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MULTIRATE STABILIZATION TECHNIQUES APPLIED TO A HYDROGENERATOR-UNIT FOR THE DESIGN OF EXCITATION CONTROLLERS

***[1]Dr. Boglou A. K., [2]Casini E. and [1]Pappas D. I.**

[1]International Hellenic University (IHU), Department of Chemistry, Ag. Loukas, 654 04 Kavala, Greece.

[2]Automate Solutions Builder, Viale Beethoven, 63,00144 Rome, Italy.

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***Corresponding Author**
Dr. Boglou A. K.
International Hellenic University (IHU), Department of Chemistry, Ag. Loukas, 654 04 Kavala, Greece.

ABSTRACT

An optimal control strategy is used to design, in this paper, a desirable excitation controller of a hydrogenerator system, in order to enhance its dynamic stability characteristics. The proposed control strategy based on Two-Point-Multirate Controllers (TPMRCs) is readily applicable in cases where the state variables of the controlled plant are not available for feedback. In the TPMRCs based scheme, the control is constrained to a certain piecewise constant signal, while each of the controlled plant outputs is detected many times over a fundamental sampling period T_0. On the basis on this strategy, the original problem is reduced to an associate discrete-time linear quadratic (LQ) regulation problem for the performance index with cross product terms, for which a fictitious static state feedback controller is needed to be computed. Simulation results for the actual 117 MVA hydrogenerator unit, show the effectiveness of the proposed method which has a quite satisfactory performance.

KEYWORDS: Optimal output-feedback, multirate controllers, discrete system representation, synchronous machines.

I. INTRODUCTION

The dynamic stability enhancement of an open-loop power system model, linearized about its nominal operating point may be achieved by designing a suitable excitation controller and thus obtaining a closed-loop system with desired dynamic stability characteristics. The actual design of such controllers may be accomplished by using various modern control methods. Among them, linear quadratic (LQ) optimal control methods have received considerable attention in the past (Al-rahmani and Franklin, 1990; Chen et al., 2004; Heidarinejad et al., 2011; Polushin et al. 2004; Sagfor et al., 1998, Srinivasarao et al., 2007). Most of these optimal control techniques, however, suffer from several serious disadvantages.

It is pointed out that the used TPMRCs (Arvanitis, 1998; Arvanitis et al., 2000). technique reduced the original LQ regulation problem to an associated discrete-time LQ regulation problem for the performance index with crossed product terms, for which is computer a fictitious static state feedback controller. In addition thus technique offers more flexibility in choosing the sampling rates and provides a power design computed method.

On the other hand, multi rate controllers are in general time-varying. Thus multi rate control systems can achieve what single rate cannot; e.g. gain improvement, simultaneous stabilization and decentralized control. Finally, multi rate controllers are normally more complex than single rate ones; but often they are finite-dimensional and periodic in a certain sense and hence can be implemented on microprocessors via difference equations with finitely many coefficients. Therefore, like single rate controllers, multi rate controllers do not violate the finite memory constraint in microprocessors. In particular, the control strategy presented here is essentially a combination of the control strategies reported in (Arvanitis, 1998; Al-rahmani et al., 1990; Polushin et al., 2004; Sagfors et al., 1998). The control is constrained to a certain piecewise constant signal, while the controlled plant output is detected many times over a fundamental sampling period. The proposed control strategy relies on solving the continuous LQ regulation problem.

TPMRCs provide the ability of the exact reconstruction of action of the state feedback without resorting to the design of state estimators, and without introducing high order exogenous dynamics in the control loop. Based on this strategy, the original problem is reduced to an associated discrete-time LQ regulation problem for the performance index with cross product terms, for which a fictitious static state feedback controller is needed to be computed. Thus, the present technique essentially resort to the computation of simple gain controllers in a digital environment, rather than to the computation of state observers, as compared to known techniques. Finally, the designed TPMRCs based LQ optimal regulators can possess any prescribed degree of stability, since there is the possibility to choose the transition matrices of the controllers arbitrarily.

In this paper, the proposed optimal control strategy is used to design a desirable excitation controller of a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The particular hydrogenerator studied in the paper, is a 117 MVA hydrogenerator unit of the Greek Electric Utility Power System (Boglou and Papadopoulos, 1995; Smith et al., 1998), which supplies power through a step-up transformer and a transmission line to an infinite grid.

The proposed optimal control design is based on linear state space models of the hydrogenerator, obtained by linearizing its nonlinear Park's equation (Park, 1929; Shackshaft, 1963), about a particular operating point. Simulation results regarding the application of the proposed technique to the linearized state-space model of the hydrogenerator unit clearly show the effectiveness of the method and a significant improvement of the dynamic performance of the system.

II. OVERVIEW OF RELEVANT MATHEMATICAL CONSIDERATIONS

The general description of the controllable and observable continuous, linear, time-invariant, multivariable mimo dynamical open-loop system expressed in state-space form is

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(k) \end{aligned} \tag{1}$$

where: $\mathbf{x}(t) \in \mathbf{R}^n$, $\mathbf{u}(t) \in \mathbf{R}^m$, $\mathbf{y(t)} \in \mathbf{R}^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

The associated general discrete description of the system (1) is as follows

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{aligned} \tag{2}$$

where: $\mathbf{x}(k) \in \mathbf{R}^n$, $\mathbf{u}(k) \in \mathbf{R}^m$, $\mathbf{y}(k) \in \mathbf{R}^p$ are state, input and output vectors respectively; and **A**, **B** and **C** are real constant system matrices with proper dimensions.

III. LQ REGULATION USING TWO-POINT MULTIRATE CONTROLLERS

This method with H_o and H_N being zero-order holds and with holding times T_o and T_N, respectively (see Fig. 1) is presented here in a concise manner, whereas the details are found (Arvanitis, 1998; Al-rahmani, 1990).

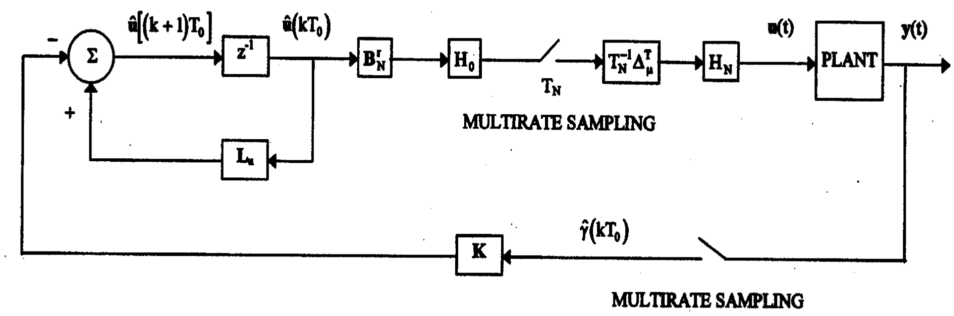


Figure 1: Simplified representation of power system under investigation in discrete form.

Starting with the general linear state space system description in continuous form

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t)\end{aligned} \tag{3}$$

where $\mathbf{x}(\mathrm{t}) \in \mathrm{R}_{\mathrm{n}}$, $\mathbf{u}(\mathrm{t}) \in \mathrm{R}^{\mathrm{m}}$ and $\mathbf{y}(\mathrm{t}) \in \mathrm{R}^{\mathrm{p}}$ are the state, input and output vectors respectively.

The associated discrete system description is obtained by letting $\mathrm{n_i}, \mathrm{i} \in \mathrm{J_p} = \{1, 2,..., \mathrm{p}\}$, be used of observability indices of the pair **(A,C),** and $\mathrm{T_o} \in \mathrm{R}^{+}$ be a sampling period. Also, by letting

$$\Phi = \exp(\mathbf{A}\mathrm{T}_0) \tag{4}$$

and $\mathbf{B}_{\mathrm{N}} \in \mathrm{R}^{\mathrm{nxp_N}}$ be the full rank matrix defined by

$$\mathbf{B}_{\mathrm{N}}\mathbf{B}_{\mathrm{N}}^{\mathrm{T}} = \mathbf{W}_{\mathrm{N}}(\mathrm{T}_0,0) \geq 0$$

with the generalized reachability Grammian of ord N in the interval $[0, \mathrm{T}_0]$ being

$$\mathbf{W}_N(T_0,0) = T^{*-1}\sum_{\mu=0}^{N_0-1}\Delta_\mu\Delta_\mu^T,\ p_N = rank\mathbf{W}_N(T_0,0) \text{ and}$$

$$\mathrm{T}^* = \mathrm{T}_0 / \mathrm{N}_0,\ \Delta_\mu = \hat{\mathrm{A}}_{\mathrm{N}}^{\mathrm{N}_0-\mu-1}\hat{\mathrm{B}}_{\mathrm{T}^*}$$

$$\hat{\mathbf{A}}_N = \exp(\mathbf{A}T^*),\ \hat{\mathbf{B}}_{T^*} = \int_0^{T^8}\exp(\mathbf{A}\lambda)\mathbf{B}d\lambda \tag{5}$$

Next follows the application of the TPMRCs technique to the above descriptions. The input of the plant are constrained to the following piecewise constant control

$$\mathbf{u}(kT_0+\mu T^*+\varsigma)=T^{*-1}\Delta_\mu^T\mathbf{B}_N^\tau\hat{u}(kT_0),\qquad \hat{\mathbf{u}}(kT_0)\in R^{p_N} \tag{6}$$

for

$t=kT_0+\mu T^*,\ \mu=0,\ldots,N_0-1,\ k>>0$

and $J\in[0,T^*)$, where $B_N^\tau=B_N(B_N^TB_N)^{-1}$.

The ith plant output $\mathbf{y_i(t)}$ is detected at every $T_i=T_0/M_i$, such that

$$y_i(kT_0+\rho T_i)=c_i^Tx(kT_0+\rho T_i),\qquad \rho=0,1,\ldots,M_i-1 \tag{7}$$

where $M_i\in Z^+, i\in J_p$ are the output multiplicities of the sampling. In general $M_i\neq N$. The sampled values of the plant outputs obtained over $[kT_0,(k+1)T_0)$ are stored in the M^*-dimensional column vector $\hat{\gamma}(kT_0)$ of the form

$$\hat{\gamma}(kT_0)=[y_1(kT_0)\ldots y_1(kT_0+(M_1-1)T_1)\ldots y_p(kT_0)\ldots y_p(kT_0+(M_p-1)T_p)]^T \tag{8}$$

where $M^*=\sum_{i=1}^{p}M_i$. The vector $\hat{\gamma}(kT_0)$ is used in the control law of the form

$$\hat{\mathbf{u}}[(k+1)T_0]=\mathbf{L}_u\hat{\mathbf{u}}(kT_0)-\mathbf{K}\hat{\gamma}(kT_0) \tag{9}$$

where $\mathbf{L}_u\in\mathbf{R}^{p_N x p_N}$, $\mathbf{K}\in\mathbf{R}^{p_N x M^*}$.

Finally one scetcs a controller in the form of (5) and (7) which, nowen applied to system (1), minimizes the following performance index

$$J=\frac{1}{2}\int_0^\infty[\mathbf{y}^T(t)\mathbf{Q}\mathbf{y}(t)+\mathbf{u}^T(t)\mathbf{R}\mathbf{u}(t)]dt \tag{10}$$

where $\mathbf{Q}\in R^{pxp}$ *and* $\mathbf{R}\in R^{mxm}$ are symmetric matrices with $\mathbf{Q}\geq 0$, $\mathbf{R}>0$ while $(\mathbf{AC^TQC})$ is an observable pair.

The above problem is equivalent to the problem of designing a control law of the form of equation 9, in order to minimize the following index:

$$J=\frac{1}{2}\sum_{k=0}^{\infty}\begin{bmatrix}\mathbf{x}^T(kT_0) & \hat{\mathbf{u}}^T(kT_0)\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{Q}}_N & \tilde{\mathbf{G}}_N\\ \tilde{\mathbf{G}}_N^T & \tilde{\mathbf{\Gamma}}_N\end{bmatrix}\begin{bmatrix}\mathbf{x}(kT_0)\\ \hat{\mathbf{u}}(kT_0)\end{bmatrix} \tag{11}$$

for the system

$$\mathbf{x}\left[(k+1)T_0\right]=\Phi\,\mathbf{x}(kT_0)+B_N\,\hat{u}(kT_0) \tag{12}$$

Where $\tilde{\mathbf{Q}}_N$, $\tilde{\mathbf{G}}_N$, $\tilde{\Gamma}_N$ are giver explicitly (Cimino, et al., 2010; Van Loan, 1978).

Theorem. The following basic formula of the multirate sampling mechanism holds

$$\mathbf{Hx}[(\mathrm{k}+1)\mathrm{T}_0]=\hat{\gamma}(\mathrm{kT}_0)-\mathbf{D}\hat{\mathbf{u}}(\mathrm{kT}_0)\ ,\ \mathrm{k}\geq 0 \tag{13}$$

where, matrices $\mathbf{x}(\mathrm{kT}_0+\rho\mathrm{T}_i)=\hat{\mathbf{A}}_i^{\rho-\mathrm{M}_i}\mathbf{x}[(\mathrm{k}+1)\mathrm{T}_0]+\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(\mathrm{kT}_0)$ are defined as follows

$$\mathbf{H}=\begin{bmatrix}\mathbf{c}_1^{\mathrm{T}}\left(\hat{\mathbf{A}}_1^{\mathrm{M}_1}\right)^{-1}\\ \vdots\\ \mathbf{c}_1^{\mathrm{T}}\hat{\mathbf{A}}_1^{-1}\\ \vdots\\ \mathbf{c}_p^{\mathrm{T}}\left(\hat{\mathbf{A}}_p^{\mathrm{M}_p}\right)^{-1}\\ \vdots\\ \mathbf{c}_p^{\mathrm{T}}\hat{\mathbf{A}}_p^{-1}\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}\mathbf{c}_1^{\mathrm{T}}\hat{\mathbf{B}}_{1,0}\\ \vdots\\ \mathbf{c}_1^{\mathrm{T}}\hat{\mathbf{B}}_{1,\mathrm{M}_1-1}\\ \vdots\\ \mathbf{c}_p^{\mathrm{T}}\hat{\mathbf{B}}_{p,0}\\ \vdots\\ \mathbf{c}_p^{\mathrm{T}}\hat{\mathbf{B}}_{p,\mathrm{M}_p-1}\end{bmatrix} \tag{14}$$

and where,

$$\mathrm{y}_i(\mathrm{kT}_0+\rho\mathrm{T}_i)=\mathbf{c}_i^{\mathrm{T}}\hat{\mathbf{A}}_i^{\rho-\mathrm{M}_i}\mathbf{x}[(\mathrm{k}+1)\mathrm{T}_0]+\mathbf{c}_i^{\mathrm{T}}\hat{\mathbf{B}}_{i,\rho}\hat{\mathbf{u}}(\mathrm{kT}_0) \tag{15}$$

The ultimate expressions for the control law optimal gain matrices $\mathbf{L}_u$ and $\mathbf{K}$ are as follows

$$\mathbf{L}_u=(\tilde{\mathbf{R}}_N+\mathbf{B}_N^T\mathbf{P}\mathbf{B}_N)^{-1}(\tilde{\mathbf{G}}_N+\mathbf{B}_N^T\mathbf{P}\boldsymbol{\Phi})\mathbf{H}^{-1}\mathbf{D} \tag{16}$$

$$\mathbf{K}=(\tilde{\mathbf{R}}_N+\mathbf{B}_N^T\mathbf{P}\mathbf{B}_N)^{-1}(\tilde{\mathbf{G}}_N+\mathbf{B}_N^T\mathbf{P}\boldsymbol{\Phi})\mathbf{H}^{-1} \tag{17}$$

where $\tilde{\mathbf{R}}_N, \tilde{\mathbf{G}}_N$ *and* $\mathbf{H}$ are defined in (Ramamoorty et al., 1971; Srinivasarao, et al., 2007). The resulting discrete closed-loop system matrix $(\mathbf{A}_{cl/d})$ takes the following

$$\mathbf{A}_{cl/d}=\mathbf{A}_{ol/d}-\mathbf{B}_N\mathbf{K}\mathbf{H} \tag{18}$$

where cl=closed-loop, ol=open-loop and d=discrete.

IV. HYDROGENERATOR SYSTEM MODEL AND SIMULATION RESULTS

In the present work, the aforementioned optimal control strategy is used to design a desirable excitation controller of a hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The hydrogenerator system studied, is an 117 MVA hydrogenerator unit of the Greek Electric Utility Power System, and which supplies power through a step-up transformer and a transmission line to an infinite grid. A linear model of the hydrogenerator can be obtained by linearizing its nonlinear Park's equations (Park, 1929; Shackshaft, 1963) about the operating point, v_t=1.0 p.u., P_t=1.1 p.u., Q_t=0.5 p.u.

Based on the state variables Figure 2 and the values of the parameters and the operating point (see Appendix A), the system of Figure 2 may be described in state-space form, in the form of system 3, where:

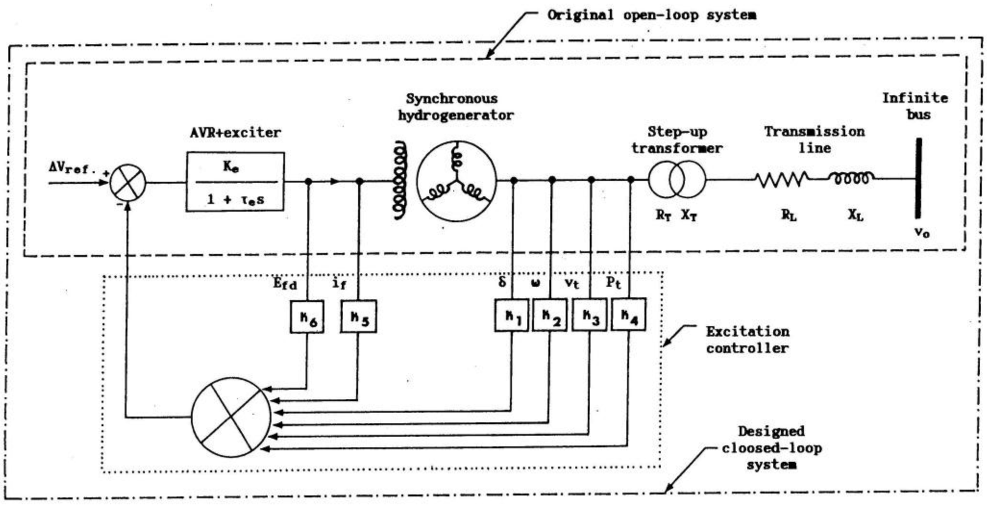


Figure 2: Simplified representation of hydrogenerator system supplying power by an infinite grid.

$$\mathbf{x} = \begin{bmatrix} \Delta\delta & \Delta\omega & \Delta v_t & \Delta P_t & \Delta i_f & \Delta E_{fd} \end{bmatrix}^T$$

$u = \Delta V_{ref.}$, $\mathbf{y} = \mathbf{x}$. , and **A**, **B**, **C**, are real constant system matrices with proper dimensions.

The eigenvalues of the original continuous open-loop power system models and the simulated responses of the output variables $\left(\Delta\delta, \Delta\omega, \Delta v_t, \Delta P_t, \Delta i_f, \Delta E_{fd}\right)$, are shown in Table 1 and Figure 3, respectively.

Table 1: Eigenvalues of original open-loop power system models.

Original open-loop power system model	λ	-25.6139 0.0931+7.7898i 0.0931-7.7898i -8.1191+6.2036i -8.1191-.2036i -6.4021

As it can be easily checked the above linear state space model is unstable, since matrix **A** has two unstable complex eigenvalues at $\lambda_{1,2}$=0.0931±j7.7898.

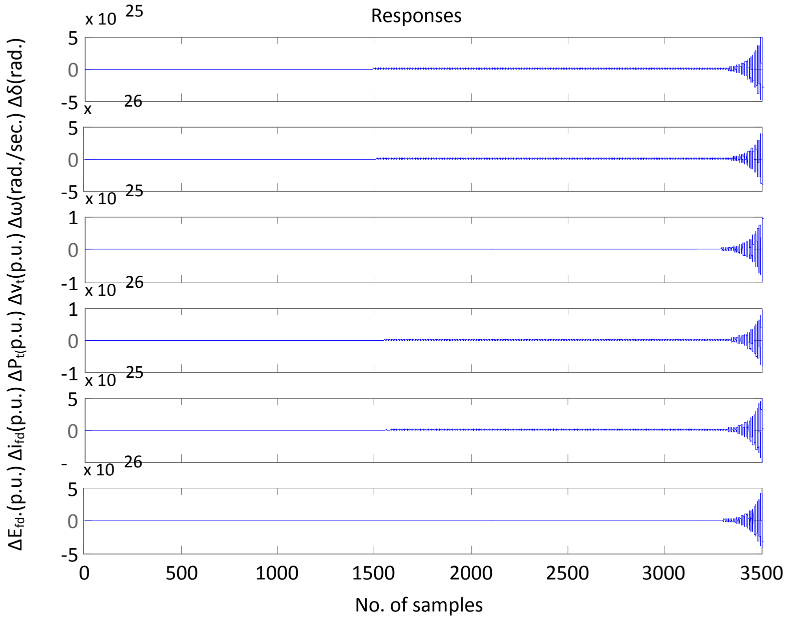


Figure 3: Responses of the output variables of the original continuous open-loop power system models to step input change: $\Delta V_{ref.}$=0.05.

The computed discrete linear open-loop power system model, based on the associated linearised continuous open-loop system model is given below in terms of its matrices with sampling period T_o =0.189 sec.

$$\mathbf{A}_{ol/d} = \begin{bmatrix} -0.6669 & 0.1232 & -1.8627 & 0.3742 & -0.1015 & -0.0034 \\ -12.2050 & 0.0555 & -11.4420 & 2.4032 & -1.0276 & -0.0395 \\ 0.2174 & -0.0146 & 0.5835 & -0.0919 & 0.0814 & 0.0035 \\ -0.9495 & 0.2065 & -2.6607 & 0.7181 & -0.0069 & 0.0015 \\ 0.2534 & 0.1030 & -0.5463 & 0.0331 & 0.1225 & 0.0081 \\ -6.3238 & 0.6747 & -30.5630 & 3.4351 & -4.0444 & -0.1392 \end{bmatrix}$$

$$\mathbf{B}_{ol/d} = [-0.18573 \quad -3.3992 \quad 0.4871 \quad 0.7278 \quad 2.4377 \quad 32.6050]^T$$

$$\mathbf{C}_{ol/d} = \mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The computed magnitude of the eigenvalues of the discrete open-loop power system models and the simulated responses of the output variables $(\Delta\delta, \Delta\omega, \Delta v_t, \Delta P_t, \Delta i_f, \Delta E_{fd})$, are shown in Table 2 and Figure 3, respectively.

Table 2: Magnitude of eigenvalues of discrete original open-loop and designed closed-loop power system models.

Original discrete 6th-order open-loop power system model		$\lvert\lambda\rvert$	1.0178 1.0178 0.2982 0.2156 0.2156 0.0079
Designed discrete 6th-order closed-loop power system model	with matrices $\mathbf{Q}_1,\mathbf{R}_1$	$\lvert\hat{\lambda}\rvert$	0.3727 0.3727 0.3697 0.2358 0.2358 0.0077
	with matrices $\mathbf{Q}_2,\mathbf{R}_2$	$\lvert\hat{\lambda}\rvert$	0.7413 0.7413 0.3192 0.0066 0.1781 0.1781

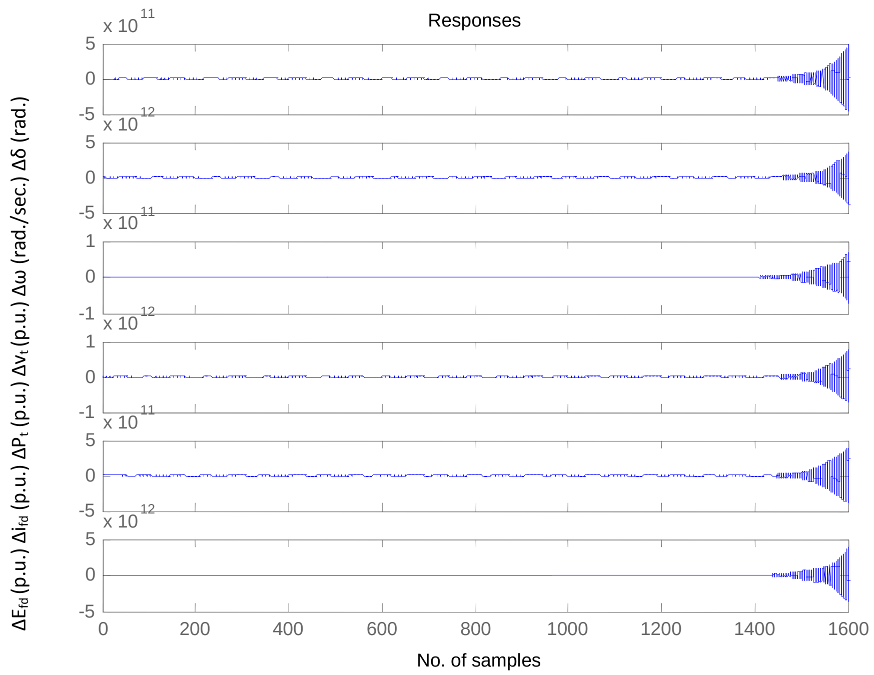


Figure 4: Responses of the output variables of the original discrete open-loop power system models to step input change: $\Delta V_{ref.} = 0.05$.

In the present simulation, our control objective is to solve the problem of minimizing the performance index equation 9, (considering two distinct cases), using the selected weighing matrices:

a) $\mathbf{Q}_1$ = diag(0.01 0.01 0.1 0.1 0.00001 0.00001)

$\mathbf{R}_1$ = 1.

The **K** and $\mathbf{L}_u$ feedback matrices were computed as:

$$\mathbf{K}=\begin{bmatrix} -0.3449 & -0.0061 & -0.4028 & 0.0821 & -0.0372 & -0.0015 \\ -0.0248 & -0.0183 & 0.1184 & -0.0144 & -0.0050 & -0.0004 \\ -0.1045 & 0.0307 & -0.1998 & 0.0335 & -0.0057 & 0.0 \\ -0.0663 & 0.0163 & -0.1998 & 0.0335 & -0.0058 & 0.0 \\ 0.0098 & -0.0024 & 0.0296 & -0.0050 & 0.0008 & 0.0 \\ -0.0371 & 0.0091 & -0.1122 & 0.0188 & -0.0032 & 0.0 \end{bmatrix}$$

$$\mathbf{L}_u=\begin{bmatrix} 0.0979 & 0.0234 & 0.0758 & -0.3829 & -3.5873 & -0.0008 \\ -0.0676 & 0.1767 & -0.4307 & -3.4352 & -28.0740 & -0.0001 \\ 0.1525 & -0.2589 & 0.6988 & 5.3527 & 43.6560 & 0.0001 \\ 0.0837 & -0.1338 & 0.3672 & 2.7668 & 22.5430 & 0.0 \\ -0.0123 & 0.0196 & -0.0541 & -0.4089 & -3.3321 & 0.0 \\ 0.0465 & -0.0738 & 0.2036 & 1.5413 & 12.5620 & 0.0 \end{bmatrix}$$

and

b) $\mathbf{Q}_2$=diag(0.01 0.001 0.001 0.01 0.001 0.0001)

$\mathbf{R}_2$=1.

The **K** and $\mathbf{L}_u$ feedback matrices were computed as:

$$\mathbf{K}=\begin{bmatrix} -0.1104 & 0.0011 & -0.1119 & 0.0267 & -0.0125 & -0.0008 \\ -0.0206 & -0.0088 & 0.0619 & -0.0050 & -0.0013 & -0.0001 \\ -0.0201 & 0.0148 & -0.1455 & 0.0210 & -0.0023 & -0.0001 \\ -0.0160 & 0.0077 & -0.0819 & 0.0122 & -0.0016 & 0.0 \\ -0.0023 & 0.0011 & 0.0119 & -0.0018 & 0.0002 & 0.0 \\ -0.0086 & 0.0043 & -0.0450 & 0.0067 & -0.0009 & 0.0 \end{bmatrix}$$

$$\mathbf{L}_u=\begin{bmatrix} 0.0979 & 0.0250 & 0.0577 & 0.2113 & 0.2917 & 0.0640 \\ -0.0676 & 0.3911 & -1.4294 & -0.0588 & 0.2173 & 0.0561 \\ 0.1525 & -0.3110 & 0.9196 & 0.0567 & -0.0919 & -0.0238 \\ 0.0837 & -0.1245 & 0.3775 & 0.0266 & -0.0341 & -0.0084 \\ -0.0123 & -0.1093 & 0.3133 & 0.0221 & -0.0263 & -0.0066 \\ 0.0465 & -0.0554 & 0.1599 & 0.0113 & -0.0135 & -0.0034 \end{bmatrix}$$

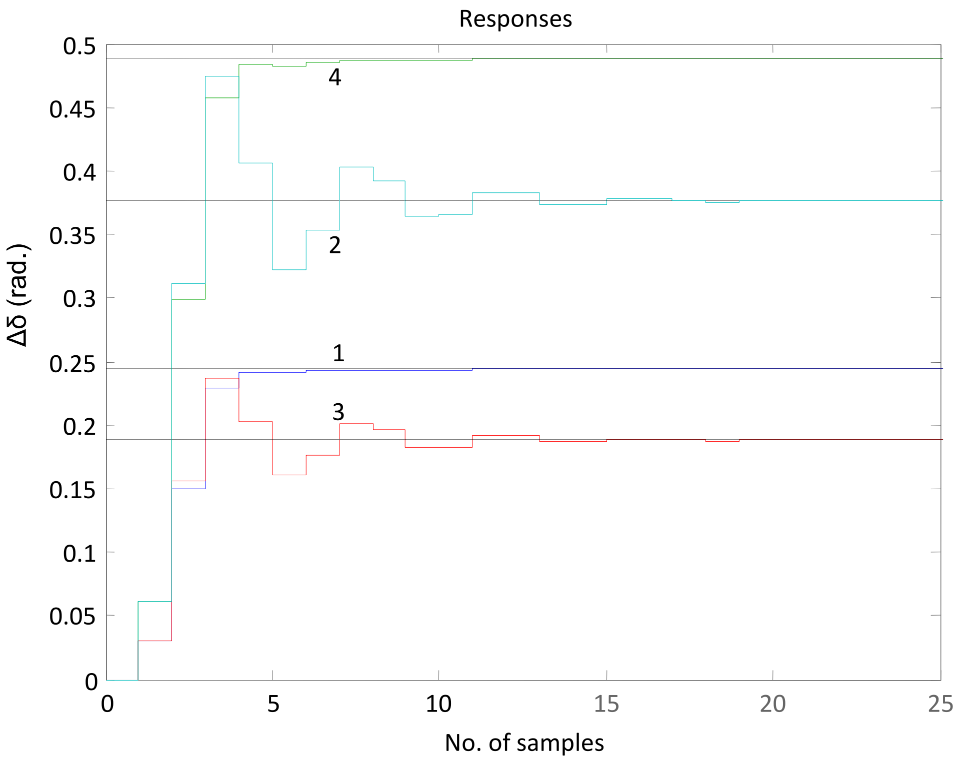
The output multiplicities, (for the two cases a and b), of the sampling are chosen as $\mathbf{M}_i$=[2 4 6 7 8 12], whereas the input multiplicity of the sampling is taken as N_o=8.

The simulation results of the discrete open-loop and closed-loop power system models i.e. eigenvalues, eigenvectors, Riccati solutions, responses of system variables etc, for zero initial conditions and with input disturbances ($\Delta V_{ref.}$=0.05 & $\Delta V_{ref.}$=0.10) were obtained using a special software program, which is based on the theory of § III and runs on MATLAB program environment.

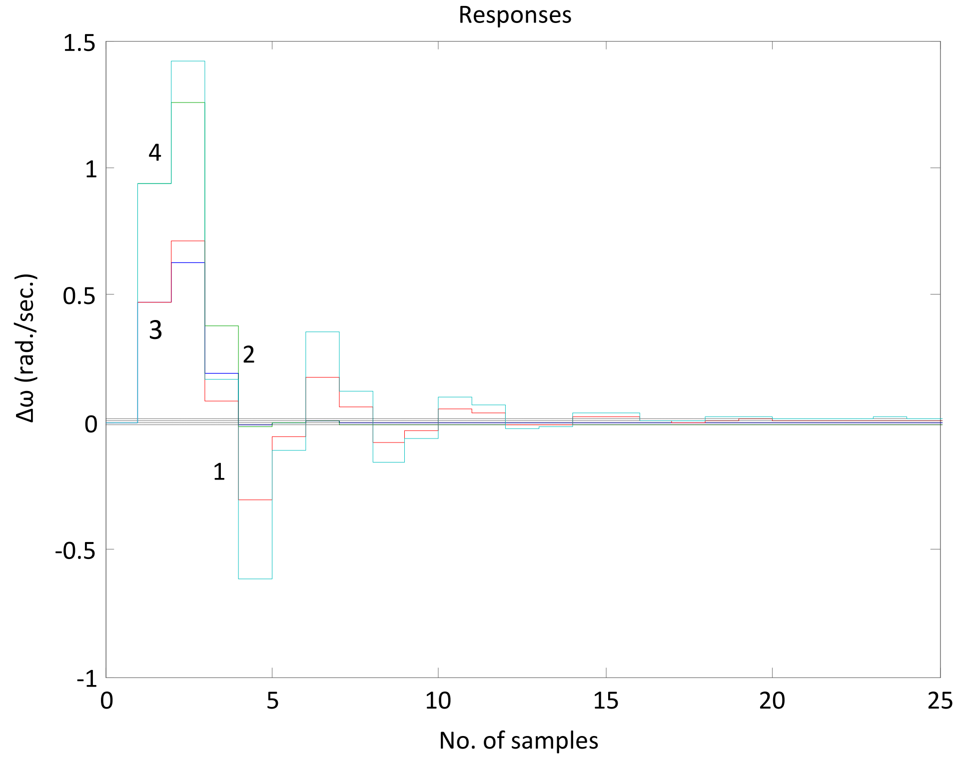
The associate discrete closed-loop system model matrices $\mathbf{A}_{cl/d}$, $\mathbf{B}_{cl/d}$, is also computed and is given in Appendix B.

By comparing the eigenvalues of the designed closed-loop power system models to those of the original open-loop power system models the resulting enhancement in dynamic system is judged as being remarkable.

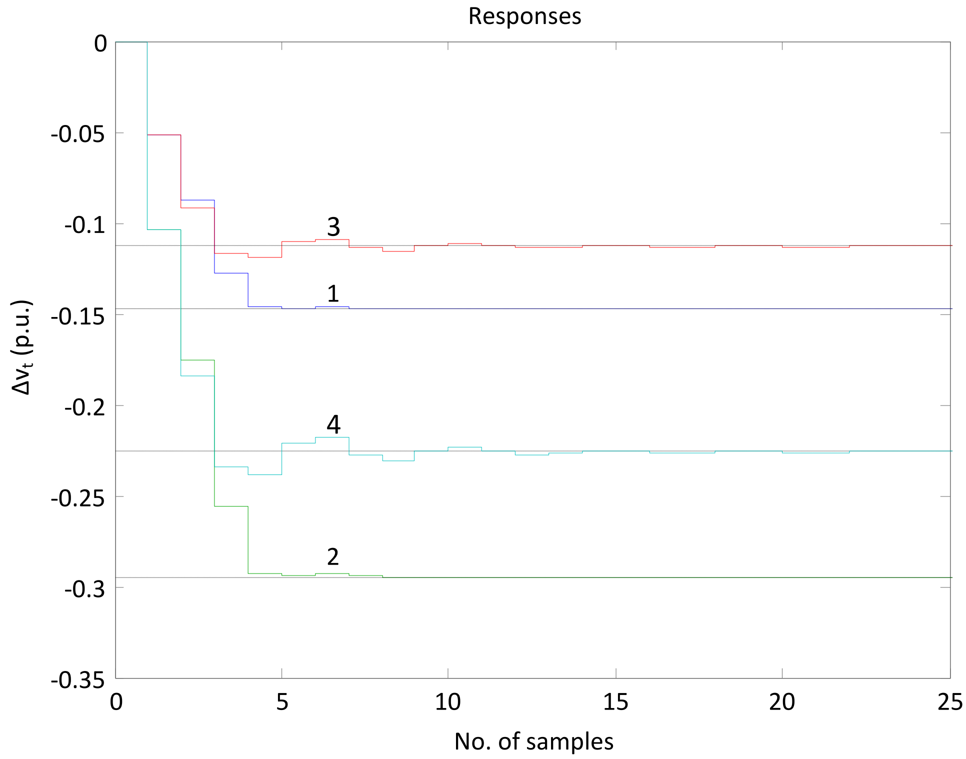
The responses of the output variables ($\Delta\delta$, $\Delta\omega$, Δv_t, ΔP_t, Δi_f, ΔE_{fd}) of the designed closed-loop power system models for zero initial conditions and unit step input disturbance are shown in Figure 5.



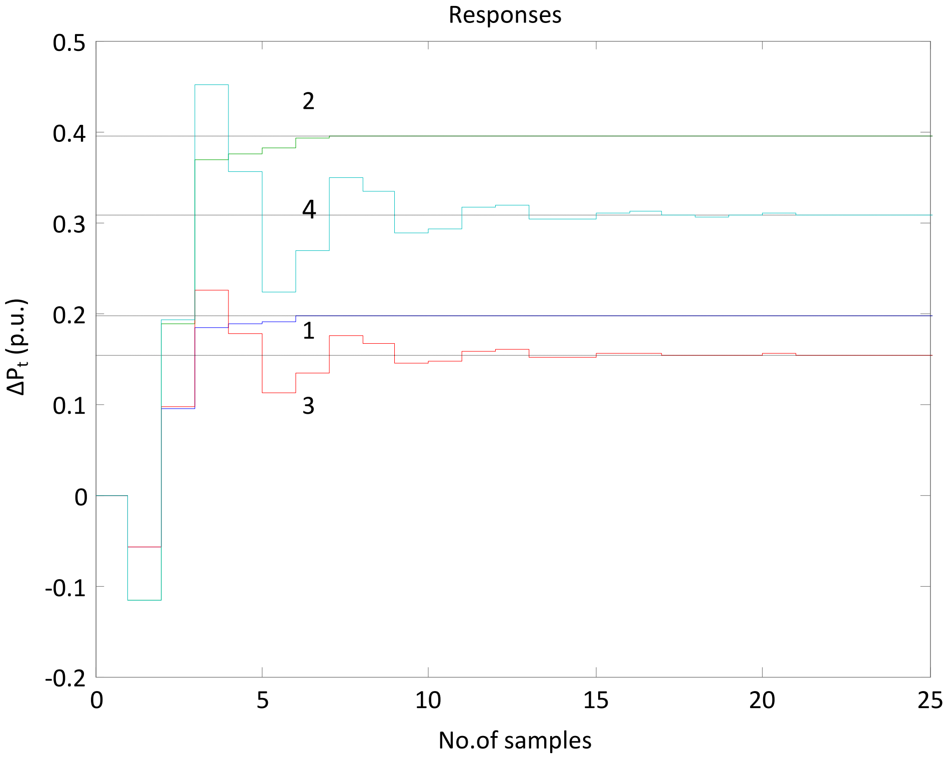
(A)


Responses
1.5
1
0.5
0
-0.5
-1
Δω (rad./sec.)
4
3
2
1
0
5
10
15
20
25
No. of samples

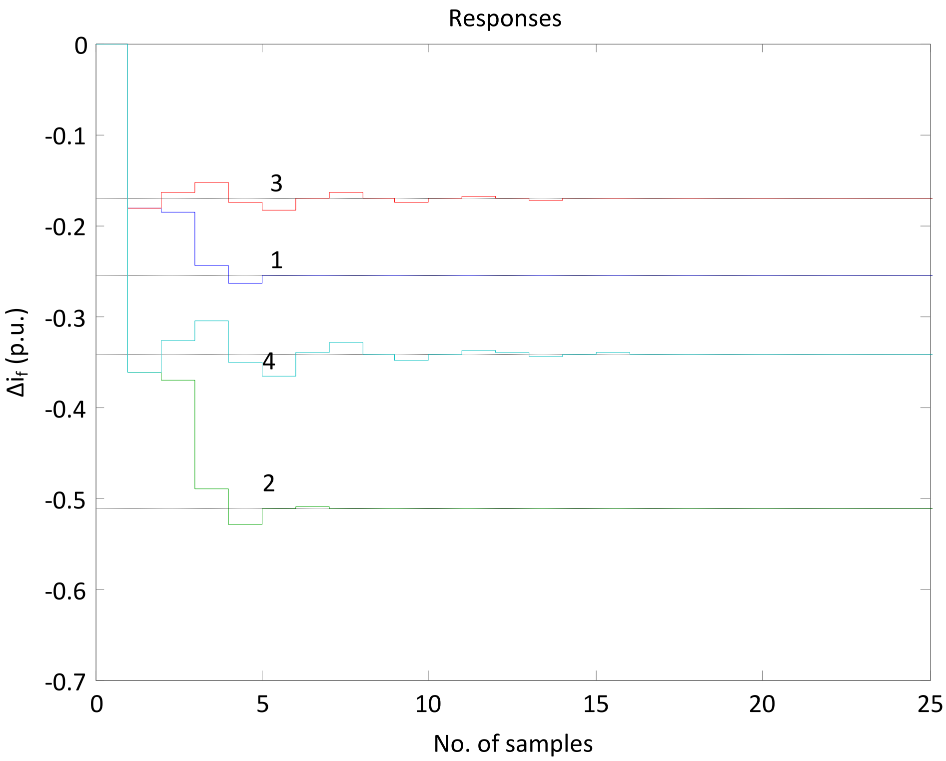
(B)


Responses
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
Δv_t (p.u.)
3
1
4
2
0
5
10
15
20
25
No. of samples

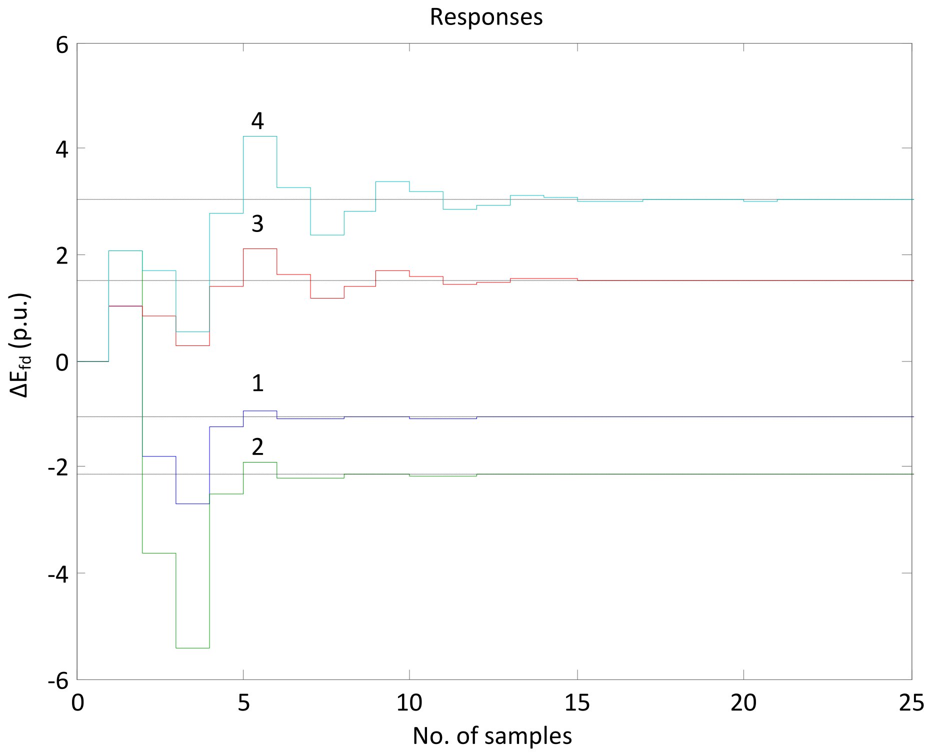
(C)


Responses
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
25
No.of samples
ΔPt (p.u.)
1
2
3
4

(D)


Responses
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
5
10
15
20
25
No. of samples
Δif (p.u.)
1
2
3
4

(E)



(F)

Figure 5: (A), (B), (C), (D), (E), (F): Responses of $\Delta\delta$, $\Delta\omega$, Δv_t, ΔP_t, Δi_f, ΔE_{fd}, of the designed discrete closed-loop-model: (1),(3): with weighting matrices Q_1, R_1 and to step input changes: $\Delta V_{Ref.}$ =0.05 & $\Delta V_{Ref.}$=0.10, rispectively. (2),(4): with weighting matrices Q_2, R_2 and to step input changes: $\Delta V_{Ref.}$=0.05 & $\Delta V_{Ref.}$=0.10, rispectively.

From Figure 6 it is clear that the dynamic stability characteristics of the designed discrete closed- loops systems-models are far more syperior than the correspondig, ones of the original open-loop model, which attests in favour of the proposed TPMRCs-control technique.

In Figure 6, the variation of the optimal average cost J_{opt} (Al-rahmani and Franklin, 1990) with respect to the fundamental sampling period T_o is depicted in the case where (No, M1, M2, M3, M4, M5, M6)=(8, 2, 4, 6, 7, 8, 12) is depicted. The optimal average cost obtained by, $J_{opt} = \frac{1}{2}\mathbf{x}(0)^T \mathbf{P}\mathbf{x}(0)$, where **P** is the Riccati solution (Al-rahmani and Franklin; Chu, et al., 2007; Komarofff, 1979).

For the case where T_o=0.189 sec. the variation of the optimal average cost J_{opt} (Al-rahmani and Franklin; Ramamoorty and Arumugan, 1971; Cimino and Pagilla, 2010), with respect to N_o is given in Figure 7, wherein the optimal average cost $\hat{J}$ (=116.62) of the respective continuous-time design, is also depicted. From Figure 5, it becomes clear that $J_{opt} \rightarrow \hat{J}$ as $N_o \rightarrow +\infty$. Analogous results can be easily obtained for other values of the sampling period T_o.

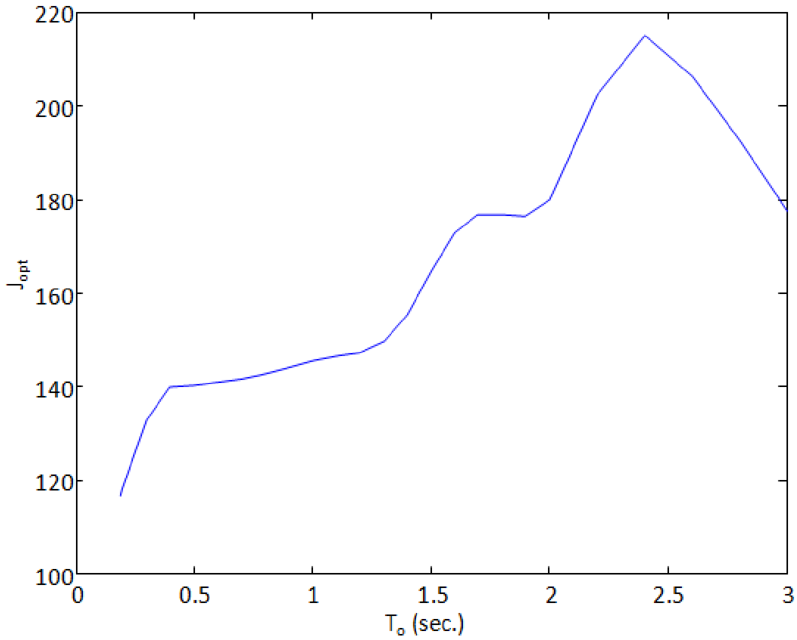


Figure 6: Discrete-time optimal average cost J_{opt} and continuous-time optimal average cost $\hat{J}$ versus N_o for T_o=0.189 sec.

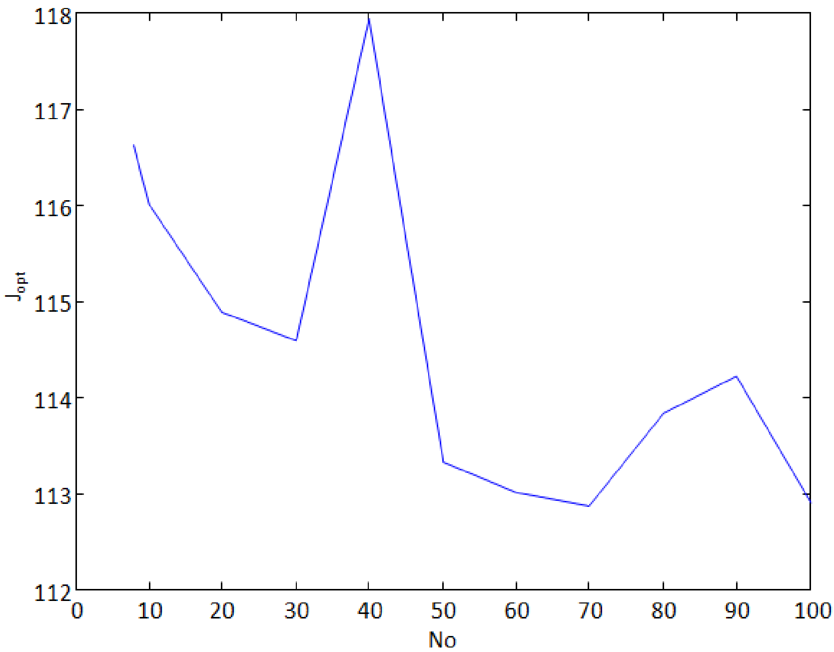


Figure 7: Discrete-time optimal average cost J_{opt} versus sampling period T_o.

V. CONCLUSION

An optimal digital control strategy based on Two-Point-Multirate Controllers has been used in this paper in order to design a desirable excitation controller of an unstable hydrogenerator system, for the purpose of enhancing its dynamic stability characteristics. The proposed method offers acceptable closed loop response as well as more design flexibility (particularly in cases where the system states are not measurable), and its performance is at least comparable to known LQ optimal regulation methods. The clear simplicity of the TPMRCs used makes it an appropriate and reliable tool for the design of such implementable controllers.

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