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VARIOUS ELECRICAL-AND-THERMOELECTRIC LAWS, RELATIONS AND COEFFICIENTS IN NEW n(p)-TYPE DEGENERATE "COMPENSATED" GaAs(1-x)P(x)-CRYSTALLINE ALLOY, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT LAW, ACCURATE FERMI ENERGY, AND ELECRICAL CONDUCTIVITY MODEL (XI)

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ABSTRACT

In the $n^{+}(p^{+}) - p(n)$ $\mathbf{X(x)} \equiv \mathbf{GaAs_{1-x}P_x}$- crystalline alloy, $0 \leq x \leq 1$, various electrical-and-thermoelectric laws, relations and coefficients, enhanced by our static dielectric constant law given in Equations (1a, 1b), being due to the effects of the size of donor (acceptor) d(a)-radius $r_{d(a)}$ and the x-concentration, by our accurate Fermi energy given in Eq. (11), and finally by our electrical conductivity model given in Eq. (14), are now investigated, basing on the same physical model and mathematical treatment method, as those used in our recent works.[1, 2] It should be noted here that, for x=0, these obtained numerical results are reduced to those given in the n(p)-type degenerate GaAs-crystal.[4] Then, some remarkable results could be cited in the following. In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Further, one notes in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N) one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of the figure of merit ZT show a same maximum

$(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same obtained results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$).**

KEYWORDS: Electrical conductivity, Seebeck coefficient (S), Figure of merit (ZT), First Van-Cong coefficient (VC1), Second Van-Cong coefficient (VC2), Thomson coefficient (Ts), Peltier coefficient (Pt)

INTRODUCTION

In the $\mathbf{n^+(p^+) - p(n)\ X(x) \equiv GaAs_{1-x}P_x}$ - crystalline alloy, $0 \leq x \leq 1$, the electrical-and-thermoelectric laws, relations, and various coefficients, enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b) and our electrical conductivity model, in Eq. (14), and also by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), are now investigated, by basing on the same physical model and mathematical treatment method, as those used in our recent works.[1, 2] It should be noted here that for x=0, these obtained numerical results may be reduced to those given in the n(p)-type degenerate GaAs-crystal.[3-7] Then, some remarkable results could be noted in the following.

(1) The generalized Mott criterium in the metal-insulator transition (**MIT**) is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N^{EBT}_{CDn(CDp)}$, obtained with a precision of the order of $\mathbf{2.92 \times 10^{-7}}$, as given in our recent work (Van Cong, 2024), and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^*.

(3) The Fermi energy for any N and T, $E_{Fn(Fp)}$, determined in Eq. (11) with a precision of the order of 2.11×10^{-4} [7], and it is present in all the expressions of electrical-and-thermoelectric coefficients.

(4) Our expressions for the electrical conductivity, σ, and for the Seebeck coefficient, S, determined respectively in Equations (14, 19) are the basic expressions, used to determine all the following electrical-and-thermoelectric coefficients.

(5) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and further in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). Furthermore, one notes in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a same minimum $(S)_{min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K}\right)$, those of the figure of merit ZT show a same maxi**mum** $(ZT)_{max.} = 1$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the first Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$. It seems that these same results could represent **a new law for the thermoelectric properties, obtained in the degenerate case ($\xi_{n(p)} \geq 0$)**.

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the $\mathbf{n^+(p^+) - p(n)\ X(x)}$- crystalline alloy at T=0 K, we denote the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $r_{do(ao)} = r_{As(Ga)}$, the unperturbed relative effective electron (hole) mass in conduction (valence) bands by: $m_{c(v)}(x)/m_o$, the unperturbed relative static dielectric constant by: $\varepsilon_o(x)$, and the intrinsic band gap by: $E_{go}(x)$. Then, their values are reported in Table 1 in Appendix 1.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as: $E_{do(ao)}(x) = \frac{13600 \times [m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2}$ meV, and then, the isothermal bulk modulus, by: $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right) \times \left(r_{do(ao)}\right)^3}$.

Our Static Dielectric Constant Law

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective relative dielectric constant $\varepsilon(r_{d(a)}, x)$, developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$, $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, for the pressure p, $p_o = 0$, and for the deformation potential energy (or the strain energy) α, $\alpha_o = 0$. Further, the two important equations, used to determine the α -variation, $\Delta\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by: $\frac{dp}{dV} = -\frac{B}{V}$ and $p = -\frac{d\alpha}{dV}$, giving rise to: $\frac{d}{dV}(\frac{d\alpha}{dV}) = \frac{B}{V}$. Then, by an integration, one gets:

$$[\Delta\alpha(r_{d(a)}, x)]_{n(p)} = B_{do(ao)}(x) \times (V - V_{do(ao)}) \times \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to the increase (the decrease) in the energy gap $E_{gn(gp)}(r_{d(a)}, x)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}(r_{d(a)}, x)$ in absolute values, obtained in the effective Bohr model, which is represented respectively by: $\pm [\Delta\alpha(r_{d(a)}, x)]_{n(p)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = +[\Delta\alpha(r_{d(a)}, x)]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -[\Delta\alpha(r_{d(a)}, x)]_{n(p)}.$$

Therefore, one obtains the expressions for relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and energy band gap $E_{gn(gp)}(r_{d(a)}, x)$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new $\boldsymbol{\varepsilon(r_{d(a)}, x)}$-law**,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (1a)$$

according to the increase in both $E_{gn(gp)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with increasing $r_{d(a)}$ and for a given x, and

(ii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}, x) = \frac{\varepsilon_o(x)}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being a **new $\boldsymbol{\varepsilon(r_{d(a)}, x)}$-law**,

$$E_{gno(gpo)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (1b)$$

corresponding to the decrease in both $E_{gno(gpo)}(r_{d(a)}, x)$ and $E_{d(a)}(r_{d(a)}, x)$, with decreasing $r_{d(a)}$ and for a given x.

It should be noted that, in the following, all the electrical-and-thermoelectric properties strongly depend on this **new $\boldsymbol{\varepsilon}(\mathbf{r}_{\mathbf{d(a)}}, \mathbf{x})$-law.**

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)}, x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}, x) \equiv \frac{\varepsilon(r_{d(a)},x)\times\hbar^2}{m_{c(v)}(x)\times m_o\times q^2} = 0.53\times 10^{-8}\ \text{cm}\times\frac{\varepsilon(r_{d(a)},x)}{m_{c(v)}(x)}. \qquad (2)$$

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(NDp)}(r_{d(a)}, x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as[2]:

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3}\times a_{Bn(Bp)}(r_{d(a)}, x) = M_{n(p)},\ \ M_{n(p)} = 0.25, \qquad (3)$$

depending thus on our **new $\boldsymbol{\varepsilon}(\mathbf{r}_{\mathbf{d(a)}}, \mathbf{x})$-law**.

This excellent one can be explained from the definition of the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, by:

$$r_{sn(sp)}(N, r_{d(a)}, x) \equiv \left(\frac{3}{4\pi N}\right)^{1/3}\times\frac{1}{a_{Bn(Bp)}(r_{d(a)},x)} = 1.1723\times 10^{8}\times\left(\frac{1}{N}\right)^{1/3}\times\frac{m_{c(v)}(x)\times m_o}{\varepsilon(r_{d(a)},x)}, \qquad (4)$$

being equal to, in particular, at N= $N_{CDn(CDp)}(r_{d(a)}, x)$: $r_{sn(sp)}(N_{CDn(CDp)}(r_{d(a)}, x), r_{d(a)}, x) =$ **2.4813963**, for any $(r_{d(a)}, x)$-values. Then, from Eq. (4), one also has :

$$N_{CDn(CDp)}(r_{d(a)}, x)^{1/3}\times a_{Bn(Bp)}(r_{d(a)}, x) = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}}\times\frac{1}{\mathbf{2.4813963}} = \mathbf{0.25} = (\mathbf{WS})_{\mathbf{n(p)}} = \mathbf{M}_{\mathbf{n(p)}}, \qquad (5)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\boldsymbol{\mathcal{H}}_{\mathbf{n(p)}} = \mathbf{0.47137}$, as those given in our previous work[2], we have also showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $\mathbf{N}^{\mathbf{EBT}}_{\mathbf{CDn(CDp)}}$, with a precision of the order of $\mathbf{2.92\ \times 10^{-7}}$.[2]

It shoud be noted that the values of $M_{n(p)}$ and $\mathcal{H}_{n(p)}$ could be chosen so that those of $N_{CDn(CDp)}$ and $N^{EBT}_{CDn(CDp)}$ are found to be in good agreement with their experimental results.

Therefore, the density of electrons (holes) given in parabolic conduction (valence) bands can be defined, as that given in compensated materials:

$N^*(N, r_{d(a)}, x) \equiv N - N_{CDn(NDp)}(r_{d(a)}, x) = N^*$, for a presentation simplicity. (6)

In summary, as observed in Table 6 of our previous paper[2], one remarks that, for a given x and an increasing $r_{d(a)}$, $\varepsilon(r_{d(a)}, x)$ decreases, while $E_{gno(gpo)}(r_{d(a)}, x)$, $N_{CDn(NDp)}(r_{d(a)}, x)$ and $N^{EBT}_{CDn(CDp)}(r_{d(a)}, x)$ increase, affecting strongly all electrical-and-thermoelectric properties, as those observed in following Sections.

PHYSICAL MODEL

In the $n^+(p^+) - p(n)$ **X**(**x**)- crystalline alloy, if denoting the Fermi wave number by: $k_{Fn(Fp)}(N^*) \equiv \left(\frac{3\pi^2 N^*}{g_{c(v)}}\right)^{\frac{1}{3}}$, the reduced effective Wigner-Seitz (**WS**) radius $r_{sn(sp)}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by N^*, is now defined by:

$$\gamma \times r_{sn(sp)}(N^*) \equiv \frac{k^{-1}_{Fn(Fp)}}{a_{Bn(Bp)}} < 1,$$

being proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k^{-1}_{Fn(Fp)}$ means the averaged distance between ionized donors (acceptors), and $a_{Bn(Bp)}(r_{d(a)}, x)$ is determined in Eq. (2).

Then, the ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K is defined by:

$$R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}} = \frac{k^{-1}_{Fn(Fp)}}{k^{-1}_{sn(sp)}} = R_{snWS(spWS)} + \left[R_{snTF(spTF)} - R_{snWS(spWS)}\right]e^{-r_{sn(sp)}} < 1, \quad (7)$$

being valid at any N^*.

Here, these ratios, $R_{snTF(spTF)}$ and $R_{snWS(spWS)}$, can be determined as follows.

First, for $N \gg N_{CDn(NDp)}(r_{d(a)}, x)$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{snTF(spTF)}(N^*)$ is reduced to

$$R_{snTF(spTF)}(N^*) \equiv \frac{k_{snTF(spTF)}}{k_{Fn(Fp)}} = \frac{k^{-1}_{Fn(Fp)}}{k^{-1}_{snTF(spTF)}} = \sqrt{\frac{4\gamma r_{sn(sp)}}{\pi}} \ll 1, \quad (8)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{CDn(NDp)}(r_{d(a)})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{snWS(snWS)}$ is respectively reduced to

$$R_{sn(sp)WS}(N^*) \equiv \frac{k_{sn(sp)WS}}{k_{Fn}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d\left[r_{sn(sp)}^2 \times E_{CE}(N^*)\right]}{dr_{sn(sp)}}\right), \quad (9)$$

where $E_{CE}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{CE}(N^*) = \frac{-0.87553}{0.0908 + r_{sn(sp)}} + \frac{\frac{0.87553}{0.0908+r_{sn(sp)}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(r_{sn(sp)}) - 0.093288}{1 + 0.03847728 \times r_{sn(sp)}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\eta_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1, \quad \eta_{n(p)}(N^*) \equiv \frac{\sqrt{2\pi N^*}}{\varepsilon(r_{d(a)})} \times q^2 k_{sn(sp)}^{-1/2}, \quad (10)$$

which gives: $A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$.

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy

Here, for a presentation simplicity, we change all the sign of various parameters, given in the $p^+ - X(x)$- crystalline alloy in order to obtain the same one, as given in the $n^+ - X(x)$- crystalline alloy, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N, r_{d(a)}, x, T)$, obtained in our previous paper[7], obtained with a precision of the order of 2.11×10^{-4}, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, \ A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

where u is the reduced electron density, $u(N, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T,x)}$,

$N_{c(v)}(T, x) = 2g_{c(v)} \times \left(\frac{m_{c(v)}(x) \times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} (cm^{-3})$, $g_{c(v)} = 1$, $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq Ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times Ln(u)$ as **u ⟶ 0, the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times a u^{\frac{2}{3}}\left(1 + b u^{-\frac{4}{3}} + c u^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)}(x) \times m_o}$ as **u ⟶ ∞ , the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, at T=0K, since $u^{-1} = 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m_{c(v)}(x) \times m_o}$, being proportional to $(N^*)^{2/3}$, and also equal to 0 at $N^* = 0$, according to the MIT.

In the following, it should be noted that all the electrical-and-thermoelectric properties strongly depend on such the accurate expression of $\xi_{n(p)}(N, r_{d(a)}, x, T)$.[2]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

So, the average of E^p, calculated using the FDDF-method, as developed in our previous works[1, 3] is found to be given by:

$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}.$

Further, one notes that, at 0 K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fno(Fpo)})$, $\delta(E - E_{Fno(Fpo)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fno(Fpo)}) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p(E_{Fn(Fp)}) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times (k_B T\gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E^{-\beta}_{Fn(Fp)} \times I_{\beta}$, where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{(e^{\gamma/2} + e^{-\gamma/2})^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains: .

$I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma$.

Now, using an identity $(1 + e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function:

$\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n}-2) \times \pi^{2n} \times |B_{2n}|$. So, from above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case the following ratio:

$$G_p\left(E_{Fn(Fp)}\right) \equiv \frac{\langle E^p \rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\dots(p-2n+1)}{(2n)!} \times (2^{2n}-2) \times |B_{2n}| \times y^{2n} \equiv G_{p\geq 1}(y), \quad (12)$$

where $y \equiv \frac{\pi}{\xi_{n(p)}(N^*,T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*,T)}$.

Then, some usual results of $G_{p\geq 1}(y)$ are given in Table 2 in Appendix 1, being needed to determine all the following electrical-and-thermoelectric properties.

ELECTRICAL-AND-THERMOELECTRIC PROPERTIES

Here, if denoting, for majority electrons (holes), the electrical conductivity by $\sigma(N, r_{d(a)}, x, T)$ expressed in $\text{ohm}^{-1} \times \text{cm}^{-1}$, the thermal conductivity by $\kappa(N, r_{d(a)}, x, T)$ in $\frac{W}{cm \times K}$, and the Lorenz number L defined by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W \times ohm}{K^2}\right) = 2.4429637 \times 10^{-8}\ (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\kappa}{\sigma}$, is proportional to the temperature T(K), as:

$$\frac{\kappa(N, r_{d(a)}, x, T)}{\sigma(N, r_{d(a)}, x, T)} = L \times T. \quad (13)$$

We now determine the general form of σ in the following.

First of all, it is expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{Cn(Cp)} \times m_o}$, or the wave number k, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times \left[k \times a_{Bn(Bp)}\right] \times \left(\frac{E_k}{\eta_{n(p)}}\right)^{1/2},$$

which is thus proportional to ${E_k}^2$.

Then, for $\underline{E \geq 0}$, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2(y = \frac{\pi k_B T}{E_{Fn(Fp)}}) \times E^2_{Fn(Fp)}$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}\left(N, r_{d(a)}, x, T\right)$ for a presentation simplicity.

Therefore, one obtains[1]:

$\sigma(N, r_{d(a)}, x, T) \equiv \left[\frac{q^2}{\pi \times \hbar} \times \frac{k_{Fn(Fp)}(N^*)}{R_{sn(sp)}(N^*)} \times [k_{Fn(Fp)}(N^*) \times a_{Bn(Bp)}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*)}\right] \times \left[G_2(N, r_{d(a)}, x, T) \times \left(\frac{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}{E_{Fno(Fpo)}(N^*)}\right)^2\right] \left(\frac{1}{ohm \times cm}\right)$, $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}\ ohm^{-1}$, $A_{n(p)}(N^*) = \frac{E_{Fno(Fpo)}(N^*)}{\eta_{n(p)}(N^*)}$, $R_{sn(sp)}(N^*) \equiv \frac{k_{sn(sp)}}{k_{Fn(Fp)}}$, (14)

which can be used to define the resistivity as: $\rho(N, r_{d(a)}, x, T) \equiv 1/\sigma(N, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$. This $\sigma(N, r_{d(a)}, x, T)$-result is an essential one in this paper, being used to determine other electrical-and-thermoelectric properties.

In Eq. (14), one notes that at T= 0 K, $\sigma(N, r_{d(a)}, x, T = 0K)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^*)^{\frac{4}{3}}$. Thus, $\sigma(N = N_{CDn(NDp)}, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Electrical Coefficients

The relaxation time τ is related to σ by[1]:

$\tau(N, r_{d(a)}, x, T) \equiv \sigma(N, r_{d(a)}, x, T) \times \frac{m_{c(v)}(x) \times m_o}{q^2 \times N^*}$. Therefore, the mobility μ is given by:

$$\mu(N, r_{d(a)}, x, T) \equiv \mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N, r_{d(a)}, x, T)}{m_{c(v)}(x) \times m_o} = \frac{\sigma(N, r_{d(a)}, x, T)}{q \times N^*}\ (\frac{cm^2}{V \times s}). \quad (15)$$

Here, at T= 0K, $\mu(N^*, r_{d(a)}, T)$ is thus proportional to $(N^*)^{1/3}$, since $\sigma(N^*, r_{d(a)}, T = 0K)$ is proportional to $(N^*)^{4/3}$. Thus, $\mu(N^* = 0, r_{d(a)}, T = 0K) = 0$ at $N^* = 0$, at which the metal-insulator transition (MIT) occurs.

Then, since τ and σ are both proportional to $E_{Fn(Fp)}(N^*, T)^2$, as given above, the Hall factor is defined by:

$r_H(N, r_{d(a)}, x, T) \equiv \frac{\langle \tau^2 \rangle_{FDDF}}{[\langle \tau \rangle_{FDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N, r_{d(a)}, x, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N, r_{d(a)}, x, T)}$, and therefore, the Hall mobility yields:

$$\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T) \times r_H(N^*, T)\ (\frac{cm^2}{V \times s}), \quad (16)$$

Noting that, at T=0K, since $r_H(N, r_{d(a)}, x, T) = 1$, one then gets: $\mu_H(N, r_{d(a)}, x, T) \equiv \mu(N, r_{d(a)}, x, T)$.

Our generalized Einstein relation

Our generalized Einstein relation is found to be defined as[1]:

$$\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3\times L}{\pi^2}} \times T \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right), \ \frac{k_B}{q} = \sqrt{\frac{3\times L}{\pi^2}} \qquad (17)$$

where $D(N, r_{d(a)}, x, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), and the mobility $\mu(N, r_{d(a)}, x, T)$ is determined in Eq. (15). Then, by differentiating this function $\xi_{n(p)}(u)$ with respect to u, one thus obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (17) can also be rewritten as:

$$\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} = \frac{k_B \times T}{q} \times u\frac{V'(u)\times W(u) - V(u)\times W'(u)}{W^2(u)},$$

Where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}}e^{-du}(1 - du) + \frac{2}{3}Au^{B-1}F(u)\left[\left(1 + \frac{3B}{2}\right) + \frac{4}{3}\times\frac{bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}}{1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}}\right]$.

One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and therefore: $\frac{D_{n(p)}(u)}{\mu} \simeq \frac{k_B \times T}{q}$, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2u^{2B}$ and $u[V' \times W - V \times W'] \approx \frac{2}{3}au^{2/3}A^2u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, the **above generalized Einstein relation** is reduced to the **usual Einstein one**: $\frac{D(N,r_{d(a)},x,T=0\ K)}{\mu(N,r_{d(a)},x,T=0\ K)} \approx \frac{2}{3}E_{Fno(Fpo)}(N^*)/q$. In other words, **Eq. (17) verifies the correct limiting conditions**.

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (17) gives:

$$\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}\right)}{\left(1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}\right)}\right], \qquad (18)$$

where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

In Tables 3n(3p) given in Appendix 1, for given x, $N > N_{CDn}$ and T(=4.2 K and 77 K), and from Equations (14, 15, 16, 17), the numerical results of the coefficients: σ, μ, μ_H and D are found to be decreased with increasing $r_{d(a)}$, respectively.

Thermoelectric Coefficients

First of all, from Eq. (14), obtained for $\sigma(N, r_{d(a)}, x, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S, is found to be given by:

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q>0} \times k_B T \times \left.\frac{\partial \ln\sigma(E)}{\partial E}\right]_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln\sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

Then, using Eq. (11), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting $F_S(N, r_{d(a)}, x, T) \equiv \left[1 - \frac{y^2}{3\times G_2\left(y=\frac{\pi}{\xi_{n(p)}}\right)}\right]$,

$$S(N, r_{d(a)}, x, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2F_{Sb}(N^*,T)}{\xi_{n(p)}} = -\sqrt{\frac{3\times L}{\pi^2}} \times \frac{2\times\xi_{n(p)}}{\left(1+\frac{3\times\xi_{n(p)}{}^2}{\pi^2}\right)} = -2\sqrt{L} \times \frac{\sqrt{(ZT)_{Mott}}}{1+(ZT)_{Mott}} \left(\frac{V}{K}\right) < 0, \quad (ZT)_{Mott} = \frac{\pi^2}{3\times\xi_{n(p)}^2}, \qquad (19)$$

according to:

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3\times L}{\pi^2}} \times 2 \times \frac{\frac{3\times\xi_{n(p)}{}^2}{\pi^2} - 1}{\left(1+\frac{3\times\xi_{n(p)}{}^2}{\pi^2}\right)^2} = \sqrt{\frac{3\times L}{\pi^2}} \times 2 \times \frac{(ZT)_{Mott}\times[1-(ZT)_{Mott}]}{[1+(ZT)_{Mott}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \to +\infty$ or $\xi_{n(p)} \to +0$, one has a same limiting value of S: $S \to -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit, ZT, is found to be defined by:

$$ZT(N, r_{d(a)}, x, T) \equiv \frac{S^2\times\sigma\times T}{\kappa} = \frac{S^2}{L} = \frac{4\times(ZT)_{Mott}}{[1+(ZT)_{Mott}]^2}. \qquad (20)$$

Here, one notes that: (i) $\frac{\partial(ZT)}{\partial\xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial\xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial(ZT)}{\partial\xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{max.} = 1$, and $(ZT)_{Mott} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $(ZT)_{Mott} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient, VC1, can be defined by:

$$VC1(N, r_{d(a)}, x, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial\xi_{n(p)}} \times -\frac{\partial\xi_{n(p)}}{\partial N^*},$$ being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, (21)

and the second Van-Cong coefficient, VC2, as:

$$VC2(N, r_{d(a)}, x, T) \equiv T \times VC1\ (V), \qquad (22)$$

the Thomson coefficient, Ts, by:

$$\mathrm{Ts}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T}) \equiv \mathrm{T} \times \frac{\mathrm{dS}}{\mathrm{dT}} \left(\frac{\mathrm{V}}{\mathrm{K}}\right) = \mathrm{T} \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial \mathrm{T}}, \text{ being equal to 0 for } \xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}, \quad (23)$$

and the Peltier coefficient, Pt, as:

$$\mathrm{Pt}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T}) \equiv \mathrm{T} \times \mathrm{S}\ (\mathrm{V}). \quad (24)$$

One notes here that in next Tables 5n(p) and 6n(p) given in Appendix 1, obtained with such given physical conditions N(or T) for the decreasing $\xi_{n(p)}$, since $\mathrm{VC1}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})$ and $\mathrm{Ts}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})$ are expressed in terms of $\frac{-\mathrm{dS}}{\mathrm{dN^*}}$ and $\frac{\mathrm{dS}}{\mathrm{dT}}$, one has: $[\mathrm{VC1}, \mathrm{Ts}] < 0$ for $\xi_{n(p)} > \sqrt{\frac{\pi^2}{3}}$, $[\mathrm{VC1}, \mathrm{Ts}] = 0$ for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$, and $[\mathrm{VC1}, \mathrm{Ts}] > 0$ for $\xi_{n(p)} < \sqrt{\frac{\pi^2}{3}}$, stating also that for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$:

(i) S, determined in Eq. (19), thus presents **a same minimum** $(\mathrm{S})_{\mathrm{min.}} = -\sqrt{\mathrm{L}} \simeq -1.563 \times 10^{-4} \left(\frac{\mathrm{V}}{\mathrm{K}}\right)$,

(ii) ZT, determined in Eq. (20), therefore presents **a same maximum:** $(\mathrm{ZT})_{\mathrm{max.}} = 1$, since the variations of ZT are expressed in terms of $[\mathrm{VC1}, \mathrm{Ts}] \times \mathrm{S}, \quad \mathrm{S} < 0$.

Furthermore, it is interesting to remark that the (VC2)-coefficient is related to our generalized Einstein relation (17) by:

$$\frac{\mathrm{k_B}}{\mathrm{q}} \times \mathrm{VC2}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T}) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\mathrm{D}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})}{\mu(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})} \left(\frac{\mathrm{V^2}}{\mathrm{K}}\right), \qquad \frac{\mathrm{k_B}}{\mathrm{q}} = \sqrt{\frac{3 \times \mathrm{L}}{\pi^2}}, \quad (25)$$

according, in this work, with the use of our Eq. (21), to:

$$\mathrm{VC2}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T}) \equiv -\frac{\mathrm{D}(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})}{\mu(\mathrm{N}, \mathrm{r_{d(a)}}, \mathrm{x}, \mathrm{T})} \times 2 \times \frac{(\mathrm{ZT})_{\mathrm{Mott}} \times [1 - (\mathrm{ZT})_{\mathrm{Mott}}]}{[1 + (\mathrm{ZT})_{\mathrm{Mott}}]^2} \ (\mathrm{V}).$$

Of course, our relation (25) is reduced to: $\frac{\mathrm{D}}{\mu}$, VC1 and VC2, being determined respectively by Equations (17, 21, 22).

Now, in the degenerate n(p)-type $\mathrm{X(x)}$ – alloy, and for N> $\mathrm{N_{CDn(CDp)}}$, and for T=3K (80K), the numerical results of various thermoelectric coefficients are reported in Tables 4n(4p) in Appendix 1, noting that their variations with increasing $\mathrm{r_{d(a)}}$ are represented by the arrows: ↗ (increase), and ↘ (decrease), respectively.

Then, in Tables 5n(5p) given Appendix 1 for a given N and with increasing T, and in Tables 6n(6p) given Appendix 1 for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and various thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘).

CONCLUDING REMARKS

Here, some concluding remarks can be given as follows.

(1) In the $\mathbf{n^+(p^+)-p(n)\ X(x)}$- crystalline alloy, $0 \le x \le 1$, the electrical-and-thermoelectric laws, relations, and various coefficients are found to be enhanced by our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, being, for a given x, decreased with increasing $r_{d(a)}$, as given in Equations (1a, 1b) and also given in Table 6 of our recent work[2], by our accurate Fermi energy, $E_{Fn(Fp)}$, given in Eq. (11), and in particular by our electrical conductivity model given in Eq. (14).

(2) The generalized Mott criterium in the MIT is expressed in Equations (3, 5, 6), stating that the critical impurity density $N_{CDn(CDp)}$ is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$, obtained with a precision of the order of $\mathbf{2.92 \times 10^{-7}}$, as given in our previous work[2], and the effective electron (hole)-density can be defined by: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, as that observed in the compensated crystals.

(3) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid for any density N^*.

(4) In Tables 5n(5p) given Appendix 1, for a given impurity density N and with increasing temperature T, and then in Tables 6n(6p) given Appendix 1, for a given T and with decreasing N, the reduced Fermi-energy $\xi_{n(p)}$ decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows by: (increase: ↗, decrease: ↘). One remarks in these Tables that, for any given x, $r_{d(a)}$ and N (or T), with increasing T (or decreasing N), one obtains: (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of the Seebeck coefficient S present a **same minimum** $\mathbf{(S)_{min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)}$, those of the figure of merit ZT show a **same maximum** $\mathbf{(ZT)_{max.} = 1}$, (ii) for $\xi_{n(p)} = 1$, the numerical results of S, ZT, the Mott figure of merit $(ZT)_{Mott}$, the Van-Cong coefficient VC1, and the Thomson coefficient Ts, present the same results: $\mathbf{-1.322 \times 10^{-4} \frac{V}{K}}$, **0.715, 3.290,** $\mathbf{1.105 \times 10^{-4} \frac{V}{K}}$, **and** $\mathbf{1.657 \times 10^{-4} \frac{V}{K}}$, respectively, and finally (iii) for $\xi_n \simeq 1.8138$, $\mathbf{(ZT)_{Mott} = 1}$. It seems that these same results could represent **a new law given for the thermoelectric properties, obtained in the degenerate case**.

(5) Finally, our electrical-and-thermoelectric relation is given in Eq. (25) by:

$\frac{k_B}{q} \times VC2(N, r_{d(a)}, x, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \left(\frac{V^2}{K}\right)$, $\frac{k_B}{q} = \sqrt{\frac{3\times L}{\pi^2}}$, according, in this work, to:

$VC2(N, r_{d(a)}, x, T) \equiv -\frac{D(N,r_{d(a)},x,T)}{\mu(N,r_{d(a)},x,T)} \times 2 \times \frac{(ZT)_{Mott}\times[1-(ZT)_{Mott}]}{[1+(ZT)_{Mott}]^2}$ (V), being reduced to: $\frac{D}{\mu}$, VC1 and VC2, determined respectively in Equations (17, 21, 22). This should be **a new result**.

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APPENDIX 1: Tables

Table 1: The values of energy-band-structure parameters are given in the following.

In the $\mathbf{X(x) \equiv GaAs_{1-x}P_x}$**-crystalline alloy**, in which $r_{do(ao)}$=$r_{As(Ga)}$=0.118 nm (0.126 nm), we have: $g_{c(v)}(x) = 1 \times x + 1 \times (1-x)$, $m_{c(v)}(x)/m_o = 0.13\ (0.5) \times x + 0.066\ (0.291) \times (1-x)$, $\varepsilon_o(x) = 11.1 \times x + 13.13 \times (1-x)$, $E_{go}(x) = 1.796 \times x + 1.52 \times (1-x)$.

Table 2: Expressions for $G_{p\geq 1}(y \equiv \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, noting that $G_{p=1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}) = 1$, used to determine the electrical-and-thermoelectric coefficients.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}+\frac{7y^4}{640}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}-\frac{7y^4}{384}\right)$	$(1+y^2)$	$\left(1+\frac{35y^2}{24}+\frac{49y^4}{384}\right)$	$\left(1+2y^2+\frac{7y^4}{15}\right)$	$\left(1+\frac{21y^2}{8}+\frac{147y^4}{128}\right)$

Table 3n: Here, one notes that, for given x, $N > N_{CDn}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm \times cm}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^2 \times cm^2}{s}\right)$, decrease with increasing r_d.

Donor	P	As	Sb	Sn
r_d (nm) ↗	0.110	0.118	0.136	0.140
For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{18} cm^{-3})				
3	1.23, 2.566, 2.566, 1.96	1.19, 2.499, 2.499, 1.91	1.05, 2.189, 2.190, 1.67	0.98, 2.062, 2.063, 1.57
10	3.10, 1.936, 1.936, 3.30	3.01, 1.884, 1.884, 3.22	2.64, 1.648, 1.648, 2.81	2.48, 1.553, 1.553, 2.65
40	9.34, 1.458, 1.458, 6.28	9.07, 1.415, 1.415, 6.09	7.83, 1.223, 1.223, 5.27	7.34, 1.147, 1.147, 4.94
70	14.8, 1.324, 1.324, 8.28	14.4, 1.284, 1.284, 8.03	12.4, 1.103, 1.013, 6.90	11.6, 1.031, 1.031, 6.45
For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{18} cm^{-3})				
3	0.82, 1.729, 1.730, 0.88	0.79, 1.684, 1.684, 0.86	0.69, 1.470, 1.471, 0.74	0.64, 1.382, 1.383, 0.70
10	2.07, 1.301, 1.301, 1.49	2.02, 1.267, 1.267, 1.45	1.77, 1.111, 1.112, 1.27	1.66, 1.048, 1.048, 1.20
40	6.12, 0.957, 0.957, 2.77	5.95, 0.930, 0.930, 2.69	5.17, 0.809, 0.809, 2.34	4.86, 0.760, 0.760, 2.20
70	9.63, 0.859, 0.859, 3.62	9.35, 0.834, 0.834, 3.51	8.08, 0.721, 0.721, 3.03	7.57, 0.676, 0.676, 2.85
For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{18} cm^{-3})				
3	0.56, 1.229, 1.230, 0.46	0.54, 1.196, 1.196, 0.45	0.46, 1.040, 1.040, 0.38	0.43, 0.976, 0.976, 0.36
10	1.47, 0.931, 0.931, 0.80	1.43, 0.907, 0.907, 0.78	1.24, 0.795, 0.795, 0.68	1.17, 0.750, 0.750, 0.64
40	4.30, 0.673, 0.673, 1.47	4.18, 0.655, 0.655, 1.43	3.65, 0.572, 0.572, 1.25	3.43, 0.539, 0.539, 1.17
70	6.69, 0.598, 0.598, 1.89	6.50, 0.581, 0.581, 1.84	5.65, 0.505, 0.505, 1.60	5.31, 0.475, 0.475, 1.50
For **x=0**, the values of (σ, μ, μ_H, D) at **77 K**				
N (10^{18} cm^{-3})				
3	1.25, 2.615, 2.728, 1.99	1.22, 2.547, 2.657, 1.94	1.07, 2.232, 2.329, 1.69	1.00, 2.102, 2.193, 1.60
10	3.11, 1.943, 1.960, 3.31	3.03, 1.891, 1.908, 3.23	2.65, 1.655, 1.669, 2.82	2.49, 1.559, 1.573, 2.66
40	9.35, 1.459, 1.461, 6.28	9.07, 1.416, 1.418, 6.10	7.84, 1.224, 1.225, 5.27	7.35, 1.147, 1.149, 4.94
70	14.8, 1.324, 1.325, 8.28	14.4, 1.284, 1.285, 8.03	12.4, 1.103, 1.014, 6.90	11.6, 1.031, 1.032, 6.45
For **x=0.5**, the values of (σ, μ, μ_H, D) at **77 K**				

N (10^{18} cm^{-3})				
3	0.85, 1.805, 1.975, 0.91	0.83, 1.757, 1.923, 0.89	0.72, 1.535, 1.682, 0.77	0.67, 1.443, 1.582, 0.72
10	2.09, 1.312, 1.338, 1.50	2.04, 1.278, 1.303, 1.46	1.78, 1.121, 1.143, 1.28	1.68, 1.057, 1.078, 1.21
40	6.13, 0.958, 0.961, 2.78	5.96, 0.931, 0.934, 2.70	5.18, 0.810, 0.812, 2.34	4.87, 0.761, 0.763, 2.20
70	9.63, 0.860, 0.862, 3.62	9.35, 0.835, 0.836, 3.51	8.08, 0.722, 0.723, 3.04	7.58, 0.677, 0.678, 2.85
For **x=1**, the values of (σ, μ, μ_H, D) at **77 K**				
N (10^{18} cm^{-3})				
3	0.60, 1.329, 1.552, 0.49	0.59, 1.293, 1.511, 0.47	0.50, 1.127, 1.322, 0.41	0.46, 1.059, 1.245, 0.38
10	1.49, 0.945, 0.977, 0.81	1.45, 0.920, 0.952, 0.79	1.26, 0.808, 0.836, 0.69	1.19, 0.761, 0.788, 0.65
40	4.31, 0.675, 0.678, 1.47	4.19, 0.657, 0.660, 1.43	3.66, 0.574, 0.577, 1.25	3.44, 0.540, 0.543, 1.18
70	6.70, 0.599, 0.600, 1.90	6.51, 0.582, 0.583, 1.84	5.66, 0.506, 0.507, 1.60	5.31, 0.476, 0.477, 1.51

Table 3p: Here, one notes that, for given x, $N > N_{CDp}$ and T(=4.2 K and 77 K), the functions: σ, μ, μ_H, D, expressed respectively in $\left(\frac{10^3}{ohm\times cm}, \frac{10^2\times cm^2}{V\times s}, \frac{10^2\times cm^2}{V\times s}, \frac{10\times cm^2}{s}\right)$, decrease with increasing r_a.

Acceptor	Ga	Mg	In	Cd
r_a (nm) ↗	0.126	0.140	0.144	0.148
For x=**0**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{19} cm^{-3})				
3	4.50, 9.743, 9.744, 7.65	4.12, 8.973, 8.974, 7.02	3.89, 8.531, 8.532, 6.65	3.65, 8.040, 8.040, 6.24
5	7.03, 8.982, 8.982, 10.0	6.43, 8.244, 8.245, 9.18	6.08, 7.821, 7.821, 8.69	5.69, 7.351, 7.351, 8.14
8	10.6, 8.404, 8.404, 12.9	9.69, 7.693, 7.693, 11.8	9.16, 7.285, 7.286, 11.1	8.57, 6.833, 6.833, 10.4
10	12.9, 8.162, 8.162, 14.6	11.8, 7.464, 7.464, 13.3	11.1, 7.063, 7.063, 12.6	10.4, 6.618, 6.618, 11.8
For x=**0.5**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{19} cm^{-3})				
3	2.39, 5.654, 5.655, 3.08	2.17, 5.265, 5.267, 2.82	2.04, 5.043, 5.044, 2.66	1.89, 4.798, 4.799, 2.49
5	3.76, 5.063, 5.063, 4.01	3.43, 4.688, 4.689, 3.68	3.24, 4.473, 4.474, 3.49	3.02, 4.234, 4.235, 3.27
8	5.67, 4.633, 4.634, 5.12	5.18, 4.273, 4.273, 4.70	4.90, 4.066, 4.066, 4.45	4.58, 3.836, 3.837, 4.18
10	6.88, 4.458, 4.459, 5.76	6.29, 4.105, 4.105, 5.28	5.95, 3.902, 3.902, 5.00	5.57, 3.676, 3.677, 4.69
For x=**1**, the values of (σ, μ, μ_H, D) at **4.2K**				
N (10^{19} cm^{-3})				
3	1.27, 3.892, 3.894, 1.41	1.11, 3.699, 3.701, 1.26	1.00, 3.596, 3.598, 1.17	0.88, 3.491, 3.494, 1.07
5	2.15, 3.330, 3.331, 1.90	1.93, 3.123, 3.124, 1.73	1.80, 3.006, 3.007, 1.63	1.65, 2.879, 2.880, 1.52
8	3.34, 2.959, 2.959, 2.45	3.03, 2.755, 2.755, 2.24	2.85, 2.639, 2.639, 2.12	2.64, 2.510, 2.510, 1.99
10	4.08, 2.815, 2.815, 2.75	3.71, 2.614, 2.614, 2.53	3.50, 2.498, 2.499, 2.39	3.26, 2.371, 2.371, 2.24
For x=**0**, the values of (σ, μ, μ_H, D) at **77K**				
N (10^{19} cm^{-3})				
3	4.59, 9.921, 10.33, 7.76	4.19, 9.139, 9.516, 7.12	3.97, 8.689, 9.050, 6.74	3.71, 8.191, 8.534, 6.33
5	7.09, 9.063, 9.248, 10.1	6.48, 8.319, 8.490, 9.24	6.13, 7.892, 8.055, 8.75	5.74, 7.418, 7.572, 8.20
8	10.7, 8.444, 8.535, 12.9	9.74, 7.730, 7.814, 11.8	9.21, 7.320, 7.400, 11.2	8.61, 6.866, 6.941, 10.5
10	13.0, 8.191, 8.257, 14.6	11.8, 7.490, 7.550, 13.3	11.2, 7.088, 7.145, 12.6	10.5, 6.642, 6.695, 11.8
For x=**0.5**, the values of (σ, μ, μ_H, D) at **77K**				
N (10^{19} cm^{-3})				
3	2.49, 5.870, 6.360, 3.17	2.25, 5.474, 5.945, 2.90	2.12, 5.248, 5.711, 2.75	1.97, 4.999, 5.454, 2.57
5	3.83, 5.153, 5.360, 4.07	3.49, 4.774, 4.969, 3.73	3.30, 4.556, 4.745, 3.54	3.08, 4.314, 4.497, 3.32
8	5.72, 4.676, 4.773, 5.16	5.23, 4.313, 4.404, 4.73	4.95, 4.104, 4.192, 4.48	4.63, 3.873, 3.956, 4.21

10	6.93, 4.489, 4.557, 5.79	6.34, 4.133, 4.197, 5.30	5.99, 3.929, 3.990, 5.03	5.61, 3.702, 3.760, 4.71
For x=**1**, the values of (σ, μ, μ_H, D) at **77K**				
N (10^{19} cm^{-3})				
3	1.38, 4.229, 4.983, 1.50	1.21, 4.061, 4.866, 1.36	1.11, 3.981, 4.839, 1.27	0.99, 3.920, 4.868, 1.17
5	2.23, 3.445, 3.706, 1.96	2.01, 3.238, 3.497, 1.78	1.87, 3.121, 3.382, 1.68	1.72, 2.996, 3.261, 1.57
8	3.39, 3.008, 3.118, 2.48	3.08, 2.802, 2.908, 2.27	2.90, 2.684, 2.789, 2.15	2.69, 2.555, 2.658, 2.01
10	4.12, 2.848, 2.923, 2.78	3.76, 2.645, 2.717, 2.55	3.54, 2.529, 2.599, 2.41	3.30, 2.401, 2.469, 2.26

Table 4n: In the lightly degenerate n-type X(x) − alloy and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Donor		P	As	Sb	Sn
For **x=0** and **N=5 × 10^{17} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	132.351	132.207	131.342	130.858
$\xi_{n(T=80K)}$	↘	5.206	5.200	5.168	5.151
$\kappa_{(T=3K)}$ $\left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	2.165	2.098	1.791	1.665
$\kappa_{(T=80K)}$ $\left(\frac{10^{-4}\times W}{cm\times K}\right)$	↘	7.120	6.905	5.906	5.496
$-S_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	4.283	4.288	4.316	4.332
$-S_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	9.713	9.720	9.767	9.794
$-VC1_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	2.854	2.857	2.876	2.886
$-VC1_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	4.748	4.750	4.765	4.773
$-VC2_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	8.562	8.571	8.628	8.659
$-VC2_{(T=80K)}$ $\left(\frac{10^{-3}\times V}{K}\right)$	↘	3.798	3.800	3.812	3.818
$-Ts_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	4.281	4.286	4.314	4.330
$-Ts_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	7.122	7.126	7.147	7.159
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.285	1.286	1.295	1.299
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	7.770	7.776	7.814	7.835
$ZT_{(T=3K)}$ (10^{-4})	↗	7.510	7.526	7.625	7.682
$ZT_{(T=80K)}(10^{-1})$	↗	3.861	3.868	3.905	3.926
For **x=0.5** and **N=10^{18} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	138.849	138.526	136.577	135.481
$\xi_{n(T=80K)}$	↘	5.444	5.432	5.361	5.321
$\kappa_{(T=3K)}$ $\left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	2.433	2.355	1.992	1.842
$\kappa_{(T=80K)}$ $\left(\frac{10^{-4}\times W}{cm\times K}\right)$	↘	7.877	7.631	6.486	6.013
$-S_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	4.083	4.092	4.151	4.184
$-S_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	9.375	9.391	9.490	9.547
$-VC1_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	2.721	2.727	2.766	2.788
$-VC1_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	4.655	4.659	4.685	4.701
$-VC2_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	8.162	8.181	8.297	8.364
$-VC2_{(T=80K)}$ $\left(\frac{10^{-3}\times V}{K}\right)$	↘	3.724	3.727	3.748	3.760
$-Ts_{(T=3K)}$ $\left(\frac{10^{-6}\times V}{K}\right)$	↘	4.081	4.090	4.149	4.182
$-Ts_{(T=80K)}$ $\left(\frac{10^{-5}\times V}{K}\right)$	↘	6.982	6.988	7.028	7.051
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.225	1.228	1.245	1.255

$-Pt_{(T=80K)}(10^{-3} \times V)$	↘	7.500	7.513	7.592	7.638
$ZT_{(T=3K)}\ (10^{-4})$	↗	6.823	6.855	7.052	7.167
$ZT_{(T=80K)}(10^{-1})$	↗	3.597	3.610	3.687	3.731
For **x=1** and **N=2 $\times$ 10^{18} cm^{-3}**, one has:					
$\xi_{n(T=3K)}$	↘	162.976	162.385	158.800	156.780
$\xi_{n(T=80K)}$	↘	6.320	6.299	6.168	6.095
$\kappa_{(T=3K)}\ \left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	2.894	2.798	2.346	2.157
$\kappa_{(T=80K)}\ \left(\frac{10^{-4}\times W}{cm\times K}\right)$	↘	8.933	8.645	7.292	6.729
$-S_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	3.478	3.491	3.570	3.616
$-S_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	8.288	8.312	8.460	8.545
$-VC1_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	2.318	2.327	2.379	2.410
$-VC1_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	4.385	4.392	4.430	4.451
$-VC2_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	6.955	6.980	7.137	7.229
$-VC2_{(T=80K)}\ \left(\frac{10^{-3}\times V}{K}\right)$	↘	3.508	3.513	3.544	3.561
$-Ts_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	3.477	3.490	3.569	3.615
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	6.578	6.587	6.644	6.676
$-Pt_{(T=3K)}(10^{-5} \times V)$	↘	1.043	1.047	1.071	1.085
$-Pt_{(T=80K)}(10^{-3} \times V)$	↘	6.631	6.650	6.768	6.836
$ZT_{(T=3K)}\ (10^{-4})$	↗	4.953	4.989	5.217	5.352
$ZT_{(T=80K)}(10^{-1})$	↗	2.812	2.828	2.930	2.989

Table 4p: In the lightly degenerate p-type X(x) − alloy, in which **N=2 $\times$ 10^{19} cm^{-3}**, and for T=3K and 80K, the numerical results of various thermoelectric coefficients are reported. Further, their variations with increasing $r_{d(a)}$ are represented by the arrows: ↗ (increase), and ↘ (decrease).

Acceptor		Ga	Mg	In	Cd
For x=0, one has:					
$\xi_{n(T=3K)}$	↘	343.331	340.811	339.015	336.619
$\xi_{n(T=80K)}$	↘	12.972	12.878	12.811	12.722
$\kappa_{(T=3K)}\ \left(\frac{10^{-5}\times W}{cm\times K}\right)$	↘	23.190	21.191	20.030	18.726
$\kappa_{(T=80K)}\ \left(\frac{10^{-3}\times W}{cm\times K}\right)$	↘	6.400	5.851	5.533	5.175
$-S_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	1.651	1.664	1.672	1.684
$-S_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	4.287	4.317	4.339	4.368
$-VC1_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	1.101	1.109	1.115	1.223
$-VC1_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	2.707	2.723	2.735	2.752
$-VC2_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	3.302	3.327	3.344	3.368
$-VC2_{(T=80K)}\ \left(\frac{10^{-3}\times V}{K}\right)$	↘	2.165	2.179	2.188	2.201
$-Ts_{(T=3K)}\ \left(\frac{10^{-6}\times V}{K}\right)$	↘	1.651	1.663	1.672	1.684
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	4.060	4.085	4.103	4.128
$-Pt_{(T=3K)}(10^{-5} \times V)$	↘	0.495	0.499	0.502	0.505
$-Pt_{(T=80K)}(10^{-3} \times V)$	↘	3.430	3.454	3.471	3.494
$ZT_{(T=3K)}\ (10^{-4})$	↗	1.116	1.133	1.145	1.161
$ZT_{(T=80K)}(10^{-1})$	↗	0.752	0.763	0.770	0.781
For x=0.5, one has:					

$\xi_{n(T=3K)}$	↘	229.683	223.439	218.947	212.904
$\xi_{n(T=80K)}$	↘	8.760	8.530	8.365	8.143
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	12.009	10.797	10.075	9.243
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	3.454	3.119	2.920	2.691
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.468	2.537	2.589	2.663
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	6.206	6.359	6.474	6.634
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	1.645	1.691	1.726	1.775
$-VC1_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	3.667	3.732	3.779	3.844
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.936	5.074	5.178	5.325
$-VC2_{(T=80K)}\ \left(\frac{10^{-3}\times V}{K}\right)$	↘	2.933	2.986	3.024	3.075
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.468	2.537	2.589	2.662
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	5.500	5.598	5.669	5.766
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	0.740	0.761	0.777	0.799
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	4.965	5.087	5.179	5.307
$ZT_{(T=3K)}\ (10^{-4})$	↗	2.494	2.635	2.745	2.903
$ZT_{(T=80K)}(10^{-1})$	↗	1.577	1.655	1.715	1.801
For x=1, one has:					
$\xi_{n(T=3K)}$	↘	134.452	119.021	107.417	90.933
$\xi_{n(T=80K)}$	↘	5.283	4.406	4.241	3.483
$\kappa_{(T=3K)}\ (\frac{10^{-5}\times W}{cm\times K})$	↘	5.576	4.510	3.828	2.983
$\kappa_{(T=80K)}\ (\frac{10^{-3}\times W}{cm\times K})$	↘	1.824	1.535	1.338	1.054
$-S_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.216	4.763	5.277	6.233
$-S_{(T=80K)}\ (\frac{10^{-5}\times V}{K})$	↘	9.601	10.490	11.301	12.806
$-VC1_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	2.809	3.173	3.515	4.151
$-VC1_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	4.716	5.044	5.530	6.430
$-VC2_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	8.428	9.519	10.545	12.452
$-VC2_{(T=80K)}\ \left(\frac{10^{-3}\times V}{K}\right)$	↘	3.772	4.035	4.424	5.144
$-Ts_{(T=3K)}\ (\frac{10^{-6}\times V}{K})$	↘	4.214	4.760	5.273	6.226
$-Ts_{(T=80K)}\ \left(\frac{10^{-5}\times V}{K}\right)$	↘	7.073	7.567	8.295	9.646
$-Pt_{(T=3K)}(10^{-5}\times V)$	↘	1.265	1.429	1.583	1.870
$-Pt_{(T=80K)}(10^{-3}\times V)$	↘	7.681	8.392	9.041	10.245
$ZT_{(T=3K)}\ (10^{-4})$	↗	7.277	9.285	11.398	15.902
$ZT_{(T=80K)}(10^{-1})$	↗	3.773	4.505	5.228	6.713

Table 5n: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with

increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- X(x) – alloy, for $N = 2 \times N_{CDn}(r_P) = 2.507675 \times 10^{16}\ cm^{-3}$, one gets:

T(K)	↗	15.263		**15.5943916**		15.929		**21.2202591**		21.242
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.933	↗	0	↗	0.999	↗	23.447	↗	23.558
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−2.384	↘	−2.437	↘	−2.488	↘	−2.8047	↗	−2.8039

In the degenerate As- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 2.6660176 \times 10^{16}\ cm^{-3}$, one gets:

T(K)	↗	15.8977		**16.244123**		16.593		**22.1043894**		22.127
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.975	↗	0	↗	1.042	↗	24.424	↗	24.540
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−2.483	↘	−2.539	↘	−2.592	↘	−2.9215	↗	−2.9207

In the degenerate Sb- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 3.619757 \times 10^{16}\ cm^{-3}$, one gets:

T(K)	↗	19.493		**19.917762**		20.346		**27.103339**		27.13
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.195	↗	0	↗	1.279	↗	29.948	↗	30.084
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−3.045	↘	−3.113	↘	−3.178	↘	−3.5822	↗	−3.5813

In the degenerate Sn- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 4.152416 \times 10^{16}\ cm^{-3}$, one gets:

T(K)	↗	21.3612		**21.8267041**		22.296		**29.700956**		29.731
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.310	↗	0	↗	1.402	↗	32.818	↗	32.971
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−3.337	↘	−3.411	↘	−3.483	↘	−3.9256	↗	−3.9246

For x=0.5,

In the degenerate P- X(x) – alloy, for $N = 2 \times N_{CDn}(r_P) = 1.0450608 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	26.618		**27.197762**		27.782		**37.009689**		37.047
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.632	↗	0	↗	1.745	↗	40.894	↗	41.084
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−4.158	↘	−4.251	↘	−4.339	↘	−4.8916	↗	−4.8903

In the degenerate As- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 1.1110493 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	27.7267		**28.330942**		28.9405		**38.551677**		38.589
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.701	↗	0	↗	1.821	↗	42.597	↗	42.788
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−4.331	↘	−4.428	↘	−4.520	↘	−5.0954	↗	−5.0941

In the degenerate Sb- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 1.5085154 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	33.9971		**34.738038**		35.485		**47.270211**		47.318
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.085	↗	0	↗	2.231	↗	52.231	↗	52.475
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−5.310	↘	−5.429	↘	−5.543	↘	−6.2477	↗	−6.2461

In the degenerate Sn- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 1.7304984 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	37.2556		**38.067375**		38.886		**51.800647**		51.853
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.285	↗	0	↗	2.445	↗	57.237	↗	57.505
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-3}V)$		−5.819	↘	−5.950	↘	−6.074	↘	−6.8465	↗	−6.8447

For x=1,

In the degenerate P- X(x) – alloy, for $N = 2 \times N_{CDn}(r_P) = 3.1717192 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	42.062		**42.978508**		43.903		**58.483533**		58.543
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713

$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.580	↗	0	↗	2.762	↗	64.621	↗	64.925
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−6.570	↘	−6.717	↘	−6.858	↘	−7.7298	↗	−7.7278

In the degenerate As- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{As}) = 3.3719916 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	43.814		**44.769183**		45.732		**60.920214**		60.982
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.688	↗	0	↗	2.876	↗	67.313	↗	67.629
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−6.844	↘	−6.997	↘	−7.143	↘	−8.0518	↗	−8.0497

In the degenerate Sb- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sb}) = 4.5782858 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	53.723		**54.89382**		56.074		**74.697437**		74.7741
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−3.295	↗	0	↗	3.525	↗	82.536	↗	82.929
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−8.391	↘	−8.580	↘	−8.759	↘	−9.8728	↗	−9.8702

In the degenerate Sn- X(x) – alloy, for $N = 2 \times N_{CDn}(r_{Sn}) = 5.2519956 \times 10^{17}\ cm^{-3}$, one gets:

T(K)	↗	58.872		**60.15491**		61.448		**81.856532**		81.94
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−3.611	↗	0	↗	3.863	↗	90.447	↗	90.874
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−9.196	↘	−9.402	↘	−9.598	↘	−10.8190	↗	−10.8162

Table 5p: Here, for a given N and with increasing T, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_p = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate Ga- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Ga}) = 2.285126 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	70.095	**71.621813**	73.16	**97.46026**	97.56
ξ_p	↘	1.880	**1.8138**	1.750	**1**	0.998

$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.043	↗	0	↗	0.046	↗	1.077	↗	1.082
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−1.095	↘	−1.119	↘	−1.143	↘	−1.2881	↗	−1.2878

In the degenerate Mg- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Mg}) = 2.6994914 \times 10^{18}\ cm^{-3}$, one gets

T(K)	↗	78.331		**80.037437**		81.759		**108.911924**		109.023
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.048	↗	0	↗	0.051	↗	1.203	↗	1.209
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−1.223	↘	−1.251	↘	−1.277	↘	−1.4395	↗	−1.4391

In the degenerate In- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{In}) = 2.9940338 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	83.93		**85.758335**		87.6		**116.696705**		116.81
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.051	↗	0	↗	0.055	↗	1.289	↗	1.295
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−1.311	↘	−1.340	↘	−1.368	↘	−1.5424	↗	−1.5420

In the degenerate Cd- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Cd}) = 3.3855772 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	91.1		**93.080853**		95.08		**126.660914**		126.79
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.998	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.056	↗	0	↗	0.060	↗	1.399	↗	1.406
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−1.423	↘	−1.455	↘	−1.485	↘	−1.6741	↗	−1.6736

For x=0.5,

In the degenerate Ga- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Ga}) = 7.302887 \times 10^{18}\ cm^{-3}$, one gets:

T(K)	↗	111.897		**114.33559**		116.79		**155.583553**		155.74
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109

$VC2\left(10^{-2}\frac{V}{K}\right)$	−0.069	↗	0	↗	0.073	↗	1.719	↗	1.727
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.7478	↘	−1.7871	↘	−1.8243	↘	−2.0563	↗	−2.0558

In the degenerate Mg- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Mg}) = 8.6271302 \times 10^{18}\ cm^{-3}$, one gets

T(K) ↗	125.045		**127.770112**		130.51		**173.86475**		174.043
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$	−0.077	↗	0	↗	0.082	↗	1.921	↗	1.930
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−1.9532	↘	−1.9970	↘	−2.0386	↘	−2.2980	↗	−2.2974

In the degenerate In- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{In}) = 9.5684394 \times 10^{18}\ cm^{-3}$, one gets:

T(K) ↗	133.983		**136.90284**		139.848		**186.29222**		186.483
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$	−0.082	↗	0	↗	0.088	↗	2.058	↗	2.068
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.0928	↘	−2.1398	↘	−2.1844	↘	−2.4622	↗	−2.4616

In the degenerate Cd- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Cd}) = 1.0819748 \times 10^{19}\ cm^{-3}$, one gets:

T(K) ↗	145.43		**148.592355**		151.78		**202.19887**		202.406
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$	−0.089	↗	0	↗	0.095	↗	2.234	↗	2.245
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.2716	↘	−2.3225	↘	−2.3708	↘	−2.6725	↗	−2.6718

For x=1,

In the degenerate Ga- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Ga}) = 1.9185205 \times 10^{19}\ cm^{-3}$, one gets:

T(K) ↗	168.517		**172.18917**		175.894		**234.30852**		234.549
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$	−0.103	↗	0	↗	0.111	↗	2.589	↗	2.601
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.6322	↘	−2.6913	↘	−2.7475	↘	−3.0969	↗	−3.0960

In the degenerate Mg- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{Mg}) = 2.2664086\ \times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	188.318		**192.42153**		196.56		**261.83996**		262.1
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.115	↗	0	↗	0.124	↗	2.893	↗	2.906
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−2.9415	↘	−3.0075	↘	−3.0703	↘	−3.4607	↗	−3.4599

In the degenerate In- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{In}) = 2.5136974 \times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	201.78		**206.175403**		210.599		**280.55571**		280.84
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.124	↗	0	↗	0.132	↗	3.100	↗	3.114
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−3.1518	↘	−3.2225	↘	−3.2896	↘	−3.7081	↗	−3.7071

In the degenerate Cd- X(x) – alloy, for $N = 2 \times N_{CDp}(r_{In}) = 2.842425 \times 10^{19}\ cm^{-3}$, one gets:

T(K)	↗	219.01		**223.779792**		228.59		**304.5111**		304.82
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-2}\frac{V}{K}\right)$		−0.134	↗	0	↗	0.144	↗	3.365	↗	3.380
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-2}V)$		−3.4209	↘	−3.4977	↘	−3.5706	↘	−4.0247	↗	−4.0237

Table 6n: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_n decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_n \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563 \times 10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.} = 1$, (ii) for $\xi_n = 1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322 \times 10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105 \times 10^{-4}\frac{V}{K}$, and $1.657 \times 10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_n \simeq 1.8138$, $(ZT)_{Mott} = 1$.

For x=0,

In the degenerate P- X(x) – alloy, for T= **15.5943916 K**, one gets:

$N(10^{16}cm^{-3})$	↘	2.5488		**2.507675**		2.4683		**2.0437298**		2.04253
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.955	↗	0	↗	0.980	↗	17.231	↗	17.294
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663

$Pt\,(10^{-3}V)$		−2.436	↘	−2.437	↗	−2.436	↗	−2.0611	↗	−2.0585

In the degenerate As- X(x) – alloy, for T= **16.244123 K**, one gets

$N(10^{16}cm^{-3})$	↘	2.7098		**2.6660176**		2.6242		**2.1727774**		2.1715
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−0.996	↗	0	↗	1.020	↗	17.949	↗	18.015
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−2.537	↘	−2.539	↗	−2.537	↗	−2.1470	↗	−2.1442

In the degenerate Sb- X(x) – alloy, for T= **19.917762 K**, one gets:

$N(10^{16}cm^{-3})$	↘	3.6792		**3.619757**		3.563		**2.95006533**		2.94832
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.221	↗	0	↗	1.250	↗	22.008	↗	22.089
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−3.111	↘	−3.113	↗	−3.111	↗	−2.6325	↗	−2.6291

In the degenerate Sn- X(x) – alloy, for T=**21.8267041 K**, one gets:

$N(10^{16}cm^{-3})$	↘	4.2206		**4.152416**		4.0873		**3.384177**		3.38217
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.338	↗	0	↗	1.370	↗	24.117	↗	24.207
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−3.409	↘	−3.411	↗	−3.409	↗	−2.8848	↗	−2.8811

For x=0.5,

In the degenerate P- X(x) – alloy, for T=**27.197762 K**, one gets:

$N(10^{17}cm^{-3})$	↘	1.2096		**1.0450608**		1.0287		**0.85171401**		0.85121
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**– 1.322**	↗	– 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.668	↗	0	↗	1.705	↗	30.052	↗	30.163
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−4.2483	↘	−4.2510	↗	−4.2483	↗	−3.5947	↗	−3.5901

In the degenerate As- X(x) – alloy, for T= **28.330942 K**, one gets:

$N(10^{17}cm^{-3})$	↘	1.1293		**1.1110493**		1.0936		**0.90549396**		0.90496
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998

$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−1.737	↗	0	↗	1.781	↗	31.304	↗	31.420
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−4.4253	↘	−4.4281	↗	−4.4253	↗	−3.7445	↗	−3.7397

In the degenerate Sb- X(x) – alloy, for T=**34.738038 K**, one gets:

$N(10^{17}cm^{-3})$	↘	1.5333		**1.5085154**		1.4848		**1.2294248**		1.228695
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.130	↗	0	↗	2.187	↗	38.384	↗	38.526
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−5.4261	↘	−5.4295	↗	−5.4261	↗	−4.5913	↗	−4.5854

In the degenerate Sn- X(x) – alloy, for T=**38.067375 K** one gets:

$N(10^{17}cm^{-3})$	↘	1.7589		**1.7304984**		1.7033		**1.41033872**		1.409501
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.332	↗	0	↗	2.396	↗	42.062	↗	42.219
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−5.9461	↘	−5.9499	↗	−5.9461	↗	−5.0314	↗	−5.0249

For x=1,

In the degenerate P- X(x) – alloy, for T=**42.978508 K**, one gets:

$N(10^{17}cm^{-3})$	↘	3.2238		**3.1717192**		3.1219		**2.5849191**		2.5834
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.634	↗	0	↗	2.703	↗	47.489	↗	47.664
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\,(10^{-3}V)$		−6.7132	↘	−6.7175	↗	−6.7132	↗	−5.6805	↗	−5.6732

In the degenerate As- X(x) – alloy, for T=**44.769183 K**, one gets:

$N(10^{17}cm^{-3})$	↘	3.4274		**3.3719916**		3.3191		**2.74813903**		2.7466
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−2.746	↗	0	↗	2.812	↗	49.467	↗	49.641

$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−6.9929	↘	−6.9974	↗	−6.9929	↗	−5.9171	↗	−5.9100

In the degenerate Sb- X(x) − alloy, for T=**54.89382 K**, one gets:

$N(10^{17}cm^{-3})$	↘	4.6535		**4.5782858**		4.5064		**3.7312567**		3.72905
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−3.366	↗	0	↗	3.451	↗	60.655	↗	60.879
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−8.5744	↘	−8.5799	↗	−8.5744	↗	−7.2553	↗	−7.2460

In the degenerate Sn- X(x) − alloy, for T=**60.15491 K**, one gets:

$N(10^{17}cm^{-3})$	↘	5.3383		**5.2519956**		5.1695		**4.2803233**		4.2778
ξ_n	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**− 1.322**	↗	− 1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2\left(10^{-4}\frac{V}{K}\right)$		−3.690	↗	0	↗	3.783	↗	66.468	↗	66.713
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-3}V)$		−9.3962	↘	−9.4022	↗	−9.3962	↗	−7.9507	↗	−7.9405

Table 6p: Here, for a given T and with decreasing N, the reduced Fermi-energy ξ_p decreases, and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that with increasing T: (i) for $\xi_p \simeq 1.8138$, while the numerical results of S present a same minimum $(S)_{min.}\left(\simeq -1.563\times10^{-4}\frac{V}{K}\right)$, those of ZT show a same maximum $(ZT)_{max.}=1$, (ii) for $\xi_p=1$, those of S, ZT, $(ZT)_{Mott}$, VC1, and T_s present the same results: $-1.322\times10^{-4}\frac{V}{K}$, 0.715, 3.290, $-1.105\times10^{-4}\frac{V}{K}$, and $1.657\times10^{-4}\frac{V}{K}$, respectively, and (iii) for $\xi_p \simeq 1.8138$, $(ZT)_{Mott}=1$.

For x=0,

In the degenerate Ga- X(x) − alloy, for T=**71.621813 K**, one gets:

$N(10^{18}cm^{-3})$	↘	2.3226		**2.285126**		2.2492		**1.8623546**		1.8613
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.044	↗	0	↗	0.045	↗	0.791	↗	0.794
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
Pt $(10^{-2}V)$		−1.1187	↘	−1.1194	↗	−1.1187	↗	−0.9466	↗	−0.9455

In the degenerate Mg- X(x) − alloy, for T= **80.037437** K, one gets:

$N(10^{18}cm^{-3})$	↘	2.7438		**2.6994914**		2.6571		**2.2000582**		2.1988
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713

$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.049	↗	0	↗	0.050	↗	0.884	↗	0.887
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.25019	↘	−1.25098	↗	−1.25019	↗	−1.0578	↗	−1.0565

In the degenerate In- X(x) − alloy, for T=**85.758335 K**, one gets:

$N(10^{18}cm^{-3})$	↘	3.0432		**2.9940338**		2.947		**2.4401073**		2.4387
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.052	↗	0	↗	0.054	↗	0.947	↗	0.951
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.339	↘	−1.340	↗	−1.339	↗	−1.1335	↗	−1.1320

In the degenerate Cd- X(x) − alloy, for T=**93.080853 K**, one gets:

$N(10^{18}cm^{-3})$	↘	3.441		**3.3855772**		3.3325		**2.7592112**		2.7576
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.057	↗	0	↗	0.058	↗	1.028	↗	1.032
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.454	↘	−1.455	↗	−1.454	↗	−1.2302	↗	−1.2287

For x=0.5,

In the degenerate Ga- X(x) − alloy, for T=**114.33559** K, one gets:

$N(10^{18}cm^{-3})$	↘	7.4229		**7.302887**		7.18814		**5.9517791**		5.9483
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.070	↗	0	↗	0.072	↗	1.263	↗	1.268
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.7859	↘	−1.7871	↗	−1.7859	↗	−1.5112	↗	−1.5093

In the degenerate Mg- X(x) − alloy, for **T=127.770112 K**, one gets:

$N(10^{18}cm^{-3})$	↘	8.7689		**8.6271302**		8.4915		**7.0310238**		7.0269
ξ_p	↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$		−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT		0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$	↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$		−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$		−0.078	↗	0	↗	0.080	↗	1.412	↗	1.417
$T_s\left(10^{-4}\frac{V}{K}\right)$		−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$		−1.9958	↘	−1.9970	↗	−1.9958	↗	−1.6887	↗	−1.6866

In the degenerate In- X(x) − alloy, for T=**136.90284 K**, one gets:

$N(10^{18}cm^{-3})$ ↘	9.7257		**9.5684394**		9.4181		**7.7981815**		7.7936
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.084	↗	0	↗	0.086	↗	1.513	↗	1.518
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.1384	↘	−2.1398	↗	−2.1384	↗	−1.8094	↗	−1.8071

In the degenerate Cd- X(x) − alloy, for T=**148.592355 K**, one gets:

$N(10^{19}cm^{-3})$ ↘	1.09975		**1.0819748**		1.06498		**0.8817985**		0.8813
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.091	↗	0	↗	0.093	↗	1.642	↗	1.648
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.3210	↘	−2.3225	↗	−2.3210	↗	−1.9639	↗	−1.9615

For x=1,

In the degenerate Ga- X(x) − alloy, for T=**172.18917** K, one gets:

$N(10^{19}cm^{-3})$ ↘	1.95005		**1.9185205**		1.8884		**1.56357483**		1.56265
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.106	↗	0	↗	0.108	↗	1.902	↗	1.909
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−2.6896	↘	−2.6913	↗	−2.6896	↗	−2.2758	↗	−2.2729

In the degenerate Mg- X(x) − alloy, for T=**192.42153** K, one gets:

$N(10^{19}cm^{-3})$ ↘	2.3035		**2.2664086**		2.2308		**1.8471001**		1.84601
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.117	↗	0	↗	0.121	↗	2.126	↗	2.134
$T_s\left(10^{-4}\frac{V}{K}\right)$	−0.092	↗	0	↗	0.094	↗	**1.657**	↗	1.663
$Pt\ (10^{-2}V)$	−3.0056	↘	−3.0075	↗	−3.0056	↗	−2.5432	↗	−2.5400

In the degenerate In- X(x) − alloy, for T=**206.175403** K, one gets:

$N(10^{19}cm^{-3})$ ↘	2.55501		**2.51636974**		2.4742		**2.04863793**		2.04742
ξ_p ↘	1.880		**1.8138**		1.750		**1**		0.998
$S\left(10^{-4}\frac{V}{K}\right)$	−1.562	↘	**−1.563**	↗	−1.562	↗	**−1.322**	↗	−1.320
ZT	0.999	↗	**1**	↘	0.999	↘	**0.715**	↘	0.713
$(ZT)_{Mott}$ ↗	0.931		**1**		1.074		**3.290**		3.306
$VC1\left(10^{-4}\frac{V}{K}\right)$	−0.061	↗	0	↗	0.063	↗	**1.105**	↗	1.109
$VC2(10^{-2}V)$	−0.126	↗	0	↗	0.129	↗	2.278	↗	2.286

$T_s\left(10^{-4}\frac{V}{K}\right)$ −0.092 ↗ 0 ↗ 0.094 ↗ **1.657** ↗ 1.663
Pt $(10^{-2}V)$ −3.2204 ↘ −3.2225 ↗ −3.2204 ↗ −2.7250 ↗ −2.7215

In the degenerate Cd- X(x) − alloy, for T=**223.779792** K, one gets:

$N(10^{19}cm^{-3})$ ↘ 2.8891 **2.842425** 2.7978 **2.3165476** 2.04742
ξ_p ↘ 1.880 **1.8138** 1.750 **1** 0.998
$S\left(10^{-4}\frac{V}{K}\right)$ −1.562 ↘ **−1.563** ↗ −1.562 ↗ **−1.322** ↗ −1.320
ZT 0.999 ↗ **1** ↘ 0.999 ↘ **0.715** ↘ 0.713
$(ZT)_{Mott}$ ↗ 0.931 **1** 1.074 **3.290** 3.305
$VC1\left(10^{-4}\frac{V}{K}\right)$ −0.061 ↗ 0 ↗ 0.063 ↗ **1.105** ↗ 1.109
$VC2(10^{-2}V)$ −0.137 ↗ 0 ↗ 0.141 ↗ 2.473 ↗ 2.481
$T_s\left(10^{-4}\frac{V}{K}\right)$ −0.092 ↗ 0 ↗ 0.094 ↗ **1.657** ↗ 1.663
Pt $(10^{-2}V)$ −3.4954 ↘ −3.4977 ↗ −3.4954 ↗ −2.9577 ↗ −2.9540