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RESEARCH ON PT SYMMETRY AND SYSTEM ENERGY IN INDUCTIVELY COUPLED CIRCUITS

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ABSTRACT

The parity-time (PT) symmetric system based on inductively coupled circuits is widely used in energy transfer and passive sensing. When considering the impedance of the coupled inductor coils, the stability analysis of the PT symmetric property of the coupled circuit cannot be analytically derived from the theory of PT symmetric stability based on ideal inductors. Therefore, a method for analyzing the stability of PT symmetry based on the energy characteristics of the system is proposed. This method can effectively analyze the PT symmetric stability of ideal coupled inductors and determine the influence of the impedance of coupled inductors on the PT symmetric stability of the coupled circuit,as well as determine the conditions for re-establishing stable PT symmetry after the PT symmetric instability of the circuit system. The research results have practical guidance significance for the implementation of energy transfer systems.

KEYWORDS: Coupled inductance impedance; parity-time symmetry; system energy; inductive coupling.

1. INTRODUCTION

Since the concept of parity-time (PT) symmetry was first proposed in quantum mechanics by physicists Bender and Boettcher[1], research on the properties of PT symmetry has attracted widespread interest in many fields. In quantum systems, various aspects of PT symmetric systems have been extensively studied.[2,3] Special dynamic properties such as PT symmetric birefringence[4] and unidirectional invisibility[5,6] have attracted the attention of scholars in photonics[7,8], optomechanics[9,10], and acoustics.[11,12] PT symmetric systems also exhibit power oscillation dynamics[13-15]. Therefore, in the field of electronics[16,17], PT symmetry is mainly applied to wireless energy transfer[18] and LC passive wireless sensing.[19] Schindler et al.[20] proposed a generalized PT symmetric circuit system, based on two ideal mutually coupled LRC oscillating circuits, to study phase transitions and temporal behaviors of active binary elements, marking the first analysis of PT symmetry phenomenon from a circuit perspective; Kananian et al.[19] achieved fully passive LC resistor-type sensors and coupling-independent robust wireless sensing using a similar parallel circuit structure. Some scholars have also proposed ideal LRC resonator circuits based on asymmetric PT (APT) symmetry[21], observing the dynamics of APT symmetry-breaking transitions and the conservation of system energy differences. The PT symmetry theory was first applied to wireless power transmission technology by a Stanford University team.[18] The introduction of PT symmetry brings new ideas and methods to electronics.

To better analyze the PT symmetry in coupled circuit systems, ideal inductive coupling situations are often considered. However, in practical circuit applications, the resistance of the coupled inductors often needs to be considered. It is not yet clear whether these existing small resistances affect the PT symmetry of the coupled circuit system and to what extent. Because it has a certain impact on the application of wireless sensors and wireless energy transfer, it is necessary to explore the mechanism of the influence of small impedances in coupled inductors on the PT symmetry of the coupled circuit. Considering the inductively coupled circuit system, the impedance of the conductors in the inductor coils is taken into account, and its influence on the PT symmetric stability of the coupled circuit system is analyzed, and the conditions for establishing stable PT symmetry of the circuit are established. When the impedance of the conductors in the coupled inductors is not considered, the PT symmetric stability of the inductively coupled identical circuits can be analyzed through circuit theory, and the critical condition parameters for PT symmetry instability can be analytically obtained. However, when considering the impedance of the conductors in the coupled inductors, the parameter interval of the PT symmetric stability analyzed based on circuit theory cannot be analytically determined, and thus, the critical condition for PT symmetry instability cannot be analytically determined. By proposing a method for analyzing the stability of PT symmetry based on the energy characteristics of the system, the influence of the impedance value of the inductance on the PT symmetric stability of the coupled circuit system can be determined, and the parameter interval for ensuring PT symmetric stability can be provided. The results show that this method can not only be applied to the judgment of the PT symmetric stability parameter area of the previous ideal inductive coupled circuit but also can be effectively applied to the judgment of the PT symmetric stability area in the coupled inductive coupled circuit with impedance. In the coupled inductor, after considering the impedance of the conductors in the coil, the PT symmetric stable area of the coupled identical circuit system disappears. The stable PT symmetry can be re-established by introducing resistance parameter mismatch in two coupled circuits, and the newly established PT symmetric stable region is closely related to the parameters of the coupled circuit. The impedance of the coupled inductors will shrink the stable parameter region of PT symmetry in the coupled circuit.

2. Resonant Circuit Model

2.1 Circuit Model

Based on the coupled LRC oscillating circuit proposed by Schindler et al.[20], a model of an inductively coupled resonant circuit is established, as shown in Figure 1.

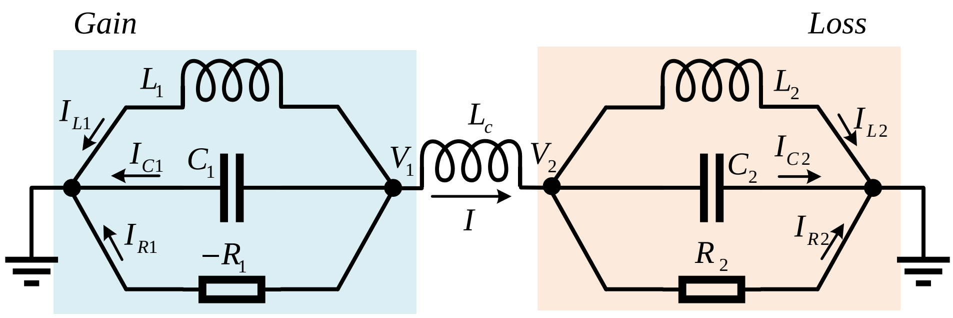

Fig. 1: Inductively coupled resonant circuit.

The coupled resonant circuit consists of two LRC resonators coupled through inductors, where the left side represents the gain end and the right side represents the loss end. The resistance component on the gain end is a negative resistance $-R_1$, composed of an active operational amplifier and a resistor, which can provide a gain mechanism. The loss end is connected to a positive resistance R_2, exhibiting linear loss properties. The two resonators are coupled through an inductor L_c, establishing coupling effects, and the energy of the system is transferred from the gain end to the loss end through the coupling inductor. This structure is a simple circuit implementation with PT symmetry properties.

2.2 Theoretical Analysis

To analyze the various standard modes of the coupled resonant circuit with PT symmetry, coupling mode equations are written separately for the gain side and the loss side based on Kirchhoff's laws of circuit theory

$$\begin{cases} \dfrac{V_1}{i\omega L_1}-\dfrac{V_1}{R_1}+i\omega C_1V_1+\dfrac{V_1-V_2}{i\omega L_c+R_L}=0 \\ \dfrac{V_2}{i\omega L_2}+\dfrac{V_2}{R_2}+i\omega C_2V_2-\dfrac{V_1-V_2}{i\omega L_c+R_L}=0 \end{cases} \quad (1)$$

where $n=1,2$ represents the sub-resonator number, L_n, C_n and R_n are the inductance, capacitance and resistance of resonator n, respectively, L_c is the coupling inductor, R_L represents the resistance of the inductor coil in the coupling inductor L_c, V_n represents the voltage across the capacitor, and ω is the system angular frequency. The equilibrium equation of the system is obtained from Equation (1)

$$\begin{pmatrix} \dfrac{1}{i\omega L_1}-\dfrac{1}{R_1}+i\omega C_1+\dfrac{1}{i\omega L_c+R_L} & -\dfrac{1}{i\omega L_c+R_L} \\ -\dfrac{1}{i\omega L_c+R_L} & \dfrac{1}{i\omega L_2}+\dfrac{1}{R_2}+i\omega C_2+\dfrac{1}{i\omega L_c+R_L} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=0 \quad (2)$$

Setting the determinant of the admittance matrix in Equation (2) to zero and solving for the complex frequency ω directly yields the exact eigenfrequencies. If $s=i\omega$ is taken as a complex frequency, this method is equivalent to Laplace domain analysis of the circuit. For simplicity, let $C_1=C_2=C$, $L_1=L_2=L$, then the natural frequency of the system is $\omega_n=1/\sqrt{L_nC_n}$, $\omega_0=\omega_1=\omega_2=1/\sqrt{LC}$, where $\gamma_1=\sqrt{L/C}/R_1$ is the system gain coefficient and $\gamma_2=\sqrt{L/C}/R_2$ is the system loss coefficient. The quality factor is $Q=R_L/L_c$, the coupling coefficient is $k=L/L_c$, and Equation (2) can be rewritten as

$$\begin{pmatrix} \dfrac{\omega_0}{\omega}-\dfrac{\omega}{\omega_0}+\dfrac{k\omega\omega_0}{\omega^2+Q^2}+i\dfrac{k\omega_0Q}{\omega^2+Q^2}-i\gamma_1 & -\dfrac{k\omega\omega_0}{\omega^2+Q^2}-i\dfrac{k\omega_0Q}{\omega^2+Q^2} \\ -\dfrac{k\omega\omega_0}{\omega^2+Q^2}-i\dfrac{k\omega_0Q}{\omega^2+Q^2} & \dfrac{\omega_0}{\omega}-\dfrac{\omega}{\omega_0}+\dfrac{k\omega\omega_0}{\omega^2+Q^2}+i\dfrac{k\omega_0Q}{\omega^2+Q^2}+i\gamma_2 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}=0 \quad (3)$$

Equation (3) only has non-trivial solutions when its coefficient determinant is zero, from which the working frequency of the system needs to satisfy

$$\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2+2\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)\frac{k\omega\omega_0}{\omega^2+Q^2}+(\gamma_1-\gamma_2)\frac{k\omega_0 Q}{\omega^2+Q^2}+\gamma_2\gamma_1$$
$$+i\left((\gamma_2-\gamma_1)\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}+\frac{k\omega\omega_0}{\omega^2+Q^2}\right)+\frac{2k\omega_0 Q}{\omega^2+Q^2}\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)\right)=0 \quad (4)$$

When the impedance of the coil in the coupling inductor is not considered, Equation (4) can be simplified to

$$\left(\frac{\omega_0^{\,2}}{\omega^2}+\frac{\omega^2}{\omega_0^{\,2}}+\frac{2k\omega_0^{\,2}}{\omega^2}-2k-2+\gamma_1\gamma_2\right)+i(\gamma_2-\gamma_1)\left(\frac{\omega_0(1+k)}{\omega}-\frac{\omega}{\omega_0}\right)=0 \quad (5)$$

The real and imaginary parts of Equation (5) are simultaneously set to zero, leading to the solution for angular frequency ω. When $\gamma_1=\gamma_2=\gamma$, the system's angular frequency has four solutions

$$\omega_{1,2}=\frac{1}{2}\omega_0\sqrt{4+4k-2\gamma^2\pm2\sqrt{\gamma^4-(4k+4)\gamma^2+4k^2}}$$
$$\omega_{3,4}=-\frac{1}{2}\omega_0\sqrt{4+4k-2\gamma^2\pm2\sqrt{\gamma^4-(4k+4)\gamma^2+4k^2}} \quad (6)$$

From Equation (6), the condition for the system to be in the PT symmetric stable region is that the angular frequency ω is real, thereby obtaining the critical point γ_{PT} between the PT symmetric stable region and the unstable region.

$$\gamma_{PT}=\sqrt{2k+2-2\sqrt{1+2k}} \quad (7)$$

When $\gamma<\gamma_{PT}$, the system has two distinct positive real eigenfrequencies , indicating that the system is in the PT symmetric stable region, and the eigenfrequencies will vary with the coupling coefficient k and the gain and loss coefficients . When $\gamma=\gamma_{PT}$, the system is at the critical point between the PT symmetric stable region and the PT symmetric broken region, and the system frequency transitions from real to complex, similarly, the spontaneous transition from PT symmetric state to PT symmetric broken state can be observed. When $\gamma>\gamma_{PT}$, the system frequency is complex conjugate, indicating that the system is in the PT symmetric broken region, generating a mode with exponential decay and a mode with exponential growth, both with the same oscillation frequency.

When considering the impedance R_L of the coil in the coupling inductor, by solving for the angular frequency in Equation (4), it is found that the coupled identical circuit ($\gamma_1 = \gamma_2 \neq 0$) does not have real solutions, indicating that the system is necessarily in the PT symmetric broken state. Therefore, the PT symmetry of the original ideal inductively coupled identical resonant circuit system is destroyed by the impedance of the coil in the coupling inductor, causing instability in PT symmetry.

From Equation (4), it can be seen that when considering the impedance of the coil in the coupling inductor, in order to establish stable PT symmetry in the coupling system, it is necessary to ensure that the gain coefficient and the loss system are not equal, namely $\gamma_1 \neq \gamma_2$. At this point, it is more difficult to analytically derive the expression of the angular frequency solution in Equation (4). Therefore, in order to determine the conditions for establishing stable PT symmetry in the coupled non-identical resonant system when considering the impedance of the coil in the coupling inductor, it is proposed to judge the PT symmetry stability based on the change characteristics of the system energy.

2.3 System Energy Characteristics

To better demonstrate the feasibility of judging the PT symmetric state based on the energy characteristics of the system, firstly, the determination of the PT symmetric stable region by analyzing the energy characteristics of the coupled system without considering the impedance of the coil in the coupling inductor is analyzed, and then compared with the theoretical analysis results. According to Kirchhoff's laws, the relationship between the voltage and current of the inductively coupled resonant circuit is as follows:

$$\begin{aligned} & V_n = L_n \frac{dI_{Ln}}{dt} = \frac{1}{C_n}\int_0^t I_{Cn}(\tau)d\tau = R_n I_{Rn} \\ & V_1 - V_2 = L_c \frac{dI}{dt} + IR_L \qquad (8) \\ & I_{Ln} + I_{Rn} + I_{Cn} + (-1)^{n+1} I = 0 \end{aligned}$$

where I_{Ln}, I_{cn} and I_{Rn} are the currents passing through the inductor, capacitor and resistor of the resonator, respectively, and I is the current passing through the coupling inductor. Without loss of generality, take the system parameters $R_1 = R_2 = 5k\Omega$, $L_1 = L_2 = 5.32mH$ and $C_1 = C_2 = 10.7nF$. For different values of the coupling inductor and the inductance of the resonator, solve the differential equation (8) of the coupled system through numerical calculation, obtain the time sequence of the voltage across the capacitor and the current passing through the inductor , and calculate the energy of the capacitor $E_n = 0.5C_nV_n^2$, the magnetic energy of the inductor $M_n = 0.5L_nI_{Ln}^2$, and then obtain the change relationship of the total energy of the resonator $T_n = E_n + M_n$ with time . When the coupling inductor is $30mH$, according to the theoretical analysis results (Equation (7)), it is known to be exactly in the PT symmetric stable region. The evolution of the current passing through the inductor and the voltage across the capacitor of resonator 1 and resonator 2 with time obtained by numerical calculation is shown in Figure 2(a)(b). Figure 2(c)(d) shows the corresponding evolution relationship of the total energy, the electric energy, and the magnetic energy of resonator 1 and resonator 2. The total energy of the gain end 1 and the loss end 2 oscillates periodically, fluctuating within a certain range, and the fluctuation amplitude of the energy shows the same steady oscillation state, where the gain and loss of the two resonator systems are balanced. At this time, the modes of the system (the inherent vibration characteristics of the system) are completely symmetrical. The system energy well reflects the PT symmetry of the coupled oscillator system.

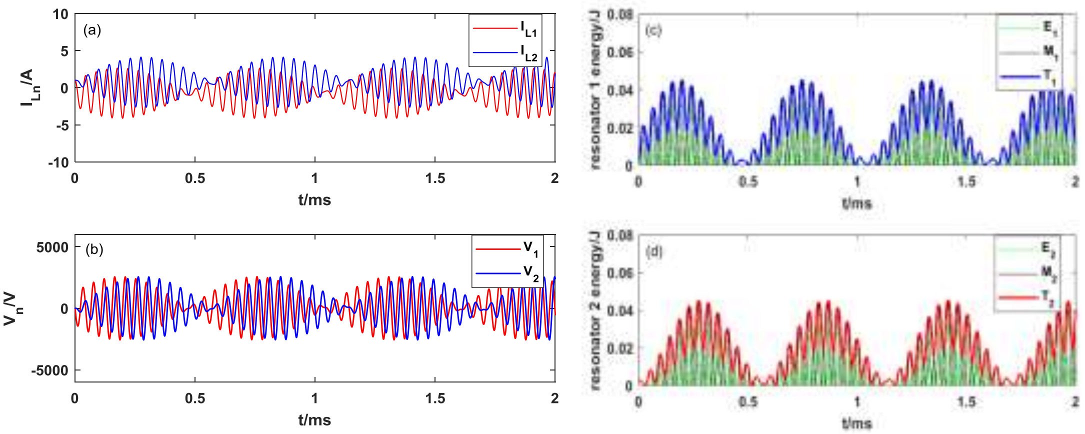

Fig. 2. Evolution of I_{Ln}、V_n、resonator 1 energy and resonator 2 energy when coupling inductance $L_c = 30mH$.

When the coupling inductor is $50mH$, according to Equation (7), the coupled system is in the PT symmetric broken region. The evolution of the current passing through the inductor and the voltage across the capacitor of resonator 1 and resonator 2 with time obtained by numerical calculation is shown in Figure 3(a)(b), and Figure 3(c)(d) displays the corresponding evolution relationship of the total energy, the electric energy, and the magnetic energy of resonator 1 and resonator 2.The energies at both the gain end 1 and the loss end 2 exhibit exponential growth, with differing growth rates. The difference in total energy between the two ends gradually increases, indicating an unequal balance between energy gain and loss in the system, corresponding to the energy characteristic of PT symmetry breaking.

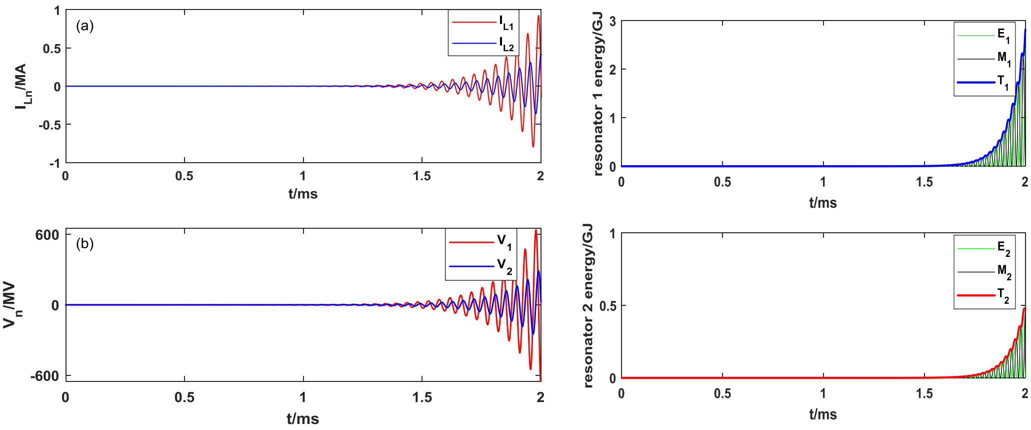

Fig. 3: Evolution of I_{Ln}、V_n、resonator 1 energy and resonator 2 energy with time when coupling inductance $L_c = 50mH$.

To further validate whether the energy characteristics can effectively determine the PT symmetry stability of the coupled oscillator system, in the case of ideal inductive coupling, the PT symmetry stable region and the broken region of the coupled identical resonant circuit in the parameter space $L_n \sim L_c$ are determined using the stability characteristics of the system energy, as shown in Figure 4. In this figure, the red region represents the PT symmetry broken region, while the blue region represents the PT symmetry stable region. The green solid line in Figure 4 represents the analytical critical line derived from the theoretical analysis for the PT symmetry stable and broken regions

$$L_c = \frac{-2L^2R^2C + 4R^3\sqrt{L^3C^3}}{4LR^2C - L^2} \qquad (9)$$

From the figure, it can be observed that the critical line for the PT symmetry stable region determined based on the energy-based method completely coincides with the theoretical analytical critical line (Equation (9)). This indicates the effectiveness of using energy-based criteria for determining PT symmetry stability.

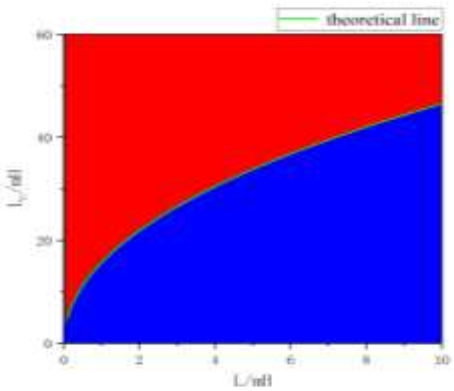

Fig. 4: If $R_1 = R_2 = 5k\Omega$、 $C_1 = C_2 = 10.7nF$, the different behaviors of the ideal inductively coupled PT symmetric system in the $L \sim L_c$ parameter space.

Next, the analysis method based on energy characteristics is employed to determine the conditions for establishing stable PT symmetry in the coupled system when considering different impedances in the coil resistance of the coupling inductors. It is essential to ensure $\gamma_1 \neq \gamma_2$. Typically, the resistance of the inductor coil is influenced by factors such as the material of the coil, the length and diameter of the wire, and the structure of the coil. The resistance varies within the range $0.1 \sim 20\Omega$. Therefore, when varying within this range, the system parameters $C_1 = C_2 = 10.7nF$, $L_1 = L_2 = 5.32mH$, $L_c = 30mH$ are taken, and the resistance parameters R_1, R_2 of the two resonator subsystems are adjusted accordingly. This ensures that for different values of the corresponding relationships between the resistance parameters of the two coupled circuits must satisfy: $R_2 = KR_1 + B$. As shown in Figure 5, with the increase of R_L, the slope remains constant at $K = 1$, while the intercept linearly B decreases.

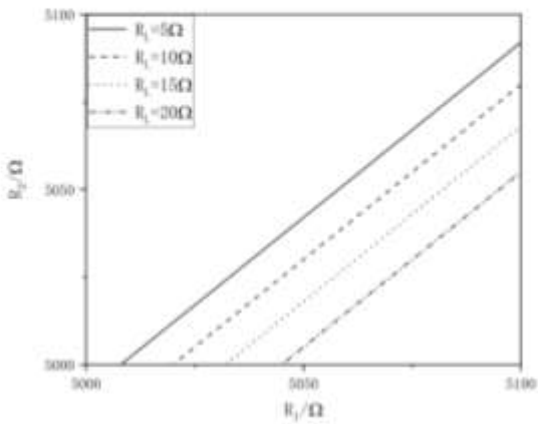

Fig. 5: Relationship between the resistor values R_1 and R_2 for re-establishing PT symmetry with different impedance values.

Similarly, using the energy characteristic analysis method, for a given value of the resistance parameters in the coil resistance of the coupling inductors $R_L = 5\Omega, 10\Omega, 15\Omega, 20\Omega$, and with the resistance parameters of the two resonator subsystems set to fixed unequal values such as $R_1 = 5020\Omega$ and $R_2 = 5000\Omega$. It is determined that in order to establish stable PT symmetry, the relationship between the coupling inductance value and the resonator inductance must satisfy: $L_c = aL^b$, where a are respectively 0.39579, 0.40134, 0.37958, 0.2655, and b are 0.4724, 0.48246, 0.49996, 0.501, as shown in Figure 6. Clearly, when there are resistance values in the coil resistance of the coupling inductors, for given circuit parameters, only the two coupled non-identical circuit systems that lie on the theoretical line satisfy strict PT symmetry.

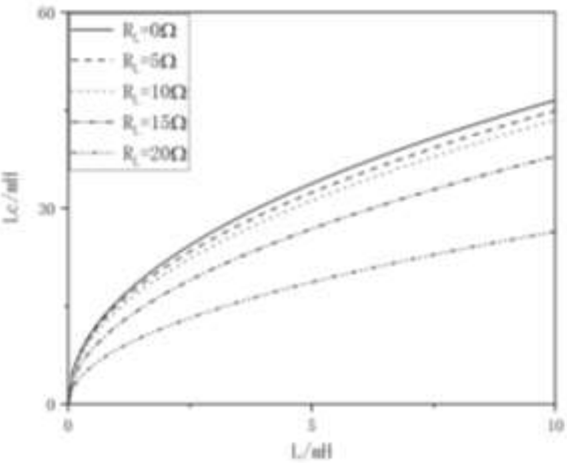

Fig. 6: Theoretical line of PT-symmetric stability for coupled inductance systems with impedance in the $L \sim L_c$ parameter space.

In practical circuit systems, due to factors such as temperature variations, the parameters in the coupling circuits cannot always strictly adhere to theoretical values. Therefore, maintaining strict and stable PT symmetry is often challenging. When PT symmetry is destabilized, the energy of the system oscillates and increases over time. In practical applications, it can be considered that situations where the energy growth rate is less than a certain value still approximately exhibit PT symmetric characteristics. To examine the sensitivity of circuit parameters to the energy growth rate, the average energy growth rate parameter for resonator subsystem 1 is defined as $\beta_1 = \Delta T_1 / \Delta t$, $\Delta T_1 = T_1(t_2) - T_1(t_1)$, where $T_1(t_1)$, $T_1(t_2)$ represent the energy of subsystem 1 at t_1, t_2 respective time instances, as shown in Figure 7.

When the system parameters lie on the theoretical line as illustrated in Figure 6, the coupled resonator subsystem exhibits stable PT symmetry, with the average energy growth rate of the subsystem being zero.

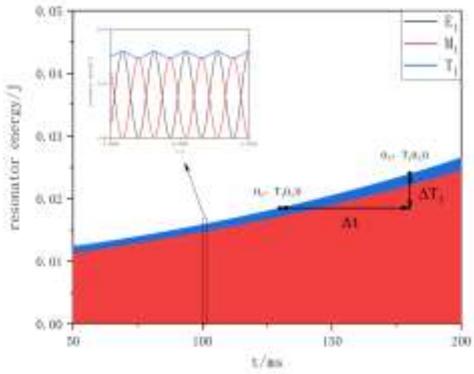

Fig. 7: Energy growth rate diagram.

When the absolute value of the energy growth rate changes, the region where the system remains in a stable PT symmetric state may also change. Figure 8 depicts a phase diagram of the distribution of PT symmetric stable and broken regions based on the system's energy response over time. In this diagram, the red region represents the PT symmetric broken region, while the blue region represents the PT symmetric stable region. When the average energy growth rate of the resonator subsystem is 1%, the range in the parameter space $L \sim L_c$ expands from the original theoretical line to the blue region shown in Figure 8(a). Furthermore, when the average energy growth rate of the resonator subsystem reaches 5%, the parameter space $L \sim L_c$ expands further, as depicted in Figure 8(b). Comparing with Figure 4, it can be observed that the PT symmetric stable regions have significantly shrunk, with most areas of the system being in a PT symmetric broken state. Furthermore, as the constraint on the absolute value of the energy growth rate increases, the range of the PT symmetric stable region also expands.

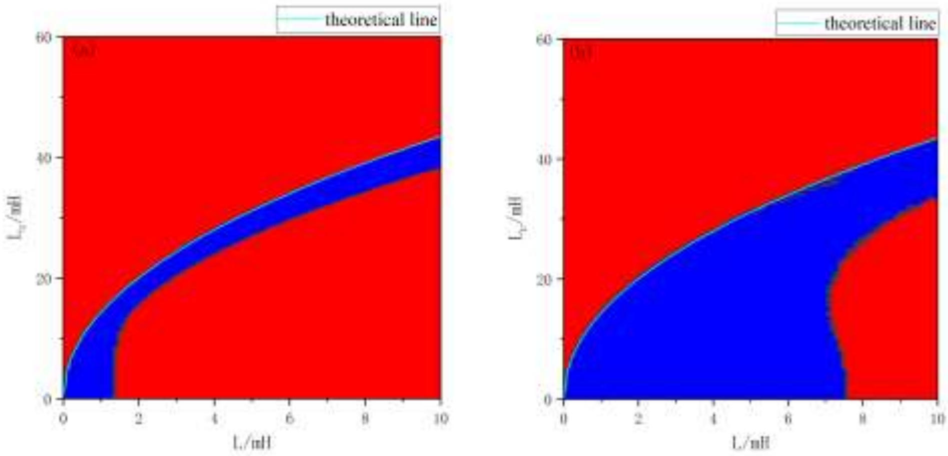

Fig. 8: If $R_1 = 5020\Omega$、 $R_2 = 5000\Omega$、 $C_1 = C_2 = 10.7nF$、 $R_L = 10\Omega$,different behaviors in coupled inductance PT symmetric systems with impedance in the $L \sim L_c$ parameter space.(a)the absolute value of the energy increase rate is less than 1% ; (b)the absolute value of the energy increase rate is less than 5%.

3. CONCLUSION

Through analyzing the variation of energy and PT symmetry in an ideal PT symmetric system under the influence of mutual inductance coupling, it was observed that when the system is in a PT symmetric stable state, the energy of the coupled system oscillates within a certain range, while in the PT symmetric broken state, the energy of the coupled system exhibits exponential growth or decay. Based on the energy characteristics of the coupled system, the PT symmetric stable region and broken region in the parameter space, as well as their critical lines, were determined. The critical lines determined based on the energy of the system are in accordance with the theoretically derived critical lines, validating the effectiveness of this method. Furthermore, based on the energy characteristics of the system, the PT symmetric stability of the coupled system considering the impedance of the coupling inductance coil was determined. The results indicate that the presence of impedance in the coupling inductance coil in practical applications will destabilize the PT symmetry of the coupled identical circuit system. When parameter mismatch is introduced into the two coupled circuit subunits, the coupled circuit system can reestablish PT symmetry under certain conditions. These findings deepen the understanding of PT symmetric circuit systems and provide important guidance for practical applications in energy transmission systems. To effectively improve the efficiency of energy transmission systems, it is important to select the parameters of the system components based on the actual resistance of the inductance coil, ensuring that the system remains in a PT symmetric state. This has significant implications for practical applications of PT symmetric energy transmission systems.

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