# THE APPLICATIONS OF THE SPECIFIC (EXTREME) STATES OF SYSTEMS 

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## 1. Critical state

A critical point of a function F is defined, a point where the value of p satisfies the below equation: (1) with the principal value of $p=1$

$$
\begin{equation*}
d F=-p \cdot F \tag{1}
\end{equation*}
$$

The equation can also be taken as a general definition similar to the method of own values / vectors of linear systems, but can also be mathematically proven starting from the tangent approximation through the Newton-Kantorovici development method.

The tangent or the Newton linearization method is applied for the calculation of the real roots of this equation

$$
P(x)=0
$$

An arbitrary point is chosen x 0 near the root, and the equation is linearized around this point, (2)

$$
\begin{equation*}
\boldsymbol{P}_{\text {lin }}(x)=P\left(x_{0}\right)+\boldsymbol{P}^{\prime}\left(x_{0}\right) \cdot\left(x-x_{0}\right) \tag{2}
\end{equation*}
$$

with the solution:

$$
\begin{gather*}
x_{1}=x_{0}-\left(P^{\prime}\left(x_{0}\right)\right)^{-1} \cdot P\left(x_{0}\right)  \tag{3}\\
\text { Or }
\end{gather*}
$$

$$
\begin{equation*}
\frac{d P\left(x_{0}\right)}{P\left(x_{0}\right)}=-1 \tag{4}
\end{equation*}
$$

$\mathrm{P}=0$ is the classic condition of extreme, it defines the condition of a systematic evolution. There are various process examples, with different types of evolutions, each of them treated differently. Therefore

The evolving systems can reach more than one critical point, between these the high-speed waveforms are the most characteristic. While near the critical points the speeds decrease, the material properties will become the priority. Considering that the undulatory behavior takes place at high speeds (at lower temporal intervals), the evolution has a jumping character between the critical points.

## Applications

| Actual Domain | Extremal values for $\mathbf{p}=1$ |
| :---: | :---: |
| $\begin{array}{lcc} \hline \text { Shrodinger's } & \text { equation } & \text { (quantum } \\ \text { mechanincs) } & i h \Psi\left(\Psi^{\prime}=H(t) . \Psi\right. & \\ \hline \end{array}$ | First approximation $i h=-H(t) d t$ |
| Thermotechnic system $\mathrm{F}=(\mathrm{P} . \mathrm{V} / \mathrm{T})$, Carnot efficiency | First approximation $\eta \approx\left(1+\left(\frac{d P}{P}\right)+\left(\frac{d V}{V}\right)\right),\left(2+\left(\frac{d P}{P}\right)+\left(\frac{d V}{V}\right)\right)$ |
| Canonic equations of Hamilton $\begin{aligned} & f\left(p_{1}, p_{2} \cdots p_{n}, q_{1}, q_{2} \cdots q_{n}\right)=0 \\ & \frac{\partial q_{i}}{\partial \tau}=\frac{\partial f}{\partial p_{i}} \ldots \frac{\partial p_{i}}{\partial \tau}=-\frac{\partial f}{\partial q_{i}} \end{aligned}$ | First approximation $\begin{aligned} & f\left(p_{i}\right) d \tau=q_{i}(\tau) \cdot d p_{i} \\ & p_{i}(\tau) \cdot d q_{i}=-f\left(q_{i}\right) d \tau \end{aligned}$ |
| Relativistic force $F=\frac{d}{d t}\left(\frac{m_{0} \cdot v}{\sqrt{1-\frac{v^{2}}{c^{2}}}}\right)$ | First approximation $\mathrm{F}=\frac{-m_{0} \cdot v}{\sqrt{1-\frac{v^{2}}{c^{2}}}} \frac{1}{t-t_{0}}$ |
| Radioactive disintegration, the probability that a core will disintegrate ( $\lambda$ ) $\lambda=-\frac{d N}{d t} \cdot \frac{1}{N}$ | First approximation $\lambda=\frac{1}{N-N_{0}}$ |
| Euler's equation $\boldsymbol{R} \cdot \boldsymbol{u}^{\prime \prime}(x)+\boldsymbol{R} \cdot \cdot \boldsymbol{u}^{\prime}(x)+\left(\boldsymbol{Q}^{\prime}-P\right) \cdot \boldsymbol{u}(x)=0$ | Second approximation $u^{\prime}=\frac{-(R+R)}{\left(Q^{\prime}-P\right) \cdot u}$ <br> Third approximation $u=\frac{-(R+R)}{Q^{-}-P}$ |
| Polinom of order $n$, application of Newton's cinematic eqation | $\begin{gathered} \frac{n \cdot a_{n}}{n \cdot a_{n} \cdot x+a_{n-1}}=\frac{-p}{x-x_{0}} \\ v=v_{0}-a \cdot t+\left(3 a^{2} / 16 v_{0}\right) \cdot t^{2}+\ldots \end{gathered}$ |


| Nonlinear automatic systems $x^{\prime}=f(x, u, t)$ | $f(x, u, t)=-p . x /\left(t-t_{0}\right)$ |
| :---: | :---: |
| $x^{\prime}=A(t) \cdot x+B(t) u$ | $u(t)=\left[-A(t)-\frac{p}{t-t_{0}}\right] \cdot B^{-1}(t) \cdot x$ |
| Pontreaghin maximum principle | $\begin{gathered} d F(x(t), u(t), t)=1 \\ H(x, u)=\omega(x, u)=1 \\ \omega(f)=\sum \frac{d f}{f} \quad \omega(x)=\sum \frac{d x}{x} \end{gathered}$ |
| The equation of elastic waves $\frac{d^{2} \Psi}{d x^{2}}=\frac{1}{u^{2}} \cdot \frac{d^{2} \Psi}{d t^{2}}$ <br> $\Psi$-wave function <br> u- phase propagation velocity <br> $\lambda$-length of the wave <br> $p_{1} \cdot p_{2}{ }^{-}$the critical coefficients of function $\Psi, \Psi$ | Second approximation $\lambda^{2}=4 \Pi^{2} \cdot c^{2} \cdot T^{2} / p_{1} \cdot p_{2}$ |
| The equation of the light emission $\frac{d^{2} \varepsilon}{d x^{2}}+\frac{d^{2} \varepsilon}{d y^{2}}+\frac{d^{2} \varepsilon}{d z^{2}}=\frac{1}{c^{2}} \frac{d^{2} \varepsilon}{d t^{2}}$ | Second approximation $\frac{1}{(d x)^{3}}+\frac{1}{(d y)^{3}}+\frac{1}{(d z)^{3}}=\frac{1}{c^{2}} \frac{1}{(d t)^{3}}$ |
| Solving differential equations $x^{(n)}+f\left(x, x, x^{\prime \prime}, \cdots, x^{n-1}, t\right)=0$ | $\begin{gathered} x_{1}^{\prime}=x_{2} \quad x_{2}^{\prime}=x_{3} \cdots \quad x_{n-1}^{\prime}=x_{n} \\ x_{n}=\mathrm{f}\left(x_{1}, x_{2}, \cdots, x_{n}, \mathrm{t}\right) \\ x_{1}=\frac{x_{2} \cdot x_{10}}{1+x_{2}} \quad x_{2}=\frac{x_{3} \cdot x_{20}}{1+x_{3}} \\ x_{n-1}=\frac{x_{n} \cdot x_{n-1,0}}{1+x_{n}} \quad x_{n}=\frac{f \cdot x_{n 0}}{1+f} \end{gathered}$ |
| Increasing entropy changes the temporal order of systems $\frac{d S}{d t} \geq 0$ | First approximation $p \frac{S}{d t} \leq 0$ |

It is especially important to define correctly the evolution equations. The results in the field of chemical kinetics emphasize that in order to obtain oscillatory evolutions, at least two equations and an additional condition is required. Some equations being even inhomogeneous. The existence of attraction zones of limited circles, mostly characteristic for
the self-organizing systems, imposes the necessity of a non-linear character existence, of a spatial and temporal order.

The lack of an interacting relationship at the complete systems can suggest the existence of the ether (the total sum graph of the actions is not closing)

Uncertainty conditions may also occur due to the insufficient number of equations. We emphasize here that the theory of catastrophes through branch studies addresses incomplete systems (system breakes/crashes) with insufficient number of equations, without equilibrium states (such as poles for the linear systems).

Critical points usually define the sense of evolution, although they are not always part of the accepted functional field. There are known evolutions (for example in chemical kinetics), where the evolution is proportional with the value of the critical point, such as:

$$
\frac{d V}{V}=-p \cdot V
$$

The hyper volume of the system can be introduced as a generating function of the system:

## $V=\prod F_{j}$ with the defining funtions of the system (5)

In some special systems the hyper volume can take the below form:

$$
\begin{equation*}
V=\prod x_{i}^{p_{i}} \tag{6}
\end{equation*}
$$

The $p_{i}$ exponentials will be missing, they can be weights or dimensional coefficients, concentrations, etc.

The relative variation of the hyper volume will be

$$
\Omega(V)=d V / V=\sum_{i=1}^{n} a_{i}\left(d x_{i} / x_{i}\right)=\sum_{i=1}^{n} a_{i} \omega_{i} \quad \omega(V)=\sum_{i=1}^{n}\left(d x_{i} / x_{i}\right)=\sum_{i=1}^{n}\left(\omega_{i}\right)(7)
$$

If the hyper volume $\mathrm{V}>0$ and $\omega(\mathrm{V}) \leq 0$ then the system is stable in Liapunov sense. So the V $>0$ and $\omega \leq 0$ conditions can be used for the stability analysis of the systems. Let us note that in this way we can analyze stability without knowing the law of evolution, only on the basis of measured or observed variables.

In literature, according to $\Omega(V)$ the systems can be classified also as conservatives (Hamiltonians) when $\Omega(V)=0$, respectively dissipatives with $\Omega(V)<0$

If the internal interaction laws complement the interactions with the external environment, we can define a complete system model.

This system - apparently isolated - similar to object-oriented systems in computer science, contains also the effects of the outside world.

The complete systems have special properties, for example, they are not sensitive to disturbances, have oscillatory properties, increase casual effects, operating laws are usually valid even on asymptotic limits, etc.

## 1. Interaction between $\mathbf{2}$ critical points (between motion and inertion)

$\mathrm{p} \leq \omega \leq q \quad S=(\omega-\mathrm{p}) .(\mathrm{q}-\omega)$
there are more cases
a. Small variations
$\omega \approx p, \omega \ll \mathrm{q} \quad S=(\omega-p) \cdot q, \quad \mathrm{~S}$ optim for $\omega=p$
b. Big variations
$\omega \gg \mathrm{p}, \omega \approx q \quad S=\omega(\mathrm{q}-\omega)$, S optim for $\omega=\mathrm{q}$
c. Medium variations
$\omega^{\omega} \gg \mathrm{p}, \omega \ll \mathrm{q} \quad \mathrm{S}={ }^{\omega} \cdot \mathrm{q}, \mathrm{S}$ optim for $\omega=0, \mathrm{p}<0$
The separation limit

$$
\begin{gathered}
\omega-p=\mathrm{q}-\omega \quad \omega=(\mathrm{p}+\mathrm{q}) / 2 \\
\mathrm{~S}=((\mathrm{q}-\mathrm{p}) / 2)^{2}
\end{gathered}
$$

There are no optimums for different p and q .

## 2. Relaxed state

The remote states of relaxation of the systems are asymptotic states, where due to large entropy (at too high or too low energies), indepence/equal probability of the components become dominant (the presystemic state, for example the brownian movement) and the validity of the initial operation laws must be checked. It can be shown that the relaxed state
for a defined system with a system of equations F1...Fn (to which asymptomatically tends the free evolution, even if it does not necessarily reach this state), can be achieved with the following system of equations

$$
\begin{gathered}
\Omega(V) \longrightarrow \omega(V), \boldsymbol{a}_{i}=\boldsymbol{x}_{i} \sum_{j}(1 / F j) .\left(\partial \boldsymbol{F}_{j} / \partial \boldsymbol{X}_{i}\right)= \pm 1 \\
\omega_{R}=\frac{d V}{V}=\sum_{i=1}^{n} \varepsilon_{i} \frac{d x_{i}}{x_{i}}=\text { const. } \\
\varepsilon_{i}=\varepsilon_{k}= \pm 1 \\
k=1, n
\end{gathered}
$$

## Applications

## Gas Dilation

$\mathrm{V}=\mathrm{V}_{0}(1+\gamma . \Delta \mathrm{t})$
$\mathrm{V}-\mathrm{V}_{0} / \mathrm{V}_{0}=\gamma \cdot \Delta \mathrm{t} \rightarrow \gamma \cdot \Delta \mathrm{t}=1$
$\mathrm{V}_{\text {max }}=2 \mathrm{~V}_{0}$

## 3. Chaotic evolution

Like the Liapunov exponents, we will define relative variations $\frac{\Delta X_{i}}{x_{i}}$ of the system variables.

We will formulate a rising sequence with these relative variations.

If all elements of the sequence keep their position, the system does not have a chaotic evolution. Reciprocally this is not true. Pairs of terms can periodically change their place, returning to normal.

If the first element changes its place, we are talking about a temporal leap. If two elements become equal with the opposite sign, and the pair disappears from the sequence, we are talking about system contraction.

## 4. The law of conserving evolution even in interacting conditions

## A system keeps its continuous evolution if it retains the shape of the evolution curve.

## Evolution curve

Similarly to the concepts of curved space in physics (including relativity theory) expressions of the evolution thresholds can be defined, after which we can characterize the movement, for example with the form of
$I(k)=1$ stable linear evolution on the initial curve (any k point of the trajectory)
$I(k)=$ const $\neq 1$ stable non-linear evolution of constant curve
$I(k) \neq I(k-1)$ variable, k as a point of discontinutity of the trajectory
A first such equation can be defined such as
$I(k)=\left(x_{k}-x_{k-1}\right) \cdot\left(x_{k}-x_{k-2}\right) /\left(x_{k+1}-x_{k-1}\right) \cdot\left(x_{k-1}-x_{k-2}\right)$

Bending ratio 4 consecutive points of the trajectories taken in equal temporal intervals. It can be verified that the above relationship has equal invariant value of 1 for the linear systems and respects the curve conditions defined above.

The above conditions can be the base for verifying a sustainable development.

Let us recall that the theory of relativity supports the preservation of the evolutionary form of equations for inertial systems. The above equation can be utilized in applications of interpolation, evolution tracking, etc.

## 5. The synthesis of evolution equations

Starting from a series of classical powers
$y=y_{0}+a_{1} \cdot h+a_{2} h^{2}+a_{3} h^{3}+\ldots \ldots . .+a_{n} h^{n}=y_{0}\left(1+\left(a_{1} / y_{0}\right) h+\left(a_{2} / y_{0} h^{2}+\ldots .+\left(a_{n} / y_{0} h^{n}\right)\right.\right.$ it is possible to express an error type equation of the form:
$\left.\left.\left(y-y_{0}\right) / y_{0}=\left(a_{1} / y_{0}\right) h+\left(a_{2} / y_{0}\right) h^{2}+\ldots .+\left(a_{n} / y_{0}\right) h^{n}\right)=b_{1} h+b_{2} h^{2}+\ldots .+b_{n} h^{n}\right)$
On the other hand an error type equation can be obtained also through expressing a Tailor type equation of the below form:
$y=y_{0}\left(1+c_{1} \cdot h\left(1+c_{2} h\left(1+c_{3} h \ldots . . . . . ..\right) \ldots ..\right)\right.$
$\left(\mathrm{y}-\mathrm{y}_{0}\right) / \mathrm{y}_{0}=\mathrm{c}_{1} . \mathrm{h}+\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{~h}^{2}+\mathrm{c}_{1} \mathrm{c}_{2} \mathrm{c}_{3} \mathrm{~h}^{3}+\quad+\mathrm{c}_{1} \ldots \ldots . . \mathrm{c}_{\mathrm{n}} \mathrm{h}^{\mathrm{n}} \quad \mathrm{c}_{\mathrm{n}}=(1 / \mathrm{n}) .\left.\left(\mathrm{f}^{\mathrm{n}} / \mathrm{f}^{\mathrm{n}-1}\right)\right|_{\mathrm{x} 0}$
$\mathrm{c}_{1}=\mathrm{a}_{1} / \mathrm{y}_{0} \quad \mathrm{c}_{1} \mathrm{c}_{2}=\mathrm{a}_{2} / \mathrm{y}_{0}$

Having recurrence relationships
$\mathrm{c}_{\mathrm{n}}=\mathrm{a}_{\mathrm{n}} / \mathrm{a}_{\mathrm{n}-\mathrm{l}}$

We recommend the latest development, which has a high convergence speed, is easy to use also in the discreet field.

