



**GLOBAL EINSTEIN -AND- VAN CONG RELATIONS, AND NEW
ELECTRICAL THERMOELECTRIC LAWS IN N(P)-TYPE
DEGENERATE, COMPENSATED AND VISCOUS HELIUM- FERMI
LIQUID, ENHANCED BY OUR STATIC DIELECTRIC CONSTANT
LAW, CONDUCTIVITY MODEL, AND ESPECIALLY ACCURATE
FERMI ENERGY EXPRESSION (XII)**

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ABSTRACT

In degenerate, compensated and viscous **n(p) – type Helium – Fermi Liquid (He – FL)**, the Global Einstein -and- Van Cong relations, as those given in Equations(25,32), and electrical thermoelectric laws, enhanced by: our static dielectric constant law given in Equations (5,6), our electrical conductivity model in Eq.,^[19] and especially our accurate Fermi energy expression in Eq.,^[17] are now investigated, by basing on the same physical model and the mathematical treatment method, as those used in our recent works.^[1-5] Their numerical results are obtained and given in next Tables in Appendix 1, suggesting an equivalence between degeneracy-and-viscosity concept, given in this **He – FL**. Here, the well-known “local” Einstein relation, being valid only at the lowest degeneracy and lowest viscosity, is now globalized by our “Global” Einstein -and- Van Cong relations, valid at any degeneracy and viscosity, as showed in Equations.(25,32)

KEYWORDS: Conductivity, Mobility, Viscosity coefficient, Diffusion coefficient, Activation energy, Fermi energy.

INTRODUCTION

In the **degenerate, compensated and viscous**

n(p) – type Helium – Fermi Liquid (He – FL), the Global Einstein -and- Van Cong relations, as those given in Equations (25, 32), and various optical-electrical-thermoelectric laws, enhanced by : (i) our static dielectric constant law given in Equations (5, 6), (ii) our electrical conductivity (σ)-model obtained in Eq. (19), and especially (iii) our accurate Fermi energy expression in Eq. (17), with a precision of the order of 2.11×10^{-4} , affecting strongly all the expressions of electrical coefficients, are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in References.^[1-18] Then, some important remarks can be summarized as follows.

(1) As observed in our recent work.^[1] the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N_{CDn(CDp)}^{EBT}$, being obtained with a precision of the order of 3.5447×10^{-7} , as that observed in **Tables 1A and 1B in Appendix 1**, and also obtained in our recent work.^[1] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N_{CDn(CDp)}^{EBT}$, N being the total impurity density, as that observed in the compensated **He-FL**.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (13), is valid at any N^* .

Therefore, by basing on this σ -expression, the Global Einstein -and- Van Cong relations defined in Equations (25, 32) and various electrical thermoelectric laws are determined. Finally, the numerical results of the electrical thermoelectric coefficients are obtained, discussed and reported in Tables (1-7) in Appendix 1, suggesting an equivalence between degeneracy-and-viscosity concept, given in this **He-FL**.

Our Static Dielectric Constant Law and Generalized Mott Criterium in the Metal-Insulator Transition (MIT)

First of all, in the n(p)-type degenerate **He-FL**.^[1] we denote : the donor (acceptor) d(a)-radius by $r_{d(a)}$, with the corresponding intrinsic one by: $r_{do(ao)} = r_{He} = \mathbf{0.46\ nm\ (covalent)}$, the static dielectric constant by: $\epsilon_0 = 1.055$,^[3] and the two values of the unperturbed **reduced** effective electron (hole) mass in conduction (valence) bands by: $\frac{m_{c(v)}}{m_0} = \mathbf{3}$ and $\mathbf{6.2}$ [2], $m_0 (= 9.109558 \times 10^{-28}\ g)$ being the free electron mass, according to those given Tables [1A, 3A, 4A, 5A, 6A] and [1B, 3B, 4B, 5B, 6B] in Appendix 1, respectively.

Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$$E_{do(ao)} = \frac{13.6 \times \frac{m_{c(v)}}{m_0}}{[\epsilon_0]^2} \text{ eV}, \quad (1)$$

and then, the isothermal bulk modulus, by :

$$B_{do(ao)} \equiv \frac{E_{do(ao)}}{\left(\frac{4\pi}{3}\right) \times (r_{do(ao)})^3} \left(\frac{\text{eV}}{\text{cm}^3}\right). \quad (2)$$

Effects of impurity size

Here, one shows that the effect of the size of donor (acceptor) d(a)-radius, $r_{d(a)}$, strongly affects the changes in all the energy-band-structure parameters, which can be represented by the effective relative static dielectric constant $\epsilon(r_{d(a)})$, in the following.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be given, for the impurity-atom volume $V = (4\pi/3) \times (r_{d(a)})^3$ and $V_{do(ao)} = (4\pi/3) \times (r_{do(ao)})^3$, by:

(i) the pressure p , as: $p_0(r_{do(ao)}) = 0$, according to the **p-pressure variation**,

$$[\Delta p(r_{d(a)})]_{n(p)} \equiv p(r_{d(a)}) - p_0(r_{do(ao)}) = p(r_{d(a)}), \text{ being equal to 0 at } r_{d(a)} = r_{do(ao)}, \text{ and}$$

(ii) the deformation potential energy (or the strain energy) σ , as: $\sigma_0(r_{do(ao)}) = 0$, according

$$\text{to the } \sigma - \text{strain-energy variation, } [\Delta \sigma(r_{d(a)})]_{n(p)} \equiv \sigma(r_{d(a)}) - \sigma_0(r_{do(ao)}) = \sigma(r_{d(a)}),$$

being equal to 0 at $r_{d(a)} = r_{do(ao)}$.

Further, we also shown that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) gives rise to: the increase (the decrease) in the effective donor (acceptor)-ionization energy

$E_{d(a)}(r_{d(a)})$ in the absolute values, being obtained from the effective Bohr model, and then such the compression (dilatation) is represented respectively by: $E_{d(a)}(r_{d(a)}) - E_{do(ao)} \equiv \pm [\Delta\sigma(r_{d(a)})]_{n(p)}$.

Furthermore, for $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the two important equations, used to determine $[\Delta p(r_{d(a)})]_{n(p)}$ and $[\Delta\sigma(r_{d(a)})]_{n(p)}$, are defined by: $\frac{dp}{dv} = \pm \frac{B_{do(ao)}}{v}$ and $p = \pm \frac{d\sigma}{dv}$, giving raise to: $\frac{dp}{dv} = \frac{d}{dv}(\frac{d\sigma}{dv}) = \frac{B_{do(ao)}}{v}$.

Then, by an integration, one gets:

$$[\Delta p(r_{d(a)})]_{n(p)} = \pm B_{do(ao)} \times \ln\left(\frac{v}{v_{do(ao)}}\right) = \pm B_{do(ao)} \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \text{ and}$$

$$[\Delta\sigma(r_{d(a)})]_{n(p)} = B_{do(ao)} \times (v - v_{do(ao)}) \times \ln\left(\frac{v}{v_{do(ao)}}\right) = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0. \quad (3)$$

In **Tables 1A-1B in Appendix 1**, the numerical results of the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta\sigma(r_{d(a)})]_{n(p)}$, are obtained and computed, using Equation (3), noting that those increase with increasing $r_{d(a)}$.

As noted above, one thus obtains for $r_{d(a)} \geq r_{do(ao)}$, according to the compression,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = [\Delta\sigma(r_{d(a)})]_{n(p)},$$

and for $r_{d(a)} \leq r_{do(ao)}$, according to the dilatation,

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{\varepsilon_o}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = -[\Delta\sigma(r_{d(a)})]_{n(p)}. \quad (4)$$

Therefore, from above Equations (3) and (4), one obtains the expressions, for $\varepsilon(r_{d(a)})$ and $E_{d(a)}(r_{d(a)})$, as:

(i)-for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 + \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0, \quad (5)$$

according to the increase in $E_{d(a)}(r_{d(a)})$, and

(iii)-for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)}) = \frac{\varepsilon_o}{\sqrt{1 - \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o$, with a condition,

given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, and

$$E_{d(a)}(r_{d(a)}) - E_{do(ao)} = -E_{do(ao)} \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0, \quad (6)$$

corresponding to the decrease in $E_{d(a)}(r_{d(a)})$.

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)})$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)}) \equiv \frac{\varepsilon(r_{d(a)}) \times \hbar^2}{m_{c(v)} \times q^2} = 0.53 \times 10^{-8} \text{ cm} \times \frac{\varepsilon(r_{d(a)})}{m_{c(v)}/m_o}, \quad (7)$$

where $-q$ is the electron charge; $q > 0$.

Generalized Mott Criterium in the MIT

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at $T=0$ K, $N_{CDn(CDp)}(r_{d(a)})$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as^[1]:

$$N_{CDn(CDp)}(r_{d(a)})^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = M_{n(p)}, \quad M_{n(p)} = 0.25, \quad (8)$$

depending thus on our **new $\varepsilon(r_{d(a)})$ -law**.

This result can be explained by using the definition of the reduced effective Wigner-Seitz (WS) radius $r_{sn(sp),M}$, in the Mott's criterium, being characteristic of interactions, by:

$$r_{sn(sp),M}(N = N_{CDn(CDp)}(r_{d(a)}), r_{d(a)}) \equiv \left(\frac{3}{4\pi N_{CDn(CDp)}(r_{d(a)})}\right)^{1/3} \times \frac{1}{a_{Bn(Bp)}(r_{d(a)})} = \left(\frac{3}{4\pi}\right)^{1/3} \times 4 \cong 2.4814, \quad (9)$$

for any $r_{d(a)}$ -value. Therefore, one also has :

$$N_{CDn(CDp)}(r_{d(a)})^{1/3} \times a_{Bn(Bp)}(r_{d(a)}) = 1/4 = 0.25 = M_{n(p)}, \quad (10)$$

explaining thus the existence of the Mott's criterium.

Furthermore, by using $M_{n(p)} = 0.25$, according to the empirical Heisenberg parameter $\mathcal{H}_{n(p)} = 0.47137$, as that given in our recent work,^[1] we have showed that $N_{CDn(CDp)}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail**, $N_{CDn(CDp)}^{EBT}$, with a precision of the order of 3.5447×10^{-7} , as also showed in **Tables 1A and 1B in Appendix 1**. So,

$$N_{\text{CDn(NDp)}}(r_{\text{d(a)}}) \cong N_{\text{CDn(CDp)}}^{\text{EBT}}(r_{\text{d(a)}}). \quad (11)$$

Here, it should be noted that the values of $M_{\text{n(p)}}$ and $\mathcal{H}_{\text{n(p)}}$ could be chosen such that those of $N_{\text{CDn(CDp)}}$ and $N_{\text{CDn(CDp)}}^{\text{EBT}}$ are found to be in good agreement with experimental results.

Therefore, the effective density of electrons (holes) given in parabolic conduction (valence) bands, N^* , can be defined, as that given in compensated materials:

$$N^*(N, r_{\text{d(a)}}) \equiv N - N_{\text{CDn(NDp)}}(r_{\text{d(a)}}) \cong N - N_{\text{CDn(CDp)}}^{\text{EBT}}(r_{\text{d(a)}}) \geq 0. \quad (12)$$

One notes here that, with increasing $r_{\text{d(a)}}$ and for a given N , $N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$ increases, as that observed in **Tables 1A and 1B in Appendix 1**, and therefore, $N^*(r_{\text{d(a)}})$ decreases, according to an **increasing compensation**.

In summary, as observed in our recent paper,^[1] for an increasing $r_{\text{d(a)}}$, $\varepsilon(r_{\text{d(a)}})$ decreases, while $N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$ increases, affecting strongly all the electrical-and-thermoelectric coefficients, as those observed in following Sections.

Physical model

In the **n(p)-type degenerate He-FL**, the reduced effective Wigner-Seitz (**WS**) radius $r_{\text{sn(sp)}}$, characteristic of interactions, being given in Eq. (9), in which N is replaced by N^* , is now defined by:

$$\gamma \times r_{\text{sn(sp)}}(N^*, r_{\text{d(a)}}) \equiv \frac{k_{\text{Fn(Fp)}}^{-1}}{a_{\text{Bn(Bp)}}} < 1, \quad r_{\text{sn(sp)}}(N^*, r_{\text{d(a)}}) \equiv \left(\frac{3}{4\pi N^*}\right)^{1/3} \times \frac{1}{a_{\text{Bn(Bp)}}(r_{\text{d(a)}})}, \quad \text{being}$$

proportional to $N^{*-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $k_{\text{Fn(Fp)}}(N^*) \equiv (3\pi^2 N^*)^{1/3}$ is the Fermi wave number.

Then, the ratio of the inverse effective screening length $k_{\text{sn(sp)}}$ to $k_{\text{Fn(kp)}}$ is defined by:

$$R_{\text{sn(sp)}}(N^*) \equiv \frac{k_{\text{sn(sp)}}}{k_{\text{Fn(Fp)}}} = \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{sn(sp)}}^{-1}} = R_{\text{snWS(spWS)}} + [R_{\text{snTF(spTF)}} - R_{\text{snWS(spWS)}}]e^{-r_{\text{sn(sp)}}} < 1, \quad (13)$$

being valid at any N^* .

Here, these ratios, $R_{\text{snTF(spTF)}}$ and $R_{\text{snWS(spWS)}}$, can be determined as follows.

First, for $N \gg N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $R_{\text{snTF(spTF)}}(N^*)$ is reduced to:

$$R_{\text{snTF(spTF)}}(N^*) \equiv \frac{k_{\text{snTF(spTF)}}}{k_{\text{Fn(Fp)}}} = \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{snTF(spTF)}}^{-1}} = \sqrt{\frac{4\gamma r_{\text{sn(sp)}}}{\pi}} \ll 1, \quad (14)$$

being proportional to $N^{*-1/6}$.

Secondly, for $N \ll N_{\text{CDn(NDp)}}(r_{\text{d(a)}})$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $R_{\text{snWS(snWS)}}$ is respectively reduced to:

$$R_{\text{sn(sp)WS}}(N^*) \equiv \frac{k_{\text{sn(sp)WS}}}{k_{\text{Fn}}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{d[r_{\text{sn(sp)}}^2 \times E_{\text{CE}}(N^*)]}{dr_{\text{sn(sp)}}} \right), \quad (15)$$

where $E_{\text{CE}}(N^*)$ is the majority-carrier correlation energy (CE), being determined by:

$$E_{\text{CE}}(N^*) = \frac{-0.87553}{0.0908 + r_{\text{sn(sp)}}} + \frac{\frac{0.87553}{0.0908 + r_{\text{sn(sp)}}} + \left(\frac{2[1 - \ln(2)]}{\pi^2} \right) \times \ln(r_{\text{sn(sp)}}) - 0.093288}{1 + 0.03847728 \times r_{\text{sn(sp)}}^{1.67378876}}.$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$$\frac{k_{\text{Fn(Fp)}}^{-1}}{a_{\text{Bn(Bp)}}} < \frac{\mathbb{U}_{n(p)}}{E_{\text{Fno(Fpo)}}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{\text{Fn(Fp)}}^{-1}}{k_{\text{sn(sp)}}^{-1}} \equiv R_{\text{sn(sp)}} < 1, \quad \mathbb{U}_{n(p)}(N^*, r_{\text{d(a)}}) \equiv \frac{\sqrt{2\pi} N^*}{\varepsilon(r_{\text{d(a)}})} \times \quad (16)$$

$$\text{which gives: } A_{n(p)}(N^*, r_{\text{d(a)}}) = \frac{E_{\text{Fno(Fpo)}}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{\text{d(a)}})}, \quad E_{\text{Fno(Fpo)}}(N^*, r_{\text{d(a)}}) \equiv \frac{\hbar^2 \times k_{\text{Fn(Fp)}}^2(N^*)}{2 \times m_{\text{c(v)}}}.$$

Here, one remarks that: (i) the generalized Thomas-Fermi energy $\mathbb{U}_{n(p)}(N^*, r_{\text{d(a)}})$ can thus be approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{\text{d(a)}})}$, C being a constant, and (ii) $\mathbb{U}_{n(p)}(r_{\text{d(a)}})$ increases with increasing $r_{\text{d(a)}}$ and for a given N , since $\varepsilon(r_{\text{d(a)}})$ decreases, as given in Ref.^[3]

Fermi Energy and Fermi-Dirac Distribution Function

Accurate Fermi Energy Expression

Here, for presentation simplicity, we change the sign of all various parameters, given in the **p-type He-FL**, in order to obtain the same one, as that given in the **n-type He-FL**, according to the reduced Fermi energy $E_{\text{Fn(Fp)}}$, $\xi_{n(p)}(N^*, r_{\text{d(a)}}, T) \equiv \frac{E_{\text{Fn(Fp)}}(N^*, r_{\text{d(a)}}, T)}{k_{\text{B}}T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N^*, r_{\text{d(a)}}, T)$, the reduced Fermi energy $\xi_{n(p)}(N^*, r_{\text{d(a)}}, T)$ or the Fermi energy $E_{\text{Fn(Fp)}}(N^*, r_{\text{d(a)}}, T)$, obtained in our previous paper,^[9] obtained with a precision of the order of 2.11×10^{-4} , is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{\text{Fn(Fp)}}(u)}{k_{\text{B}}T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}, \quad A = 0.0005372 \text{ and } B = 4.82842262, \quad (17)$$

where u is the reduced electron density, $u(N^*, r_{d(a)}, T) \equiv \frac{N^*}{N_{c(v)}(T)}$, $N_{c(v)}(T) = 2 \times \left(\frac{m_{c(v)} \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}} (\text{cm}^{-3})$, $F(u) = au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8} \left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920} \left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq \ln(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2} \left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case ($u \ll 1$), one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times \ln(u)$ as $u \rightarrow 0$, **the limiting non-degenerate condition**, and in the very degenerate case ($u \gg 1$), one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}} \left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k_{Fn(Fp)}^2 (N^*)}{2 \times m_{c(v)} \times m_o}$ as $u \rightarrow \infty$, **the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, as $T \rightarrow 0$ K, since $u^{-1} \rightarrow 0$, Eq. (11) is reduced to: $E_{Fn(Fp)}(N^*) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2 (N^*)}{2 \times m_{c(v)}}$, being proportional to $(N^*)^{2/3}$. Further, at $T=0$ K and $N^* = 0$, being physical conditions, given for the metal-insulator transition (**MIT**).

In the following, it should be noted that all the optical and electrical-and-thermoelectric properties strongly depend on such an accurate expression of $\xi_{n(p)}(N^*, r_{d(a)}, T)^{[9]}$

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1 + e^\gamma)^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

Thus, the average of E^p , calculated using the FDDF-method, as developed in our previous works.^[1, 6] is found to be given by:

$$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E_{Fn(Fp)}^p \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE, \quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^\gamma}{(1+e^\gamma)^2}.$$

Further, it should be noted that, at $T=0$ K, $-\frac{\partial f}{\partial E} = \delta(E - E_{Fn(Fp)})$, $\delta(E - E_{Fn(Fp)})$ being the Dirac delta (δ)-function. Therefore, $G_p(E_{Fn(Fp)}) = 1$.

Then, at low T , by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$$G_p(E_{Fn(Fp)}) \equiv 1 + E_{Fn(Fp)}^{-p} \times \int_{-\infty}^{\infty} \frac{e^\gamma}{(1+e^\gamma)^2} \times (k_B T \gamma + E_{Fn(Fp)})^p d\gamma = 1 + \sum_{\mu=1,2,\dots}^p C_p^\beta \times (k_B T)^\beta \times E_{Fn(Fp)}^{-\beta} \times I_\beta, \text{ where } C_p^\beta \equiv p(p-1) \dots (p-\beta+1)/\beta! \quad \text{and the integral } I_\beta \text{ is given by:}$$

$$I_\beta = \int_{-\infty}^{\infty} \frac{\gamma^\beta \times e^\gamma}{(1+e^\gamma)^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^\beta}{(e^{\gamma/2} + e^{-\gamma/2})^2} d\gamma, \text{ vanishing for odd values of } \beta. \text{ Then, for even values of } \beta = 2n, \text{ with } n=1, 2, \dots, \text{ one obtains:}$$

$$I_{2n} = 2 \int_0^{\infty} \frac{\gamma^{2n} \times e^\gamma}{(1+e^\gamma)^2} d\gamma.$$

Now, using the identity $(1 + e^\gamma)^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t} dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1} \pi^{2n} |B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n} - 2) \times \pi^{2n} \times |B_{2n}|$. From the above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case ($\xi_{n(p)} > 1$) the following ratio:

$$G_{p>1} \left(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}} < 1 \right) \equiv \frac{\langle E^p \rangle_{FDDF}}{E_{Fn(Fp)}^p} = 1 + \sum_{n=1}^p \frac{p(p-1) \dots (p-2n+1)}{(2n)!} \times (2^{2n} - 2) \times |B_{2n}| \times y^{2n},$$

which can thus be approximated by:

$$G_{p>1}(y < 1) \simeq 1 + p(p-1) \times |B_2| \times y^2, \quad B_2 = \frac{1}{6}. \quad (18)$$

Then, some usual results of $G_{p>1}(y < 1)$ are given in the **Table 2 in Appendix 1**, suggesting that, with increasing T (or decreasing T) and for given (N, r_d) , since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases).

Electrical Properties

Now, the electrical conductivity σ can be defined and expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m_{c(v)}}$, k being the wave number, as:

$$\sigma(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{Sn(sp)}} \times [k \times a_{Bn(Bp)}] \times \left(\frac{E_k}{U_{n(p)}} \right)^{\frac{1}{2}} \left(\frac{1}{\Omega \times cm} \right), \text{ which is thus proportional to } E_k^2,$$

where $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5} \text{ ohm}^{-1}$ and $U_{n(p)}(N^*, r_{d(a)})$ is determined in Eq. (16).

Then, we obtain: $\langle E^2 \rangle_{\text{FDDF}} \equiv G_2(y = \frac{\pi k_B T}{E_{\text{Fn(Fp)}}}) \times E_{\text{Fn(Fp)}}^2$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N^*, r_{d(a)}, T)$, with $y \equiv \frac{\pi}{\xi_{n(p)}}$, $\xi_{n(p)} = \xi_{n(p)}(N^*, r_{d(a)}, T)$ for simplicity. Therefore, for $E_{\text{Fn(Fp)}} > 0$ and $T = 0\text{K}$, $G_2(y = 0) = 1$.

Therefore, if denoting the function $H(N^*, r_{d(a)}, T)$ by:

$$H(N^*, r_{d(a)}, T) =$$

$$\left[\frac{k_{\text{Fn(Fp)}}(N^*)}{R_{\text{sn(sp)}}(N^*)} \times [k_{\text{Fn(Fp)}}(N^*) \times a_{\text{Bn(Bp)}}(r_{d(a)})] \times \sqrt{A_{n(p)}(N^*) = \frac{E_{\text{Fn(Fp)}}(N^*)}{U_{n(p)}(N^*, r_{d(a)})}} \right] \times G_2(N^*, r_{d(a)}, T),$$

which can be approximately expressed in terms of: $E_{\text{Fn(Fp)}}^2(N^*) \times G_2(N^*, r_{d(a)}, T) \times \frac{\sqrt{\varepsilon(r_{d(a)})}}{(N^*)^{\frac{1}{4}}}$. Thus, with increasing $r_{d(a)}$ and for given T and N , the function $H(r_{d(a)})$ therefore decreases since $\varepsilon(r_{d(a)})$ decreases, as noted in Ref.^[3]

Therefore, the electrical conductivity σ may be determined by:

$$\sigma(N^*, r_{d(a)}, T) = \frac{q^2}{\pi \times \hbar} \times H(N^*, r_{d(a)}, T) \times \left(\frac{E_{\text{Fn(Fp)}}}{E_{\text{Fn(Fp)}}} \right)^2 \left(\frac{1}{\Omega \times \text{cm}} \right), \quad (19)$$

noting that, at $T = 0\text{K}$, it can thus be approximately expressed in terms of

$$E_{\text{Fn(Fp)}}^2(N^*, r_{d(a)}, x) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{1}{4}}}, \text{ being proportional to: } \sqrt{\varepsilon(r_{d(a)}, x)} \times (N^*)^{\frac{13}{12}}, \text{ for } E_{\text{Fn(Fp)}} > 0 \text{ and } T = 0\text{K}.$$

Electrical Properties

Here, if denoting, for majority electrons (holes), the thermal conductivity by:

$$\sigma_{\text{Th.}}(N^*, r_{d(a)}, T) \text{ in } \frac{\text{W}}{\text{cm} \times \text{K}}, \text{ and the Lorenz number } L \text{ by:}$$

$$L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q} \right)^2 = 2.4429637 \left(\frac{\text{W} \times \text{ohm}}{\text{K}^2} \right) = 2.4429637 \times 10^{-8} (\text{V}^2 \times \text{K}^{-2}), \text{ then the well-}$$

known Wiedemann-Frank law states that the ratio, $\frac{\sigma_{\text{Th.}}}{\sigma}$, is proportional to the temperature

$T(\text{K})$, as:

$$\frac{\sigma_{\text{Th.}}(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} = L \times T. \quad (20)$$

Further, the resistivity is found to be given by: $\rho(N^*, r_{d(a)}, T) \equiv 1/\sigma(N^*, r_{d(a)}, T)$, noting again that $N^* \equiv N - N_{\text{CDn(NDp)}}(r_{d(a)}, x)$.

New Electrical Coefficients

The relaxation time τ is related to by:^[3]

$\tau(N^*, r_{d(a)}, T) \equiv \sigma(N^*, r_{d(a)}, T) \times \frac{m_{c(v)}}{q^2 \times N^*}$. Therefore, the mobility μ is given by:

$$\mu(N^*, r_{d(a)}, T) = \frac{q \times \tau(N^*, r_{d(a)}, T)}{m_{c(v)} \times m_0} = \frac{\sigma(N^*, r_{d(a)}, T)}{q \times N^*} \left(\frac{\text{cm}^2}{\text{V} \times \text{s}} \right), \quad (21)$$

being expressed in terms of $\frac{\sigma(N^*, r_{d(a)}, T)}{N^*}$. Further, as noted in above Eq. (19) for $\sigma(N^*, r_{d(a)}, T)$ and at $T=0\text{K}$, $\mu(N^*, r_{d(a)}, T)$ can thus be expressed in terms of:

$$E_{\text{Fno(Fp)}}^2(N^*, r_{d(a)}, x) \times \frac{\sqrt{\varepsilon(r_{d(a)})}}{N^{* \frac{5}{4}}}.$$

Then, from the well-known idea of Stokes, Einstein, Sutherland and Reynolds, we can define our viscosity coefficient, $\mathbb{V}(N^*, r_{d(a)}, T)$, and its reduced one, $R\mathbb{V}(N^*, r_{d(a)}, T)$, by:

$$\frac{\mathbb{V}(N^*, r_{d(a)}, T)}{q} \equiv \frac{1}{6\pi \times \mu(N^*, r_{d(a)}, T) \times R_{\text{WS}}(N^*)} \left(\frac{\text{V}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2} \right), \quad R\mathbb{V}(N^*, r_{d(a)}, T) \equiv \frac{\mathbb{V}(N^*, r_{d(a)}, T)}{\mathbb{V}(N^*, r_{d(a)}, T=0\text{K})}, \quad (22)$$

where $R_{\text{WS}}(N^*) \equiv \left(\frac{3}{4\pi N^*} \right)^{1/3}$ is the effective Wigner-Seitz radius, decreasing with increasing N^* .

Further, as noted above for $\mu(N^*, r_{d(a)}, T)$, $\mathbb{V}(N^*, r_{d(a)}, T)$ can thus be expressed in terms of

$$\frac{(N^*)^{\frac{19}{12}} \times [N_{\text{CDn(NDp)}}(r_{d(a)}, x)]^{\frac{1}{6}}}{E_{\text{Fn(Fp)}}^2(N^*, r_{d(a)}, T) \times G_2(N^*, r_{d(a)}, T)}, \quad \text{giving raise to some concluding remarks as follows.}$$

—**With decreasing T** and for given N and $r_{d(a)}$, and —**with increasing N** and for given T and $r_{d(a)}$, corresponding to the high degeneracy conditions, $\mathbb{V}(N)$ **thus increases, suggesting an equivalence between degeneracy-and-viscosity concept, and being used to explain the numerical results of $\mathbb{V}$, given next Tables.**

Then, the activation energy, $AE(N^*, r_{d(a)}, T)$, can be defined as:^[17]

$$AE(N^*, r_{d(a)}, T) \equiv k_B T \times \text{Ln} \left(R\mathbb{V}(N^*, r_{d(a)}, T) \right) \leq 0 \text{ eV}, \quad (23)$$

according to the reduced activation energy, $RAE(N^*, r_{d(a)}, T)$, given by:

$$RAE(N^*, r_{d(a)}, T) \equiv \frac{AE(N^*, r_{d(a)}, T)}{k_B T} \equiv \text{Ln} \left(R\mathbb{V}(N^*, r_{d(a)}, T) \right) \leq 0.$$

Furthermore, the Hall factor, $r(N^*, r_{d(a)}, T)$, is defined by:

$r(N^*, r_{d(a)}, T) \equiv \frac{\langle \tau^2 \rangle_{FDDDF}}{[\langle \tau \rangle_{FDDDF}]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N^*, r_{d(a)}, T)} = \frac{\pi k_B T}{E_{Fn(Fp)}(N^*, r_{d(a)}, T)}$, and the Hall mobility is found to be given by:

$$\mu_H(N^*, r_{d(a)}, T) \equiv \mu(N^*, r_{d(a)}, T) \times r(N^*, r_{d(a)}, T) \left(\frac{\text{cm}^2}{V \times s} \right). \quad (24)$$

Global Einstein -and- Van Cong relations

By taking into account Equations (21, 22) and for **any** N^* , $r_{d(a)}$, **and** T , our **global relations** are found to be defined by:

$$\begin{aligned} \mathbb{R}(N^*, r_{d(a)}, T) &\equiv \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \equiv \frac{D(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} \times q \times N^* \equiv D(N^*, r_{d(a)}, T) \times \frac{V(N^*, r_{d(a)}, T)}{q} \times \\ 6\pi \times R_{WS}(N^*) &\equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right) = \sqrt{\frac{3 \times L}{\pi^2}} \times T \times \left(u \frac{d\xi_{n(p)}(u)}{du} \right), \quad \frac{k_B}{q} = \\ &\sqrt{\frac{3 \times L}{\pi^2}}, \end{aligned} \quad (25)$$

where $D(N^*, r_{d(a)}, T)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (17), the mobility $\mu(N^*, r_{d(a)}, T)$ is determined in Eq. (21), the conductivity $\sigma(N^*, r_{d(a)}, T)$ is determined in Eq. (19), and the viscosity coefficient $V(N^*, r_{d(a)}, T)$ is defined in Eq. (22). By differentiating this function $\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)}$, with respect to u , being defined in Eq. (11), one obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (24) can also be rewritten as: $\mathbb{R}(u) = \frac{k_B \times T}{q} \times$

$$u \frac{V'(u) \times W(u) - V(u) \times W'(u)}{W^2(u)} \quad \text{where } W'(u) = ABu^{B-1} \text{ and } V'(u) = u^{-1} + 2^{-\frac{3}{2}} e^{-du} (1 - du) + \frac{2}{3} Au^{B-1} F(u) \left[\left(1 + \frac{3B}{2} \right) + \frac{4}{3} \times \frac{bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}}}{1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}} \right].$$

One remarks that: (i) as $u \rightarrow 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] = 1$, and therefore: $\mathbb{R}(u \rightarrow 0) = \frac{k_B \times T}{q}$, being a **well-known local (u → 0) relation given by Einstein**, and (ii) as $u \rightarrow \infty$, one has: $W^2 \approx A^2 u^{2B}$ and $u[V' \times W - V \times W'] = \frac{2}{3} au^{2/3} A^2 u^{2B}$, and therefore, in this **highly degenerate case** and at $T=0K$, our relation (24) is reduced to a correct result: $\mathbb{R}(N^*, r_{d(a)}, x, T = 0K) = \frac{2}{3} E_{Fno(Fpo)}(N^*)/q$.

It should be noted that the well-known “local” Einstein relation, being valid only at the lowest degeneracy (or $u \rightarrow 0$), is now globalized by our “Global” Einstein -and- Van Cong relations (25), valid at any degeneracy (or any u).

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (25) can be rewritten as:

$$\mathbb{R}_{(VC)}(N^*, r_{d(a)}, x, T = 0K) \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}} + 2cu^{-\frac{8}{3}} \right)}{\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}} \right)} \right],$$

where $a = [3\sqrt{\pi}/4]^{2/3}$, $b = \frac{1}{8} \left(\frac{\pi}{a} \right)^2$ and $c = \frac{62.3739855}{1920} \left(\frac{\pi}{a} \right)^4$.

Then, in **Tables 3A and 3B in Appendix 1**, for given $r_{d(a)}$, and $T=(4.2 \text{ K and } 300 \text{ K})$, the numerical results of $\mathbb{V}$, μ and D , expressed as functions of N , are obtained by using Equations (22, 21 and 25).

Further, in **Tables 4A and 4B in Appendix 1**, for given $r_{d(a)}$ and N , the numerical results of Fermi energy $E_{Fn(Fp)}(T=0K)$ (eV) [or reduced Fermi energy $\xi_{n(p)}(N^*, r_d, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_{d(a)}, T)$, expressed in $\left(\frac{\text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2} \right)$, are obtained, as functions of T , by using Equations (17, 22), respectively.

Thermoelectric Coefficients

Then, from Eq. (19), obtained for $\sigma(N^*, r_{d(a)}, T)$, the well-known Mott definition for the thermoelectric power or for the Seebeck coefficient, S , is found to be given by:

$$S(N^*, r_{d(a)}, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times k_B T \times \left. \frac{\partial \ln \sigma}{\partial E} \right|_{E=E_{Fn(Fp)}} = \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{\partial \ln \sigma(\xi_{n(p)})}{\partial \xi_{n(p)}}.$$

$$= -2\sqrt{L} \times \frac{\sqrt{ZT_{\text{Mott}}}}{1 + ZT_{\text{Mott}}} \left(\frac{V}{K} \right) < 0, \quad ZT_{\text{Mott}} = \frac{\pi^2}{3 \times \xi_{n(p)}^2},$$

Then, using Eq. (17), for the degenerate case, $\xi_{n(p)} \geq 0$, one gets, by putting

$$Y_{\text{Sb}}(N^*, r_{d(a)}, T) \equiv \left[1 - \frac{y^2}{3 \times G_2 \left(y = \frac{\pi}{\xi_{n(p)}} \right)} \right],$$

$$S(N^*, r_{d(a)}, T) \equiv \frac{-\pi^2}{3} \times \frac{k_B}{q} \times \frac{2Y_{\text{Sb}}}{\xi_{n(p)}} = -\sqrt{\frac{3 \times L}{\pi^2}} \times \frac{2 \times \xi_{n(p)}}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2} \right)} = -2\sqrt{L} \times \frac{\sqrt{ZT_{\text{Mott}}}}{1 + ZT_{\text{Mott}}} \left(\frac{V}{K} \right) <$$

$$0, \quad ZT_{\text{Mott}} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}, \quad (26) \quad \text{according to:}$$

$$\frac{\partial S}{\partial \xi_{n(p)}} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{\frac{3 \times \xi_{n(p)}^2}{\pi^2} - 1}{\left(1 + \frac{3 \times \xi_{n(p)}^2}{\pi^2} \right)^2} = \sqrt{\frac{3 \times L}{\pi^2}} \times 2 \times \frac{ZT_{\text{Mott}} \times [1 - ZT_{\text{Mott}}]}{[1 + ZT_{\text{Mott}}]^2}.$$

Here, one notes that: (i) as $\xi_{n(p)} \rightarrow +\infty$ or $\xi_{n(p)} \rightarrow +0$, one has a same limiting value of S : $S \rightarrow -0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial S}{\partial \xi_{n(p)}} = 0$, one therefore gets: a minimum $(S)_{\min.} = -\sqrt{L} \simeq -1.563 \times 10^{-4} \left(\frac{V}{K}\right)$, and (iii) at $\xi_{n(p)} = 1$ one obtains: $S \simeq -1.322 \times 10^{-4} \left(\frac{V}{K}\right)$.

Further, the figure of merit is found to be defined by:

$$ZT(N^*, r_{d(a)}, T) \equiv \frac{S^2 \times \sigma \times T}{\kappa} = \frac{S^2}{L} = \frac{4 \times ZT_{\text{Mott}}}{[1 + ZT_{\text{Mott}}]^2}. \quad (27)$$

Here, one notes that: (i) $\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 2 \times \frac{S}{L} \times \frac{\partial S}{\partial \xi_{n(p)}}$, $S < 0$, (ii) at $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, since $\frac{\partial(ZT)}{\partial \xi_{n(p)}} = 0$, one gets: a maximum $(ZT)_{\max.} = 1$, $ZT_{\text{Mott}} = 1$, and (iii) at $\xi_{n(p)} = 1$, one obtains: $ZT \simeq 0.715$ and $ZT_{\text{Mott}} = \frac{\pi^2}{3} \simeq 3.290$.

Finally, the first Van-Cong coefficient can be defined by:

$$VC1(N^*, r_{d(a)}, T) \equiv -N^* \times \frac{dS}{dN^*} \left(\frac{V}{K}\right) = N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (28)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

and the second Van-Cong coefficient as:

$$VC2(N^*, r_{d(a)}, T) \equiv T \times VC1(N^*, r_{d(a)}, T) \equiv T \times N^* \times \frac{\partial S}{\partial \xi_{n(p)}} \times -\frac{\partial \xi_{n(p)}}{\partial N^*}, \quad (29)$$

the Thomson coefficient, T_s , by:

$$T_s(N^*, r_{d(a)}, T) \equiv T \times \frac{dS}{dT} \left(\frac{V}{K}\right) = T \times \frac{\partial S}{\partial \xi_{n(p)}} \times \frac{\partial \xi_{n(p)}}{\partial T}, \quad (30)$$

being equal to 0 for $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}}$,

and the Peltier coefficient, P_t , as:

$$P_t(N^*, r_{d(a)}, T) \equiv T \times S. \quad (31)$$

Then, in **Tables 5A and 5B in Appendix 1**, for given r_d and N , the numerical results of $\xi_{n(p)}$, V , AE , $\sigma_{\text{Th.}}$, S , $VC1$, $VC2$, T_s , P_t and ZT are obtained, as functions of $T=(4.2 \text{ K}, 300 \text{ K})$, by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively.

Further, in **Tables 6A and 6B in Appendix 1**, for given $r_{d(a)}$ and $T=0K$, the numerical results of Fermi energy $E_{Fn(Fp)}$, $\mathbb{V}$, σ , μ , and D , are obtained, as functions of N , by using Equations (17, 22, 19, 21, 25), respectively.

Finally, in **Table 7 in Appendix 1**, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: $\nearrow$, decrease: $\searrow$). One notes here that **(i)** for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{\min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K} \right)$, those of ZT show a same maximum $ZT_{\max.} = 1$, **(ii)** for $\xi_{n(p)} = 1$, those of S , ZT , ZT_{Mott} , $VC1$, and Ts present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715 , 3.290 , $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and **(iii)** for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{Mott} = 1$.

Furthermore, it is interesting to remark that the VC2-coefficient, defined in Eq. (29), is related to our Global Einstein -and- Van Cong relations (25), by:

$$\frac{k_B}{q} \times VC2(N^*, r_{d(a)}, T) \equiv -\frac{\partial S}{\partial \xi_{n(p)}} \times \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \left(\frac{V^2}{K} \right), \quad \frac{k_B}{q} = \sqrt{\frac{3 \times L}{\pi^2}},$$

according, in this work, with the use of our Eq. (26),

$$\text{to:} \quad VC2(N, r_{d(a)}, T) \equiv T \times VC1(N, r_{d(a)}, T) \equiv -\frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \times 2 \times \frac{ZT_{Mott} \times [1 - ZT_{Mott}]}{[1 + ZT_{Mott}]^2} (V), \quad ZT_{Mott} = \frac{\pi^2}{3 \times \xi_{n(p)}^2}.$$

Therefore, we obtain in this work:

$$\mathbb{R}(N^*, r_{d(a)}, T) \equiv \frac{D(N^*, r_{d(a)}, T)}{\mu(N^*, r_{d(a)}, T)} \equiv \frac{D(N^*, r_{d(a)}, T)}{\sigma(N^*, r_{d(a)}, T)} \times q \times (N^*/g_{c(v)}) \equiv D(N^*, r_{d(a)}, T) \times \frac{V(N^*, r_{d(a)}, T)}{q} \times 6\pi \times R_{WS}(N^*) \equiv -\frac{VC2(N, r_{d(a)}, T)}{2} \times \frac{[1 + ZT_{Mott}]^2}{ZT_{Mott} \times [1 - ZT_{Mott}]} \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*}, \quad (32)$$

being new results.

Concluding Remarks

In the $n(p)$ – type degenerate **He – FL**, the electrical and thermoelectric coefficients, enhanced by : **(i)** our static dielectric constant law, $\varepsilon(r_{d(a)})$, given in Equations (5, 6), **(ii)** our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq.,⁽¹⁷⁾ being accurate with a precision of the order of 2.11×10^{-4} ,^[9] affecting all those coefficients, and finally **(iii)** our electrical

conductivity model, given in Eq. (19), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.

Finally, the numerical results of those coefficients are given and discussed in Tables 3A, 3B, 4A, 4B, 5A, 5B, 6A, 6B, and 7 in Appendix 1.

REFERENCES

1. Van Cong, H. New Critical Density in Metal-Insulator Transition, obtained in n(p)-Type Degenerate Helium-Fermi Liquid, being Just That of the Carriers, Localized in Exponential Band Tails. (IX). WJERT., 2026; 12(10): 215-241.
2. Landau, L. D. Theory of a Fermi Liquid. Sov. Phys. JETP., 1957; 3(6): 920-925.
3. Grebenkemper, C. J. and Hagen, J. P. The Dielectric Constant of Liquid Helium. Phys. Rev., 1950; 80(1): 89-91.
4. Van Cong, H. Same maximum figure of merit $ZT(=1)$, due to the effect of impurity size, obtained in the n(p)-type degenerate Ge -crystal ($\xi_{n(p)} \geq 1$), at same reduced Fermi energy $\xi_{n(p)} = \sqrt{\frac{\pi^2}{3}} \simeq 1.8138$, same minimum Seebeck coefficient $(S)_{\min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K} \right)$, same maximum $(ZT)_{\max.} = 1$, and same $(ZT)_{\text{Mott}} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1 \right)$. SCIREA Journal of Physics, 2023; 8(4): 407-430.
5. Van Cong, H. Same Maximal Figure of Merit $ZT(=1)$, Due to the Effect of Impurity Size, Obtained in the n(p)-Type Degenerate GaAs-Crystal ($\xi_{n(p)} \geq 1$), at Same Reduced Fermi Energy $\xi_{n(p)} (= 1.8138)$ and Same Minimum Seebeck Coefficient $S \left(= -1.563 \times 10^{-4} \frac{V}{K} \right)$, at which Same $(ZT)_{\text{Mott}} \left(= \frac{\pi^2}{3\xi_{n(p)}^2} = 1 \right)$. SCIREA Journal of Physics, 2023; 8(2): 133-157.
6. Van Cong, H. Effects of donor size and heavy doping on optical, electrical and thermoelectric properties of various degenerate donor-silicon systems at low temperatures. American Journal of Modern Physics, 2018; 7(4): 136-165.
7. Kim, H. S. et al. Characterization of Lorenz number with Seebeck coefficient measurement. APL Materials, 2015; 3(4): 041506.
8. Hyun, B. D. et al. Electrical-and-Thermoelectric Properties of 90%Bi₂Te₃ – 5%Sb₂Te₃ – 5%Sb₂Se₃ Single Crystals Doped with SbI₃. Scripta Materialia, 1998; 40(1): 49-56.

9. Van Cong, H. and Debiais, G. A simple accurate expression of the reduced Fermi energy for any reduced carrier density. *J. Appl. Phys.*, 1993; 73: 1545-1546.
10. Van Cong, H. et al. Size effect on different impurity levels in semiconductors. *Solid State Communications*, 1984; 49: 697-699.
11. Van Cong, H. Diffusion coefficient in degenerate semiconductors. *Phys. Stat. Sol. (b)*, 1984; 101: K27.
12. Van Cong, H. and Doan Khanh, B. Simple accurate general expression of the Fermi-Dirac integral $F_j(a)$ and for $j > -1$. *Solid-State Electron.*, 1992; 35(7): 949-951.
13. Van Cong, H. New series representation of Fermi-Dirac integral $F_j(-\infty < a < \infty)$ for arbitrary $j > -1$, and its effect on $F_j(a \geq 0_+)$ for integer $j \geq 0$. *Solid-State Electron.*, 1991; 34(5): 489-492.
14. Van Cong, H. and G. Mesnard. Thermoelectric effects of heavily doped semiconductors at low temperatures. *Phys. Stat. Sol. (b)*, 1972; 50(1): 53-58.
15. Van Cong, H. Fermi energy and band-tail parameters in heavily doped semiconductors. *Journal of Physics and Chemistry of Solids*, 1975; 36(11): 1237-1240.
16. Van Cong, H. Quantum efficiency and radiative lifetime in degenerate n-type GaAs. *Journal of Physics and Chemistry of Solids*, 1981; 36(11): 95-99.
17. Ewell, R. H and Eyring, H. Theory of the Viscosity of Liquids as a Function of Temperature and Pressure. *Journal of Chemical Physics*, 1937; 5: 726-736.
18. Wenhao, Z. Influence of Temperature and Concentration on Viscosity of Complex Fluids. *Journal of Physics: Conference Series*, 1965, 2021; 1: 012064.

APPENDIX 1

Table 1A. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, the numerical results of ϵ and $N_{CDn(CDp)}$ are computed, using Equations (5, 6) and (8), and those of $N_{CDn(CDp)}^{EBT}$ were given in Ref.^[1] It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.6017×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ ^[1] Further, it should be noted that, for an increasing $r_{d(a)}$, both $N_{CDn(NDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ increase, since, in Eq. (3), both the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$ increase.

Donor	He	P	As	Sb	Sn
r_d (nm)	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d) \searrow$	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in $10^{26} \text{ cm}^{-3} \nearrow$	0.024132469	4.8159728	7.4993210	17.850733	21.222714
$N_{CDn}^{EBT}(r_d)$ in $10^{26} \text{ cm}^{-3} \nearrow$	0.024132463	4.8159732	7.4993223	17.850721	21.222704
$ RD $ in 10^{-7}	2.6525	0.8945	1.7785	6.6017	4.9063
$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right) \nearrow$	0	2.3515308	2.5408863	2.9238096	3.0019950
$[\Delta \sigma(r_d)]_n (10^3 \text{ eV}) \nearrow$	0	1.2151676	1.6451213	2.9615259	3.3281077
Acceptor	He	Ga	Mg	In	Cd
r_a (nm)	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a) \searrow$	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{CDp}(r_a)$ in $10^{27} \text{ cm}^{-3} \nearrow$	0.0024132469	1.1244337	2.2139604	2.5079705	2.9471425
$N_{CDp}^{EBT}(r_a)$ in $10^{27} \text{ cm}^{-3} \nearrow$	0.0024132463	1.1244338	2.2139591	2.5079702	2.9471415
$ RD $ in 10^{-7}	2.6509	0.7982	5.7286	1.3613	3.4333
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{ eV}}{\text{cm}^3} \right) \nearrow$	0	2.7178158	3.0211923	3.0779777	3.1518784
$[\Delta \sigma(r_a)]_p (10^3 \text{ eV}) \nearrow$	0	2.1664893	3.4243355	3.7243346	4.1514861

Table 1B. In the n(p) type degenerate **He – FL**, in which $r_{do(ao)} = r_{He} = 0.046$ nm (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, the numerical results of ϵ and $N_{CDn(CDp)}$ are computed, using Equations (5, 6) and (8), and those of $N_{CDn(CDp)}^{EBT}$ were given in Ref.^[1] It should be noted that the relative deviations in absolute values are defined by: $|RD| \equiv \left| 1 - \frac{N_{CDn(CDp)}^{EBT}}{N_{CDn(CDp)}} \right|$, giving rise to their maximal value equal to 6.6372×10^{-7} . It means that such the critical d(a)-density $N_{CDn(NDp)}(r_{d(a)})$, determined in Eq. (8), is just the density of electrons (holes) localized in the EBT, $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ ^[1] Further, it should be noted that, for an increasing $r_{d(a)}$, both $N_{CDn(NDp)}(r_{d(a)})$ and $N_{CDn(CDp)}^{EBT}(r_{d(a)})$ increase, since, in Eq. (3), both the pressure variation, $[\Delta p(r_{d(a)})]_{n(p)}$, and the strain-energy variation, $[\Delta \sigma(r_{d(a)})]_{n(p)}$ increase.

Donor	He	P	As	Sb	Sn
r_d (nm)	0.046	0.110	0.118	0.136	0.140
$\epsilon(r_d) \searrow$	1.0550	0.18053	0.15576	0.11665	0.11012
$N_{CDn}(r_d)$ in $10^{27} \text{ cm}^{-3} \nearrow$	0.021301641	4.2510414	6.6196229	15.756776	18.733211
$N_{CDn}^{EBT}(r_d)$ in $10^{27} \text{ cm}^{-3} \nearrow$	0.021301635	4.2510418	6.6196241	15.756766	18.733202
$ RD $ in 10^{-7}	2.6506	0.8302	1.7531	6.6372	4.8921

$[\Delta p(r_d)]_n \left(\frac{10^{26} \text{eV}}{\text{cm}^3} \right) \nearrow$	0	4.8598303	5.2511650	6.0425397	6.2041229
$[\Delta \sigma(r_d)]_n (10^3 \text{eV}) \nearrow$	0	2.5113463	3.3999130	6.1204869	6.8780893
Acceptor	He	Ga	Mg	In	Cd
$r_a \text{ (nm)} \nearrow$	0.046	0.126	0.141	0.144	0.148
$\epsilon(r_a) \searrow$	1.0550	0.1361	0.1086	0.1041	0.0987
$N_{\text{CDP}}(r_a) \text{ in } 10^{28} \text{ cm}^{-3} \nearrow$	0.0021301641	0.99253344	1.9542547	2.2137763	2.6014318
$N_{\text{CDP}}^{\text{EBT}}(r_a) \text{ in } 10^{28} \text{ cm}^{-3} \nearrow$	0.0021301635	0.99253353	1.9542535	2.2137760	2.6014309
$ \text{RD} \text{ in } 10^{-7}$	2.6490	0.9313	5.8938	1.4370	3.5026
$[\Delta p(r_a)]_p \left(\frac{10^{26} \text{eV}}{\text{cm}^3} \right) \nearrow$	0	5.6168192	6.2437973	6.3611538	6.5138820
$[\Delta \sigma(r_a)]_p (10^3 \text{eV}) \nearrow$	0	4.4774111	7.076960	7.6969583	8.5797379

Table 2: Expressions for $G_{p>1}(y \equiv \frac{\pi k_B T}{E_{\text{Fn}}(F_p)} = \frac{\pi}{\xi_{\text{n}}(p)})$, due to the Fermi-Dirac distribution function, are used to determine the electrical-and-thermoelectric coefficients, suggesting that, with decreasing T and for given (high N, r_d), the function $G_{p>1}(y)$ decreases as: $G_{p>1}(y \rightarrow 0) \rightarrow 1$.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1 + \frac{y^2}{8}\right)$	$\left(1 + \frac{y^2}{3}\right)$	$\left(1 + \frac{5y^2}{8}\right)$	$(1 + y^2)$	$\left(1 + \frac{35y^2}{24}\right)$	$(1 + 2y^2)$	$\left(1 + \frac{21y^2}{8}\right)$

Table 3A: In the n(p) type degenerate **He – FL**, in which $r_{\text{do(ao)}} = r_{\text{He}} = 0.046 \text{ nm (covalent)}$, $m_{\text{c(v)}}/m_0 = 3$ and $\epsilon_0 = 1.055$, for a given $r_{\text{d(a)}}$, and $T = (4.2 \text{ K and } 300 \text{ K})$, the numerical results of $\mathbb{V}$, μ and D , expressed respectively in $\left(\frac{10^{10} \times \text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}; \frac{10^{-2} \times \text{cm}^2}{\text{V} \times \text{s}}; \frac{10^2 \times \text{cm}^2}{\text{s}}\right)$, as functions of N, are obtained by using Equations (22, 21 and 25), noting that, in this very degenerate case, for same physical conductions, all the corresponding physical results are found to almost be the same.

Donor	He	As	Sb	Sn
$r_d \text{ (nm)} \nearrow$	0.046	0.118	0.136	0.140
At T=4.2 K,				
$N (10^{28} \text{ cm}^{-3})$				
8.0	(1.172 ; 3.143 ; 4.727)	(34.85 ; 0.105 ; 0.157)	(55.04 ; 0.066 ; 0.098)	(60.08 ; 0.061 ; 0.090)
8.2	(1.183 ; 3.139 ; 4.800)	(35.25 ; 0.105 ; 0.160)	(55.71 ; 0.066 ; 0.100)	(60.82 ; 0.060 ; 0.091)
8.4	(1.194 ; 3.135 ; 4.872)	(35.64 ; 0.105 ; 0.162)	(56.37 ; 0.066 ; 0.101)	(61.55 ; 0.060 ; 0.092)
8.6	(1.205 ; 3.132 ; 4.943)	(36.02 ; 0.104 ; 0.164)	(57.02 ; 0.066 ; 0.102)	(62.27 ; 0.060 ; 0.093)
At T=300 K,				
$N (10^{28} \text{ cm}^{-3})$				
8.0	(1.172 ; 3.143 ; 4.727)	(34.85 ; 0.105 ; 0.157)	(55.04 ; 0.066 ; 0.098)	(60.08 ; 0.061 ; 0.090)
8.2	(1.183 ; 3.139 ; 4.800)	(35.25 ; 0.105 ; 0.160)	(55.71 ; 0.066 ; 0.100)	(60.82 ; 0.060 ; 0.091)
8.4	(1.194 ; 3.135 ; 4.872)	(35.64 ; 0.105 ; 0.162)	(56.37 ; 0.066 ; 0.101)	(61.55 ; 0.060 ; 0.092)
8.6	(1.205 ; 3.132 ; 4.943)	(36.02 ; 0.104 ; 0.164)	(57.02 ; 0.066 ; 0.102)	(62.27 ; 0.060 ; 0.093)

Acceptor	He	Mg	In	Cd
r_a (nm) ↗	0.046	0.141	0.144	0.148
At T=4.2 K,				
N (10^{28} cm^{-3})				
8.0	(1.172 ; 3.143 ; 4.727)	(61.37 ; 0.059 ; 0.088)	(65.30 ; 0.056 ; 0.082)	(70.67 ; 0.051 ; 0.075)
8.2	(1.183 ; 3.139 ; 4.800)	(62.13 ; 0.059 ; 0.089)	(66.12 ; 0.056 ; 0.083)	(71.58 ; 0.051 ; 0.076)
8.4	(1.194 ; 3.135 ; 4.872)	(62.88 ; 0.059 ; 0.090)	(66.92 ; 0.055 ; 0.084)	(72.46 ; 0.051 ; 0.077)
8.6	(1.205 ; 3.132 ; 4.943)	(63.62 ; 0.059 ; 0.091)	(67.72 ; 0.055 ; 0.085)	(73.34 ; 0.051 ; 0.078)
At T=300 K,				
N (10^{28} cm^{-3})				
8.0	(1.172 ; 3.143 ; 4.727)	(61.37 ; 0.059 ; 0.088)	(65.30 ; 0.056 ; 0.082)	(70.67 ; 0.051 ; 0.075)
8.2	(1.183 ; 3.139 ; 4.800)	(62.13 ; 0.059 ; 0.089)	(66.12 ; 0.056 ; 0.083)	(71.58 ; 0.051 ; 0.076)
8.4	(1.194 ; 3.135 ; 4.872)	(62.88 ; 0.059 ; 0.090)	(66.92 ; 0.055 ; 0.084)	(72.46 ; 0.051 ; 0.077)
8.6	(1.205 ; 3.132 ; 4.943)	(63.62 ; 0.059 ; 0.091)	(67.72 ; 0.055 ; 0.085)	(73.34 ; 0.051 ; 0.078)

Table 3B: In the n(p) type degenerate He – FL, in which $r_{do(ao)} = r_{He} = 0.046 \text{ nm}$ (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, for a given $r_{d(a)}$, and T=(4.2 K and 300 K), the numerical results of $\mathbb{V}$, μ and D, expressed respectively in $\left(\frac{10^{11} \times \text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}; \frac{10^{-3} \times \text{cm}^2}{\text{V} \times \text{s}}; \frac{10^2 \times \text{cm}^2}{\text{s}}\right)$, as functions of N, are obtained by using Equations (22, 21 and 25), noting that, in this very degenerate case, for same physical conductions, all the corresponding physical results are found to almost be the same.

Donor	He	As	Sb	Sn
r_d (nm) ↗	0.046	0.118	0.136	0.140
At T=4.2 K,				
N (10^{28} cm^{-3})				
8.0	(0.444 ; 8.302 ; 0.604)	(10.25 ; 0.349 ; 0.024)	(14.22 ; 0.241 ; 0.015)	(14.95 ; 0.225 ; 0.014)
8.2	(0.448 ; 8.290 ; 0.613)	(10.40 ; 0.347 ; 0.024)	(14.47 ; 0.239 ; 0.015)	(15.22 ; 0.224 ; 0.014)
8.4	(0.452 ; 8.277 ; 0.622)	(10.54 ; 0.346 ; 0.025)	(14.71 ; 0.238 ; 0.015)	(15.49 ; 0.222 ; 0.014)
8.6	(0.457 ; 8.265 ; 0.631)	(10.68 ; 0.344 ; 0.025)	(14.94 ; 0.236 ; 0.016)	(15.75 ; 0.220 ; 0.014)
At T=300 K,				
N (10^{28} cm^{-3})				
8.0	(0.444 ; 8.302 ; 0.604)	(10.25 ; 0.349 ; 0.024)	(14.22 ; 0.241 ; 0.015)	(14.95 ; 0.225 ; 0.014)
8.2	(0.448 ; 8.290 ; 0.613)	(10.40 ; 0.347 ; 0.024)	(14.47 ; 0.239 ; 0.015)	(15.22 ; 0.224 ; 0.014)
8.4	(0.452 ; 8.277 ; 0.622)	(10.54 ; 0.346 ; 0.025)	(14.71 ; 0.238 ; 0.015)	(15.49 ; 0.222 ; 0.014)
8.6	(0.457 ; 8.265 ; 0.631)	(10.68 ; 0.344 ; 0.025)	(14.94 ; 0.236 ; 0.016)	(15.75 ; 0.220 ; 0.014)
Acceptor	He	Mg	In	Cd
r_a (nm) ↗	0.046	0.141	0.144	0.148

At T=4.2 K,

N (10^{29} cm^{-3})

8.0	(1.085 ; 7.313 ; 2.470)	(57.63 ; 0.137 ; 0.045)	(61.37 ; 0.128 ; 0.042)	(66.49 ; 0.118 ; 0.039)
8.2	(1.095 ; 7.304 ; 2.508)	(58.34 ; 0.136 ; 0.046)	(62.13 ; 0.128 ; 0.043)	(67.32 ; 0.118 ; 0.039)
8.4	(1.106 ; 7.296 ; 2.546)	(59.03 ; 0.136 ; 0.046)	(62.88 ; 0.127 ; 0.044)	(68.15 ; 0.117 ; 0.040)
8.6	(1.116 ; 7.287 ; 2.583)	(59.72 ; 0.135 ; 0.047)	(63.62 ; 0.127 ; 0.044)	(68.96 ; 0.117 ; 0.040)

At T=300 K,

N (10^{29} cm^{-3})

8.0	(1.085 ; 7.313 ; 2.470)	(57.63 ; 0.137 ; 0.045)	(61.37 ; 0.128 ; 0.042)	(66.49 ; 0.118 ; 0.039)
8.2	(1.095 ; 7.304 ; 2.508)	(58.34 ; 0.136 ; 0.046)	(62.13 ; 0.128 ; 0.043)	(67.32 ; 0.118 ; 0.039)
8.4	(1.106 ; 7.296 ; 2.546)	(59.03 ; 0.136 ; 0.046)	(62.88 ; 0.127 ; 0.044)	(68.15 ; 0.117 ; 0.040)
8.6	(1.116 ; 7.287 ; 2.583)	(59.72 ; 0.135 ; 0.047)	(63.62 ; 0.127 ; 0.044)	(68.96 ; 0.117 ; 0.040)

Table 4A: In the n(p) type degenerate **He – FL**, in which $r_{\text{do(ao)}}=r_{\text{He}}=0.046 \text{ nm}$ (covalent), $m_{\text{c(v)}}/m_0 = 3$ and $\epsilon_0 = 1.055$, for given $r_{\text{d(a)}}$ and N, the numerical results of Fermi energy $E_{\text{Fn(Fp)}}(T=0\text{K})$ (eV) [or reduced Fermi energy $\xi_{\text{n(p)}}(N^*, r_{\text{d(a)}}, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_{\text{d(a)}}, T)$, expressed in $\left(\frac{\text{eV}}{\text{cm}} \times \frac{\text{s}}{\text{cm}^2}\right)$, are obtained, as functions of T, by using Equations (17, 22), respectively. In particular, from these numerical results of $\mathbb{V}(T)$, one observes that, for such given ($r_{\text{d(a)}}$ and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring.^[17] and complex fluids by Wenhao.^[18] and from those of $\mathbb{V}(r_{\text{d}})$, one observes that, for given (T and N), they increase with increasing r_{d} .

Donor	As	Sb	Sn
$r_{\text{d}} \text{ (nm)} [4] \nearrow$	0.118	0.136	0.140

For $N=8 \times 10^{28} \text{ cm}^{-3}$,

$N_{\text{CDn}}(r_{\text{d}})$ in $10^{26} \text{ cm}^{-3} \nearrow$	7.4993210	17.850733	21.222714
$E_{\text{Fn}}(T=0\text{K}) (10^4 \text{ eV}) \searrow$	2.25673113	2.25673112	2.25673111
$\mathbb{V}_{(0\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=4.2\text{K}) \searrow$	6.1948360×10^7	6.1407744×10^7	6.1231124×10^7
$\mathbb{V}_{(4.2\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=77\text{K}) \searrow$	3.3790014×10^6	3.3495133×10^6	3.3398795×10^6
$\mathbb{V}_{(77\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=100\text{K}) \searrow$	2.6018311×10^6	2.5791253×10^6	2.5717072×10^6
$\mathbb{V}_{(100\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=150\text{K}) \searrow$	1.7345541×10^6	1.7194168×10^6	1.7144715×10^6
$\mathbb{V}_{(150\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=200\text{K}) \searrow$	1.3009156×10^6	1.2895626×10^6	1.2858536×10^6
$\mathbb{V}_{(200\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
$\xi_{\text{n}}(T=250\text{K}) \searrow$	1.0407324×10^6	1.0316501×10^6	1.0286829×10^6
$\mathbb{V}_{(250\text{K})} \nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}

$\xi_{n(T=300K)}$	$\searrow$	8.6727704×10^5	8.5970842×10^5	8.5723574×10^5
$\mathbb{V}_{(300K)}$	$\nearrow$	3.4850277×10^{11}	5.5039642×10^{11}	6.0081717×10^{11}
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Acceptor		Mg	In	Cd
r_a (nm)	$\nearrow$	0.141	0.144	0.148
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For $N=8 \times 10^{28} \text{ cm}^{-3}$,				
$N_{CDP}(r_a)$ in 10^{27} cm^{-3}	$\nearrow$	2.2139604	2.5079705	2.9471425
$E_{FP}(T=0K) (10^4 eV)$	$\searrow$	2.2149008	2.2093161	2.2009609
$\mathbb{V}_{(0K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=4.2K)}$	$\searrow$	6.1183055×10^7	6.1028787×10^7	6.0797989×10^7
$\mathbb{V}_{(4.2K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=77K)}$	$\searrow$	3.3372575×10^6	3.3288429×10^6	3.3162540×10^6
$\mathbb{V}_{(77K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=100K)}$	$\searrow$	2.5696883×10^6	2.5632091×10^6	2.5535156×10^6
$\mathbb{V}_{(100K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=150K)}$	$\searrow$	1.7131255×10^6	1.7088060×10^6	1.7023437×10^6
$\mathbb{V}_{(150K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=200K)}$	$\searrow$	1.2848442×10^6	1.2816045×10^6	1.2767578×10^6
$\mathbb{V}_{(200K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=250K)}$	$\searrow$	1.0278753×10^6	1.0252836×10^6	1.0214062×10^6
$\mathbb{V}_{(250K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}
$\xi_{p(T=300K)}$	$\searrow$	8.5656277×10^5	8.5440302×10^5	8.5117185×10^5
$\mathbb{V}_{(300K)}$	$\nearrow$	6.1370157×10^{11}	6.5298848×10^{11}	7.0675450×10^{11}

Table 4B: In the n(p) type degenerate He – FL, in which $r_{do(ao)}=r_{He}=0.046 \text{ nm}$ (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, for given $r_{d(a)}$ and N , the numerical results of Fermi energy $E_{Fn(Fp)}(T=0K)$ (eV) [or reduced Fermi energy $\xi_{n(p)}(N^*, r_{d(a)}, T)$], and viscosity coefficient $\mathbb{V}(N^*, r_{d(a)}, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained, as functions of T , by using Equations (17, 22), respectively. In particular, from these numerical results of $\mathbb{V}(T)$, one observes that, for such given ($r_{d(a)}$ and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring.^[17] and complex fluids by Wenhao.^[18] and from those of $\mathbb{V}(r_d)$, one observes that, for given (T and N), they increase with increasing r_d .

Donor		As	Sb	Sn
r_d (nm) [4]	$\nearrow$	0.118	0.136	0.140
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For $N=8 \times 10^{28} \text{ cm}^{-3}$,				
$N_{CDn}(r_d)$ in 10^{27} cm^{-3}	$\nearrow$	6.6196229	15.756776	18.733211
$E_{Fn}(T=0K) (eV)$	$\searrow$	1.09196670×10^4	1.09196670×10^4	1.09196670×10^4
$\mathbb{V}_{(0K)}$	$\nearrow$	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}

$\xi_n(T=4.2K)$	↘	2.8476039×10^7	2.6060205×10^7	2.5248933×10^7
$V_{(4.2K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=77K)$	↘	1.5532385×10^6	1.4214657×10^6	1.3772145×10^6
$V_{(77K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=100K)$	↘	1.1959936×10^6	1.0945286×10^6	1.0604552×10^6
$V_{(100K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=150K)$	↘	7.9732909×10^5	7.2968575×10^5	7.0697012×10^5
$V_{(150K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=200K)$	↘	5.9799681×10^5	5.4726431×10^5	5.3022759×10^5
$V_{(200K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=250K)$	↘	4.7839745×10^5	4.3781145×10^5	4.2418207×10^5
$V_{(250K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}
$\xi_n(T=300K)$	↘	3.9866454×10^5	3.6484288×10^5	3.5348506×10^5
$V_{(300K)}$	↗	1.0253793×10^{12}	1.4222698×10^{12}	1.4954188×10^{12}

Acceptor		Mg	In	Cd
r_a (nm)	↗	0.141	0.144	0.148

For $N=8 \times 10^{28} \text{ cm}^{-3}$,

$N_{CDP}(r_a) \text{ in } 10^{28} \text{ cm}^{-3}$ ↗		1.9542547	2.2137763	2.6014318
$E_{FP}(T=0K)(eV)$	↘	9.0597447×10^3	8.7985858×10^3	8.4010801×10^3
$V_{(0K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=4.2K)$	↘	2.5026081×10^7	2.4304672×10^7	2.3206626×10^7
$V_{(4.2K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=77K)$	↘	1.3650590×10^6	1.3257094×10^6	1.2658160×10^6
$V_{(77K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=100K)$	↘	1.0510954×10^6	1.0207962×10^6	9.7467831×10^5
$V_{(100K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=150K)$	↘	7.0073027×10^5	6.8053081×10^5	6.4978554×10^5
$V_{(150K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=200K)$	↘	5.2554770×10^5	5.1039811×10^5	4.8733915×10^5
$V_{(200K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=250K)$	↘	4.2043816×10^5	4.0831849×10^5	3.8987132×10^5
$V_{(250K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}
$\xi_p(T=300K)$	↘	3.5036514×10^5	3.4026541×10^5	3.2489277×10^5
$V_{(300K)}$	↗	1.5121637×10^{12}	1.5579513×10^{12}	1.6066137×10^{12}

Table 5A: In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.046$ nm (covalent), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, for given $r_{d(a)}$ and N, the numerical results of $\xi_{n(p)}$, V , AE , $\sigma_{Th.}$, S , $VC1$, $VC2$, Ts , Pt and ZT are obtained, as functions of $T=(4.2$ K, 300 K), by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $V(T)$, one observes that, for such given ($r_{d(a)}$ and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring.^[17] and complex fluids by Wenhao.^[18] suggesting **an equivalence between degeneracy-and-viscosity concept**, and from those of $V(r_{d(a)})$, one observes that, for given (T and N), they increase with increasing $r_{d(a)}$.

Donor	As	Sb	Sn
r_d (nm) [4] ↗	0.118	0.136	0.140

For $N=8 \times 10^{28} \text{ cm}^{-3}$,			
$N_{CDn}(r_d)$ in 10^{26} cm^{-3} ↗	7.4993210	17.850733	21.222714
$\xi_{n(T=4.2K)} \times 10^{-7}$ ↘	6.1948360	6.1407744	6.1231124
$\xi_{n(T=300K)} \times 10^{-5}$ ↘	8.6727704	8.5970842	8.5723574
$V_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-11}$ ↗	3.4850277	5.5039642	6.0081717
$V_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-11}$ ↗	3.4850277	5.5039642	6.0081717
$-AE_{(4.2K)} (eV) \times 10^{18}$ ↗	2.4109248	2.5716531	2.4912889
$-AE_{(300K)} (eV) \times 10^{13}$ ↗	1.9796563	2.0147869	2.0264971
$\sigma_{Th. (4.2K)} \left(\frac{W}{cm \times K} \right) \times 1$ ↘	1.3732049	0.8543831	0.7781872
$\sigma_{Th. (300K)} \left(\frac{W}{cm \times K} \right) \times 0.1$ ↘	9.8086068	6.1027367	5.5584803
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12}$ ↗	9.1526729	9.2332502	9.2598834
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^{10}$ ↗	6.5376235	6.5951787	6.6142024
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12}$ ↗	6.1017819	6.1555002	6.1732556
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^{10}$ ↗	4.3584156	4.3967858	4.4094683
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11}$ ↗	2.5627484	2.5853101	2.5927673
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^7$ ↗	1.3075247	1.3190357	1.3228405
$-Ts_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12}$ ↗	9.1526729	9.2332502	9.2598834
$-Ts_{(300K)} \left(\frac{V}{K} \right) \times 10^{10}$ ↗	6.5376235	6.5951787	6.6142024
$-Pt_{(4.2K)} (V) \times 10^{11}$ ↗	3.8441226	3.8779651	3.8891510
$-Pt_{(300K)} (V) \times 10^7$ ↗	1.9612870	1.9785536	1.9842607
$ZT_{(4.2K)} \times 10^{15}$ ↗	3.4290898	3.4897329	3.5098941
$ZT_{(300K)} \times 10^{11}$ ↗	1.7495356	1.7804760	1.7907623

Acceptor	Mg	In	Cd
r_a (nm) ↗	0.141	0.144	0.148

For $N=8 \times 10^{28} \text{ cm}^{-3}$,

$N_{\text{CDP}}(r_a) \text{ in } 10^{27} \text{ cm}^{-3} \nearrow$	2.2139604	2.5079705	2.9471425
$\xi_{n(T=4.2K)} \times 10^{-7} \searrow$	6.1183055	6.1028787	6.0797989
$\xi_{n(T=300K)} \times 10^{-5} \searrow$	8.5656277	8.5440302	8.5117185
$V_{(4.2K)} \left(\frac{eV}{\text{cm}} \times \frac{s}{\text{cm}^2} \right) \times 10^{-11} \nearrow$	6.1370157	6.5298848	7.0675450
$V_{(300K)} \left(\frac{eV}{\text{cm}} \times \frac{s}{\text{cm}^2} \right) \times 10^{-11} \nearrow$	6.1370157	6.5298848	7.0675450
$-AE_{(4.2K)} (eV) \times 10^{18} \nearrow$	2.5314710	2.6520173	2.4912889
$-AE_{(300K)} (eV) \times 10^{13} \nearrow$	2.0295107	2.0399294	2.0552273
$\sigma_{\text{Th. (4.2K)}} \left(\frac{W}{\text{cm} \times K} \right) \times 10 \searrow$	7.6065382	7.1128874	6.5221656
$\sigma_{\text{Th. (300K)}} \left(\frac{W}{\text{cm} \times K} \right) \times 0.1 \searrow$	5.4332416	5.0806339	4.6586897
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12} \nearrow$	9.2671586	9.2905839	9.3258524
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^{10} \nearrow$	6.6193990	6.6361314	6.6613231
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12} \nearrow$	6.1781057	6.1937226	6.2172349
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^{10} \nearrow$	4.4129327	4.4240876	4.4408821
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11} \nearrow$	2.5948044	2.6013635	2.6112387
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^7 \nearrow$	1.3238798	1.3272263	1.3322646
$-TS_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{12} \nearrow$	9.2671586	9.2905839	9.3258524
$-TS_{(300K)} \left(\frac{V}{K} \right) \times 10^{10} \nearrow$	6.6193990	6.6361314	6.6613231
$-Pt_{(4.2K)} (V) \times 10^{11} \nearrow$	3.8922066	3.9020453	3.9168580
$-Pt_{(300K)} (V) \times 10^7 \nearrow$	1.9858197	1.9908394	1.9983969
$ZT_{(4.2K)} \times 10^{15} \nearrow$	3.5154115	3.5332063	3.5600824
$ZT_{(300K)} \times 10^{11} \nearrow$	1.7935773	1.8026563	1.8163686

Table 5B: In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.046$ nm (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, for given r_d and N, the numerical results of $\xi_{n(p)}$, V , AE , $\sigma_{Th.}$, S , $VC1$, $VC2$, Ts , Pt and ZT are obtained, as functions of $T=(4.2$ K, 300 K), by using Equations (17, 22, 23, 20, 26, 28, 29, 30, 31, and 27), respectively. In particular, from the numerical results of $V(T)$, one observes that, for such given ($r_{d(a)}$ and N), they increase with decreasing T , in good agreement with those, obtained in liquids by Ewell and Eyring.^[17] and complex fluids by Wenhao.^[18] suggesting **an equivalence between degeneracy-and-viscosity concept**, and from those of $V(r_{d(a)})$, one observes that, for given (T and N), they increase with increasing $r_{d(a)}$.

Donor	As	Sb	Sn
r_d (nm) [4] ↗	0.118	0.136	0.140

For $N=8 \times 10^{28} \text{ cm}^{-3}$,			
$N_{CDn}(r_d)$ in 10^{27} cm^{-3} ↗	6.6196229	15.756776	18.733211
$\xi_{n(T=4.2K)} \times 10^{-7}$ ↘	2.8476039	2.6060205	2.5248933
$\xi_{n(T=300K)} \times 10^{-5}$ ↘	3.9866454	3.6484288	3.5348506
$V_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-12}$ ↗	1.0253793	1.4222698	1.4954188
$V_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-12}$ ↗	1.0253793	1.4222698	1.4954188
$-AE_{(4.2K)} (eV) \times 10^{18}$ ↗	4.2191184	4.8620316	5.3040345
$-AE_{(300K)} (eV) \times 10^{13}$ ↗	9.3638022	11.1802044	11.909825
$\sigma_{Th. (4.2K)} \left(\frac{W}{cm \times K} \right) \times 10$ ↘	4.2120883	2.5432937	2.2706284
$\sigma_{Th. (300K)} \left(\frac{W}{cm \times K} \right) \times 0.1$ ↘	3.0086345	1.8166383	1.6218774
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11}$ ↗	1.9911234	2.1757045	2.2456120
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^9$ ↗	1.4222310	1.5540747	1.6040086
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11}$ ↗	1.3274156	1.4504697	1.4970747
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^{10}$ ↗	9.4815400	10.360498	10.693390
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11}$ ↗	5.5751455	6.0919727	6.2877136
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^7$ ↗	2.8444620	3.1081493	3.2080172
$-Ts_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11}$ ↗	1.9911234	2.1757045	2.2456120
$-Ts_{(300K)} \left(\frac{V}{K} \right) \times 10^9$ ↗	1.4222310	1.5540747	1.6040086
$-Pt_{(4.2K)} (V) \times 10^{11}$ ↗	8.3627183	9.1379591	9.4315705
$-Pt_{(300K)} (V) \times 10^7$ ↗	4.2666930	4.6622240	4.8120257
$ZT_{(4.2K)} \times 10^{14}$ ↗	1.6228536	1.9376834	2.0642031
$ZT_{(300K)} \times 10^{11}$ ↗	8.2798651	9.8861397	10.531649

Acceptor	Mg	In	Cd
r_a (nm) ↗	0.141	0.144	0.148

For $N=8 \times 10^{28} \text{ cm}^{-3}$,

$N_{CDP}(r_a) \text{ in } 10^{28} \text{ cm}^{-3} \nearrow$	1.9542547	2.2137763	2.6014318
$\xi_p(T=4.2K) \times 10^{-7} \searrow$	2.5026081	2.4304672	2.3206626
$\xi_p(T=300K) \times 10^{-5} \searrow$	3.5036514	3.4026541	3.2489277
$V_{(4.2K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-12} \nearrow$	1.5121637	1.5579513	1.6066137
$V_{(300K)} \left(\frac{eV}{cm} \times \frac{s}{cm^2} \right) \times 10^{-12} \nearrow$	1.5121637	1.5579513	1.6066137
$-AE_{(4.2K)} (eV) \times 10^{18} \nearrow$	5.1834883	5.2236703	5.4647628
$-AE_{(300K)} (eV) \times 10^{12} \nearrow$	1.2122991	1.2853185	1.4098112
$\sigma_{Th. (4.2K)} \left(\frac{W}{cm \times K} \right) \times 10 \searrow$	2.2060214	2.0195216	1.7853995
$\sigma_{Th. (300K)} \left(\frac{W}{cm \times K} \right) \times 0.1 \searrow$	1.5757296	1.4425154	1.2752853
$-S_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11} \nearrow$	2.2656087	2.3328563	2.4432378
$-S_{(300K)} \left(\frac{V}{K} \right) \times 10^9 \nearrow$	1.6182919	1.6663259	1.7451699
$-VC1_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11} \nearrow$	1.5104058	1.5552375	1.6288252
$-VC1_{(300K)} \left(\frac{V}{K} \right) \times 10^9 \nearrow$	1.0788613	1.1108840	1.1634466
$-VC2_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11} \nearrow$	6.3437044	6.5319977	6.8410659
$-VC2_{(300K)} \left(\frac{V}{K} \right) \times 10^7 \nearrow$	3.2365839	3.3326519	3.4903397
$-TS_{(4.2K)} \left(\frac{V}{K} \right) \times 10^{11} \nearrow$	2.2656087	2.3328563	2.4432378
$-TS_{(300K)} \left(\frac{V}{K} \right) \times 10^9 \nearrow$	1.6182919	1.6663259	1.7451699
$-Pt_{(4.2K)} (V) \times 10^{11} \nearrow$	9.5155566	9.7979965	10.261599
$-Pt_{(300K)} (V) \times 10^7 \nearrow$	4.8548758	4.9989778	5.2355096
$ZT_{(4.2K)} \times 10^{14} \nearrow$	2.1011293	2.2277116	2.4435119
$ZT_{(300K)} \times 10^{10} \nearrow$	1.0720048	1.1365875	1.2466898

Table 6A: In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.046$ nm (covalent), $m_{c(v)}/m_o = 3$ and $\epsilon_o = 1.055$, for given $r_{d(a)}$ and $T=0K$, the numerical results of Fermi energy $E_{Fn(Fp)}(10^4 \times eV)$, $\mathbb{V}(10^{10} \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^8}{\Omega \times cm})$, $\mu(\frac{10^{-2} \times cm^2}{V \times s})$, and $D(\frac{10^2 \times cm^2}{s})$, are obtained, as functions of N , by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions, ($T=0K$, $N=N_{CDn(CDp)}$ or $N^* = 0$), and (ii) those values of $E_{Fn(Fp)} \gtrsim 0$, $\mathbb{V} \gtrsim 0$, $\sigma \gtrsim 0$, $\mu \gtrsim 0$, and $D \gtrsim 0$, obtained for $N \gtrsim N_{CDn(CDp)}$, thus define the properties of **this degenerate (or viscous) He-FL**, given in the Mott MIT. In particular, from these numerical results of $\mathbb{V}$, one observes that, for such given ($r_{d(a)}$ and $T=0K$), they **increase with increasing N (or increasing $E_{Fn(Fp)}$)**, in good agreement with those, obtained in complex fluids by Wenhao,^[18] suggesting an equivalence between degeneracy-and-viscosity concept.

$N \gtrsim N_{CDn} \quad \cong N_{CDn}$	8.2	8.4	8.6
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Here, N is expressed in 10^{28} cm^{-3}

For $r_d = r_{He}$ and $N_{CDn}=2.4132469 \times 10^{24} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	2.2941	2.3313	2.3681
$\mathbb{V}$	$\nearrow$	$\cong 0$	1.1836	1.1946	1.2054
σ	$\nearrow$	$\cong 0$	4.1239	4.2194	4.3149
μ	$\searrow$	$\cong 0$	3.1390	3.1353	3.1316
D	$\nearrow$	$\cong 0$	4.7998	4.8717	4.9430

For $r_d = r_{As}$ and $N_{CDn}=7.4993210 \times 10^{26} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	2.2802	2.3174	2.3544
$\mathbb{V}$	$\nearrow$	$\cong 0$	35.246	35.637	36.022
σ	$\nearrow$	$\cong 0$	0.1368	0.1398	0.1427
μ	$\searrow$	$\cong 0$	0.1051	0.1048	0.1045
D	$\nearrow$	$\cong 0$	0.1597	0.1618	0.1640

For $r_d = r_{Sb}$ and $N_{CDn}=1.7850733 \times 10^{27} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	2.2608	2.2982	2.3353
$\mathbb{V}$	$\nearrow$	$\cong 0$	55.708	56.367	57.017
σ	$\nearrow$	$\cong 0$	0.0851	0.0869	0.0887
μ	$\searrow$	$\cong 0$	0.0662	0.0660	0.0657
D	$\nearrow$	$\cong 0$	0.0998	0.1010	0.1023

For $r_d = r_{Sn}$ and $N_{CDn}=2.1222714 \times 10^{27} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	2.2544	2.2919	2.3291
$\mathbb{V}$	$\nearrow$	$\cong 0$	60.823	61.554	62.275
σ	$\nearrow$	$\cong 0$	0.0775	0.0791	0.0808
μ	$\searrow$	$\cong 0$	0.0605	0.0603	0.0601
D	$\nearrow$	$\cong 0$	0.0910	0.0921	0.0933

$N \gtrsim N_{CDP}$	$\cong N_{CDP}$	8.2	8.4	8.6
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Here, N is expressed in 10^{28} cm^{-3}

For $r_a = r_{He}$ and $N_{CDP} = 2.4132469 \times 10^{24} \text{ cm}^{-3}$,

E_{Fp}	$\nearrow$	$\cong 0$	2.2941	2.3313	2.3681
$\mathbb{V}$	$\nearrow$	$\cong 0$	1.1836	1.1946	1.2054
σ	$\nearrow$	$\cong 0$	4.1239	4.2194	4.3149
μ	$\searrow$	$\cong 0$	3.1390	3.1353	3.1316
D	$\nearrow$	$\cong 0$	4.7998	4.8717	4.9430

For $r_a = r_{Mg}$ and $N_{CDP} = 2.2139604 \times 10^{27} \text{ cm}^{-3}$,

E_{Fp}	$\nearrow$	$\cong 0$	2.2527	2.2902	2.3274
$\mathbb{V}$	$\nearrow$	$\cong 0$	62.131	62.880	63.619
σ	$\nearrow$	$\cong 0$	0.0757	0.0773	0.0790
μ	$\searrow$	$\cong 0$	0.0592	0.0590	0.0588
D	$\nearrow$	$\cong 0$	0.0890	0.0901	0.0912

For $r_a = r_{In}$ and $N_{CDP} = 2.5079705 \times 10^{27} \text{ cm}^{-3}$,

E_{Fp}	$\nearrow$	$\cong 0$	2.2472	2.2847	2.3219
$\mathbb{V}$	$\nearrow$	$\cong 0$	66.118	66.925	67.721
σ	$\nearrow$	$\cong 0$	0.0708	0.0723	0.0738
μ	$\searrow$	$\cong 0$	0.0556	0.0554	0.0552
D	$\nearrow$	$\cong 0$	0.0833	0.0844	0.0854

For $r_a = r_{Cd}$ and $N_{CDP} = 2.9471425 \times 10^{27} \text{ cm}^{-3}$,

E_{Fp}	$\nearrow$	$\cong 0$	2.2389	2.2765	2.3138
$\mathbb{V}$	$\nearrow$	$\cong 0$	71.577	72.465	73.341
σ	$\nearrow$	$\cong 0$	0.0649	0.0663	0.0677
μ	$\searrow$	$\cong 0$	0.0513	0.0511	0.0509
D	$\nearrow$	$\cong 0$	0.0765	0.0775	0.0784

Table 6B: In the n(p) type degenerate **He – FL**, in which $r_{do(ao)}=r_{He}=0.046$ nm (covalent), $m_{c(v)}/m_o = 6.2$ and $\epsilon_o = 1.055$, for given $r_{d(a)}$ and $T=0K$, the numerical results of Fermi energy $E_{Fn(Fp)}(10^3 \times eV)$, $V(10^{11} \times \frac{eV}{cm} \times \frac{s}{cm^2})$, $\sigma(\frac{10^5}{\Omega \times cm})$, $\mu(\frac{10^{-5} \times cm^2}{V \times s})$, and $D(\frac{cm^2}{s})$, are obtained, as functions of N , by using Equations (17, 22, 19, 21, 25), respectively. It should be noted here that (i) they are cancelled at the MIT-conditions, ($T=0K$, $N=N_{CDn(CDp)}$ or $N^* = 0$), and (ii) those values of $E_{Fn(Fp)} \geq 0$, $V \geq 0$, $\sigma \geq 0$, $\mu \geq 0$, and $D \geq 0$, obtained for $N \geq N_{CDn(CDp)}$, thus define the properties of **this degenerate (or viscous) He-FL**, given in the Mott MIT. In particular, from these numerical results of V , one observes that, for such given ($r_{d(a)}$ and $T=0K$), they **increase with increasing N (or increasing $E_{Fn(Fp)}$)**, in good agreement with those, obtained in complex fluids by Wenhao,^[18] suggesting an equivalence between degeneracy-and-viscosity concept.

$N \geq N_{CDn} \cong N_{CDn}$	8.2	8.4	8.6
Here, N is expressed in 10^{28} cm^{-3}			

For $r_d = r_{He}$ and $N_{CDn}=2.1301641 \times 10^{25} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	11.099	11.279	11.457
V	$\nearrow$	$\cong 0$	0.4481	0.4524	0.4567
σ	$\nearrow$	$\cong 0$	1088.8	1113.7	1138.5
μ	$\searrow$	$\cong 0$	828.97	827.72	826.50
D	$\nearrow$	$\cong 0$	61.324	62.223	63.114

For $r_d = r_{As}$ and $N_{CDn}=6.6196229 \times 10^{27} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	10.495	10.680	10.863
V	$\nearrow$	$\cong 0$	10.399	10.542	10.684
σ	$\nearrow$	$\cong 0$	41.955	42.856	43.753
μ	$\searrow$	$\cong 0$	34.739	34.567	34.401
D	$\nearrow$	$\cong 0$	2.4300	2.4606	2.4908

For $r_d = r_{Sb}$ and $N_{CDn}=15.756776 \times 10^{27} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	9.6289	9.8217	10.013
V	$\nearrow$	$\cong 0$	14.466	14.706	14.943
σ	$\nearrow$	$\cong 0$	25.386	25.982	26.575
μ	$\searrow$	$\cong 0$	23.919	23.763	23.613
D	$\nearrow$	$\cong 0$	1.5351	1.5556	1.5758

For $r_d = r_{Sn}$ and $N_{CDn}=18.733211 \times 10^{27} \text{ cm}^{-3}$,

E_{Fn}	$\nearrow$	$\cong 0$	9.3383	9.5340	9.7278
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$\nabla \nearrow$	$\cong 0$	15.225	15.492	15.756
$\sigma \nearrow$	$\cong 0$	22.687	23.240	23.790
$\mu \searrow$	$\cong 0$	22.381	22.225	22.074
$D \nearrow$	$\cong 0$	1.3930	1.4123	1.4312

$N \gtrsim N_{CDP}$	$\cong N_{CDP}$	8.2	8.4	8.6
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Here, N is expressed in 10^{29} cm^{-3}

For $r_a = r_{He}$ and $N_{CDP} = 2.1301641 \times 10^{25} \text{ cm}^{-3}$,

$E_{Fp} \nearrow$	$\cong 0$	51.525	52.359	53.187
$\nabla \nearrow$	$\cong 0$	1.0958	1.1060	1.1160
$\sigma \nearrow$	$\cong 0$	9596.2	9818.7	10040.9
$\mu \searrow$	$\cong 0$	730.44	729.58	728.74
$D \nearrow$	$\cong 0$	250.85	254.61	258.34

For $r_a = r_{Mg}$ and $N_{CDP} = 1.9542547 \times 10^{28} \text{ cm}^{-3}$,

$E_{Fp} \nearrow$	$\cong 0$	50.704	51.545	52.379
$\nabla \nearrow$	$\cong 0$	58.339	59.034	59.720
$\sigma \nearrow$	$\cong 0$	174.56	178.27	181.97
$\mu \searrow$	$\cong 0$	13.611	13.561	13.514
$D \nearrow$	$\cong 0$	4.5997	4.6591	4.7179

For $r_a = r_{In}$ and $N_{CDP} = 2.2137763 \times 10^{28} \text{ cm}^{-3}$,

$E_{Fp} \nearrow$	$\cong 0$	50.594	51.436	52.271
$\nabla \nearrow$	$\cong 0$	62.128	62.877	63.616
$\sigma \nearrow$	$\cong 0$	163.20	166.67	170.12
$\mu \searrow$	$\cong 0$	12.767	12.719	12.673
$D \nearrow$	$\cong 0$	4.3052	4.3605	4.4152

For $r_a = r_{Cd}$ and $N_{CDP} = 2.6014318 \times 10^{28} \text{ cm}^{-3}$,

$E_{Fp} \nearrow$	$\cong 0$	50.430	51.274	52.110
$\nabla \nearrow$	$\cong 0$	67.326	68.151	68.964
$\sigma \nearrow$	$\cong 0$	149.63	152.80	155.96
$\mu \searrow$	$\cong 0$	11.762	11.716	11.672
$D \nearrow$	$\cong 0$	3.9535	4.0040	4.0540

Table 7: In the n(p) type degenerate **He – FL**, for given physical conditions: $r_{d(a)}$, N (or T), the same values of $\xi_{n(p)}$ decrease, according to the increasing T (or to the decreasing N), and other thermoelectric coefficients are in variations, as indicated by the arrows as: (increase: ↗, decrease: ↘). One notes here that (i) for $\xi_{n(p)} \simeq 1.8138$, while the numerical results of S present a same minimum $S_{\min.} \left(\simeq -1.563 \times 10^{-4} \frac{V}{K} \right)$, those of ZT show a same maximum $ZT_{\max.} = 1$, (ii) for $\xi_{n(p)} = 1$, those of S, ZT, ZT_{Mott} , VC1, and Ts present the same results: $-1.322 \times 10^{-4} \frac{V}{K}$, 0.715, 3.290, $1.105 \times 10^{-4} \frac{V}{K}$, and $1.657 \times 10^{-4} \frac{V}{K}$, respectively, and (iii) for $\xi_{n(p)} \simeq 1.8138$, $(ZT)_{\text{Mott}} = 1$.

$\xi_{n(p)} \searrow$	1.880 [1.880]	1.8138 [1.8138]	1.750 [1.750]	1 [1]	0.998 [0.998]
$S \left(10^{-4} \frac{V}{K} \right)$	-1.562 [-1.562] ↘	-1.563 [-1.563] ↗	-1.562 [-1.562] ↗	-1.322 [-1.322] ↗	-1.320 [-1.320]
ZT	0.999 [0.999] ↗	1 [1] ↘	0.999 [0.999] ↘	0.715 [0.715] ↘	0.713 [0.713]
$(ZT)_{\text{Mott}} \nearrow$	0.931 [0.931]	1 [1]	1.074 [1.074]	3.290 [3.290]	3.306 [3.306]
$VC1 \left(10^{-4} \frac{V}{K} \right)$	-0.061 [-0.061] ↗	0 [0]	0.063 [0.063] ↗	1.105 [1.105] ↗	1.109 [1.109]
$Ts \left(10^{-4} \frac{V}{K} \right)$	-0.092 [-0.092] ↗	0 [0]	0.094 [0.094] ↗	1.657 [1.657] ↗	1.663 [1.663]