



AN M/M/C ENCOURAGED ARRIVAL FEEDBACK QUEUING MODEL WITH CUSTOMER IMPATIENCE

Bhupender Kumar Som^a, Vivek Kumar Sharma^b and Sunny Seth^{*c}

^aProfessor, Lloyd Business School, Greater Noida, Uttar Pradesh – 201306.

^bProfessor, JaganNath University, Jaipur, Rajasthan – 302022.

^cAssistant Professor, Jagan Institute of Management Studies, New Delhi – 110085.

Article Received on 06/04/2020

Article Revised on 27/04/2020

Article Accepted on 17/05/2020

*Corresponding Author

Sunny Seth

Assistant Professor, Jagan
Institute of Management
Studies, New Delhi – 110085

ABSTRACT

Understanding the behavior of the customers is the most challenging task which firms have to face in today's scenario of cut-throat competition. In order to gain competitive edge firms often introduce lucrative offers and discounts due to which the customers get attracted

towards particular firm these customers are termed as *encouraged arrivals*. As a consequence of encouraged arrivals more and more customers join the system hereby increasing the time, a customer will have to spend in waiting for the service which results in customer impatience. These impatient customers may leave the system without service. Firms often deploy various strategies to retain these renegeing customers for sustainable growth. Encouraged arrivals also put service facility under pressure due to which some customers, dissatisfied with the service join the queue again as *feedback customers* for satisfactory service. In this paper we develop a queuing model with all issues as discussed above and solve the model in steady state to derive various probabilistic and performance measures and we also present numerical illustration for better understanding of the applicability of the model. Having known these performance measures in advance various strategies can be designed for smooth administration and functioning of the system.

KEYWORDS: Encouraged arrivals, multi-server queuing model, feedback queue, renegeing, retention, customer impatience.

1 INTRODUCTION

In today's scenario of fierce competition every firm strives for sustainable growth and in order to design sustainability strategies, understanding the functioning of the overall system and uncertain customers' behavior is of utmost importance & need. Easy access to global markets at their fingertips with smartphones, customers have enormous choices available to them and they choose the one offering them best deals and discounts. So to attract customers firms often introduce lucrative offers and announce sale. The customers who thus get attracted towards the firm are termed as *encouraged arrivals*. During the sale period firms often experience heavy rush and customers have to wait for longer in the queues to avail service. Some customers come with a threshold limit of time and having waited for longer time, they leave the system. Customers thus leaving can prove to be detrimental for the reputation of the firm and result in loss of revenue as well as goodwill. This can hamper the sustainable growth of the firm as customers with easy access to various social media platforms can post negative comments which can hamper the reputation and goodwill of the firm, so firms deploy various strategies to retain these customers to the best possible extent. Heavy rush at times puts limited service facility under pressure and that in turn can result in dissatisfaction among certain customers. Customers dissatisfied with the service join the queue again as *feedback customers* for satisfactory service. This paper is an extension of the work by authors in,^[9] with infinite capacity system. As now-a-days with various online e-commerce platforms there is no restriction of physical space needed for customers, and innately infinite number of customers can join the system through various online e-commerce platforms. This elaborates the significance of studying infinite capacity model. Detailed literature can be seen in Som and Seth.^[9]

Rest of the paper is organized as follows: Formulation of the model is presented in section 2. Steady-state probabilities of the models are derived recursively in section 3. Various performance measures are presented in section 4. Section 5 presents a real-life numerical illustration and conclusion and future scope is given in section 6.

2 Mathematical Model Formulation

Model is formulated with same assumptions as in Som and Seth,^[9] with an exception that capacity of the system is considered to be innately infinite here.

Steady-State difference equations governing the model are given by:

$$0 = -\lambda(1 + \eta)P_0 + \mu p P_1; n = 0 \tag{1}$$

$$0 = \lambda(1 + \eta)P_{n-1} - \{\lambda(1 + \eta) + n\mu p\}P_n + (n + 1)\mu p P_{n+1}; 1 \leq n \leq c - 1 \tag{2}$$

$$0 = \lambda(1 + \eta)P_{n-1} - \{\lambda(1 + \eta) + c\mu p + (n - c)\xi p'\}P_n + \{c\mu p + (n + 1 - c)\xi p'\}P_{n+1}; n \geq c \tag{3}$$

3 Steady-state solution

On solving (1) - (3) recursively we get;

$$P_n = \text{Pr}\{n' \text{ customers in the system}\} \\ = \begin{cases} \frac{1}{n!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^n P_0; & 1 \leq n \leq c \\ \frac{1}{c!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^c \prod_{k=c+1}^n \left(\frac{\lambda(1 + \eta)}{[c\mu p + (k - c)\xi p']}\right) P_0; & n > c \end{cases} \tag{4}$$

Using condition of normality $\sum_{n=0}^{\infty} P_n = 1$, we get

$$P_0 = \text{Pr}\{\text{no customer in the system}\} \\ = \left[\sum_{n=0}^c \frac{1}{n!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^n + \frac{1}{c!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^c \sum_{n=c+1}^{\infty} \left\{ \prod_{k=c+1}^n \left(\frac{\lambda(1 + \eta)}{[c\mu p + (k - c)\xi p']}\right) \right\} \right]^{-1} \\ = \left[\sum_{n=0}^c \frac{1}{n!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^n + \frac{1}{c!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^c e^{\frac{\lambda(1 + \eta)}{\xi p'}} \left(\frac{\lambda(1 + \eta)}{\xi p'}\right)^{-\frac{c\mu p}{\xi p'}} \left[\Gamma\left(\frac{c\mu p + \xi p'}{\xi p'}\right) - \Gamma\left(\frac{c\mu p}{\xi p'} + 1, \frac{\lambda(1 + \eta)}{\xi p'}\right) \right] \right]^{-1} \tag{5}$$

4 Measures of Performance

1. Probability that a customer has to wait (P_b):

$P_b = \text{Pr}\{\text{all servers are busy}\}$

$$= \sum_{n=c}^{\infty} P_n = \left[\frac{1}{c!} \left(\frac{\lambda(1 + \eta)}{\mu p}\right)^c \left\{ 1 + e^{\frac{\lambda(1 + \eta)}{\xi p'}} \left(\frac{\lambda(1 + \eta)}{\xi p'}\right)^{-\frac{c\mu p}{\xi p'}} \left[\Gamma\left(\frac{c\mu p + \xi p'}{\xi p'}\right) - \Gamma\left(\frac{c\mu p}{\xi p'} + 1, \frac{\lambda(1 + \eta)}{\xi p'}\right) \right] \right\} \right] P_0 \tag{6}$$

2. Length of the queue (L_q)

$L_q = \sum_{n=c}^{\infty} (n - c)P_n$

$$= \sum_{n=c+1}^{\infty} (n - c) \prod_{k=c+1}^n \frac{a}{b + (k - c)d} = \\ \frac{1}{d(b + d)} \left(\frac{a}{d} \right)^{-\frac{b}{d}} \left(a^2 \left(\frac{a}{d} \right)^{b/d} + d^2 e^{a/d} \Gamma\left(\frac{b + d}{d}\right) - d^2 e^{a/d} \Gamma\left(\frac{b + 2d}{d}\right) - \right. \\ \left. d^2 e^{a/d} \Gamma\left(\frac{b}{d} + 1, \frac{a}{d}\right) + d^2 e^{a/d} \Gamma\left(\frac{b}{d} + 2, \frac{a}{d}\right) + b d e^{a/d} \Gamma\left(\frac{b + d}{d}\right) + \right. \\ \left. a d e^{a/d} \Gamma\left(\frac{b + 2d}{d}\right) - b d e^{a/d} \Gamma\left(\frac{b + 2d}{d}\right) - b d e^{a/d} \Gamma\left(\frac{b}{d} + 1, \frac{a}{d}\right) - \right. \\ \left. a d e^{a/d} \Gamma\left(\frac{b}{d} + 2, \frac{a}{d}\right) + b d e^{a/d} \Gamma\left(\frac{b}{d} + 2, \frac{a}{d}\right) \right)$$

Where $a = \lambda(1 + \eta)$, $b = c\mu p$, $d = \xi p'$, $\Gamma(x)$ is gamma function and $\Gamma(x, y)$ is an incomplete gamma function.

3. Average time a customer has to wait in the queue (W_q):

$$W_q = \frac{L_q}{\lambda(1 + \eta)} \quad (8)$$

4. Average time a customer has to wait in the system (W_s):

$$W_s = W_q + \frac{1}{c\mu} \quad (9)$$

5. Length of the system (L_s):

$$L_s = \lambda(1 + \eta)W_s \quad (10)$$

5 Numerical Illustration

Customers visit a particular firm in accordance to Poisson process with an average rate of 3 customers per unit time. In order to attract customers and to stay ahead of competitors firm wants to introduce lucrative deals and discounts and is expecting 75% increase in the number of customers visiting the firm on the basis of data when firm offered similar deals in the past. There are 3 servers providing service in accordance to exponential distribution with average service rate of 4 customers per unit time. The probability that a dissatisfied customer joins the queue again is given as 0.4 and an impatient customer may leave the queue without taking service with probability 0.2. Firm deployed certain strategies to retain the leaving customers and is expecting that retention probability is 0.3. So as to design strategies for smooth functioning of the system firms needs to measure its performance. Calculate the following:

1. Probability that there is no customer in the system
2. Probability that a customer has to wait
3. Length of the queue
4. Average time a customer has to wait in the queue
5. Average time a customer has to wait in the system
6. Length of the system

Solution: Following are the parameters of the above problem:

$\lambda = 3; \eta = 0.75; c = 3; \mu = 4; \xi = 0.2; p = 1 - q = 1 - 0.4 = 0.6; p' = 1 - q' = 1 - 0.3 = 0.7$; Using equations (5) to (10), we get:

1	Probability that there is no customer in the system	0.0892506
2	Probability that a customer has to wait	0.5019746
3	Length of the queue	1.0159273
4	Average time a customer has to wait in the queue	0.1935100
5	Average time a customer has to wait in the system	0.2768433
6	Length of the system	1.4534273

6 Conclusions and Future Scope

Since the paper compiles realistic customer behavior, the model can predict the behavior of system. The probabilistic outputs can be used for effective business management. Performance measures derived can prove to be helpful for firms experiencing issues discussed in this paper and can enable firms to develop effective strategies for smooth and efficient functioning of the system.

In future the cost model can be developed and optimization of number of servers or service rate can be performed for efficiency of the system.

REFERENCES

1. Santhakumaran, and V. Thangaraj, "A Single Server Queue with Impatient and Feedback Customers", *Information and Management Science*, 2000; 11: 71-79.
2. K. Som, "A Stochastic Feedback Queuing Model with Encouraged Arrivals and Retention of Impatient Customers", 5th IIMA International Conference on Advanced Data Analysis, Business Analytics and Intelligence, April 08-09, 2017.
3. K. Som, A Markovian feedback queuing model with encouraged arrivals and customers impatience, 2nd International conference on evidence based management (ICEBM), March 17th – 18th, 2017.
4. B. K. Som, and S. Seth, "An M/M/1/N Queuing system with Encouraged Arrivals", *Global Journal of Pure and Applied Mathematics*, 2017; 17: 3443-3453.
5. B. K. Som, and S. Seth, "An M/M/2/N Queuing System with Encouraged Arrivals, Heterogenous Service and Retention of Impatient Customers", *Advanced Modelling and Optimization*, 2017; 19: 97-104.
6. B. K. Som, and S. Seth, "Effective Business Management in Uncertain Business Environment Using Stochastic Queuing System with Encouraged Arrivals and Impatient Customers", *Proceedings of International Conference on Strategies in Volatile and Uncertain Environment for Emerging Markets*, IIT Delhi, July 14th - 15th, 2017; 479-488.
7. B. K. Som, and S. Seth, "An M/M/c Feedback Queuing System with Encouraged Arrivals", *Siddhant*, 2017; 17(3): 252-255.
8. B. K. Som, and S. Seth, "Stochastic Priority Queuing Model with Encouraged Arrivals for Managing the Healthcare Facility", 3rd International conference on advances in healthcare management, organized by IIM – A, December 9 &10, 2017.

9. B. K. Som, and S. Seth, "M/M/c/N queuing systems with encouraged arrivals, renegeing, retention and feedback customers", *Yugoslav Journal of Operations Research*, 2018; 28(3): 333–344.
10. J. Ancker Jr, and A. V. Gafarian, "Some Queuing Problems with Balking and Reneging – I", *Operations Research*, 1963; 11: 88-100.
11. C. J. Ancker Jr, and A. V. Gafarian, "Some Queuing Problems with Balking and Reneging –II", *Operations Research*, 1963; 11: 928-937.
12. Y. Barrer, "Queuing with Impatient Customers and Indifferent Clerks", *Operations Research*, 1957; 5: 644-649.
13. Robert, "Reneging Phenomenon of Single Channel Queues", *Mathematics of Operations Research*, 1979; 4(2): 162-178.
14. A. Haight, "Queuing with renegeing", *Metrika*, 1959; 2: 186-197.
15. F. A. Haight, "Queueing with balking", *Biometrika*, 1957; 44: 360-369.
16. F. Baccelli, I. P. Boyer, and G. Hebuterne, "Single-server Queues with Impatient Customers", *Advances in Applied Probability*, 1984; 16: 887-905.
17. R. Davignon, and R. L. Disney, "Single Server Queue with State Dependent Feedback", *INFOR*, 1976; 14: 71-85.
18. Bae and S. Kim, "The Stationary Workload of the G/M/1 Queue with Impatient Customers", *Queueing Systems*, 2010; 64: 253-265.
19. Wang, N. Li, and Z. Jiang, (2010). "Queueing System with Impatient Customers: A Review", 2010 IEEE International Conference on Service Operations and Logistics and Informatics, 15-17 July, 2010; 82-87.
20. Takacs, "A Single Server Queue with Feedback", *The Bell System Tech. Journal*, 1963; 42: 134-149.
21. P. Y. Liao, "The Queuing Model with Balking Index and Reneging Rate", *International Journal of Services and Operations Management*, 2011; 10(1): 1-12.
22. R. Kumar, and S. K. Sharma, "M/M/1/N Queuing System with Retention of Reneged Customers", *Pakistan Journal of Statistics and Operation Research*, 2012; 8(4): 719-735.
23. R. Kumar, and S. K. Sharma, "An M / M / 1 / N Queuing Model with Retention of reneged customers and Balking", *American Journal of Operational Research*, 2012; 2(1): 1-5.
24. R. Kumar, and S. K. Sharma, "A Markovian Feedback Queue with Retention of Reneged Customers", *AMO-Advanced Modelling and Optimization*, 2012; 14(3): 673-679.

25. R. Kumar, and S. K. Sharma, "A Markovian Feedback Queue with Retention of Reneged Customers and balking", AMO-Advanced Modelling and Optimization, 2012; 14(3): 681-688.
26. R. Kumar, and S. K. Sharma, "A Single-Server Markovian Queuing System with Discouraged Arrivals and Retention of Reneged Customers", Yugoslav Journal of Operations Research, 2014; 24: 119-126.
27. R. Kumar, and S. K. Sharma, "An M/M/c/N queuing system with renegeing and retention of reneged customers", International Journal of Operational Research, 2013; 17: 333-340.