  depended only on  $X_n$ , and not on any  $X_i$ ,  $1 \leq i \leq n$ . (1)

$P(X_{n+1} = S_i / X_{n1}, X_{n+1}, \dots X_1, X_0) = P(X_{n+1} = S_i / X_n = S_i)$  (2)

A correlation matrix was first run to ascertain the level of significance of the chosen variables and their corporate effect on the drainage system. The level of significance for each variable are shown below.

**Table 1: Rotated Components matrix of urban drainage Variables.**

Variables	Components		
	I	II	III
Velocity	0.48	<b>0.72</b>	0.41
Drainage area	<b>0.95</b>	-0.14	0.03
Cross Sectional Area	0.86	0.26	-0.32
Artificial drainage density	-0.43	0.65	0.419
Degree of imperviousness	0.28	-0.34	<b>0.820</b>
Gradient	-0.002	-0.65	0.32

### 3.0 RESULTS AND DISCUSSIONS

Tables 2-6 show the transition probability interval, the observed frequency, measured discharge, velocity, depth of flow, size percentage of the correlation matrix, equilibrium state probabilities and the expected length of spell per location, while figure 1 shows the plots of velocity of flow for all sampled points.

The results of the rotated component analysis showed that drainage area gave the highest loading to component one, velocity for component two and degree of imperviousness for component three. Hence, the list of these factors can be reduced to three. Therefore, cross sectional area, velocity and degree of impervious can be used to study the process.

Markov assumption is that future evaluations only depend on the current state. A three state Markov chain approach was applied on the pattern of occurrence of flood. In this research, a three state of flood condition was considered (Concardo et al, 2008). The velocity of flow was used to classify flood into this three conditions: if velocity of flow is  $>1.5\text{m/s}$ , it is said to be high,  $0.5\text{m/s} < V < 1.5\text{m/s}$  medium and  $0.1\text{m/s} < V < 0.5\text{m/s}$  low. Because of the uniqueness of each location, this model was developed for each of the 10 locations selected within the Metropolis viz: Ediba one, Ediba two, Ibom Layout, Mayne Avenue, Big Qua, Wapi Junction axis, Mary Slessor, Yellow-Duke/Inyang, Marian and Marina.

Daily velocity of flow on these areas were collected and based on this criteria, a three state Markov chain model was developed.

$$P = (P_{ij}) = P(d/1) \text{ where; } i, 1 \in S. \quad (3)$$

**Table 2: Transition Probability Interval.**

		Current day		
		High (H)	Medium (M)	Low (L)
Previous day	High (H)	$P_{HH}$	$P_{HM}$	$P_{HL}$
	Medium (M)	$P_{MH}$	$P_{MM}$	$P_{ML}$
	Low (L)	$P_{LH}$	$P_{LM}$	$P_{LL}$

Where;

$P_{HH}=P_{HH}$  : Probability of a day with high flood preceded by another day with high flood.

$P_{HM}=P_{MH}$ : Probability of medium flood preceded by high flood day.

$P_{HL}=P_{LH}$  : Probability of experiencing low flood in a day preceded by high flood.

$P_{ML}=P_{LM}$  : Probability of a day having low flood preceded by a day with medium flood and so forth subject to the condition that the sum of probabilities of each row in one i.e.

$$P_{HH} + P_{HM} + P_{HL} = 1 \quad (4)$$

$$P_{MH} + P_{MM} + P_{ML} = 1 \quad (5)$$

$$P_{LH} + P_{LM} + P_{LL} = 1 \quad (6)$$

For three state Markov chain approach, the values of  $P_{01} = (0, 1 = H, M, L)$  are required (Wang & Maritz, 1990). These values can be estimated from 3 x 3 order observed frequency table 3.6 below:

**Table 3: Observed Frequency Table.**

Previous day		Current Day			
		High (H)	Medium (M)	Low (L)	Total
	H	$n_{HH}$	$n_{HM}$	$n_{HL}$	$n_H$
	M	$n_{MH}$	$n_{MM}$	$n_{ML}$	$n_M$
	L	$n_{LH}$	$n_{LM}$	$n_{LL}$	$n_L$

Where;  $n_{ij} = (i, j = H, M, L)$  is observed frequency i.e.

$n_{HH}$  : Number of day with high flood preceded by a day with high flood.

$n_{HM}$  : Number of day with medium flood preceded by a day with high flood.

$n_{HL}$  : Number of day with low flood preceded by a day with high flood.

$n_{MH}$  : Number of day with high flood proceeds with medium flood.

$n_{MM}$  : Number of day with medium flood preceded by a day with medium flood.

$n_{ML}$  : Number of day with low flood preceded by a medium flood day.

$n_{LH}$  : Number of day with high flood given that the previous day flood was high.

$n_{LM}$  : Number of day with medium flood preceded by a day with low flood.

$n_{LL}$  : Number of day with low flood given that the preceding day is low.

$n_H : n_{HH} + n_{HL} =$  Total number of high flood.

$n_M : n_{MH} + n_{MM} + n_{ML} =$  Total number of medium flood.

$n_L : n_{LH} + n_{LM} + n_{LL} =$  Total number of low flood.

The maximum likelihood estimate of  $p_{ij}$  ( $I, j = H, L, M$ ) are

$$\hat{p}_{ij} = \frac{n_{ij}}{n_{i\cdot}} \quad (7)$$

**Table 4: Measured Discharge, Velocity, Flow Depth per time of Measurement for Locations 1-10.**

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location I	5 min	1.52	0.26	0.88	1.34
	10 min	3.10	0.33	0.88	2.73
	15 min	4.82	0.38	0.88	4.24
	20 min	5.65	0.42	0.88	4.97
	30 min	9.79	0.58	0.88	8.62
	40 min	10.69	0.61	0.88	9.41
	50 min	7.63	0.45	0.88	6.71
	60 min	5.32	0.40	0.88	4.68
	90 min	4.15	0.36	0.88	3.65
	120min	2.42	0.37	0.88	2.13

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 2	5 min	1.77	0.44	2.125	3.76
	10 min	3.62	0.48	2.125	7.69
	15 min	5.01	0.56	2.125	10.65
	20 min	7.04	0.61	2.125	14.96
	30 min	10.32	0.71	2.125	21.93
	40 min	11.57	0.85	2.125	24.59
	50 min	8.60	0.65	2.125	18.28
	60 min	6.23	0.59	2.125	13.24
	90 min	4.25	0.52	2.125	9.03
	120 min	2.7	0.46	2.125	5.74

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 3	5 min	0.78	0.12	0.63	0.49
	10 min	1.64	0.14	0.63	1.03
	15 min	2.28	0.17	0.63	1.44
	20 min	3.28	0.23	0.63	2.07
	30 min	4.80	0.49	0.63	3.02
	40 min	5.40	0.61	0.63	3.40
	50 min	3.90	0.54	0.63	2.46
	60 min	2.66	0.19	0.63	1.68
	90 min	2.03	0.15	0.63	1.28
	120 min	1.16	0.13	0.63	0.73

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 4	5 min	0.82	0.36	0.63	0.52
	10 min	1.74	0.45	0.63	1.10
	15 min	2.49	0.50	0.63	1.57
	20 min	3.52	0.58	0.63	2.22
	30 min	5.40	0.62	0.63	3.40
	40 min	5.72	0.65	0.63	3.60
	50 min	3.0	0.55	0.63	2.84
	60 min	2.20	0.53	0.63	1.89
	90 min	1.03	0.47	0.63	1.39
	120 min	1.16	0.42	0.63	0.82

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 5	5 min	0.59	0.15	0.33	0.19
	10 min	1.21	0.22	0.33	0.40
	15 min	1.84	0.27	0.33	0.61
	20 min	2.68	0.41	0.33	0.88
	30 min	3.77	0.62	0.33	1.24
	40 min	3.67	0.50	0.33	1.21
	50 min	2.68	0.43	0.33	0.88
	60 min	2.16	0.32	0.33	0.71
	90 min	1.49	0.25	0.33	0.49
	120 min	1.0	0.19	0.33	0.33

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 6	5 min	1.77	0.21	6.075	10.75
	10 min	3.10	0.25	6.075	18.83
	15 min	4.34	0.32	6.075	27.58
	20 min	6.23	0.83	6.075	37.84
	30 min	9.88	1.14	6.075	60.02
	40 min	9.27	1.08	6.075	56.32
	50 min	7.25	0.97	6.075	44.04
	60 min	5.22	0.52	6.075	31.71
	90 min	3.64	0.29	6.075	22.11
	120 min	2.10	0.23	6.075	12.76

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 7	5 min	0.95	0.31	0.556	0.53
	10 min	1.92	0.37	0.556	1.08
	15 min	3.09	0.46	0.556	1.72
	20 min	4.62	0.60	0.556	2.57
	30 min	6.41	0.68	0.556	3.56
	40 min	5.93	0.65	0.556	3.30
	50 min	4.80	0.62	0.556	2.67
	60 min	3.20	0.51	0.556	1.78
	90 min	2.36	0.42	0.556	1.31
	120 min	1.59	0.34	0.556	0.87

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 8	5 min	0.93	0.11	3.08	2.86
	10 min	1.74	0.15	3.08	5.36
	15 min	2.62	0.19	3.08	8.07
	20 min	3.63	0.46	3.08	11.18
	30 min	5.48	0.82	3.08	16.88
	40 min	5.98	0.94	3.08	18.42
	50 min	4.61	0.79	3.08	14.20
	60 min	2.99	0.32	3.08	9.21
	90 min	2.24	0.16	3.08	6.90
	120 min	1.27	0.14	3.08	3.91

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location 9	5 min	0.79	0.21	0.22	0.17
	10 min	1.70	0.29	0.22	0.37
	15 min	2.45	0.38	0.22	0.54
	20 min	3.32	0.46	0.22	0.73
	30 min	4.90	0.58	0.22	1.08
	40 min	5.54	0.74	0.22	1.22
	50 min	4.36	0.52	0.22	0.96
	60 min	3.12	0.44	0.22	0.69
	90 min	2.29	0.32	0.22	0.50
	120 min	1.43	0.26	0.22	0.31

Locations	$\Delta t$ - time interval (Mins)	$\Delta V$ - velocity (m/s)	$\Delta h$ - depth (m)	Cross Sectional Area (A) ( $m^2$ )	$\Delta Q$ (discharge) measured $m^3/s$
Location10	5 min	0.92	0.23	1.4	1.29
	10 min	1.94	0.26	1.4	2.72
	15 min	2.70	0.31	1.4	3.78
	20 min	3.80	0.36	1.4	5.32
	30 min	5.22	0.54	1.4	7.31
	40 min	5.79	0.85	1.4	8.11
	50 min	4.40	0.42	1.4	6.16
	60 min	3.05	0.33	1.4	4.27
	90 min	2.28	0.27	1.4	3.19
	120 min	1.35	0.24	1.4	1.89

### 3.1: Test of Goodness of Fit

Here, the test of the velocity of the three state Markov chain approaches that one day dependences could be considered as explaining the behaviour of flood was done. This was achieved by applying the traditional chi-square test. According to Wang and Maritz (1990), for applying these tests, the following were obtained:

**H<sub>0</sub>**: Flood on conservative day is independent against the alternative hypothesis.

**H<sub>1</sub>**: Flood on conservative day is not independent.

The traditional chi-square test statistic is 
$$\chi^2 = \sum_{i=1}^n \left( \frac{O_i - E_i}{E_i} \right)^2 \quad (8)$$

Where;

$O_i$  are the observed frequencies

$E_i$  are the corresponding expected frequencies

$i$  = Number of observation (1, 2 ...n).

The critical region for testing null hypothesis is

$$\chi_c^2 \geq \chi^2(n-1) \quad (9)$$

where  $\chi_c^2$  is the calculated value of chi-square with (n-1) degrees of freedom at level of significance. In 1990, Wang and Maritz developed a test statistic for testing the hypothesis thus:

$$W_S = \frac{S_A + S_B - 1}{\sqrt{V(S_A + S_B - 1)}} \rightarrow N(0, P) \quad (10)$$

$$V(S_A + S_B - 1) = 2\Pi_1\Pi_2\Pi_3 \left( \frac{1}{nH \bullet nM} + \frac{1}{nM \bullet nL} + \frac{1}{nL \bullet nH} \right) \quad (11)$$

Where;

$$S_A = P_{HH} + P_{MM} + P_{LL}$$

$$S_B = P_{HM}P_{MH} + P_{LM}P_{ML} + P_{ML}P_{LM} - P_{HH}P_{MM} - P_{HH}P_{LL} - P_{MM}P_{LL}. \quad (12)$$

$\Pi_1, \Pi_2, \Pi_3$ , are stationary probabilities

### 3.2 Flood Prediction

To predict flood, the steady state probabilities  $\Pi_1, \Pi_2$  and  $\Pi_3$  were calculated from the formula as developed by Wang & Maritz (1990):

$$F_L = \Pi \times P \quad (13)$$

Where;  $F_L$  is the probability of flood per sample location

$P$  is the transition probability matrix of the location.

### 3.3: Computation of Steady State or Equality Probabilities

Steady state probability was computed using the formula:

$$\Pi_1 = \Pi_2 P_{ij} \quad (14)$$

#### For Location 1

Calculated value of  $\chi^2$  was greater than that of the table value. Hence, we reject the null hypothesis and conclude that the Markovian property holds for the sampled locations.

$$\Pi_1 = 0.94\Pi_1 + 0.06\Pi_2 \quad (15)$$

$$\Pi_2 = 0.11\Pi_1 + 0.89\Pi_2 \quad (16)$$

$$\Pi_1 + \Pi_2 = 1$$

Solving this equation

$$\Pi_1 = 0.5 \text{ and } \Pi_2 = 0.5$$

This means that the probability of high flood is 0.5 and medium flood is also 0.5 while that of low flood is 0 for location 1.

### Steady State Probability for Location 2

$$\Pi_1 = \Pi_1 + 0\Pi_2 + 0\Pi_3$$

$$\Pi_2 = \Pi_1 + 4.0\Pi_2 + 0\Pi_3$$

$$\Pi_1 = 0\Pi_1 + 0\Pi_2 + 0\Pi_3$$

$$\Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_1 = 0.6, \Pi_2 = 0.5 \text{ and } \Pi_3 = 0$$

### Location 3

$$\Pi_1 = 0\Pi_1 + \Pi_2 + 0\Pi_3$$

$$\Pi_2 = 0\Pi_1 + \Pi_2 + 0\Pi_3$$

$$\Pi_3 = 0$$

$$\Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_1 = \Pi_2$$

$$\Pi_1 + \Pi_2 = 1$$

Hence,  $\Pi_1 = 0.5$  and  $\Pi_2 = 0.5$

### Location 4

$$\Pi_1 = 0$$

$$\Pi_2 = 0.99\Pi_2 + 0.01\Pi_3$$

$$\Pi_3 = \Pi_2$$

$$\Pi_2 + \Pi_3 = 1$$

$$\Pi_2 + \Pi_3 = 1$$

$$\Pi_2 = 0.5$$

$$\Pi_3 = 0.5$$

$$\Pi_1 = 0$$

**Location 5**

$$\Pi_1 = 0$$

$$\Pi_2 = 0.96\Pi_2 + 0.04\Pi_3$$

$$\Pi_3 = \Pi_2$$

$$\Pi_2 + \Pi_3 = 1$$

$$\Pi_2 + \Pi_3 = 1$$

$$\Pi_2 = 0.5$$

$$\Pi_3 = 0.5$$

**Location 6**

$$\Pi_1 = 0.93\Pi_1 + 0.07\Pi_2$$

$$\Pi_2 = 0.04\Pi_1 + 0.96\Pi_2$$

$$\Pi_3 = 0$$

$$\Pi_1 + \Pi_2 = 1$$

Solving this equation

$$\Pi_1 = 0.5$$

$$\Pi_2 = 0.5$$

$$\Pi_3 = 0$$

**Location 7**

$$\Pi_1 = \Pi_1 + 0\Pi_2 + 0\Pi_3$$

$$\Pi_2 = 0\Pi_1 + 0.99\Pi_2 + 0.004\Pi_3$$

$$\Pi_3 = 0\Pi_1 + \Pi_2 + 0\Pi_3$$

$$\Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_1 = 1$$

$$\Pi_2 = 0.99\Pi_2 + 0.004\Pi_3$$

$$\Pi_3 = \Pi_3$$

$$\Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_1 = 0$$

$$\Pi_2 = 0.71$$

$$\Pi_3 = 0.29$$

**Location 8**

$$\Pi_1 = 0$$

$$\Pi_2 = 1.0$$

$$\Pi_3 = 0$$

**Location 9**

$$\Pi_1 = 0$$

$$\Pi_2 = 1$$

$$\Pi_3 = 0$$

**Location 10**

$$\Pi_1 = 0.75\Pi_1 + 0.25\Pi_2$$

$$\Pi_2 = 0.004\Pi_1 + 0.996\Pi_2$$

$$\Pi_3 = 0$$

$$\Pi_1 + \Pi_2 + \Pi_3 = 1$$

$$\Pi_1 = \Pi_2$$

$$\Pi_1 = 0.5$$

$$\Pi_2 = 0.5$$

**Table 5: Size, percentages of the correlation matrix of the first three principal components of Urban Drainage Variables.**

Component	Eigen value	Percentage total variance	Cumulative Percentage
I	2.121	35.35	35.35
II	1.552	61.22	61.22
III	1.221	81.57	81.57

**Table 6: Equilibrium State Probabilities, And Expected Length of Different Spells Per Location.**

Location	Expected Length Of						
	$\Pi_1$	$\Pi_2$	$\Pi_3$	High flood	Medium flood	Low flood	Flood cycle
1	0.5	0.5	0	17	1	1	19
2	0.5	0.5	0	Undefined	1	1	Undefined
3	0.5	0.5	0	1	Undefined	1	Undefined
4	0	0.5	0.5	1	1	1	3
5	0	0.5	0.5	1	1	1	3
6	0.5	0.5	0	14	1	1	16
7	0	0.71	0.29	Undefined	1	1	Undefined
8	0	1.0	0	1	1	1	3
9	0	1	0	1	1	1	3
10	0.5	0.5	0	4	1	1	6

$\Pi_1$ ,  $\Pi_2$  and  $\Pi_3$  are the equilibrium probabilities.

#### Expected length of different spells

- a) A high spell of length  $d$  is defined as sequence of consecutive high preceded and followed by medium or low flood.

$$E(H) = \frac{1}{1 - P_{HH}} \quad (17)$$

$$E(M) = \frac{1}{2 - P_{MM}} \quad (18)$$

$$E(L) = \frac{1}{3 - P_{LL}} \quad (19)$$

- b) A low spell is defined as sequence of consecutive low preceded and followed by high or medium flood.
- c) A medium spell is the sequence of consecutive medium flood preceded and followed by a high or low flood.

$E(H)$ ,  $E(M)$  and  $E(L)$  are the expected length of high flood, medium and low flood respectively while  $P_{HH}$ ,  $P_{MM}$ , and  $P_{LL}$  are their transition probabilities.

d) Flood cycle (FC) is given as:

$$E(FC) = E(H) + E(M) + E(L) \quad (20)$$

#### 4.0: CONCLUSION AND RECOMMENDATIONS

The perennial flooding in some parts of the Calabar Metropolis drainage basin has been a thing of grave concern to all residents and stake holders in recent times.

Ugbong (2000) had shown that the artificial drainage density in the catchment is grossly inadequate when the basin area is related to the channel network.

Runoff discharges, according to Antigha, *et al* (2014) defines the mean velocity of surface water passing through a given cross sectional area in a given unit of time. This, according to them, is paramount in the determination of flood peaks since flood has been observed to occur when the volume of flow can no longer be accommodated within the margins of its normal channel. Runoff itself is a product of rainfall whose intensity exceeds the infiltration capacity of the soil.

In urban storm drainage systems studies, rainfall-runoff processes are normally analysed by the application of mathematical models sometimes in combination with other various water quantity and quality sampling techniques. Urbanization has been shown to increase surface runoff, by creating more impervious surfaces such as pavement and structures that impede percolation. When this happens, the water instead is forced to flow directly into streams or storm water runoff drains, where erosion and siltation can be major problems, even when flooding is not.

It is recommended that more drainage facilities be injected into the over-tasked drainage system in the Metropolis in order to curb the incidence of flooding. Additionally, the drainage design consideration should embrace the delineation of each sub-basin into its major mother basin. Designers should endeavour to ensure that the watershed boundary is tenaciously adhered to such that each basin carries its discharge to a safe outlet without a cross-carpet.

#### 5.0 SUMMARY

Urbanization has been shown to increase surface runoff, by creating more impervious surface such as pavement and structures that impede percolation.

The Markovian approach was applied to study and classify flood in the Calabar Metropolis. From the study, location one was observed to have the highest flood cycle of 19, followed by location six with 16. Locations two, three and seven had undefined flood cycle.

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