wjert, 2020, Vol. 6, Issue 5, 266-276. *Original Article* **ISSN 2454-695X**

World Journal of Engineering Research and Technology

WJERT

www.wjert.org **SJIF Impact Factor: 5.924**

MATHEMATICAL MODEL OF BUOYANCY DRIVEN HYDROMAGNETIC TURBULENT FLUID FLOW OVER A VERTICAL INFINITE PLATE USING TURBULENT PRANDTL NUMBER

W. O. Mukuna*, E. Chepkemoi and J. K. Rotich

Department of Mathematics and Computer Science, University of Kabianga, P. O. Box 2030 – 20200, Kericho, Kenya.

Article Received on 01/08/2020 **Article Revised on 22/08/2020** **Article Accepted on 12/09/2020**

***Corresponding Author**

W. O. Mukuna

Department of Mathematics and Computer Science, University of Kabianga, P. O. Box 2030 – 20200, Kericho, Kenya.

ABSTRACT

A mathematical model of hydromagnetic buoyancy driven turbulent fluid flow over an infinite vertical plate using Turbulent Prandtl number is analysed. The fluid is considered to exhibit a turbulent flow over a vertical infinite plate in within a magnetic field. The fluid flow is modeled using Reynolds time averaged conservation equations of momentum and energy. This gives rise to the governing equations relating to primary velocity, secondary velocity and temperature. The governing equations are subsequently non-dimensionalised leading to the inclusion of non-dimensional parameters. The turbulence in energy equation is resolved using turbulent Prandtl number. An analytic solution for the model equations is not feasible due to nonlinearity hence the approximate numerical solution for the model equations is computed by use of the finite difference scheme. The finite difference scheme is run on a computer using MATLAB software given much computation involved. The results are displayed in graphs. The various non-dimensional parameters are examined of their effects on the velocities and temperature profiles. It is established that the primary velocity increases with decreasing magnetic parameter while it increases with increase in Hall parameter and Grashoff number.

KEYWORDS: buoyancy, hydromagnetic, turbulent flow, turbulent Prandtl number, vertical plate, finite difference.

u,v,w	*Cartesian velocity components* (ms^{-1})
x,y,z	*Cartesian coordinate variables*
t	*Time (s)*
p	*Pressure* (Nm^{-2})
T	*Temperature (K)*
μ	*Dynamic viscosity* ($Kgm^{-2}s$)
υ	*Kinematic viscosity* (m^2s^{-1})
k	*Thermal conductivity* ($wm^{-1}k^{-1}$)
$\boldsymbol{\nabla}$	*Gradient operator*, $\boldsymbol{\nabla} = \mathrm{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$
ρ	*Density* (Kg/m^3)
m	*Mass of the fluid particle (Kg)*
E	*Electric field* (Vm^{-1})*, internal energy (J)*
Gr	*Thermal Grashoff number*
g	*Acceleration due to gravity* (ms^{-2})
Pr	*Prandtl number*
Pr_t	*Turbulent Prandtl number*
M	*Magnetic parameter*
H_o	*External applied transverse magnetic field intensity*(wbm^{-2})
ϕ	*Viscous dissipative rate*
α	*Thermal diffusivity*
β	*Thermal expansion coefficient* (K^{-1})
C_p	*Specific heat at constant pressure* ($JKg^{-1}K^{-1}$)
u', v' w'	*Fluctuating components of velocity*
$\overline{u}$, $\overline{v}$, $\overline{w}$	*Mean velocities*
a	*Acceleration* (m/s^2)
Q	*Heat (J)*
W	*Work (J)*
$\frac{D}{Dt}$	*Material derivative given by* $\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$
$\frac{\partial}{\partial t}$	*Partial derivative with respect to time*
σ	*Electrical conductivity* ($\Omega^{-1}m^{-1}$)
L	*Characteristic length (m)*
μ_e	*Electron permeability (H/m)*

B *Magnetic flux density (wbm^{-2})*

H *Magnetic field intensity (wbm^{-2})*

J *Current density vector*

1. INTRODUCTION

The study of fluid flow in the presence of magnetic field is called magnetohydrodynamics or hydromagnetics. These studies have continued to be center of interest in computational fluid dynamics (CFD). Hydromagnetics is significant in areas like meteorology and astrophysics, biological, aerospace and aeronautical engineering applications among others.

Umamaheswar *et. al*. (2016) analyzed numerically the solutions of unsteady hydromagnetic convection flow of a fluid past a vertical semi-infinite plate which was impulsively started under the influence of a uniform transverse magnetic field. From the experiment it was established that, an increase in magnetic field parameter, Prandtl number and Schmidt number caused a decrease in velocity. On the other hand, Grasshoff number, Soret number and visco-elastic parameter increased with increase in velocity.

Rency (2015) studied an incompressible turbulent fluid flow past a vertical semi-infinite rotating plate. The flow was considered in the presence of a strong inclined constant magnetic field. She then solved the governing equations using finite difference method to obtain numerical solutions. She obtained velocity and temperature profiles results and presented them graphically. The effects of non-dimensional parameters and the angle inclined by the magnetic field on the flow changes were analysed. It was then noted that increasing Eckert number and rotational parameter resulted in decrease of the primary velocity. Also, the secondary velocity was increased when the Hall parameter and rotational parameter were increased.

Kwanza *et.al.* (2010) carried out a mathematical model of convective turbulent fluid flow past a vertical infinite plate with Hall current. The effects of Hall current in a heat turbulent boundary layer was observed. Prandtl mixing length hypothesis was used to resolve Reynolds stress which arose due to turbulence in the momentum equations. The governing equations were then solved by use of the finite difference method. They analyzed the effects of viscous dissipation, time and Hall current on temperature and velocity profiles and presented them graphically. It was concluded that an increase in Hall parameter increases the primary velocity.

Mukuna *et. al.* (2017) carried out an analysis of heat and mass transfer rates of hydromagnetic turbulent fluid flow over an immersed infinite horizontal cylinder. In the study the cylinder was placed in cross flow with the fluid. The fluid flow was impulsively started and the flow problem subsequently analysed. The flow was modeled using the momentum, energy and concentration conservation equations. The Reynolds stresses arising due to turbulence in the conservation equations were resolved using Prandtl mixing length hypothesis. The equations were then transformed into a finite difference scheme and solved using a computer program. The effects of flow parameters on the primary velocity, secondary velocity, temperature and concentration profiles were investigated. It was established that velocity profiles increase with increase in Hall parameter and temperature and concentration profiles increased with increase in magnetic parameter.

Kaya (2011) numerically investigated heat and mass transfer from a horizontal slender cylinder within a magnetic field perpendicular to the cylinder. The nonlinear partial differential equations governing the flow were transformed into similar boundary layer equations, which were then solved numerically using the Keller box method. In the investigation the transverse curvature parameter, the magnetic parameter, the Prandtl number and the Schmidt number were the main parameters. For various values of flow parameters, the local skin friction, heat transfer and mass transfer parameters were obtained. It was observed that the local skin friction coefficient, the local heat transfer coefficient and the local mass transfer coefficient increase with an increase in the magnetic parameter and transverse curvature parameter.

Sugunamma and Sandeep (2011) studied unsteady hydromagnetic free convective flow through porous medium past a vertical plate with constant heat generation. The temperature and velocity profiles of the flow were illustrated graphically. It was concluded that when the plate was heated by free convection current, increasing viscous dissipation resulted in a decrease in the skin friction. On the other hand, for cooling of the plate by free convection current, there was a decrease in the skin friction when the viscous dissipation was increased.

The current work involves an investigation of buoyancy driven hydromagnetic turbulent fluid flow over a vertical infinite plate using turbulent Prandtl number.

2. Mathematical model

A two-dimensional turbulent fluid flow is considered for this study. The fluid flows along an infinite vertical plate. The plate is taken to be along the x-axis and the horizontal is the y-axis while the z+-axis is taken normal to the plate. The fluid is assumed to be incompressible and viscous without an external electric field applied to it. A strong magnetic field of uniform strength H_0 is applied normal to the direction of the flow. The induced magnetic field is considered negligible hence H = (0, 0, H_0), as shown in the figure below. The temperature of the plate and the fluid are assumed to be the same initially. At time t*>0 the plate is stationary and the fluid starts moving impulsively in its plane with velocity U_o and at the same time the temperature of the plate is instantaneously raised to T_w^* which is higher than the surrounding and is maintained constant later on.

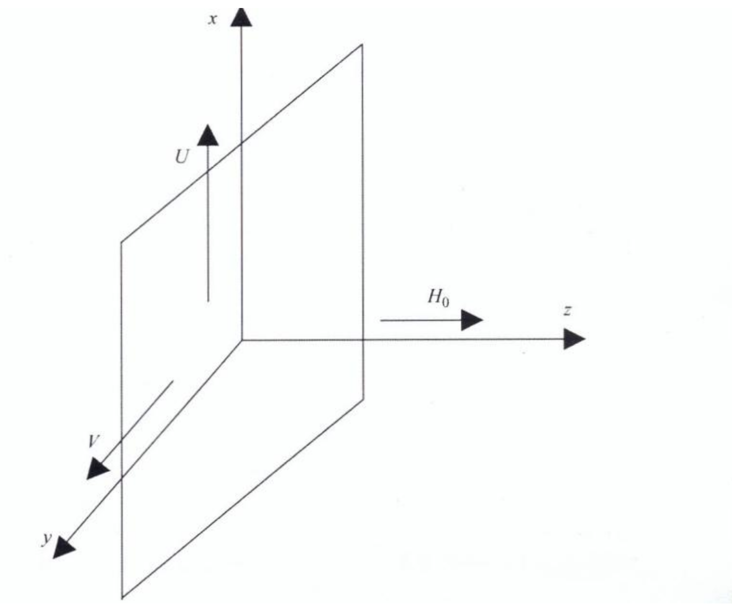

Figure 1: Schematic diagram for the fluid flow

The above flow is governed by the following turbulent fluid flow equations:

$$\frac{\partial \overline{U^*}}{\partial t} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + v\frac{\partial^2 \overline{U^*}}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} + \rho g + \mathbf{J} \times \boldsymbol{B} \qquad (1)$$

$$\frac{\partial \overline{V^*}}{\partial t} = v\frac{\partial^2 V^*}{\partial z^2} - \frac{\partial(\overline{u'w'})}{\partial z} + \mathbf{J} \times \boldsymbol{B} \qquad (2)$$

$$\frac{\partial \overline{T^*}}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 \overline{T^*}}{\partial z^2} - \frac{\partial(\overline{w'T'})}{\partial z} \qquad (3)$$

The boundary and initial conditions are:

$t^* < 0 :\ \overline{U^*} = 0, \overline{V^*} = 0, \overline{T^*} = \overline{T_\infty^*}\ everywhere$

$t^* \geq 0: \overline{U^*} = 0, \overline{V^*} = 0, \overline{T^*} = \overline{T_w^*}$ at

$Z = 0$ (4)

$\overline{U^*} \to U_o, \overline{V^*} \to 0, \overline{T^*} \to \overline{T_\infty^*}, as\ Z \to \infty$

Solving the electromagnetic force term $J \times B$:

$$(\mathbf{J} \times \mathbf{B})_{x+} = \frac{\sigma \mu^2{}_o H^2{}_0 (mV^* - U^*)}{1+m^2} \quad (5)$$

$$(\mathbf{J} \times \mathbf{B})_{y+} = \frac{-\sigma \mu^2{}_o H^2{}_0 (mU^* + V^*)}{1+m^2} \quad (6)$$

Hence the x and y components of the momentum equation are respectively:

$$\frac{\partial U^*}{\partial t} = V \frac{\partial^2 U^*}{\partial z^{*2}} - \frac{\partial u^* w^*}{\partial z^*} + g\beta(T^* - T^*{}_\infty) + \frac{\sigma \mu^2{}_o H^2{}_0 (mV^* - U^*)}{1+m^2} \quad (7)$$

$$\frac{\partial V^*}{\partial t} = V \frac{\partial^2 V^*}{\partial z^{*2}} - \frac{\partial v^* w^*}{\partial z^*} + g\beta(T^* - T^*{}_\infty) - \frac{\sigma \mu^2{}_o H^2{}_0 (mU^* + V^*)}{1+m^2} \quad (8)$$

To non-dimensionalize equations (3), (7) and (8), the following scaling variables are applied:

$$t = \frac{t^* U_0^2}{v};\ z = \frac{z^* U_0}{v};\ U = \frac{U^*}{U_0};\ V = \frac{V^*}{U_0}\ \theta = \frac{T^* - T_w^*}{T_w^* - T_\infty^*}; \quad (9)$$

Using the scaling variables above yields:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial z^2} - \frac{\partial \overline{uw}}{\partial z} + Gr\theta + \frac{M^2(mV - U)}{1+m^2} \quad (10)$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial z^2} - \frac{\partial \overline{vw}}{\partial z} - \frac{M^2(mU + V)}{1+m^2} \quad (11)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} - Pr\left(\frac{\partial \overline{wT}}{\partial z}\right) \quad (12)$$

Boundary and Initial Conditions

$t < 0, U = 0, V = 0, \theta = 0$, everywhere

$t \geq 0, U = 0, V = 0, \theta = 1$, at $z = 0$ (13)

$U \to 1, V \to 0, , \theta \to 0$, as $z \to \infty$

Resolving turbulent stress using Prandlt Mixing length hypothesis and turbulent Prandtl number gives rise to:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial z^2} + 2k^2 z \left(\frac{\partial U}{\partial z}\right)^2 + 2k^2 z^2 \left(\frac{\partial^2 U}{\partial z^2}\right)\left(\frac{\partial U}{\partial z}\right) + Gr\theta + \frac{M^2(mU + V)}{1+m^2} \quad (14)$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial z^2} + 2k^2 z\left(\frac{\partial V}{\partial z}\right)^2 + 2k^2 z^2\left(\frac{\partial^2 V}{\partial z^2}\right)\left(\frac{\partial V}{\partial z}\right) - \frac{M^2(mV-U)}{1+m^2} \qquad (15)$$

Given that the turbulent Prandtl number is given by $Pr_t = \frac{\varepsilon_M}{\varepsilon_H}$

Where $\varepsilon_M = -2k^2z^2\frac{\partial \bar{u}}{\partial z}$ then $\overline{wT} = -\frac{2k^2z^2}{\varepsilon_M}\frac{\partial \bar{u}}{\partial z}\frac{\partial \theta}{\partial z}$, substituting this in equation (12) gives

$$Pr\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial z^2} + Pr\left(\frac{2k^2z^2}{Pr_t}\frac{\partial \bar{u}}{\partial z}\frac{\partial \theta}{\partial z}\right) \qquad (16)$$

Finite difference scheme

The equivalent finite difference scheme for equations 3.55, 3.56 and 3.57 are respectively:

$$\frac{U(i,j+1)-U(i,j)}{\Delta t} = \frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)^2 + 0.32(i\Delta z)^2\left(\frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2}\right)\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right) + Gr\,\theta(i,j) + M^2\frac{mU(i,j)+V(i,j)}{1+m^2} \qquad (17)$$

$$\frac{V(i,j+1)-V(i,j)}{\Delta t} = \frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right)^2 + 0.32(i\Delta z)^2\left(\frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2}\right)\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right) + M^2\frac{mV(i,j)-U(i,j)}{1+m^2} \qquad (18)$$

$$Pr\frac{\theta(i,j+1)-\theta(i,j)}{\Delta t} = \frac{\theta(i+1,j)-2\,\theta(i,j)+\theta(i-1,j)}{(\Delta z)^2} + 0.32(i\Delta z)^2\frac{Pr}{Pr_t}\left\{\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)\left(\frac{\theta(i+1,j)-\theta(i,j)}{\Delta z}\right)\right\} \qquad (19)$$

Where i and j refer to z and t respectively. The values for *k* have been substituted as 0.4 and z have been substituted with $i\Delta z$.

The boundary and initial conditions 4.45 now take the form:

$$U(i,j) = 0;\ V(i,j) = 0; \theta(i,j) = 0 \text{ everywhere for } \theta < 0$$

$$\theta \geq 0; U(i,j) = 0;\ V(i,j) = 0; \theta(i,j) = 1 \text{ for } i = 0 \qquad (20)$$

$$U(i,j) = 1;\ V(i,j) = 0; \theta(i,j) = 0 \text{ for } i = \infty$$

Using the boundary and initial conditions (20) we compute values for consecutive grid points for primary and secondary velocities and temperature that is:

$U(i,j+1)$; $V(i,j+1)$ and $\theta(i,j+1)$ which are given as:

$$U(i,j+1) = U(i,j) + \Delta t\left\{\frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)^2 + 0.32(i\Delta z)^2\left(\frac{U(i+1,j)-2U(i,j)+U(i-1,j)}{(\Delta z)^2}\right)\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right) + Gr\,\theta(i,j) + M^2\frac{mU(i,j)+V(i,j)}{1+m^2}\right\} \qquad (21)$$

$$V(i,j+1) = V(i,j) + \Delta t\left\{\frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2} + 0.32i\Delta z\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right)^2 + 0s.32(i\Delta z)^2\left(\frac{V(i+1,j)-2V(i,j)+V(i-1,j)}{(\Delta z)^2}\right)\left(\frac{V(i+1,j)-V(i,j)}{\Delta z}\right) + M^2\frac{mV(i,j)-U(i,j)}{1+m^2}\right\} \quad (22)$$

$$\theta(i,j+1) = \theta(i,j) + \frac{\Delta t}{Pr}\left[\frac{\theta(i+1,j)-2\,\theta(i,j)+\theta(i-1,j)}{(\Delta z)^2} + 0.32(i\Delta z)^2\frac{Pr}{Pr_t}\left\{\left(\frac{U(i+1,j)-U(i,j)}{\Delta z}\right)^2\left(\frac{\theta(i+1,j)-\theta(i,j)}{\Delta z}\right)\right\}\right] \quad (23)$$

	M^2	m	Gr
I	50	0.1	10
II	50	2	10
III	5	0.1	10
IV	50	0.1	1000

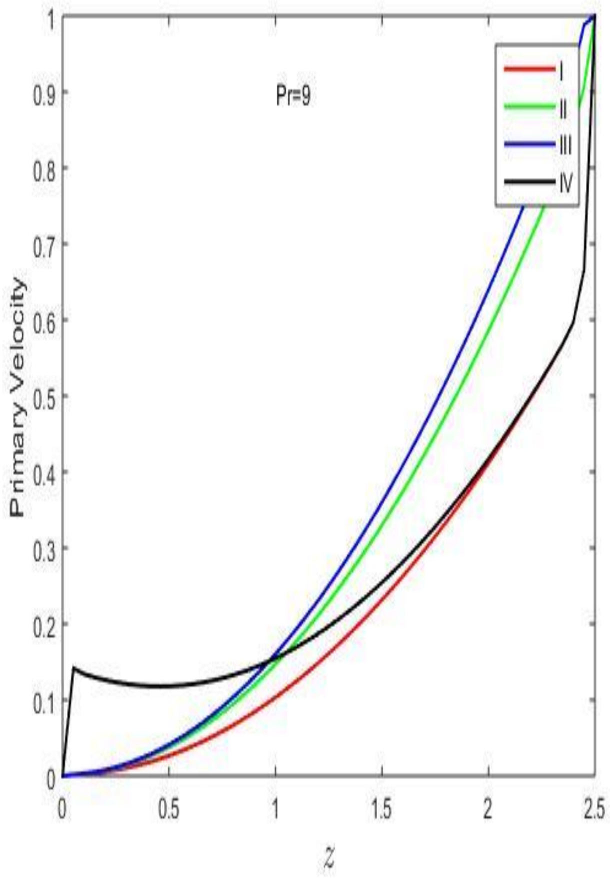

Figure 2: Primary velocity profile.

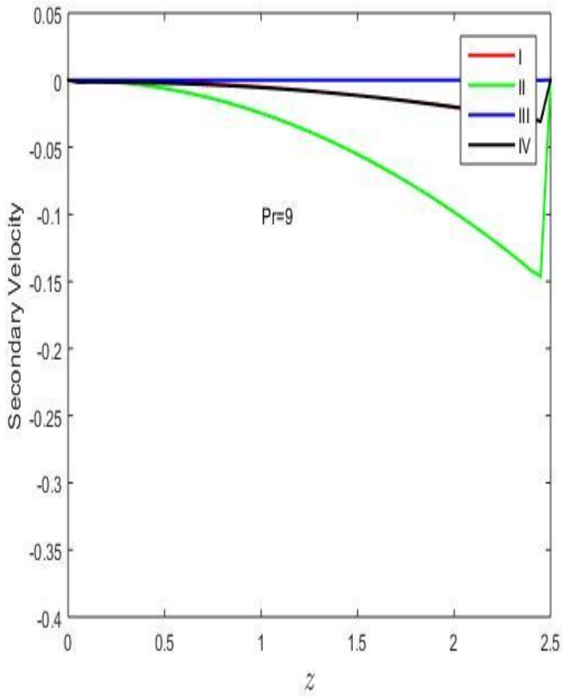

Figure 3: Secondary velocity profile.

	M^2	m	Gr
I	50	0.1	10
II	50	2	10
III	5	0.1	10
IV	50	0.1	1000

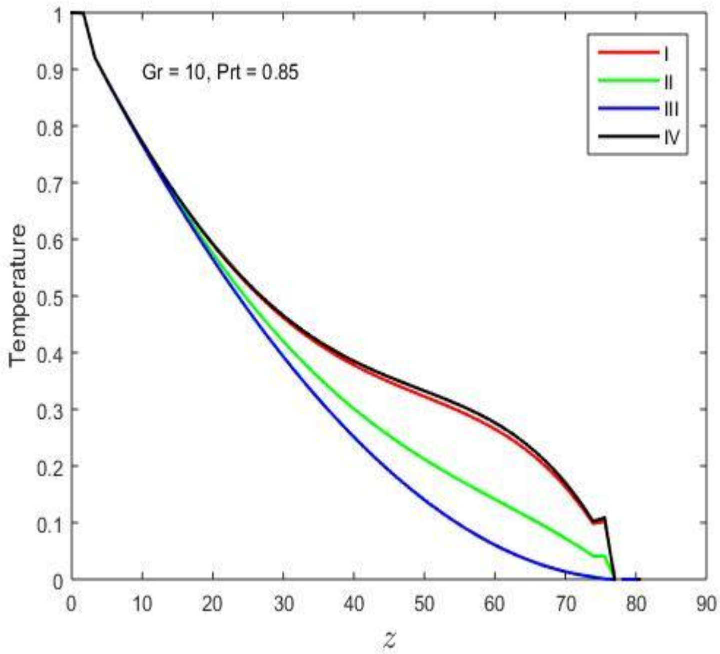

Figure 4: Temperature profile.

	M^2	Pr	M
I	50	9	1
II	50	9	2
III	5	9	1
IV	50	1	1

3. DISCUSSION OF RESULTS

The numerical results obtained from the computer program are presented in the given graphs. The trends of various fluid flow parameters are discussed and explained as they were observed upon varying.

The positive values of *Gr* in this case means that the plate is at a higher temperature than its surrounding fluid hence the plate is being cooled by the surrounding.

3.1 Primary velocity

From figure 2 it is observed that

1. When there is decrease in Magnetic parameter results in increase in the primary velocity profile. The presence of magnetic field in an electrically conducting fluid introduces a force which acts against the flow if the magnetic field is applied hence the effect in the primary velocity.
2. An increase in Hall parameter leads to increase in the primary velocity
3. Also an increase in Grashoff parameter increases the primary velocity. The Grashoff number shows the relative effect of the buoyancy force to the viscous force in the boundary layer.

3.2 Secondary velocity

From figure 3 it is observed that

1. For a decrease in Magnetic parameter there is a resulting increase in the secondary velocity. The presence of magnetic field in an electrically conducting fluid introduces a force which acts against the flow if the magnetic field is applied hence the change in the secondary velocity.
2. When the Hall parameter is increased, there is a decrease in the secondary velocity.
3. An increase in the Grashoff number does not affect the secondary velocity.

3.3 Temperature

From figure 4 it is observed that:

1. Increase Hall parameter decreases the temperature profiles.

2. Decrease in Prandtl number increases the temperature profiles. Physically, decrease in Prandtl number leads to an increase in thermal boundary layer and rise in the average temperature within boundary layer.
3. For a decrease in Magnetic parameter there is a resulting decrease in the temperature profiles.

4. CONCLUSION

In conclusion it is established that the velocity profiles increases with decreasing magnetic parameter while temperature profile is directly propotional to magnetic parameter. An increase in Hall parameter and Grashoff number causes an increase in primary velocity profiles. Temperature profiles decreases with increasing Prandtl number while they decrease with increasing Hall parameter.

REFERENCES

1. Umamaheswar M., Raju M.C., Varma S.V. and Gireeshkumar J. Numerical investigation of MHD free convection flow of non-Newtonian fluid past an impulsively started vertical plate in the presence of thermal diffusion and radiation absorption. *Alexandrian Engineering Journal: Physics and Mathematics,* 2016; *2(4): 273-380.*
2. Rency C.M. MHD turbulent flow in presence of inclined magnetic field past a rotating semi-infinite plate. *International Journal of Engineering, Science and Innovation,* 2015; *4(2): 344-360.*
3. Kwanza J.K., Mukuna W.O. and Kinyajui M. Mathematical Model of Turbulent convective fluid flow past a infinite vertical plate with hall current. *International Journal of Modelling and Simulation,* 2010; *30*: 376-383.
4. Mukuna W.O., Kwanza J.K., Sigey J.K. and Okello J.A. Analysis of heat and mass transfer rates of hydromagnetic turbulent fluid flow over an immersed infinite horizontal cylinder with Hall current. *Journal of Mathematical Theory and Modelling,* 2017; *7(4):* 45 – 64.
5. Kaya Ahmet Heat And mass transfer from a horizontal slender cylinder with a magnetic field effect, *Journal of Thermal Science and Technology,* 2011; 2(31): 73-78.
6. Sugunamma V. and Sandeep N. Unsteady hydromagnetic natural free convection flow of a dissipative and radiating fluid past a vertical porous media with constant heat flux. *International Journal of Mathematics Archive,* 2011; *2(8): 1242-1248.*