



OSCILLATIONS OF CURVED PIPELINE UNDER THE ACTION OF VARIABLE INTERNAL PRESSURE

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Annotation

1. This paper deals with fluctuations oscillations of a curved polymer pipeline and an incompressible fluid enclosed in it relative to the longitudinal axis passing through the supports. When the flow of fluid in the pipeline is taken into account, in addition to the internal

forces of the inertial force of the pipeline and fluid, as well as the buoyancy force of Archimedes, the force of resistance to movement of the pipe element, which is determined by the Stokes formula. When calculating the velocity, the motion of the fluid is neglected. The obtained numerical results of stresses and permixes taking into account the influence of the above parameters.

KEYWORDS: Pipeline, liquid, internal forces, Archimedes force, soil.

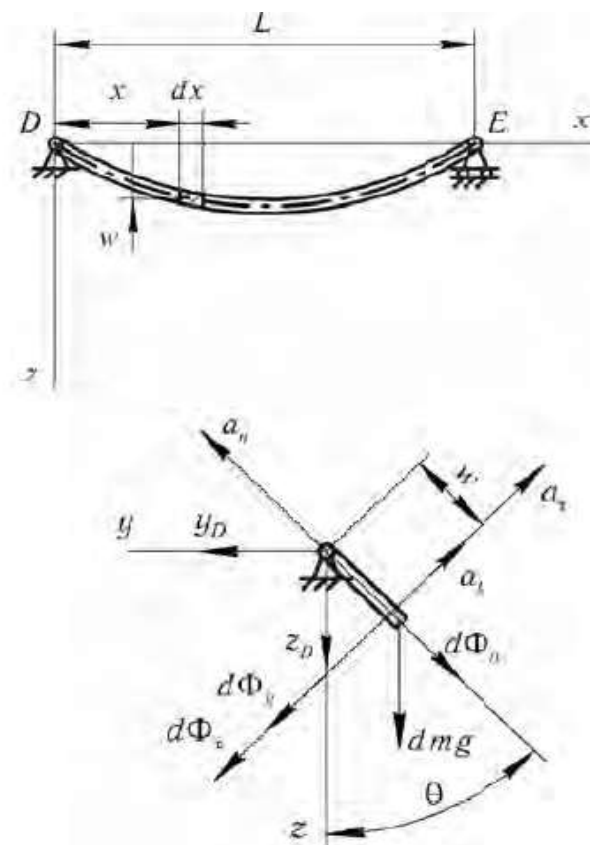
Annotation

Gas, water, and oil pipelines, containers and columns, submarine hulls, rocket engines, and aircraft fuselages are an incomplete list of structures where pipelines with a flowing fluid are a supporting element.^[1-5]

The use of pipelines from a fluid in seismic hazardous zones is associated with the need to address the issues of strength and durability of their elements. Such diverse designs, as well as the types of operating operational loads in the types of materials used, led to the creation of numerous theories describing the behavior of pipelines using differential equations and the development of methods for solving these equations. Analysis of the existing main theories of

seismic resistance of structures makes it possible to note that, when calculating structures for seismic resistance depending on soil conditions and design features of a structure, the static theory of seismic resistance of structures can be applied with small (up to 15%) errors, only in cases where the physical and mechanical properties structures and soil are close enough.^[7,8]

All these factors, as a rule, are not taken into account when designing pipeline systems. For example, when designing trunk pipelines,^[9] all loads acting on the pipeline are taken into account - temperature change, backfill weight, wind and snow loads, etc., except for the dynamic nature, the pipeline walls are loaded during operation. Regulatory documents of various industries mainly regulate permissible vibration levels of pipelines. Thus, according to the norms, the emergency vibration level is estimated by the value of vibration velocity $V_e = 18 \text{ mm / s}$, and the warning level - by exceeding $V_e = 41 \text{ mm / s}$. It should be noted that in many industry regulations there are not only restrictions on pressure pulsations, but also restrictions on vibrations.



Picture 1: Design scheme.

At the same time, in recent years, the replacement of worn-out pipelines has been proceeding at extremely low rates. In the absence of regulatory restrictions on permissible dynamic loads, this leads to an annual increase in the number of accidents on pipelines by 7-10% (according to annual reports on the state of the environment in the Russian Federation). To determine the stresses in the walls of the pipeline, we will rely on that in addition to the constant working pressure.

In this paper, we study the dynamic behavior of a curved pipeline during fluid flow. A solution technique and algorithm for obtaining numerical results are proposed. Discussed received numerical results based on the proposed techniques and algorithms.

2. Statement of the problem and methods of solution

The spatial oscillations of a curved pipeline and the incompressible fluid enclosed in it are considered relative to the Oz axis (Figure 1) passing through the supports. It is assumed that the pipeline is under the action of variable internal pressure. The velocity of the fluid movement is neglected. **The length** of the pipeline is $-l$, its wall thickness is h , and the total mass of the homogeneous pipeline and liquid $m = m_1 + m_2$. In this formulation of the problem, we neglect the longitudinal inertia forces as compared with the transverse ones. Element of the pipeline dz and weight $dm = (m/l)dz$. The lateral distributed load on the pipeline is expressed by the formula

$$q_n = -\frac{m}{l} \left(\frac{\partial^2 w}{\partial t^2} - g \cos \theta \right) + p_i F_i \frac{\partial^2 w}{\partial z^2} \quad (1)$$

$F_i = \pi R_i^2$, $p_i = p_0 + p_v \sin(\Omega t + \varphi)$, w - deflection of the pipeline element, $\Omega, \varphi, p_0, p_v$ - values of the circular frequency, initial phase, static and amplitude of the dynamic components of variable internal pressure p_i in the pipeline R_i, F_i internal radius and cross-sectional area of the pipeline, is time t . The magnitude of the buoyancy force of Archimedes acting dF_A on a pipe element in length dz is equal to $dF_A = \rho_c \pi R_k^2 g dz$, $R_k = R_i + h$, where is the density ρ_c of the liquid, $g = 9.8 m/c^2$, the resistance force $d\vec{F}_c$ of the movement of the pipe element is determined by the Stokes formula.^[9]

$$d\vec{F}_c = -\mu \vec{V}_a dz$$

where is the absolute velocity of the element, and are drag coefficients depending on the viscosity of the fluid and the shape of the inner surface of the pipe. According to the addition

speed theorem, where and are the relative and portable velocities of the pipe element. In this case, the latter is determined by the formula:

$$V_1 = \frac{\partial w}{\partial t}, V_2 = \frac{\partial \theta}{\partial t} .$$

Thus, the resistance $d\vec{F}_c$ can be represented as:

$$dF_{c1} = \mu \frac{\partial w}{\partial t} dz, dF_{c2} = \mu w \frac{\partial \theta}{\partial t} dz ,$$

Where is the angle of rotation of the pipe as a solid with respect to Oz.

The total moment M_z of recovery forces (or visco-elasticity) in the supports is directly proportional to the angle of rotation of the pipe as a solid relative to the axis Oz:

$$M_z = c_0 \left[\theta(t) - \int_0^t R_c(t-\tau) \theta(\tau) d\tau \right]$$

where is $R_c^{(t-r)}$ the core of relaxation; C_0 - instantaneous modulus of elasticity.

Tangent a_r to the trajectory, normal a_n and Coriolis a_k acceleration of the selected element of the pipeline are equal:

$$a_r = w \frac{d^2 \theta}{dt^2}, a_n = w \left(\frac{d\theta}{dt} \right)^2, a_k = 2 \frac{d\theta}{dt} \frac{\partial w}{\partial t} .$$

Thus, the inertial forces dF_τ, dF_n, dF_k of the selected element of the pipeline will be recorded

$$dF_\tau = dm \cdot w \cdot \frac{d^2 \theta}{dt^2}, dF_n = dm \cdot w \cdot \left(\frac{d\theta}{dt} \right)^2, dF_k = 2dm \cdot \frac{d\theta}{dt} \frac{dw}{dt} .$$

Equilibrium equation of the pipeline as the sum of the moments of all applied forces and forces of inertia about the axis Oz

$$- \int_{(m)} (dmg - dF_A) w \sin \theta - \int_{(m)} w dF_\tau - \int_{(m)} w dF_k - M_z = 0 \quad (2)$$

Where g - is gravitational acceleration.

Equation (2) after some transformations and taking into account

$$T = \frac{\tilde{E}F}{2l} \int_0^l \left(\frac{\partial w}{\partial z} \right)^2 dz ,$$

takes on

$$\frac{\partial^2 w}{\partial t^2} + \frac{\tilde{E}Jl}{m} \frac{\partial^4 w}{\partial z^4} - \frac{T_i}{m} \frac{\partial^2 w}{\partial z^2} - g_1 \cos \theta - w \left(\frac{d\theta}{dt} \right)^2 = 0 \quad (3)$$

Where $T_i = T - p_i F_i$, $J = \pi R_i^3 h$ - the axial moment of inertia of the mercy of the cross-section of the pipeline, $g_1 = g - F_k / m$. Pipe bending displacements satisfying boundary conditions

$$w(0,t) = 0; \quad \frac{\partial^2 w(0,t)}{\partial z^2} = 0; \quad w(l,t) = 0; \quad \frac{\partial^2 w(l,t)}{\partial z^2} = 0 \quad (4)$$

take in the form

$$w = W_0 \sin \frac{\pi z}{l} + \sum_{k=1}^{\infty} w_k(t) \sin \frac{k\pi z}{l} \quad (5)$$

where W_0 and $w_k(t)$ are the amplitudes of the static and dynamic components of bending displacements.

Substituting solution (5) into equations (3) and (4) and applying to the Bubnov – Galerkin procedure,^[10] after simple transformations we obtain ($k = 0$)

$$\begin{aligned} \frac{d^2 \theta}{dt^2} (W_0 + w_0(t))^2 + \frac{2\mu}{m} \frac{d\theta}{dt} + 2.0(W_0 + w_0(t)) \left(\frac{2g_1}{\pi} \sin \theta + \frac{d\theta}{dt} \frac{dw_0}{dt} \right) = 0; \\ \frac{d^2 w_0}{dt^2} + \frac{\mu l}{m} \frac{dw_0}{dt} + \frac{(J\pi^4) \tilde{E}}{l^3 m} (W_0 + w_0(t)) = \frac{4g_1}{\pi} \cos \theta + (W_0 + w_0(t)) \left(\frac{d\theta}{dt} \right)^2 - \\ - \frac{\pi^2}{ml} \left[\frac{\tilde{E}F_i \pi^2}{4l^2} (W_0 + w_0(t))^2 - F_i (p_0 + p_v \sin(\Omega t)) \right] (W_0 + w_0(t))^2. \end{aligned} \quad (6)$$

The system of equations (6) is solved under the following initial conditions

$$\begin{aligned} t = 0: \theta = \theta_0, \quad \dot{\theta} = d\theta/dt = \omega_0; \\ w_0 = 0, \quad dw/dt = 0. \end{aligned} \quad (7)$$

There is θ_0, ω_0 the initial angle of rotation and the angular velocity of the deviation of the pipeline from the vertical plane. In the case $\theta(t) = 0, w_0(t) = 0, p_v = 0$, then we obtain the following nonlinear integral equation for determining the quasistatic component of the deflection of the pipeline W_0

$$B_1 W_0^3 + (B_2' + B_2'') W_0 - B_2 \int_0^t R_E(t-\tau) W_0(\tau) d\tau - B_1 \int_0^t R_E(t-\tau) W_0^3(\tau) d\tau - B_3 = 0,$$

Where $B_1 = \frac{\pi^4 E_0 F_i}{4L^2}$, $B_2' = \frac{\pi^4 E_0 J}{L^2}$, $B_2'' = \pi^2 F_i p_0$, $B_3 = \frac{4gmL}{\pi}$.

If $R_E(t - \tau) = 0$ then the results of calculations are obtained.^[11]

If $\theta = const$, then (6) takes the following form

$$\frac{d^2 \bar{w}_0}{dt^2} + A \bar{w}_0(t) + B \varepsilon (\bar{w}_0(t))^2 - A \varepsilon \int_0^t R(t - \tau) w(\tau) d\tau - B \varepsilon \int_0^t R(t - \tau) (w(\tau))^2 d\tau = f(t), \quad (8)$$

Where $\bar{w}_0(t) = W_0 + w(t)$, $A = \frac{(J\pi^4)E_0}{l^3 m}$, $B = \frac{E_0 F_i \pi^4}{4l^3 m}$, $f(t) = \frac{4g}{\pi} \cos \theta + \frac{\pi^2}{ml} [F_i (p_0 + p_v \sin(\Omega t))]$

the system of integro-differential equations (8) is solved by the perturbation method.

Generally, the system of integro-differential equations (6) in the elastic formulation ($R(t - \tau) = 0$) is given in.^[12]

3. Consider the free vibrations of the pipeline. To this end, it is assumed.

$$p_v = 0, \Omega = 0, \varphi = 0$$

Linearize the system of differential equations (6), then we obtain the following system of equations:

$$\begin{aligned} \frac{d^2 \theta}{dt^2} + \frac{\mu}{m} \frac{d\theta}{dt} + (C_1 + C_2) \theta(t) - C_1 \int_0^t R_c(t - \tau) \theta(\tau) d\tau &= 0, \\ \frac{d^2 w_0}{dt^2} + \frac{\mu l}{m} \frac{dw_0}{dt} + (d_1 - d_2) w_0(t) - d_1 \int_0^t R_E(t - \tau) w_0(\tau) d\tau &= 0, \end{aligned}$$

Where $C_1 = \frac{2c_0}{W_0^2 m}$, $C_2 = \frac{4g_1}{\pi W_0}$, $d_1 = \frac{\pi^2}{ml} \left(\frac{\pi E_0 J}{l^2} + \frac{3\pi^2 E_0 F_i}{4l^2} W_0^2 \right)$, $d_2 = F_i p_0 \frac{\pi^2}{ml}$

This formula is consistent with the results obtained in.^[13]

In the case, $R(t - \tau) = 0$ then the frequency ω_1 and ω_2 natural oscillations of the pipeline will be determined by the formulas:

$$\omega_1 = \frac{2}{W_0^2} \left(\frac{c_0}{m} + \frac{2g_1}{\pi} W_0 \right), \quad \omega_2 = \frac{\pi^2}{m_0 l^2} \left(\pi^2 \frac{E_0 J}{l^2} + \frac{3\pi^2 E_0 F_i}{4l^2} W_0^2 - F_i p_0 \right) \quad (9)$$

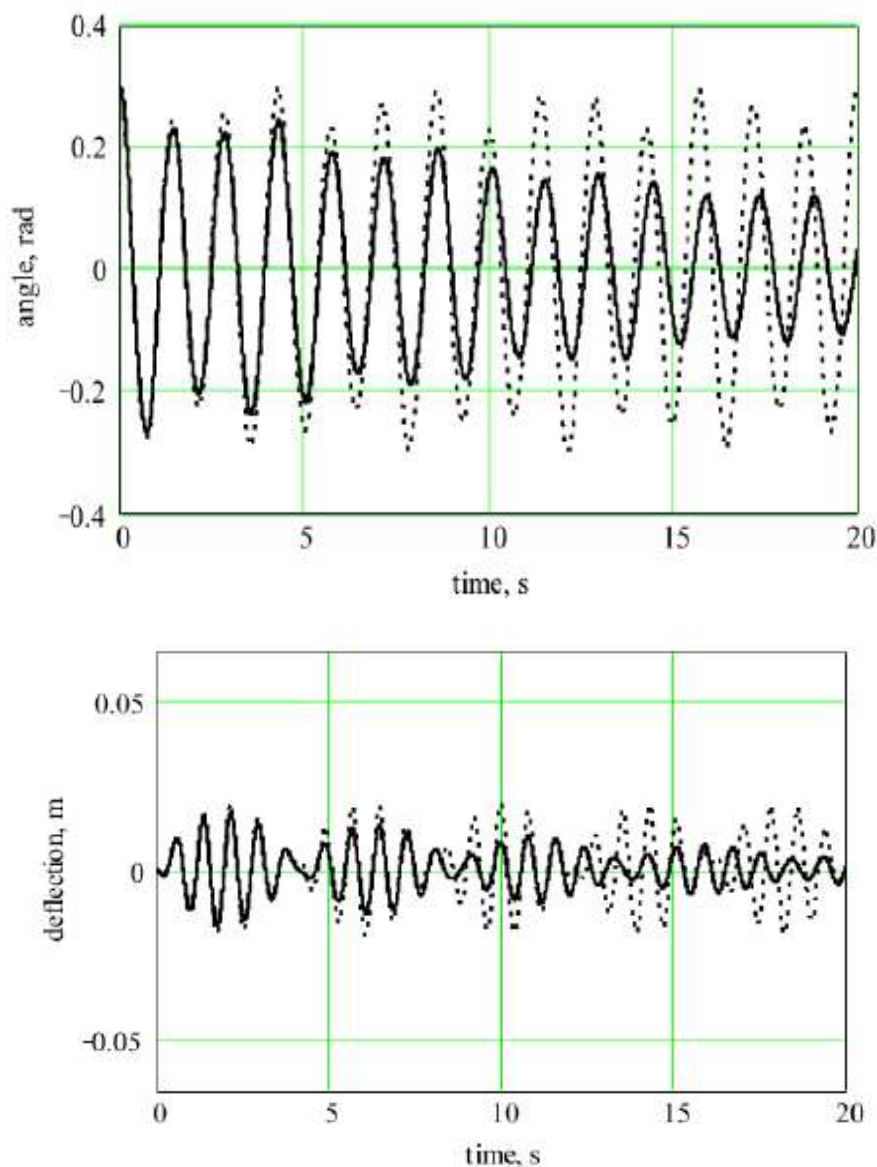


Figure 2: Dependencies of the angle of rotation and deflection of the midpoint of the pipe's span over time with $p_0 = 50 \text{ бар}$, $\mu = 25 \text{ Пас}$, $\rho_0 = 800 \text{ кг} / \text{м}^3$.

When the viscoelastic properties of pipelines are taken into account, then (9) is expressed using the transcendental equation for pipeline angular and flexural vibrations.

$$\omega^2 - \frac{4}{W_0^2} \left\{ \frac{c_0}{m} [1 - \Gamma_{c_0}^c(\omega_R) - i\Gamma_{c_0}^s(\omega_R)] + \frac{2g_1}{\pi} W_0 \right\}^2 = 0,$$

$$\omega^2 - \frac{\pi^4}{m^2 l^2} \left\{ \left(\frac{\pi^2 E_0 J}{l^2} + \frac{3\pi^2 E_0 F_l}{4l^2} \right) [1 - \Gamma_E^c(\omega_R) - i\Gamma_E^s(\omega_R)] - F_i p_0 \right\}^2 = 0,$$

Where $\omega = \omega_R + i\omega_I$ - complex frequency,

$$\Gamma_{c_0}^c(\omega_R) = \int_0^{\infty} R_{c_0}(\tau) \cos \omega_R \tau d\tau, \quad \Gamma_E^c(\omega_R) = \int_0^{\infty} R_E(\tau) \cos \omega_R \tau d\tau,$$

$$\Gamma_{c_0}^s(\omega_R) = \int_0^{\infty} R_{c_0}(\tau) \sin \omega_R \tau d\tau, \quad \Gamma_E^s(\omega_R) = \int_0^{\infty} R_E(\tau) \sin \omega_R \tau d\tau$$

Respectively, the cosine and sine Fourier images of the relaxation core of the material.

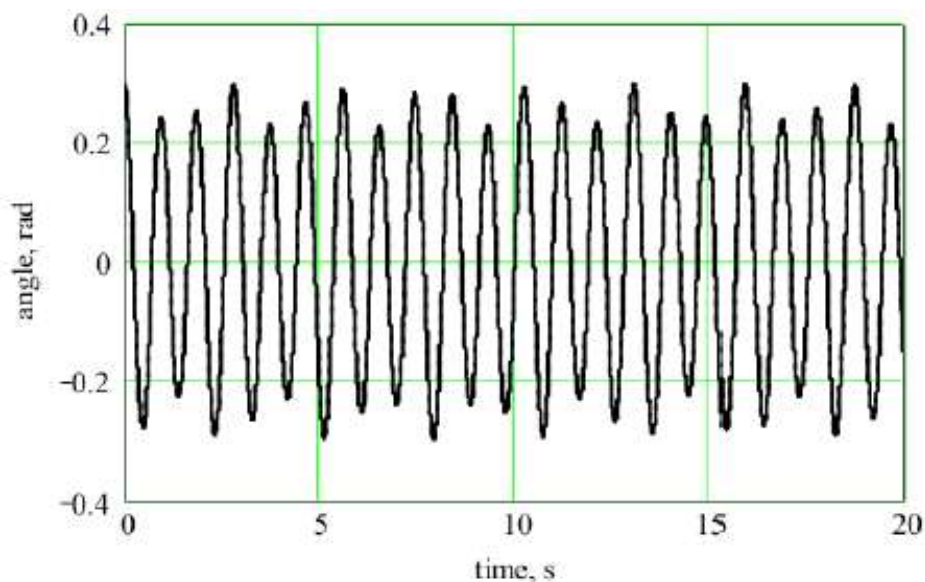
We study the influence of the Archimedes pushing force, Coriolis inertia forces, resistance force and the magnitude of the static component of the internal pressure in a fluid, as well as the geometric and physico-mechanical parameters of the pipe on its free oscillatory movements.^[14,15]

4. Numerical results. The numerical solution of problem (2.16) was determined by the Runge – Kutt method.^[16,17]

The results of calculations for the following values of the main parameters:

$$l = 3m, \quad c = 0, \quad R_i = 0.29m, \quad h = 0.006, \quad \theta_0 = 0.3rad, \\ E = 2.1 \times 10^{11} Pa, \quad \omega_0 = 0 pa\delta / cek, \quad m = 6.142 \times 10^3 kg$$

Figure 2.3 shows the graphs of the angle of rotation and dynamic deflection of the middle point of the tube's span from time t , respectively.



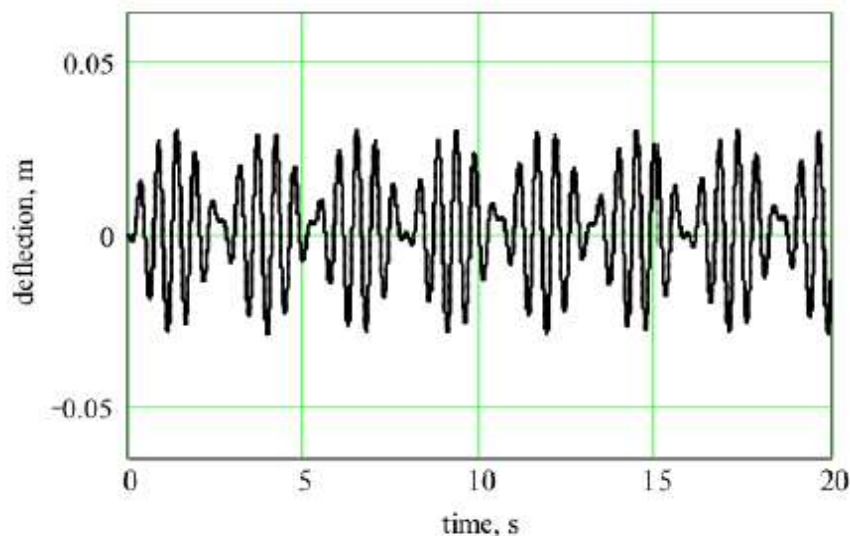


Figure 3: Dependencies of the angle of rotation and deflection of the midpoint of the tube's span of the pipe on time at. $p_0 = 50 \text{ бар}$, $\mu = 0.025 \text{ Пас}$, $\rho_0 = 1.25 \text{ кг/м}^3$.

5. Flexural vibrations of polymer pipes of variable cross-section with a fluid flowing inside

$$\frac{\partial^2}{\partial x^2} \left[E_0 J \frac{\partial^2}{\partial x^2} \left(w(x,t) - \int_0^t R(t-\tau) w(x,\tau) d\tau \right) \right] = q(x,t) \quad (10)$$

In the study of bending vibrations of a pipe with a fluid flowing inside, we use the model in the form of a non-prismatic beam and the hypothesis of flat sections. In this case, for the analysis of oscillations, the following differential equations are valid.^[18]

Where $w(x,t)$ is the equation of the elastic axis of the beam relative to its undeformed state under the action of a transverse specified load $q(x,t)$. Let the flexural rigidity EJ , the mass per unit length of the pipe m_1 and the mass of the fluid m_2 volume filling the unit length of the pipe be unchanged along the pipe axis. In accordance with the principle of the d'Alembert, the inertia forces arising from the oscillations (Fig. 4) can be considered as a transverse load for a beam:

$$q(x,t) = -m_1 \frac{\partial \vartheta_1}{\partial t} - m_2 \frac{\partial \vartheta_2}{\partial t}$$

where ϑ_1 is the absolute velocity of the element of the pipe, and is the absolute velocity of the element of the flowing fluid.

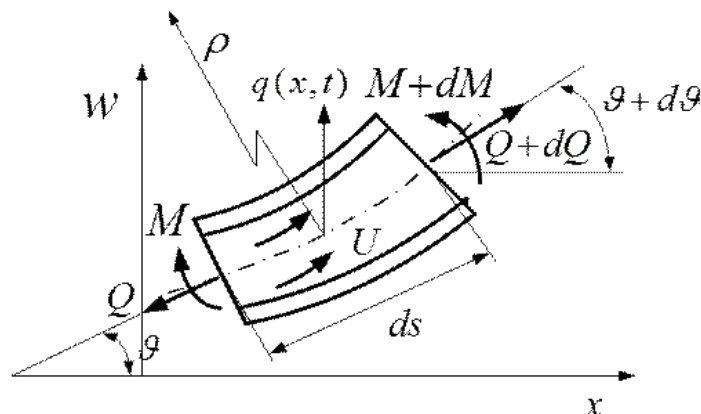


Figure 4: Design scheme.

For a stationary flow, the pressure along the pipe axis does not change, and the velocity of the fluid V does not depend on the pipe oscillations. Then the inertial load can be written as:

$$q(x,t) = -m_1 \frac{\partial^2 w}{\partial t^2} - m_2 \frac{\partial^2 w}{\partial t^2} - m_2 V^2 \frac{\partial^2 w}{\partial x^2} - 2m_2 V \frac{\partial^2 w}{\partial x \partial t} \quad (11)$$

Here, the first term is the inertia force of a pipe element arising from its transverse vibrations; since the fluid element makes a complex movement (portable movement v_i with the speed of the pipe element, relative - with speed V), the remaining terms in (11) reflect its inertial forces - the inertial force of the portable movement, the normal component of the inertial force of relative motion and Coriolis inertia, respectively. When calculating the components of the acceleration of the fluid element, it is taken into account that the curvature of the beam

$$k = \frac{1}{\rho} = \frac{\partial^2 w}{\partial x^2}, \text{ the angle of rotation of the element } \vartheta = \frac{\partial w}{\partial x}, \text{ and its angular velocity } \dot{\vartheta} = \frac{\partial^2 w}{\partial x \partial t}.$$

Substituting (11) into (10), we obtain the differential equation of transverse oscillations of the axis of the pipeline with respect to the initial straight-line position:

$$\frac{\partial^2}{\partial x^2} \left[E_0 J \frac{\partial^2}{\partial x^2} \left(w(x,t) - \int_0^t R_E(t-\tau) w(x,\tau) d\tau \right) \right] = -(m_1 + m_2) \frac{\partial^2 w}{\partial t^2} - m_2 V^2 \frac{\partial^2 w}{\partial x^2} - 2m_2 V \frac{\partial^2 w}{\partial x \partial t} \quad (12)$$

The solution of equation (12) can be obtained by one of the approximate analytical methods. We use the Bubnov – Galerkin method, presenting the solution of the equation as a product of two functions:

$$w(x,t) = X(x)e^{i\omega t}. \quad (13)$$

Solution (12) should satisfy four boundary conditions corresponding to the variants of fixing the ends of the pipeline.

Substituting (13) into (12) will allow for the function $X(x)$ to obtain an ordinary differential equation:

$$\frac{d^4 X}{dx^4} - a_z \omega^2 X + c_z \frac{d^2 w}{dx^2} = 0 \quad (14)$$

Where $a_z = \frac{m_1 + m_2}{EJ}$, $c_z = \frac{m_2}{EJ} V^2$, $\bar{E}\varphi = E[1 - \Gamma^C(\omega_R) - i\Gamma^S(\omega_R)]\varphi$

The solution of equation (14) will be sought in the form

$$X(x) = \sum_{k=1}^{\infty} A_k \sin\left(\frac{k\pi x}{l}\right)$$

substitution of which into equations (14) with subsequent multiplication

on $\sin\left(\frac{\pi x}{l}\right)$, $\sin\left(\frac{2\pi x}{l}\right)$ and integration under boundary conditions from $x = 0$ to $x = 1$

Leads to a system of linear homogeneous equations with respect to the unknown constants A_1, A_2, A_3, \dots . Equating the determinant of the system to zero, we obtain an equation for determining the e frequency of the pipeline oscillations. In the study of emerging oscillations, it is of interest to answer the question about the value of the critical flow rate of a fluid (the flow rate at which the pipeline may lose static stability). This value can be found from the condition that the first oscillation frequency is zero (which, in turn, takes place when the term that does not contain a frequency in the equation for determining frequencies is equal to zero).

As an example, consider a section of a viscoelastic pipeline with hinged supports at the ends; an ideal incompressible fluid $V = 10M/ceK$ flows at a constant speed. The average diameter of the cross section of the pipeline $D = 0.09M$, the wall thickness of the pipeline $\delta = 0.025M$, the length of the section $l = 1M$, the density of the material $\rho_1 = 2700 \text{ kg}/M^3$, the modulus of elasticity $E = 75 \text{ ГПа}$. Mass of fluid $\rho_2 = 15.64 \text{ kg}/M^3$. Determine the first two frequencies of transverse oscillations of a pipeline with fluid at rest and flowing without taking into account the effect of static weight forces. We use the differential equation (12), where is $m_1 = \rho_1 \pi D \delta$ the mass of a unit of length of the pipe, $m_2 = \rho_2 \pi (D - \delta)^2$ and is the mass of a unit of length of the liquid. To obtain a solution, we use the Bubnov-Galerkin method. Assuming

that, after $w(x,t) = X(x)e^{i\omega t}$ substituting the proposed solution in (14) and performing fairly simple transformations, for equation (14) we obtain the values of the coefficients $a = 0.014, b = 0.08V, c = 0.004V^2$. The solution of equation (14) must satisfy the boundary conditions:

$$x = 0; X(0) = 0; \frac{d^2 X}{dx^2} = 0, \quad x = l; X(l) = 0; \frac{d^2 X}{dx^2} = 0 .$$

We are looking for a solution to equation (14) in the form

$$X = A_1 \sin(\pi x/l) + A_2 \sin(2\pi x/l) .$$

We substitute this solution in (14) and successively multiply the resulting expression by $\sin(\pi x/l)$ and $\sin(2\pi x/l)$. We integrate the obtained relations in the interval from $x = 0$ to $x = l$ with allowance for the boundary conditions. As a result of the actions performed, we arrive at an algebraic system of two linear homogeneous equations with respect to the unknowns A_1, A_2 :

$$\begin{aligned} A_1 \left[(\pi/l)^4 - a\omega^2 - c(\pi/l)^2 \right] - A_2 (8bi/3l)\omega &= 0; \\ A_1 \left[(i8b/3l)\omega \right] + A_2 \left[16(\pi/l)^4 - a\omega^2 - 4c(\pi/l)^2 \right] &= 0 \end{aligned}$$

Equating to zero the determinant of this system, we obtain the equation of the form $\omega^4 + a_1\omega_2 + a_2 = 0$ for determining two frequencies of transverse vibrations of the pipeline.^[20]

6. Numerical results. When $V = 0$, $\omega_1 = 24.9 \text{ pad/сек}, \omega_2 = 98.9 \text{ pad/сек}$. At $V = 10 \text{ m/сек}, \omega_1 = 24.2 \text{ pad/сек}, \omega_2 = 101.7 \text{ pad/сек}$. The critical velocity of a fluid flow occurs when one of the oscillation frequencies is zero, which takes place at

$$a_2 = \left[(\pi/l)^2 - c \right] \left[4(\pi/l)^2 - c \right] = 0, \quad \text{where } c = \frac{m_2}{EJ} V_{kp}^2 .$$

Hence the value of the minimum critical

speed (V_{cr}) will be $V_{kp} = \frac{\pi}{l} \sqrt{\frac{EJ}{m_2}}$. The results of calculations of bending oscillations along the z axis and time t are shown in Fig.5.

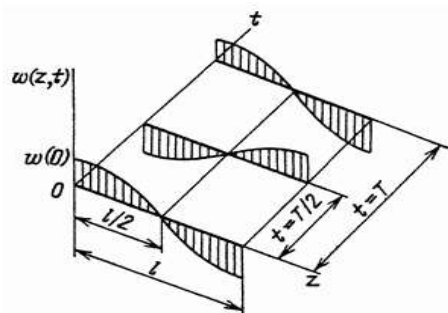


Figure 5: Moving the cross-sectional point of a pipeline as a function of distance and time.

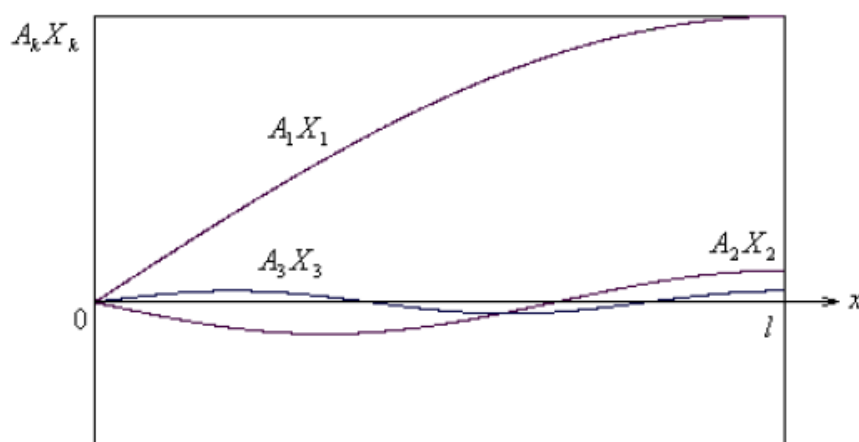


Figure 6: Moving the cross-sectional point of the cantilever pipeline as a function of distance.

7. CONCLUSIONS

On the basis of the developed approximate mathematical model of flexural-rotational oscillatory movements of the pipeline, its free vibrations were investigated. It has been established that with an increase in the static component of the internal pressure, an increase in the amplitude of free bending vibrations and an increase in the frequency of free rotational vibrations of the pipe occur simultaneously.

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