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α-ZERO SETS AND THEIR PROPERTIES IN TOPOLOGY

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ABSTRACT

In 1990, Malghan et al. have defined and studied the concepts of almost p-regular, p-completely regular and almost p-completely regular spaces. In 1997 & 2004, Malghan et al. have defined and studied the concepts of almost s-completely regular spaces and s-completely regular spaces. In 2010, Navalagi introduced the concepts of pre-zero sets and co-pre-zero sets to characterize the concepts of p-completely regular spaces and almost p-completely regular spaces. In this paper, we offer some new concepts of α-zero sets, co-α-zero sets, α-completely regular spaces and almost α-completely regular spaces. We also characterize their basic properties via α-zero sets.

INTRODUCTION

In the literature zero sets and co-zero sets due to Gilman and Jerison.[9] were used to characterize the concepts like completely regular spaces and almost completely regular spaces by Singal, Arya and Mathur in topology (See 21 & 22). In,[11] and,[12] Malghan et al have defined and studied the concepts of semi-zero sets and co-semi-zero sets in topology to characterize the properties of s-completely regular spaces and almost s-completely regular spaces using semicontinuous functions due to N.Levine,[10] In,[17] Navalagi has defined and studied the concepts pre-zero sets and co-pre zero sets in topology by using precontinuous functions due to Mashhour et al,[14] to characterize the properties of p-completely regular spaces and almost p-completely regular spaces due to Malghan et al,[13] In this paper, present author define and study the α-zero sets and co-α-zero sets using α-continuous functions due to Mashhour et al,[15] to characterize the properties of newly introduced spaces, α-completely regular spaces and almost α-completely regular spaces.

Preliminaries

Throughout this paper, we let (X, τ) and (Y,σ) be topological spaces (or simply X and Y be spaces) on which no separation axioms are assumed unless explicitly stated.Let A be a subset of a space X. Let Cl(A) and Int(A) denote the closure and the interior of subset A.

We need the following definition and results in the sequel of the paper.

DEFINITION 2.1: A subset A of a space X is said to be:

(i) Preopen[14] if $A \subset$ Int Cl(A).

(ii) Semiopen[10] if $A \subset$ Cl Int(A).

(iii)Regular open[23] if A = Int Cl(A).

(iv)α-open[18] if $A \subset$ Int Cl Int(A).

(v) δ-open set[24] if for each $x \in A$,there exists a regular open set G such that $x \in G \subset A$.

The complement of a preopen (resp. semiopen, regular open, α-open, δ-open) set of a space X is called preclosed,[6] (resp. semiclosed,[2] regular closed,[23] α-closed,[15] δ-closed,[24]) set. The family of all preopen (resp. semiopen, regular open, α-open and δ-open) sets of X is denoted by PO(X) (resp. SO(X), RO(X), αO(X) and δO(X)) and that of preclosed (resp. semiclosed, regular closed, α-closed, β-closed and δ-closed) sets of X is denoted by PF(X) (resp. SF(X), RF(X), αF(X) and δF(X))

DEFINITION 2.2: A function f X $\rightarrow$Y is called

1. Precontinuous[14] if the inverse image of each open set U of Y is preopen set in X.
2. Semicontinuous[10] if the inverse image of each open set U of Y is semiopen set in X.
3. α-Continuous[15] if the inverse image of each open set U of Y is α-open set in X.

DEFINITION 2.3: A space X is said to be

(i) P-regular[6] if for each closed set F and each point x not in F, there exist disjoint preopen sets U and V such that $x \in U$ and $F \subset V$.

(ii) P-completely regular[13] (resp. s-completely regular[11]) if for each closed set F and each point $x \in (X \setminus F)$, there exists a precontinuous (resp.a semicontinuous) function f : X$\rightarrow$ [0,1] such that f(x) = 0 and f(y) = 1 for each $y \in F$.

(iii) almost p-completely regular[13] (resp. almost s-completely regular[12]) if for each regular closed set F and each point $x \in (X \setminus F)$, there exists a precontinuous(resp. a semicontinuous) function $f : X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

(iv) Submaximal[3] if every dense subset of it is open (i.e. if $PO(X) = \tau$[8]).

(v) An extremally disconnected (E.D.)[25] if closure of each open set is open in it (i.e. if $A \in \tau$ for each $A \in RF(X)$.

(vi) α-space[7] if every α-open set of X is open in X(i.e. $\tau = \alpha O(X)$).

(vii) α-regular[4] if for every closed set F and a point $x \notin F$,there exist disjoint α-open sets A and B such that $x \in A$ and $F \subset B$.

It is well-known that a subset A of a space X is called a zero set[9] if there exists a continuous functions $f: X \rightarrow \mathbf{R}$ such that $A = \{x \in X \mid f(x) = 0\}$. The complement of a zero set of a space X is called a co-zero set of X.

REMARK 2.4: If $f: X \rightarrow \mathbf{R}$ is continuous function may be denoted by Z(f). Thus, we write $Z(f) = \{x \in X \mid f(x) = 0\}$. Thus, Z(f) is a zero set of X. Therefore, it is clear that if A is a zero set in X then it can be expressed as $A = Z(f)$, where f is continuous function.

DEFINITION 2.5: A subset A of a space X is said to be semi-zero set[11] of X if there exists a semicontinuous function $f : X \rightarrow \mathbf{R}$ such that $A = \{x \in X \mid f(x) = 0\}$

DEFINITION 2.6: A subset A of a space X is said to be co-semizero set[11] of X if its complement is a semi-zero set.

REMARK 2.7: If $f: X \rightarrow \mathbf{R}$ is semicontinuous function may be denoted by SZ(f). Thus, we write $SZ(f) = \{x \in X \mid f(x) = 0\}$. Thus, SZ(f) is a semi-zero set of X. Therefore, it is clear that if A is a semi-zero set in X then it can be expressed as $A = SZ(f)$, where f is semicontinuous function.

DEFINITION 2.8: A subset A of a space X is said to be pre-zero set[17] of X if there exists a precontinuous function $f : X \rightarrow \mathbf{R}$ such that $A = \{ x \in X \mid f(x) = 0\}$.

DEFINITION 2.9: A subset A of a space X is said to be co-prezero set [17] of X if its complement is a pre-zero set.

REMARK 2.10: If f: X→**R** is precontinuous function may be denoted by PZ(f). Thus, we write PZ(f) = {x∈ X | f(x) = 0}. Thus, PZ(f) is a pre-zero set of X. Therefore, it is clear that if A is a pre-zero set in X then it can be expressed as A = PZ(f), where f is precontinuous function.

RESULT 2.14[15]: If A is preopen set in X and B is an α-open set in X, then A∩B is an α-open set in the subspace $(A,\tau_{|A})$.

RESULT 2.15[20]: If A is semiopen set and B is an α-open set in X, then A∩B is an α-open set in the subspace $(A,\tau_{|A})$.

1. α-ZERO SETS

We define the following.

DEFINITION 3.1: A subset A of a space X is said to be α-zero set of X, if there exists a α-continuous function f: X → **R** such that A = { x ∈ X | f(x) = 0}.

A subset A of a space X is said to be co-α-zero set of X if its complement is α-zero set.

NOTE 3.2: Every zero set in X is a α-zero set in X.

REMARK 3.3: Let X be a space. If f: X → **R** is a α-continuous function then the set {x ∈ X | f(x) = 0} is a α-zero set. If g: X → **R** is also a α-continuous function then {x ∈ X | g(x) = 0} is also a α-zero set of X.

REMARK 3.4: If f: X→**R** is α-continuous function may be denoted by αZ(f). Thus, we write αZ(f) = {x∈ X | f(x) = 0 }. Thus, αZ(f) is a α-zero set of X. Therefore, it is clear that if A is a α-zero set in X then it can be expressed as A = αZ(f), where f is α-continuous function.

In view of Remark- 2.4,2.7,2.10 and 3.4, we have : zero set →α-zero set→semi-zero set & Zero set →α-zero set → pre-zero set, since continuity →α-continuity →semi-continuity & continuity → α-continuity →pre-continuity.

LEMMA 3.5: If X is a α- space then a function f: X→ Y is α-continuous then the inverse image of each member of a basis for Y is α-open set in X.

LEMMA 3.6: Let X be a α- space. A function f: X → **R** is precontinuous iff for each b∈**R** both the sets $f^{-1}(b, \infty)$ and $f^{-1}(-\infty, b)$ are α-open sets.

LEMMA 3.7: Let X be an α- space then the following are equivalent:

(i) F: X→ **R** is α-continuous.

(ii) For each b ∈ **R**, $f^{-1}(-\infty, b)$ and $(-f)^{-1}(-\infty,-b)$ are α-open sets in X.

(iii) For each b ∈ **R**, $f^{-1}(b,\infty)$ and $(-f)^{-1}(-b, \infty)$ are α-open sets in X.

PROOF: Since (b, ∞) and (-∞, b) are subbasic open sets for the usual topology on R, thus the proof follows from Lemma – 3.6 above.

We need the following.

LEMMA 3.8: Let X be an α- space Let f, g: X→ **R** are α-continuous then,

(i) $|f|^{\alpha}$ is α-continuous for each α ≥ 0.

(ii) (af + bg) is α-continuous for each pair of reals a and b.

(iii) f. g is α-continuous.

(iv) 1/ f is α-continuous whenever f ≠ 0 on X.

These results can be proved by using the proofs of Lemmas: 2.5, 2.6 and 2.7. See [5, p.84].

LEMMA 3.9: If X is an α- space and if $\{ f_i : X \to \mathbf{R} \}_{i=1}^{k}$ is a finite family of α-continuous functions, then the functions M, m: X→ **R** defined by M(x) = Max $\{f_i(x)\}^{k}_{i=1}$ and m(x) = Min $\{f_i(x)\}_{i=1}^{k}$ are also α-continuous.

Proof is straight forward and hence omitted.

LEMMA 3.10: In an α-space X, the following statements hold for real valued functions:

(i) If A is a α-zero set in X then there exists a α-continuous function g : X→ **R** such that g(x) ≥ 0 for each x ∈ X and A = αZ(g).

(ii) If A is a α-zero set in X then there is a α-continuous function h : X→ [0,1] such A = αZ(h).

(iii) Finite union of α-zero sets in X is a α-zero set in X.

(iv) Finite intersection of α-zero sets in a α-zero set in X.

(v) If a ∈ R and f :X→ **R** is a α-continuous function, then the sets A={x∈X | f(x) ≥ a} and B = { x ∈ X | f(x) ≤ a} are α-zero sets in X.

(vi) If a ∈ R and f :X→ **R** is a α-continuous function then the sets A = {x ∈ X | f(x) < a} and B = x ∈ X | f(x) > a} are co-α-zero sets in X.

These results can be proved by using Lemma- 2.8 and 2.9. See [19, p. 18]. Next, we give the following.

THEOREM 3.11: If A and B are disjoint α-zero sets of an α- space X, there exist disjoint co-α-zero sets U and V such that A ⊂ U and B ⊂ V. We, prove the following.

THEOREM 3.12: In an α-space X every α-zero (resp. co-α-zero) set is α-closed (resp. α-open) set.

PROOF: If A is α-zero set in X then by Lemma -3.10, we have A = αZ(g), where g : X → **R** is α-continuous and g(x) ≥ 0 for all x ∈ X. Then, g(x) = 0 for all x ∈ A. Hence, $g^{-1}(\{0\}) = A$. Since {0} is closed in **R** and g is α-continuous, it follows that A is α-closed set in X. The second part is proved similarly.

2. α-COMPLETE REGULARITY AND α-ZERO SETS

We, need the following.

DEFINITION 4.1 [16]: Let A be a subset of a space X. Then a subset V of a space X is said to be a α-neighbourhood of A if there exist a α-open set U of X such that A ⊂ U ⊂ V.

If A = {x} for some x ∈ X then V in the above definition is the α-neighbourhood of the point x.

DEFINITION 4.2: A space X is said to be α-completely regular if for each closed set F and each point x ∈ (X \ F), there exists a α-continuous function f : X→ [0,1] such that f(x) = 0 and f(y) = 1 for each y∈ F.

Clearly, every completely regular space is α-completely regular, every α-completely regular space is s-completely regular space as well as p-completely regular space and every α-completely regular space is α-regular space.

Next, we prove the following.

THEOREM 4.3: Every preopen subspace of an α-completely regular space is α-completely regular.

PROOF : Let X be an α-completely regular space and Y be an preopen subspace of X.Let F be a closed set in Y and $x \in Y$ such that $x \notin F$.Hence, $x \notin Cl_X(F)$.Since X is α-completely regular, there exists a α-continuous function $f:X \rightarrow [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in Cl_X(F)$. Since the restriction of a α-continuous function to a preopen subspace is α-continuous in view of Result 2.14 and by Th.1.3 in [15], it follows that $f/Y : Y \rightarrow [0,1]$ is α-continuous such that $(f/Y)(x) = 0$ and $(f/Y)(y) = 1$ for each $y \in F$. Hence Y is α-completely regular.

On similar lines of Th.4.2 above and Result-2.15 [20], one can prove the following.

THEOREM 4.4: Every semiopen subspace of an α-completely regular space is α-completely regular.

THEOREM 4.5: Every neighbourhood of a point in an α-space α-completely regular space X contains a α-zero set α-neighbourhood of the point.

PROOF: Let x_o be a point of an α-space α-completely regular space X and N be a neighbourhood of x_o. Then there exists a α-continuous function $f : X \rightarrow [0,1]$ such that $f(x_o) = 0$ and $f(x) = 1$ for each $x \in X \backslash N$. Then, $V = \{ x \in X \mid f(x) \geq ½\}$, then V is a α-zero set α-neighbourhood of x_o such that $V \subset N$, as $x_o \in \{ x \in X \mid f(x) < ½\}$ is α-open by above Lemma- 3.10 above.

Now, we need the following.

DEFINITION 4.6 [1]: A family σ of subsets of a space X is a net for X if each open set is the union of a family of elements of σ.

Now, we give the following.

THEOREM 4.7: For an α-space X, the following statement are equivalent

1. X is α-completely regular space.
2. Every closed set A of X is the intersection of α-zero sets which are α- neighbourhoods of A.
3. The family of all co-α-zero sets of X is a net for the space X.

PROOF. (i)⇒(ii) : Let A be a closed set in X and $x \notin A$. Then from (i), there is a α-continuous function $f_x : X \to [0,1]$ such that $f_x(x) = 0$ and $f_x(A) = \{1\}$. Let $G = \{y \in X \mid f_x(y) \geq 1/3\}$ and $H_x = \{y \in X \mid f_x(y) < 1/3\}$. Then, $A \subset H_x \subset G_x$, where H_x is α-open and G_x is α-zero set which is α-neighbourhood of A. Further, $A = \bigcap_{x \notin A} G_x$.

(ii)⇒ (iii): Let G be an open set of X. Then, X \ G is closed set in X. Let $X \setminus G = \bigcap\{B_\lambda \mid \lambda \in \wedge\}$, where B_λ is α-zero set α-neighbourhood of X \ G, for each $\lambda \in \wedge$. Hence, $G = \cup\{X \setminus B_\lambda \mid \lambda \in \wedge\}$, where $X \setminus B_\lambda$ is a co-α-zero for each $\lambda \in \wedge$. Hence, (iii) holds.

(iii) ⇒ (i) : Let A be a closed set and $x_o \in X \setminus A$. Then, from (iii), as X \ A is open there is a co-α-zero set U such that $x_o \in U \subset X \setminus A$. Let $U = X \setminus \alpha Z(g)$, for some α-continuous function $g : X \to [0,1]$. As $x_o \notin \alpha Z(g)$, $| g(x) | = r > 0$. If we define, $f : X \to [0,1]$ by $f(x) = \text{Max}\{ 0, 1-r^{-1}|g(x)|\}$ for some $x \in X$, then f is α-continuous by Lemma -3.9 and 3.10 above and $f(x_o) = 0$ and $f(x) = 1$ for $x \in A$. Hence, X is α-completely regular space.

3. ALMOST α-COMPLETE REGULARITY AND α-ZERO SETS

In this section, we characterize the almost p-completely regular spaces using the concepts of pre-zero sets and co-prezero sets in the following.

We define the following.

DEFINITION 5.1: A space X is said to be almost α-completely regular if for each regular closed set F and each point $x \in (X \setminus F)$, there exists a α-continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for each $y \in F$.

Obviously every almost completely regular space is almost α-completely regular and every α-completely regular space is almost α-completely regular.

We, prove the following.

THEOREM 5.2: A space X is almost α-completely regular iff for each δ-closed set F and a point $x \in (X \setminus F)$, there is a α-continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(F)=\{1\}$.

PROOF: Let X be almost α-completely regular space and let A be a δ-closed set not containing a point x.Then there exists an open set G containing x such that Int $Cl(G) \cap A = \varnothing$.Now, (X-Int Cl(G)) is a regular closed set not containing x. Since X is almost α-completely regular, there exists a α-continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(X\text{-Int } Cl(G)) = \{1\}$.Since $A \subset (X\text{-Int } Cl(G))$, it follows that $f(A) = \{1\}$.

Converse follows immediately since every regular closed set is δ-closed.

THEOREM 5.3: For an α- space X the following are equivalent:

(i) X is almost α-completely regular space.

(ii) Every δ-closed subset A of X is expressible as the intersection of some α-zero sets which are α-neighbourhood of A.

(iii)Every δ-closed subset A of X is identical with the intersection of all α-zero sets which are α-neighbourhoods of A.

(iv)Every δ-open subset of X containing a point contains a co-α-zero set containing that point.

PROOF. (i) ⇒(ii) : Let X be an almost α-completely regular space. Let A be a δ-closed set and $x \notin A$. Then there exists a α-continuous function f_x on X into [0,1] such that $f_x(x)= 0$ and $f_x(A) = \{1\}$ by Theorem -2.6. Let $G_x = \{y \in X \mid f_x(y) \geq 2/3\}$ for every $x \notin A$; G_x is α-neighbour-hood of A. Lastly, $A = \cap_{x \notin A} G_x$: We have $A \subset G_x$, for each $x \notin A$, which implies that $A \subset \cap_{x \notin A} G_x$. Further, we claim that $\cap_{x \notin A} G_x \subset A$: Let $z \notin A$. This implies that there is a α-continuous function $f_z : X \to [0,1]$ such that $f_z(z) = 0$ and $f_z(A) = \{1\}$. Also, $G_z = \{y \in X \mid f_z(y) \geq 2/3\}$.Now, $f_z(z) = 0 < 2/3$. Therefore, $z \notin G_z$. This implies that $z \notin \cap_{x \notin A} G_x$. Therefore,

$z \notin A \Rightarrow z \notin \cap_{x \notin A} G_x$. Therefore, $\cap_{x \notin A} G_x \subset A$. Hence, $A = \cap_{x \notin A} G_x$. Therefore, (i) $\Rightarrow$ (ii) is true.

(ii) ⇒ (iii): Let us suppose that (ii) holds. Let $A = \cap \{G_\lambda \mid \lambda \in \wedge\}$, where G_λ is a α-zero set which is α-neighbourhood of A for each $\lambda \in \wedge$. Let ρ be the family of all α-zero sets which are α-neighbourhoods of A. Therefore, $\{G_\lambda \mid \lambda \in \wedge\} \subset \rho$. Therefore, $\cap_{B \in \rho} B \subset \cap_{\lambda \in \wedge} G_\lambda \Rightarrow \cap_{B \in \rho} B \subset A$. Next, we prove that $A \subset \cap_{B \in \rho} B$: Now, B is a α-zero set which is α-neighbourhood of A for each $B \in \rho$ which implies that $A \subset \cap_{B \in \rho} B$. Therefore, $A = \cap_{B \in \rho} B$. Thus, (iii) holds.

(iii) ⇒ (iv): Suppose (iii) holds.Let G be a δ-open set and $x \in G$. Then, $X \setminus G$ is δ-closed set and $x \notin X \setminus G$. This implies that $X \setminus G = \cap_{\lambda \in \wedge} B_\lambda$ where $\{B_\lambda \mid \lambda \in \wedge\}$ is family of all α-zero sets which are α-neighbourhoods of X\G. Now, $x \notin X \setminus G \Rightarrow x \notin B_{\lambda o}$ for some $\lambda_o \in \wedge$, which implies that $x \in X \setminus B_{\lambda o}$. Also, we have $\mathbf{X \setminus G} = \cap_{\lambda \in \wedge} B_\lambda \Rightarrow G = X \setminus \cap_{\lambda \in \wedge} B_\lambda = \cup_{\lambda \in \wedge}(X \setminus B_\lambda)$. Therefore, $(X \setminus B_{\lambda o}) \subset \cap_{\lambda \in \wedge} (X \setminus B_\lambda) = G$. Therefore, $x \in X \setminus B_{\lambda o} \subset G$. Since $B_{\lambda o}$ is α-zero set, $X \setminus B_{\lambda o}$ is a co-α-zero set. Therefore, (iv) holds.

(iv) ⇒ (i): Suppose (iv) holds. Now, to prove that X is almost α-completely regular space : Let A be a δ-closed set and $x_o \notin A$. Then $X \setminus A$ is a δ-open set containing x_o. Then by (iv), there exists a co-α-zero set U such that $x_o \in U \subset X \setminus A$. Thus, $X \setminus U$ is a α-zero set. Therefore, there exists a α-continuous function $f : X \rightarrow [0,1]$ such that $X \setminus U = \alpha Z(f)$. Hence, $X \setminus U = \alpha Z(f) = \{ x \in X \mid f(x) = 0\}$. As $x_o \in U$, it follows that $f(x_o) \neq 0$. Hence, $| f(x_o) | = r > 0$. Now, we define $g : X \rightarrow [0,1]$ by $g(y) = \text{Min}\{ 1, \frac{1}{r}|f(y)|\}$, for each $y \in X$. Then g is α-continuous function. Also, $g(x_o) = 1$ and $g(z) = 0$, for each $z \in A$. Let $h = 1/g$. As X is an α-space, by Lemma- 3.8, $h: X \rightarrow [0,1]$ is α-continuous such that $h(x_o) = 0$ and $h(a) = \{1\}$.Hence, X is almost α-completely regular. Hence the theorem.

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