



## SYNTHESIS OF PLANAR MECHANISMS, PART VII: SIX BAR – ONE SLIDER MECHANISM (DESIGN I)

**Prof. Dr. Galal Ali Hassaan\***

Emeritus Professor, Department of Mechanical Design & Production, Faculty of Engineering,  
Cairo University, Egypt.

Article Received on 20/10/2015

Article Revised on 08/11/2015

Article Accepted on 30/11/2015

**\*Correspondence for**

**Author**

**Prof. Dr. Galal Ali  
Hassaan**

Emeritus Professor,  
Department of Mechanical  
Design and Production,  
Faculty of Engineering,  
Cairo University, Egypt.

### ABSTRACT

This research paper aims at introducing an optimal synthesis for a six bar – one slider planar mechanism. The objective of the synthesis is to maximize the time ratio of the mechanism for specific normalized stroke. The quality of the designed mechanism is controlled through Grashof's criteria for crank-rocker four bar mechanism and the transmission angle of the 6 bar mechanism. The MATLAB optimization toolbox is used to the time ratio of the mechanism as an objective functions. Two functional constraints are used to control the

performance of the mechanism. A normalized stroke between 0.1 and 0.7 is covered. The resulting minimum transmission angle for one revolution motion of the crank is between 66.6 and about 84 degrees while the maximum transmission angle is between 93.3 and 96.8 degrees.

**KEYWORDS:** Planar mechanism synthesis, six bar – one slider mechanism, optimal synthesis, mechanism transmission angle.

### INTRODUCTION

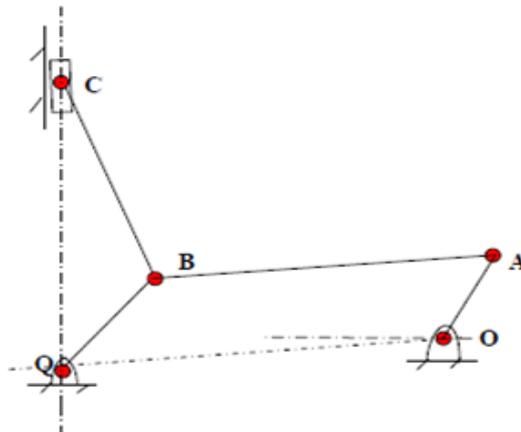
This is the seventh paper in a series of research papers aiming at optimal synthesis of planar mechanisms. The papers are expected to help machine designers to select proper linkages and dimensions for their machines of various applications.

Shiakolar, Koladiya and Kebrle, 2005 presented a methodology combining differential evolution, evolutionary optimization and geometrical centroid of precision positions for mechanism synthesis. They applied the proposed methodology to the synthesis of 6-bar linkages for dwell and dual dwell mechanism with timing and transmission angle constraints. Vashista, Jere, Rane, Grover and Sah, 2007 described the synthesis of a new mechanism for a sheep shearing machine. They presented various ideas including 6-bar linkage with adjustable driven link oscillation. They studied the kinematics of the selected mechanism in terms of displacement, velocity and acceleration analysis. Mehdigholi and Akbarnejad, 2008 considered the optimal synthesis of Watt's 6-bar mechanism generating straight and parallel leg motion. They used genetic algorithm optimization for the optimal synthesis of the mechanism where nonlinear coupler equations existed. Makhsudyan, Djavakhyan and Arakelian, 2009 presented a comparative analysis and synthesis of 6-bar mechanism formed by two spherical and planar 4-bar linkage. Their comparative analysis showed that the spherical linkage had better properties than the planar linkage. Perkins, 2011 presented the formulation for the synthesis of coupler-driven SDOF planar and spherical mechanisms. He presented a general formulation for the kinematic analysis of the coupler-driven 4-bar mechanism.

Bulatovic and Dordevic, 2012 presented the optimal dimensional synthesis of a 6-bar linkage with rotational constraints. Their objective was to bring the generated path as close to the given path as possible. They used the differential evolution optimization technique. Shivdas, Bansode, Kulkarni, Parlikar and Ghodekar, 2013 applied and designed a 6-bar mechanism for having a dual role for articulation from 0 to 90 degrees and self replenishment system for missile container avoiding the use of external crane. They manufactured and realized the synthesized mechanism and carried out the functional tests. Hassaan, 2014 formulated the synthesis problem for a 6-bar mechanism for a single dwell. He used an error based objective function and three functional constraints controlling the performance of the linkage. He could generate a single dwell during 60 degrees of crank rotation with a maximum error less than 0.23 %. Agarwal and Badduri, 2015 introduced an approach for the synthesis of planar 6 bar linkage via multi-objective numerical optimization. They applied their approach to a Stephenson III 6-bar mechanism. Hassaan, 2015 synthesized 6-bar mechanisms including number of sliding links for the purpose of maximum time ratio and specific strokes.

## MECHANISM ANALYSIS

A line diagram of the 6 bar-single slider mechanism is shown in Fig.1.

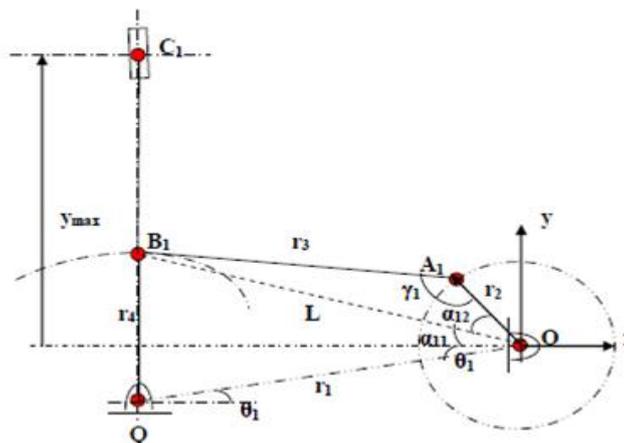


**Fig.1. The 6 bar – single slider mechanism.**

It consists of a four bar chain OABQ and a dyad BC ended by a slider which is the output link of the mechanism.

## MECHANISM ANALYSIS

**Mechanism stroke:** In order to assign the time ratio and stroke of the mechanism, it is drawn in its two limiting positions corresponding to the output slider extreme positions. The output slider is in its extreme positions when the rocker QB is collinear with the link BC as shown in Fig.2. This is true only if the limiting positions of the joint B of the 4-bar linkage OABQ is in both sides of the vertical line through joint Q.

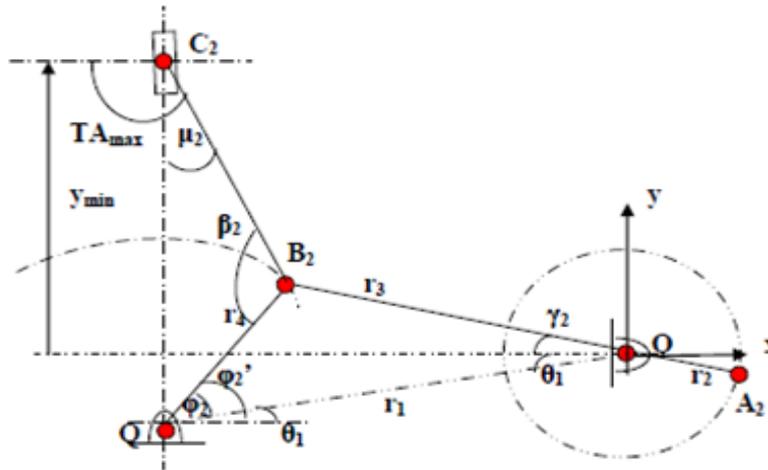


**Fig.2. The mechanism with slider in its upper limiting position.**

Using Fig.2, the maximum slider position relative to the fixed frame of reference Oxy,  $y_{\max}$  is given by:

$$y_{\max} = r_4 + r_5 - r_1 \sin \theta_1 \quad (1)$$

The lowest position of the slider with dimension  $y_{\min}$  from Q is obtained from the limiting position of the 4 bar linkage OABQ as shown in Fig.3.



**Fig. 3. The mechanism with slider in its lower limiting position.**

The lowest slider position with dimension  $y_{\min}$  is obtained as follows using Fig.4:

Angle  $\mu_2$ :

$$\mu_2 = \sin^{-1}\{(r_4/r_5)\sin(90 - \varphi_2')\}$$

where:

$$\varphi_2' = \varphi_2 + \theta_1$$

$$\varphi_2 = \cos^{-1}\{[r_1^2 + r_4^2 - (r_3 - r_2)^2] / (2r_1r_4)\}$$

Angle  $\beta_2$ :

$$\beta_2 = 180 - \mu_2 - (90 - \varphi_2')$$

$$y_{\min} = (r_4 \sin \beta_2 / \sin \mu_2) - r_1 \sin \theta_1 \quad (2)$$

Using Eqs.1 and, the slider stroke is:

$$S = y_{\max} - y_{\min} \quad (3)$$

### **Mechanism time ratio:**

In Fig.3:

$$\alpha_2 + \theta_1 = \cos^{-1}\{[r_1^2 + (r_3 - r_2)^2 - r_4^2] / [2r_1(r_3 - r_2)]\} \quad (4)$$

In Fig.2:

$$L = \sqrt{\{(r_4 - r_1 \sin \theta_1)^2 + (r_1 \cos \theta_1)^2\}}$$

$$\alpha_{11} = \sin^{-1}\{(r_4 - r_1 \sin \theta_1) / L\}$$

$$\gamma_1 = \cos^{-1}\{(r_2^2 + r_3^2 - L^2) / (2r_2r_3)\}$$

$$\alpha_{12} = \sin^{-1}\{(r_3 \sin \gamma_1) / L\}$$

Now:

$$\alpha_3 = \alpha_{11} + \alpha_{12} \quad (5)$$

The crank angle between the two positions corresponding to the lowest and upper position of the slider defining its stroke,  $\Theta$  is:

$$\Theta = 180 - \alpha_3 + \alpha_2 \quad (6)$$

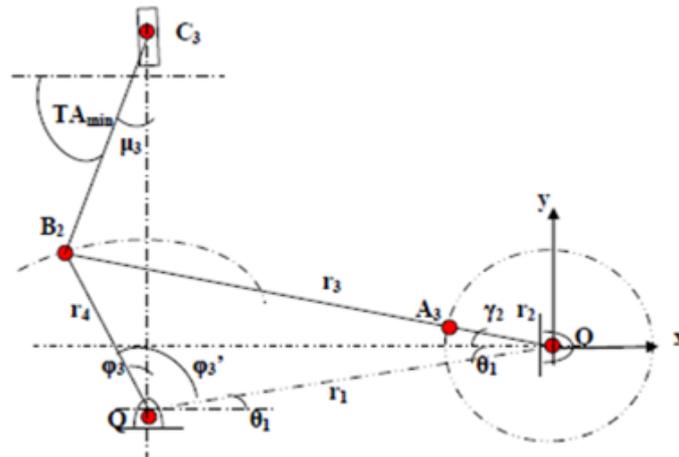
Where  $\alpha_2$  and  $\alpha_3$  are given respectively by Eqs.4 and 5.

The time ratio of the mechanism, TR is given by:

$$TR = (360 - \Theta) / \Theta \quad (7)$$

### ***Mechanism minimum and maximum transmission angles***

The minimum and maximum transmission angles of the mechanism occur at the mechanism positions corresponding to the limiting positions of the 4 bar linkage OABQ. Fig.4 shows the mechanism in its second limiting position corresponding to the 4 bar linkage OABQ.



**Fig.4. The mechanism in the second limiting position of the 4 bar linkage OABQ.**

Angle  $\varphi_3'$ :

$$\varphi_3' = \cos^{-1} \{ [r_1^2 + r_4^2 - (r_3 + r_2)^2] / (2r_1r_4) \}$$

Angle  $\varphi_3$ :

$$\varphi_3 = \varphi_3' - (90 - \theta_1)$$

Angle  $\mu_3$ :

$$\mu_3 = \sin^{-1} \{ (r_4/r_3) \sin \varphi_3 \}$$

The minimum transmission angle,  $TA_{\min}$  is (Fig.4):

$$TA_{\min} = 90 - \mu_3 \quad (8)$$

Using Fig.3, the maximum transmission angle  $TA_{\max}$  is:

$$TA_{\max} = 90 + \mu_2 \quad (9)$$

### Optimal Mechanism Synthesis

The following procedure is used to optimize the dimensions of the 6 bar mechanism under study:

1. An objective function is defined as the time ratio of the mechanism given by Eq.1. This leads to maximizing the time ratio of the mechanism.
2. The performance of the mechanism is controlled through four functional constraints:
  - (i) The basic 4-bar chain OABO has to satisfy Grashof's condition for a crank rocker mechanism. This defines the first functional constraint  $c_1$  as:

$$c_1 = S + L - Q - P \quad (10)$$

$r_2$  is the shortest.

Where  $S$  = shortest link.

$L$  = longest link.

$Q$  and  $P$  = the other two links.

- (ii) The stroke  $S$  of the mechanism has to equal a specific value  $\lambda$ . This defines the second functional constraint  $c_2$  as:

$$c_2 = S - \lambda \quad (11)$$

- (iii) The minimum transmission angle  $TA_{\min}$  has to be greater than 45 degrees [Wilson, Sadler and Michels, 1983]. This defines the third functional constraint as:

$$c_3 = 45 - TA_{\min} \quad (12)$$

- (iv) The maximum transmission angle  $TA_{\max}$  has to be less than 135 degrees [Wilson, Sadler and Michels, 1983]. This defines the fourth functional constraint as:

$$c_4 = TA_{\max} - 135 \quad (13)$$

3. The MATLAB optimization toolbox is used through its command '*fmincon*' to maximize the objective function of Eq.7 subject to the functional constraints in Eqs.10 through 13 [Lopez, 2014].
4. The mechanism has four normalized dimensions to be synthesized:  $r_{1n}$ ,  $r_{3n}$ ,  $r_{4n}$  and  $r_{5n}$ . The four parameters are bounded as follows:

$$1.1 \leq r_{ni} \leq 10 \quad (14)$$

5. The orientation of the ground link  $OQ$ ,  $\theta_1$  is assumed as an input parameter to investigate its effect on the optimal mechanism parameters for the same stroke.
6. The application of the above 6 steps procedure reveals the optimal parameters of the mechanism for a specific stroke and ground link orientation. Table 1 presents a sample of the optimization MATLAB outputs for a 5 degrees ground link orientation.

Table 1: Optimal mechanism dimensions and performance functions for  $\theta_1 = 5^\circ$ .

Desired normalized stroke, $\lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$r_{1n}$	1.9416	1.9487	1.9157	1.8574	1.7802	1.8078	1.8976
$r_{3n}$	4.9206	3.2452	2.6374	2.2859	2.0346	1.9298	1.9382
$r_{4n}$	3.9790	2.2965	1.7216	1.4284	1.2563	1.1219	1.1000
$r_{5n}$	10	10	10	10	10	10	8.4329
TR	1.6702	2.1091	2.4496	2.2422	2.1648	2.3070	2.4095
$S_n$	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$TA_{min}$ (degrees)	66.6471	76.7749	80.1243	81.8186	82.8217	83.5827	82.9035
$TA_{max}$ (degrees)	94.3122	94.8845	95.2079	95.4107	95.5391	95.5495	96.8066

- The optimal time ratio of the mechanism for  $\theta_1 = 5, 10, 15$  and  $20$  degrees is shown graphically in Fig.5.
- The optimal normalized stroke of the mechanism for  $\theta_1 = 5, 10, 15$  and  $20$  degrees is shown graphically in Fig.6.
- The optimal minimum and maximum transmission angles of the mechanism for  $\theta_1 = 5, 10, 15$  and  $20$  degrees are shown graphically in Figs.7 and 8.

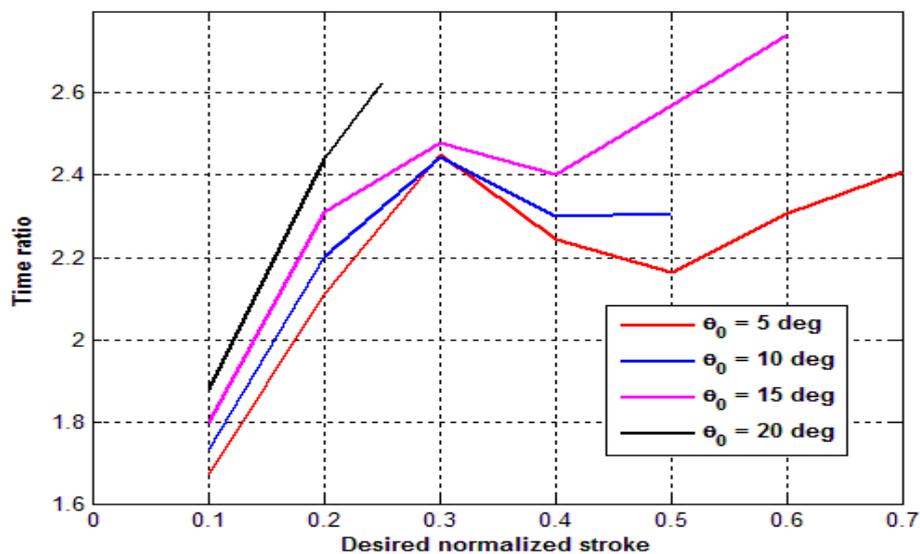


Fig. 5. Mechanism optimal time ratio.

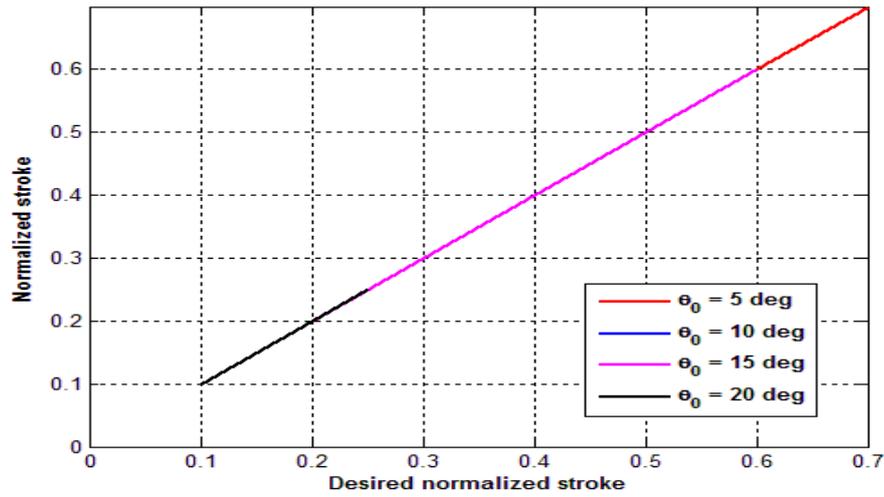


Fig.6. Mechanism optimal stroke.

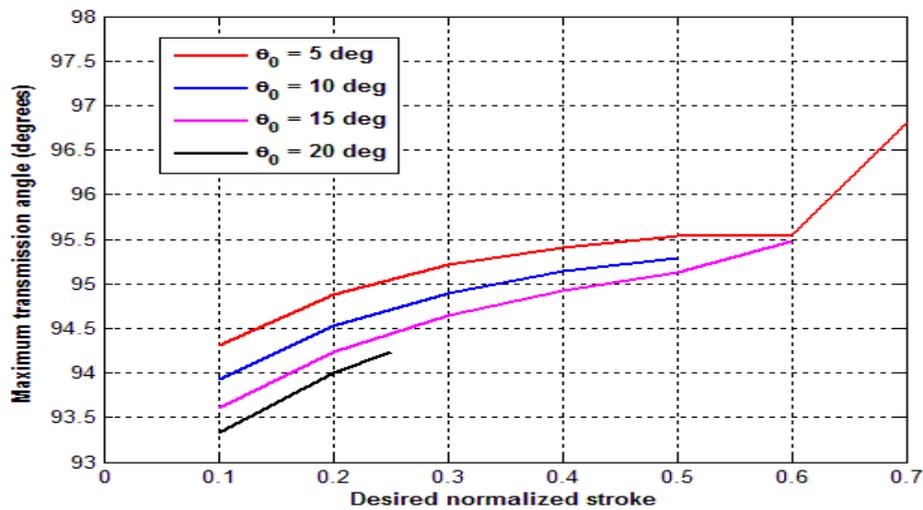


Fig.7. Mechanism optimal minimum transmission angle.

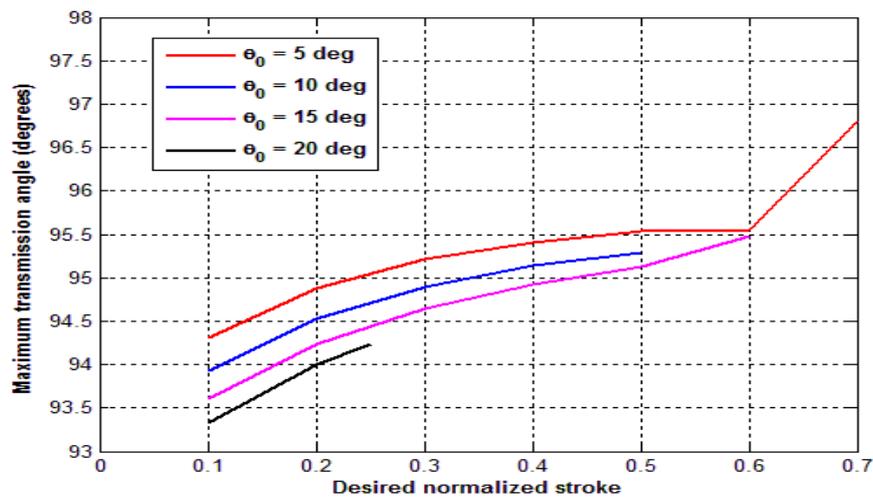


Fig.8 Mechanism optimal maximum transmission angle.

## CONCLUSION

- The objective of this research work was to optimally synthesize a 6 bar – 1 slider mechanism.
- The required relationships were derived relating the mechanism dimensions to its performance functions.
- The mechanism had four dimensionless parameters to be assigned.
- The time ratio of the mechanism was used as the purpose of the synthesis was to maximize the time ratio of the mechanism.
- Four functional constraints were used to control the performance of the synthesized mechanism during operation.
- The design parameters of the mechanism were constrained not to avoid certain limits.
- The MATLAB optimization toolbox was used to maximize the objective function subjected to the parameters and functional constraints.
- The ground orientation was considered as an input parameter changing from 5 to 20 degrees.
- It was possible to attain a dimensionless stroke between 0.1 and 0.8.
- For this type of planar mechanisms it was possible to increase the time ratio to up to 2.74.
- The minimum transmission angle of the mechanism did not decrease than 66.6 degrees (> 45 degrees).
- The maximum transmission angle of the mechanism did not increase than 96.8 degrees (< 135 degrees).
- The synthesis process for accurate and successful.

## REFERENCES

1. Agrawal, S. and Badduri, J. “Optimal synthesis of six-bar function generators”, The 14<sup>th</sup> IFToMM World Congress, Taipei, Taiwan, 25-30 October 2015; pp. 10.
2. Bulatovic, R. and Dordevic, S. (2012), “Optimal synthesis of a path generator six-bar linkage”, Journal of Mechanical Science and Technology, 2012; 26(12): 4027-4040.
3. Hassaan, G. A. “Optimal synthesis of a single dwell 6-bar linkage”, International Journal of Computational Engineering, 2014; 4(2): 50-56.
4. Hassaan, G. A. (2015), “Synthesis of planar mechanisms, Part V: Six bar – three slider mechanism”, International Journal of Advanced Research in Management, Architecture, Technology and Engineering, Vol.1, Issue 4, Accepted for publication.

5. Hassaan, G. A. (2015), “Synthesis of planar mechanisms, Part VI: Six bar – two slider mechanism”, International Journal of Science and Engineering, Vol.1, Issue 7, Accepted for publication.
6. Lopez, C. (2014), “MATLAB optimization techniques”, Springs.
7. Makhsudyan, N., Djavakhyan, R. and Arakelian, V. (2009), “Comparative analysis and synthesis of six-bar mechanisms formed by two serially connected spherical and planar four-bar linkages”, Mechanics Research Communication, 2009; 36: 162-168.
8. Mehdigholi, H. and Akbarnajad, S. “Optimization of Watt’s six-bar linkage to generate straight and parallel leg motions”, Journal of Humanoids, 2008; 1(1): 11-16.
9. Perkins, D. (2011), “Synthesis techniques for coupler-driven planar and spherical SDOF mechanism”, Ph.D. Thesis, School of Engineering, University of Dayton, December.
10. Shiakolar, P., Koladiya, D. and Kebrle, J. “On the optimum synthesis of six-bar linkages using differential evolution and the geometric centroid of precision positions technique”, Mechanism and Machine Theory, 2005; 40: 319-335.
11. Shivdas, P., Bansode, A., Kulkarni, A., Parlikar, V. and Ghedkar, M. “Use of six bar mechanisms for reduction in force and stroke requirement as against four bar mechanisms”, 1<sup>st</sup> International Conference on Machines and Vibrations, Indian Institute of Technology, Bourke, India, 18-20 December 2013; pp. 980-986.
12. Vashista, V., Jere, A., Rane, S., Grover, V. and Saha, S. (2007), “Synthesis and analysis of a new mechanism for sheep shearing machine”, 13<sup>th</sup> National Conference on Mechanisms and Machines, Bangalore, India, 12-13 December, 7 pages.
13. Wilson, C., Sadler, J. and Michels, W. (1983), “Kinematics and dynamics analysis of machinery”, Harper Row Publishers, 1983; p. 24.

## BIOGRAPHY

### Prof. Galal Ali Hassaan:

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.



- Research on Automatic Control, Mechanical Vibrations , Mechanism Synthesis and History of Mechanical Engineering.
- Published more than 100 research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Chief Justice of International Journal of Computer Techniques.
- Member of the Editorial Board of a number of International Journals including the WJERT journal.
- Reviewer in some international journals.
- Scholars interested in the author's publications can visit:

**<http://scholar.cu.edu.eg/galal>**