



α -ZERO SETS AND THEIR PROPERTIES IN TOPOLOGY

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ABSTRACT

In 1990, Malghan et al. have defined and studied the concepts of almost p-regular, p-completely regular and almost p-completely regular spaces. In 1997 & 2004 Malghan et al. have defined and studied the concepts of almost s-completely regular spaces and s-

completely regular spaces. In 2010, Navalagi introduced the concepts of pre-zero sets and co-pre-zero sets to characterize the concepts of p-completely regular spaces and almost p-completely regular spaces. In this paper, we offer some new concepts of α -zero sets, co- α -zero sets, α -completely regular spaces and almost α -completely regular spaces. We also characterize their basic properties via α -zero sets.

1. INTRODUCTION

In the literature zero sets and co-zero sets due to Gilman and Jerison,^[9] were used to characterize the concepts like completely regular spaces and almost completely regular spaces by Singal, Arya and Mathur in topology (See 21 & 22). In,^[11] and,^[12] Malghan et al have defined and studied the concepts of semi-zero sets and co-semi-zero sets in topology to characterize the properties of s-completely regular spaces and almost s-completely regular spaces using semicontinuous functions due to N.Levine,^[10] In,^[17] Navalagi has defined and studied the concepts pre-zero sets and co-pre zero sets in topology by using precontinuous functions due to Mashhour et al,^[14] to characterize the properties of p-completely regular spaces and almost p-completely regular spaces due to Malghan et al,^[13] In this paper, present author define and study the α -zero sets and co- α -zero sets using α -continuous functions due to Mashhour et al,^[15] to characterize the properties of newly introduced spaces, α -completely regular spaces and almost α -completely regular spaces.

2. Preliminaries

Throughout this paper, we let (X, τ) and (Y, σ) be topological spaces (or simply X and Y be spaces) on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X . Let $Cl(A)$ and $Int(A)$ denote the closure and the interior of subset A

We need the following definition and results in the sequel of the paper.

DEFINITION 2.1: A subset A of a space X is said to be:

- (i) Preopen^[14] if $A \subset Int Cl(A)$.
- (ii) Semiopen^[10] if $A \subset Cl Int(A)$.
- (iii) Regular open^[23] if $A = Int Cl(A)$.
- (iv) α -open^[18] if $A \subset Int Cl Int(A)$.
- (v) δ -open set^[24] if for each $x \in A$, there exists a regular open set G such that $x \in G \subset A$.

The complement of a preopen (resp. semiopen, regular open, α -open, δ -open) set of a space X is called preclosed^[6] (resp. semiclosed,^[2] regular closed,^[23] α -closed [15], δ -closed [24]) set. The family of all preopen (resp. semiopen, regular open, α -open and δ -open) sets of X is denoted by $PO(X)$ (resp. $SO(X)$, $RO(X)$, $\alpha O(X)$ and $\delta O(X)$) and that of preclosed (resp. semiclosed, regular closed, α -closed, β -closed and δ -closed) sets of X is denoted by $PF(X)$ (resp. $SF(X)$, $RF(X)$, $\alpha F(X)$ and $\delta F(X)$)

DEFINITION 2.2: A function $f: X \rightarrow Y$ is called

- (i) Precontinuous^[14] if the inverse image of each open set U of Y is preopen set in X .
- (ii) Semicontinuous^[10] if the inverse image of each open set U of Y is semiopen set in X .
- (iii) α -Continuous^[15] if the inverse image of each open set U of Y is α -open set in X .

DEFINITION 2.3: A space X is said to be

- (i) P-regular^[6] if for each closed set F and each point x not in F , there exist disjoint preopen sets U and V such that $x \in U$ and $F \subset V$.
- (ii) P-Completely regular [13] (resp. s-completely regular [11]) if for each closed set F and each point $x \in (X \setminus F)$, there exists a precontinuous (resp. a semicontinuous) function $f: X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

- (iii) Almost p -completely regular^[13] (resp. almost s -completely regular^[12]) if for each regular closed set F and each point $x \in (X \setminus F)$, there exists a precontinuous (resp. a semicontinuous) function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.
- (iv) Submaximal^[3] if every dense subset of it is open (i.e. if $PO(X) = \tau$ ^[8]).
- (v) An extremally disconnected (E.D.)^[25] if closure of each open set is open in it (i.e. if $A \in \tau$ for each $A \in RF(X)$).
- (vi) α -space^[7] if every α -open set of X is open in X (i.e. $\tau = \alpha O(X)$).
- (vii) α -regular^[4] if for every closed set F and a point $x \notin F$, there exist disjoint α -open sets A and B such that $x \in A$ and $F \subset B$.

It is well-known that a subset A of a space X is called a zero set^[9] if there exists a continuous functions $f : X \rightarrow \mathbf{R}$ such that $A = \{ x \in X \mid f(x) = 0 \}$. The complement of a zero set of a space X is called a co-zero set of X .

REMARK 2.4 : If $f : X \rightarrow \mathbf{R}$ is continuous function may be denoted by $Z(f)$. Thus, we write $Z(f) = \{ x \in X \mid f(x) = 0 \}$. Thus, $Z(f)$ is a zero set of X . Therefore, it is clear that if A is a zero set in X then it can be expressed as $A = Z(f)$, where f is continuous function.

DEFINITION 2.5: A subset A of a space X is said to be semi-zero set^[11] of X if there exists a semicontinuous function $f : X \rightarrow \mathbf{R}$ such that $A = \{ x \in X \mid f(x) = 0 \}$.

DEFINITION 2.6: A subset A of a space X is said to be co-semizero set [11] of X if its complement is a semi-zero set.

REMARK 2.7: If $f : X \rightarrow \mathbf{R}$ is semicontinuous function may be denoted by $SZ(f)$. Thus, we write $SZ(f) = \{ x \in X \mid f(x) = 0 \}$. Thus, $SZ(f)$ is a semi-zero set of X . Therefore, it is clear that if A is a semi-zero set in X then it can be expressed as $A = SZ(f)$, where f is semicontinuous function.

DEFINITION 2.8: A subset A of a space X is said to be pre-zero set^[17] of X if there exists a precontinuous function $f : X \rightarrow \mathbf{R}$ such that $A = \{ x \in X \mid f(x) = 0 \}$.

DEFINITION 2.9: A subset A of a space X is said to be co-prezero set^[17] of X if its complement is a pre-zero set.

REMARK 2.10: If $f: X \rightarrow \mathbf{R}$ is precontinuous function may be denoted by $PZ(f)$. Thus, we write $PZ(f) = \{x \in X \mid f(x) = 0\}$. Thus, $PZ(f)$ is a pre-zero set of X . Therefore, it is clear that if A is a pre-zero set in X then it can be expressed as $A = PZ(f)$, where f is precontinuous function.

RESULT 2.14^[15]: If A is preopen set in X and B is an α -open set in X , then $A \cap B$ is an α -open set in the subspace $(A, \tau|_A)$.

RESULT 2.15^[20]: If A is semiopen set and B is an α -open set in X , then $A \cap B$ is an α -open set in the subspace $(A, \tau|_A)$.

3. α -ZERO SETS

We define the following.

DEFINITION 3.1: A subset A of a space X is said to be α -zero set of X , if there exists a α -continuous function $f: X \rightarrow \mathbf{R}$ such that $A = \{x \in X \mid f(x) = 0\}$.

A subset A of a space X is said to be co- α -zero set of X if its complement is α -zero set.

NOTE 3.2: Every zero set in X is a α -zero set in X .

REMARK 3.3: Let X be a space. If $f: X \rightarrow \mathbf{R}$ is a α -continuous function then the set $\{x \in X \mid f(x) = 0\}$ is a α -zero set. If $g: X \rightarrow \mathbf{R}$ is also a α -continuous function then $\{x \in X \mid g(x) = 0\}$ is also a α -zero set of X .

REMARK 3.4: If $f: X \rightarrow \mathbf{R}$ is α -continuous function may be denoted by $\alpha Z(f)$. Thus, we write $\alpha Z(f) = \{x \in X \mid f(x) = 0\}$. Thus, $\alpha Z(f)$ is a α -zero set of X . Therefore, it is clear that if A is a α -zero set in X then it can be expressed as $A = \alpha Z(f)$, where f is α -continuous function.

In view of Remark- 2.4,2.7,2.10 and 3.4, we have : zero set \rightarrow α -zero set \rightarrow semi-zero set & Zero set \rightarrow α -zero set \rightarrow pre-zero set, since continuity \rightarrow α -continuity \rightarrow semi-continuity & continuity \rightarrow α -continuity \rightarrow pre-continuity.

LEMMA 3.5: If X is a α -space then a function $f: X \rightarrow Y$ is α -continuous then the inverse image of each member of a basis for Y is α -open set in X .

LEMMA 3.6: Let X be a α - space. A function $f: X \rightarrow \mathbf{R}$ is α -continuous if for each $b \in \mathbf{R}$ both the sets $f^{-1}(b, \infty)$ and $f^{-1}(-\infty, b)$ are α -open sets.

LEMMA 3.7: Let X be an α - space then the following are equivalent :

- (i) $f: X \rightarrow \mathbf{R}$ is α -continuous.
- (ii) For each $b \in \mathbf{R}$, $f^{-1}(-\infty, b)$ and $(-f)^{-1}(-\infty, -b)$ are α -open sets in X .
- (iii) For each $b \in \mathbf{R}$, $f^{-1}(b, \infty)$ and $(-f)^{-1}(-b, \infty)$ are α -open sets in X .

PROOF: Since (b, ∞) and $(-\infty, b)$ are subbasic open sets for the usual topology on \mathbf{R} , thus the proof follows from Lemma – 3.6 above.

We need the following.

LEMMA 3.8: Let X be an α - space .Let $f, g : X \rightarrow \mathbf{R}$ are α -continuous then,

- (i) $|f|^\alpha$ is α -continuous for each $\alpha \geq 0$.
- (ii) $(af + bg)$ is α -continuous for each pair of reals a and b .
- (iii) f, g is α -continuous.
- (iv) $1/f$ is α -continuous whenever $f \neq 0$ on X .

These results can be proved by using the proofs of Lemmas : 2.5 , 2.6 and 2.7 . See [5 , p.84].

LEMMA 3.9 : If X is an α - space and if $\{ f_i : X \rightarrow \mathbf{R} \}_{i=1}^k$ is a finite family of α -continuous functions, then the functions $M, m : X \rightarrow \mathbf{R}$ defined by $M(x) = \text{Max} \{f_i(x)\}_{i=1}^k$ and $m(x) = \text{Min} \{f_i(x)\}_{i=1}^k$ are also α -continuous.

Proof is straight forward and hence omitted.

LEMMA 3.10: In an α -space X , the following statements hold for real valued functions :

1. If A is a α -zero set in X then there exists a α -continuous function $g : X \rightarrow \mathbf{R}$ such that $g(x) \geq 0$ for each $x \in X$ and $A = \alpha Z(g)$.
2. If A is a α -zero set in X then there is a α -continuous function $h : X \rightarrow [0,1]$ such $A = \alpha Z(h)$.
3. Finite union of α -zero sets in X is a α -zero set in X .

4. Finite intersection of α -zero sets in X is a α -zero set in X .
5. If $a \in \mathbf{R}$ and $f: X \rightarrow \mathbf{R}$ is a α -continuous function, then the sets $A = \{x \in X \mid f(x) \geq a\}$ and $B = \{x \in X \mid f(x) \leq a\}$ are α -zero sets in X .
6. If $a \in \mathbf{R}$ and $f: X \rightarrow \mathbf{R}$ is a α -continuous function then the sets $A = \{x \in X \mid f(x) < a\}$ and $B = \{x \in X \mid f(x) > a\}$ are co- α -zero sets in X .

These results can be proved by using Lemma- 2.8 and 2.9. See [19, p. 18].

Next, we give the following.

THEOREM 3.11: If A and B are disjoint α -zero sets of an α -space X , there exist disjoint co- α -zero sets U and V such that $A \subset U$ and $B \subset V$.

We, prove the following.

THEOREM 3.12: In an α -space X every α -zero (resp. co- α -zero) set is α -closed (resp. α -open) set.

PROOF: If A is α -zero set in X then by Lemma -3.10, we have $A = \alpha Z(g)$, where $g: X \rightarrow \mathbf{R}$ is α -continuous and $g(x) \geq 0$ for all $x \in X$. Then, $g(x) = 0$ for all $x \in A$. Hence, $g^{-1}(\{0\}) = A$. Since $\{0\}$ is closed in \mathbf{R} and g is α -continuous, it follows that A is α -closed set in X . The second part is proved similarly.

4. α -COMPLETE REGULARITY AND α -ZERO SETS

We, need the following.

DEFINITION 4.1^[16]: Let A be a subset of a space X . Then a subset V of a space X is said to be a α -neighbourhood of A if there exist a α -open set U of X such that $A \subset U \subset V$.

If $A = \{x\}$ for some $x \in X$ then V in the above definition is the α -neighbourhood of the point x .

We define the following.

DEFINITION 4.2: A space X is said to be α -completely regular if for each closed set F and each point $x \in (X \setminus F)$, there exists a α -continuous function $f: X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

Clearly, every completely regular space is α -completely regular, every α -completely regular space is s -completely regular space as well as p -completely regular space and every α -completely regular space is α -regular space.

Next, we prove the following.

THEOREM 4.3: Every preopen subspace of an α -completely regular space is α -completely regular.

PROOF : Let X be an α -completely regular space and Y be an preopen subspace of X . Let F be a closed set in Y and $x \in Y$ such that $x \notin F$. Hence, $x \notin Cl_X(F)$. Since X is α -completely regular, there exists a α -continuous function $f: X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in Cl_X(F)$. Since the restriction of a α -continuous function to a preopen subspace is α -continuous in view of Result 2.14 and by Th.1.3 in^[15] it follows that $f/Y : Y \rightarrow [0,1]$ is α -continuous such that $(f/Y)(x) = 0$ and $(f/Y)(y) = 1$ for each $y \in F$. Hence Y is α -completely regular.

On similar lines of Th.4.2 above and Result-2.15^[20] one can prove the following.

THEOREM 4.4: Every semiopen subspace of an α -completely regular space is α -completely regular.

THEOREM 4.5: Every neighbourhood of a point in an α -space α -completely regular space X contains a α -zero set α -neighbourhood of the point.

PROOF: Let x_0 be a point of an α -space α -completely regular space X and N be a neighbourhood of x_0 . Then there exists a α -continuous function $f : X \rightarrow [0,1]$ such that $f(x_0) = 0$ and $f(x) = 1$ for each $x \in X \setminus N$. Then, $V = \{ x \in X \mid f(x) \geq 1/2 \}$, then V is a α -zero set α -neighbourhood of x_0 such that $V \subset N$, as $x_0 \in \{ x \in X \mid f(x) < 1/2 \}$ is α -open by above Lemma- 3.10 above.

Now, we need the following.

DEFINITION 4.6 [1]: A family σ of subsets of a space X is a net for X if each open set is the union of a family of elements of σ .

Now, we give the following.

THEOREM 4.7: For an α -space X , the following statements are equivalent :

- (i) X is α -completely regular space.
- (ii) Every closed set A of X is the intersection of α -zero sets which are α -neighbourhoods of A .
- (iii) The family of all co- α -zero sets of X is a net for the space X .

PROOF. (i) \Rightarrow (ii) : Let A be a closed set in X and $x \notin A$. Then from (i), there is a α -continuous function $f_x : X \rightarrow [0,1]$ such that $f_x(x) = 0$ and $f_x(A) = \{1\}$. Let $G = \{y \in X \mid f_x(y) \geq 1/3\}$ and $H_x = \{y \in X \mid f_x(y) < 1/3\}$. Then, $A \subset H_x \subset G_x$, where H_x is α -open and G_x is α -zero set which is α -neighbourhood of A . Further, $A = \bigcap_{x \notin A} G_x$.

(ii) \Rightarrow (iii): Let G be an open set of X . Then, $X \setminus G$ is closed set in X . Let $X \setminus G = \bigcap \{B_\lambda \mid \lambda \in \Lambda\}$, where B_λ is α -zero set α -neighbourhood of $X \setminus G$, for each $\lambda \in \Lambda$. Hence, $G = \bigcup \{X \setminus B_\lambda \mid \lambda \in \Lambda\}$, where $X \setminus B_\lambda$ is a co- α -zero for each $\lambda \in \Lambda$. Hence, (iii) holds.

(iii) \Rightarrow (i) : Let A be a closed set and $x_0 \in X \setminus A$. Then, from (iii), as $X \setminus A$ is open there is a co- α -zero set U such that $x_0 \in U \subset X \setminus A$. Let $U = X \setminus \alpha Z(g)$, for some α -continuous function $g : X \rightarrow [0,1]$. As $x_0 \notin \alpha Z(g)$, $|g(x_0)| = r > 0$. If we define, $f : X \rightarrow [0,1]$ by $f(x) = \text{Max}\{0, 1 - r^{-1}|g(x)|\}$ for some $x \in X$, then f is α -continuous by Lemma 3.9 and 3.10 above and $f(x_0) = 0$ and $f(x) = 1$ for $x \in A$. Hence, X is α -completely regular space.

5. ALMOST α -COMPLETE REGULARITY AND α -ZERO SETS

In this section, we characterize the almost- α -completely regular spaces using the concepts of α -zero sets and co- α -zero sets in the following.

We define the following.

DEFINITION 5.1 : A space X is said to be almost α -completely regular if for each regular closed set F and each point $x \in (X \setminus F)$, there exists a α -continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

Obviously every almost completely regular space is almost α -completely regular and every α -completely regular space is almost α -completely regular.

We, prove the following.

THEOREM 5.2: A space X is almost α -completely regular iff for each δ -closed set F and a point $x \in (X \setminus F)$, there is a α -continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(F) = \{1\}$.

PROOF : Let X be almost α -completely regular space and let A be a δ -closed set not containing a point x . Then there exists an open set G containing x such that $\text{Int Cl}(G) \cap A = \emptyset$. Now, $(X - \text{Int Cl}(G))$ is a regular closed set not containing x . Since X is almost α -completely regular, there exists a α -continuous function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(X - \text{Int Cl}(G)) = \{1\}$. Since $A \subset (X - \text{Int Cl}(G))$, it follows that $f(A) = \{1\}$.

Converse follows immediately since every regular closed set is δ -closed.

THEOREM 5.3 : For an α -space X the following are equivalent :

- (i) X is almost α -completely regular space.
- (ii) Every δ -closed subset A of X is expressible as the intersection of some α -zero sets which are α -neighbourhood of A .
- (iii) Every δ -closed subset A of X is identical with the intersection of all α -zero sets which are α -neighbourhoods of A .
- (iv) Every δ -open subset of X containing a point contains a co- α -zero set containing that point.

PROOF. (i) \Rightarrow (ii) : Let X be an almost α -completely regular space. Let A be a δ -closed set and $x \notin A$. Then there exists a α -continuous function f_x on X into $[0,1]$ such that $f_x(x) = 0$ and $f_x(A) = \{1\}$ by Theorem -2.6. Let $G_x = \{y \in X \mid f_x(y) \geq 2/3\}$ for every $x \notin A$; G_x is α -neighbourhood of A . Lastly, $A = \bigcap_{x \notin A} G_x$: We have $A \subset G_x$, for each $x \notin A$, which implies that $A \subset \bigcap_{x \notin A} G_x$. Further, we claim that $\bigcap_{x \notin A} G_x \subset A$: Let $z \notin A$. This implies that there is a α -continuous function $f_z : X \rightarrow [0,1]$ such that $f_z(z) = 0$ and $f_z(A) = \{1\}$. Also, $G_z = \{y \in X \mid f_z(y) \geq 2/3\}$. Now, $f_z(z) = 0 < 2/3$. Therefore, $z \notin G_z$. This implies that $z \notin \bigcap_{x \notin A} G_x$.

Therefore, $z \notin A \Rightarrow z \notin \bigcap_{x \notin A} G_x$. Therefore, $\bigcap_{x \notin A} G_x \subset A$. Hence, $A = \bigcap_{x \notin A} G_x$. Therefore, (i) \Rightarrow (ii) is true.

(ii) \Rightarrow (iii) : Let us suppose that (ii) holds. Let $A = \bigcap \{G_\lambda \mid \lambda \in \Lambda\}$, where G_λ is a α -zero set which is α -neighbourhood of A for each $\lambda \in \Lambda$. Let ρ be the family of all α -zero sets which are α -neighbourhoods of A . Therefore, $\{G_\lambda \mid \lambda \in \Lambda\} \subset \rho$. Therefore, $\bigcap_{B \in \rho} B \subset \bigcap_{\lambda \in \Lambda} G_\lambda \Rightarrow \bigcap_{B \in \rho} B \subset A$. Next, we prove that $A \subset \bigcap_{B \in \rho} B$: Now, B is a α -zero set which is α -neighbourhood of A for each $B \in \rho$ which implies that $A \subset \bigcap_{B \in \rho} B$. Therefore, $A = \bigcap_{B \in \rho} B$. Thus, (iii) holds.

(iii) \Rightarrow (iv) : Suppose (iii) holds. Let G be a δ -open set and $x \in G$. Then, $X \setminus G$ is δ -closed set and $x \notin X \setminus G$. This implies that $X \setminus G = \bigcap_{\lambda \in \Lambda} B_\lambda$ where $\{B_\lambda \mid \lambda \in \Lambda\}$ is family of all α -zero sets which are α -neighbourhoods of $X \setminus G$. Now, $x \notin X \setminus G \Rightarrow x \notin B_{\lambda_0}$ for some $\lambda_0 \in \Lambda$, which implies that $x \in X \setminus B_{\lambda_0}$. Also, we have $X \setminus G = \bigcap_{\lambda \in \Lambda} B_\lambda \Rightarrow G = X \setminus \bigcap_{\lambda \in \Lambda} B_\lambda = \bigcup_{\lambda \in \Lambda} (X \setminus B_\lambda)$. Therefore, $(X \setminus B_{\lambda_0}) \subset \bigcap_{\lambda \in \Lambda} (X \setminus B_\lambda) = G$. Therefore, $x \in X \setminus B_{\lambda_0} \subset G$. Since B_{λ_0} is α -zero set, $X \setminus B_{\lambda_0}$ is a co- α -zero set. Therefore, (iv) holds.

(iv) \Rightarrow (i) : Suppose (iv) holds. Now, to prove that X is almost α -completely regular space: Let A be a δ -closed set and $x_0 \notin A$. Then $X \setminus A$ is a δ -open set containing x_0 . Then by (iv), there exists a co- α -zero set U such that $x_0 \in U \subset X \setminus A$. Thus, $X \setminus U$ is a α -zero set. Therefore, there exists a α -continuous function $f : X \rightarrow [0,1]$ such that $X \setminus U = \alpha Z(f)$. Hence, $X \setminus U = \alpha Z(f) = \{x \in X \mid f(x) = 0\}$. As $x_0 \in U$, it follows that $f(x_0) \neq 0$. Hence, $|f(x_0)| = r > 0$. Now, we define $g : X \rightarrow [0,1]$ by $g(y) = \text{Min}\{1, 1/r \mid f(y)\}$, for each $y \in X$. Then g is α -continuous function. Also, $g(x_0) = 1$ and $g(z) = 0$, for each $z \in A$. Let $h = 1/g$. As X is an α -space, by Lemma- 3.8, $h : X \rightarrow [0,1]$ is α -continuous such that $h(x_0) = 0$ and $h(a) = \{1\}$. Hence, X is almost α -completely regular. Hence the theorem.

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