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FEATURES OF OPERATION IN THE VAN DER POL TRANSISTOR OSCILLATOR

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ABSTRACT

The Van der Pol Oscillator, which is a classic example of a self-oscillating device, is usually analyzed using the noted Van der Pol equation. The paper shows that being a good model for describing self-oscillations of a very different nature, this equation, however, does not accurately describe the processes in the device itself. It should be noted that Van der Pol equation was actually obtained for oscillators based on tube amplifiers, which were rather imperfect and obsolete technology and long ago replaced by transistor amplifiers. Accordingly, in this paper, a more accurate mathematical model of a transistor-based oscillator is studied, using the Matlab-Simscape program where the obtained results are compared with the conventional ones. The proposed model does not refute the Van der Pol equation, rather clarifies the processes that develops and yields a plausible new aspect of study on synchronizing self-oscillations, the processes of transferring active power and the impact of reactive power on the synchronization of oscillations. Using a physical model, the results of theoretical study were confirmed by experimental verification.

KEYWORDS: Non-linear dynamics, Oscillations, Synchronization, Reactive Power.

1. INTRODUCTION

The van der Pol equation has become a cornerstone and plays a significant role in the theory of nonlinear dynamics (Steven,1994). Initiated and developed by radio engineer and mathematician B. Van der Pol (1889-1959) to describe the functioning of his proposed tube oscillator, it determined the possibility of obtaining a continuous self-sustained periodic oscillations without the affect of a periodic source. It is clear that Van der Pol equation has no analytical solution whereas can be solved by numerical methods - currently with the assistance of modern computer programs (Phillipson, et al., 2009).

Various approximate methods are used to solve such equation. One of which is the small parameter method, that was proposed and applied by Van der Pol (Marinca, et al., 2011). Other studies are devoted for applying of alternative methods and approaches to approximate differential equations solution, such as harmonic balance (Abul Kashem Mondal, et al. 2019) , averaging methods (Hale, 1963). In (Kyzioł, et al., 2015) periodic steady-state solutions, of the corresponding equation are determined using the Krylov-Bogoliubov- Mitropolsky approach. In (Jifeng Cui, et al., 2016). the solution is obtained by employing the Homotopy Analysis Method (HAM), high accuracy frequency response curve and the stable/unstable periodic solutions of the Van der Pol-Duffing forced oscillator, with the variation of the forced frequency, is studied. Miscellaneous different numerical methods are presented, for instance, in (Solomon M. Antoniou, 2016) attaining solution by applying numerical technics, using standard programs, such as - Mathcad, Matlab, etc. Applying programs as means of visualization, for instance, Matlab-Simulink, so in this way the Van der Pol equation is solved using operational amplifiers and integrators, that directly simulate the equation (Brian D. Hahn, et al., 2019, Ahmed, et al., 2019).

Describing the possibility of attaining self-excited oscillations, the Van der Pol equation, which arose based on electrical system, turned out to be applicable for many other systems and phenomena. Examined in (Cercek, et al., 1996) is a nonlinear dynamics of an instability which is triggered by a positive electrode, in a weakly magnetized discharge plasma column. Mechanical vibrating system - in (Warminski, 2012), biological populations -Predator-Prey Interaction (Marinca, et al., 2011, Martha L. Abell, et al., 2014), mathematical model of an oscillatory chemical reaction - the brusselator (Nicolis, 1971). Few phenomena found in nonequilibrium systems, including oscillatory phenomena, order-formation processes, and pattern formation – (Shuichi Kinoshita, 2013*)*. In (Shovan Dutta, et al., 2019) quantum version of a driven Van der Pol oscillator explores as efficient sensors, due to a strongly nonlinear response.

This review bolsters the significance of Van der Pol equation, which comprehend an important place in the study of nonlinear dynamics processes, for a wide variety of phenomena in physics, chemistry, biology, and technology. As stated, the Van der Pol equation arose to describe an electric oscillator based on a vacuum tube triode. At the time, vacuum tube triode were rather imperfect and even though the nonlinearity current-voltage characteristic of the devices were not pronounced properly, it turned out to be sufficient to obtain a laconic and fairly universal mathematical model, which describes a wide range of issues. In modern amplification technology, vacuum devices have not been in use for a long time and have been replaced by more advanced semiconductor, such as transistors. The transistors are characterized by a greater pronounced nonlinearity of current-voltage characteristics, compared with the tube triode. Referring to literature (Landa, 1997), the description of the processes in the oscillator, assumes that the lamp-vacuum tube and the transistor are interchangeable, despite the significant difference in their current-voltage characteristics. Thus, there is a great interest to consider a detailed scope of study of the Van der Pol oscillator processes, utilized by semiconductor transistor, and to evaluate the consequences of the modifications due to the replacement of the amplifying devices.

The paper is divided into six sections. In section II, the traditional Van der Pol equation are derived and the solutions are given using the Matlab program, to which a brief characteristics is given. In section III, the complete electrical circuit of the oscillator is presented and simulated, using the Matlab Simscape program. For the simulated model all the characteristics of the transistor, were taken into account and so the voltages and currents of the circuit elements were considered as equivalent to the physical description, which is introduced and compared with the results of the previous section. An important factor is emphasized in this section, the possibility of assessing the flow of both active and reactive power of the circuit. Section IV evaluates the well-known method of synchronizing oscillator oscillations, by introducing a sinusoidal source which displays, based on the power circulation, in the case study that the oscillator doesn't serve its original purpose. Therefore, in section V a different method of synchronization is suggested, while maintaining the prime functionality of the circuit. The experimental results are introduced in section VI. Ending with Conclusions, the obtained results are briefly evaluated and final conclusions are introduced.

2. The Van der Pol transistor oscillator circuit and the classical description of its operation

The oscillator circuit is introduced in Fig. 1. It is assembled of two circuits - primary one, which includes a power supply V_{in} with an internal resistance R_c, an inductance L_1 and a transistor Q and secondary one consists of oscillatory circuit L-C with a series load resistance R. Both circuits are connected by mutual inductance, which is induced by inductors L_1-L mutually, with a coupling coefficient fixed to approximately one. The oscillations that have originated from it, for example, due to the initial voltage drop of the capacitor, leads to periodic commutation of the transistor and the establishment of stationary oscillations in the circuit. The frequency of such oscillations is different from the frequency of a L-C-R oscillatory circuit due to a large extent of dependence on the mutual inductance coefficient of the inductances L-L_1. As aforementioned and well emphasized above, the early version proposed directly by B. Van der Pol, consists of an electronic lamp instead of transistor. Such component imperfection characteristics, in comparison with the transistor's characteristics, paved the way for such theoretical analysis choice. We shell consider it in its classical presentation.

Taking into account the designations of voltages and currents in the scheme Fig. 1, we can write:

$$L\frac{di}{dt}+v+Ri=M\frac{di_1}{dt} \tag{1}$$

Considering that $i=C\dfrac{dv}{dt}$

$$LC\frac{d^2v}{dt^2}+RC\frac{dv}{dt}+v=M\frac{di_1}{dt} \tag{2}$$

where M is the coefficient of the mutual inductance between the inductors L and L_1.

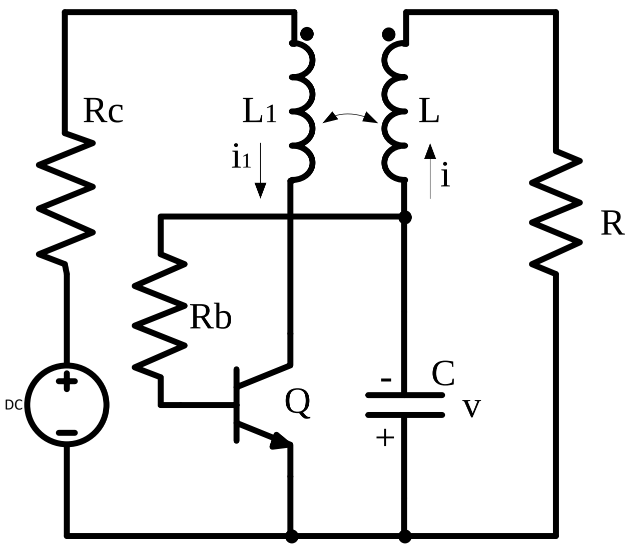


Fig. 1: The circuit of a Van der Pol transistor oscillator.

Further, it is assumed that a nonlinear dependence takes place between the control voltage of the transistor's base *v* and it's collector current i_1, which can be expressed by an odd powers polynomial of the variable *v*, namely:

$$i_1 = a_o v + a_1 v^3 + a_2 v^5 + a_3 v^7 \qquad (3)$$

Differentiating the right-hand side of (3) and discarding the terms of even powers "4" and "6", due to their little effect on the processes in the circuit [20], we obtain:

$$\frac{d^2v}{dt^2} - \left(\frac{a_0 M}{LC} - \frac{R}{L} + \frac{3a_1 M}{LC} v^2\right)\frac{dv}{dt} + \frac{1}{LC} v = 0 \qquad (4)$$

Following the notations $\frac{a_0 M}{LC} - \frac{R}{L} = \eta$ and $\frac{3a_1 M}{\eta LC} = -\alpha$ (whereas the coefficient a_1 of the polynomial will always turn out to be negative), equation (4) takes the form:

$$\frac{d^2v}{dt^2} - \eta\left(1 - \alpha v^2\right)\frac{dv}{dt} + \frac{1}{LC} v = 0 \qquad (5)$$

Following additional designations such as: $\omega_0^2 = \frac{1}{LC}$, $\mu = \frac{\eta}{\omega_0}$ and $\tau = \omega_0 t$, we obtain the final form of Van der Pol equation:

$$\frac{d^2x}{d\tau^2} - \mu\left(1 - \alpha x^2\right)\frac{dx}{d\tau} + x = 0 \qquad (6)$$

As is well known, this equation is solved using special computer programs, particularly, Matlab. For determining the coefficients in equation (6), of the numerical solution of this equation, we will use the following parameters, of the oscillator circuit: inductance L_1=L=

100μH, coupling coefficient between inductors *k = 0.99*, capacitor value *C =10μF* and load *R=1Ω*. The coefficients of the polynomial (3) are defined for the collector current function I_{CE}, which depends on the base voltage drop V_B, of the transistor. Referring to the BJT model of the Matlab-Simscape program (Fig. 2), the coefficients are equal to $a_0=0.168$, $a_1=-0.922$ (referring to Fig. 2, I_{CE} - is the collector current, V_B - is the voltage drop at the base input, i.e., the voltage drop across the capacitor *C*, R_b is the additional resistance at the base circuit, $R_b=100\Omega$).

The calculated results of equation's (6) parameters are as follows: natural frequency of the loop $\omega_0 = 3.162 \cdot 10^4 s^{-1}$, period of the oscillations $T = 201.223 \cdot 10^{-6} s$, coefficients $\mu = 0.21$ and $\alpha = 41.3$. Let us pay attention to the fact that the period of oscillatory modes, in the solution of equation (6) is ω, which is several times longer in comparison to the physical system.

To perform the calculation, equation (6) is represented by a system of equations in the Cauchy form, for two variables: $y(1) = x$ and $y(2) = \dfrac{dx}{dt} = \dfrac{dy(1)}{dt}$,

$$\begin{aligned} \frac{dy(1)}{dt} &= y(2) \\ \frac{dy(2)}{dt} &= \mu\left(1 - \alpha y(1)^2\right) y(2) - y(1) \end{aligned} \tag{7}$$

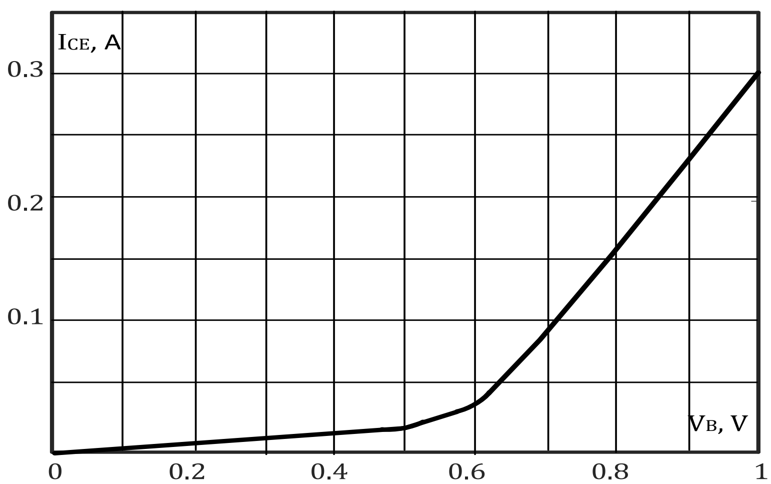


Fig. 2: Characteristics $I_{CE} = f(V_B)$ of the transistor model in the Matlab-Simscape program.

In accordance with the procedure of solving differential equations in Matlab, a user function is compiled in a script file called Vdp_0.m, in the following form:

```
Function dydt = Vdp_0(t,y)
dydt = [y(2); 0.21*(1-41.3*y(1)^2)*y(2)-y(1)];
```

and then the system itself is solved (dydt):

```
[t,y] = ode23t(@Vdp_0,[0 100],[0.5 0]);
```

Whereas the matrix [0 100] is set to be the range of time variation from 0 to 100s, and the matrix [0.5 0] – are the initial conditions for the variables $y(1)$ and $y(2)$, respectively.

The graphs in Fig. 3a, b shows the variation of values $y(1)$ and $y(2)$ with respect to time. It can be observed that the solutions of the equation reflects a steady-state periodic oscillations with the value of $T_\tau = 6.27$ or in terms of time, within the physical circuit, to $T = T_\tau / \omega_0$, $T = 198.3 \cdot 10^{-6} s$, which practically coincides with the period of oscillations in the *R-L-C* circuit.

Furthermore, an increase of value L_1 to $L_1 = 1000 \mu H$ yields to an increase in the period of oscillations, which is calculated by equation (6) and results the value of $T_\tau = 6.94$, or with respect to time within the physical circuit, to $T = T_\tau / \omega_0 = 219.5 \mu s$. A further increase in L_1 and, in general case, in the value of μ leads even to a greater increase in the value T_τ. For instance, at $\mu = 100$ and $\alpha = 1$, the period $T_\tau = 162.8$, i.e., for the physical circuit, $T = 5.1 ms$.

A comparison of voltage and current amplitude values, which are obtained from equation (6) and referring to physical quantities, are reviewed and discussed below. Let us dwell on the main features of the above analysis.

1. The equation (6) presents no explicit power source of the oscillator circuit – i.e. the voltage source V_{in}. On the other hand, it does present the power source indirectly, using an indirect form of coefficients, obtained through approximation expression. Therefore makes it difficult to understand the physical processes of the circuit. Furthermore, as presented below, it limits the possible options for synchronizing the oscillations, through changing the voltage waveform of the supply source.

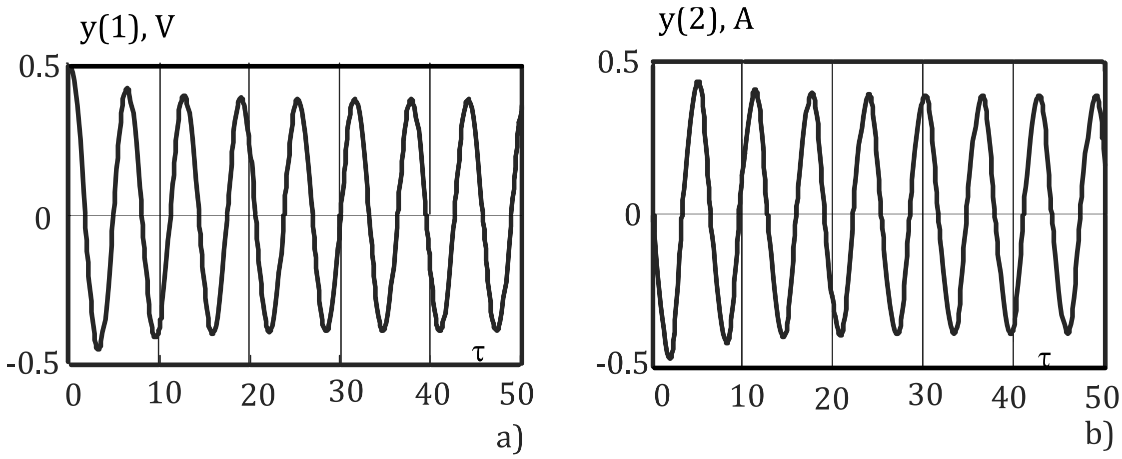


Fig. 3: Diagrams of the voltage across the capacitor *C* - (y(1)) and the inductance current *L* - (y(2)), obtained from equation (6).

2. As presented in the circuit, the transistor (or lamp) does not conduct current i_1 at a negative voltage drop, across the base of the transistor (lamp grid), i.e., a negative voltage across the capacitor. Nevertheless the approximation of characteristic (6) does not reflect this fact; it is actually symmetric with respect to the origin of coordinates, which demonstrate a flow of current i_1 in both directions.
3. Discarding terms of power higher than three, in expression (6), essentially limits the approximation to two terms a_0v-a_1v^3 , while considering the negative scale of the second term (a_1), results that the two-term approximation is valid only for x <1, whereas at an increase of x, the value of the expression decreases rather than increasing.
4. Further analysis will show, that the listed features of the analysis, lead to the distortion of voltage and particularly current waveforms in the oscillator.

3. Simulation of the complete transistor oscillator circuit

To eliminate the mentioned shortcomings, we will based on system of equations that considers the full current- voltage characteristics of the transistor, - i.e. the dependence of the collector current I_{CE} on the collector-emitter voltage V_{CE}, at different voltage values, based on V_B. The characteristics, obtained for a transistor model using the Simscape program, is presented in Fig. 4. Consequently the system of equations, describing the functioning of the oscillator, will take the following form:

$$\begin{aligned}
&L_1\frac{di_1}{dt}-M\frac{di}{dt}+R_c i_1+v_{CE}=V_{in}\\
&L\frac{di}{dt}-M\frac{di_1}{dt}+v+Ri=0\\
&C\frac{dv}{dt}=i \qquad\qquad (8)\\
&i_{CE}=i_1\\
&v_B=v\\
&v_{CE}=f(i_{CE},v_B)
\end{aligned}$$

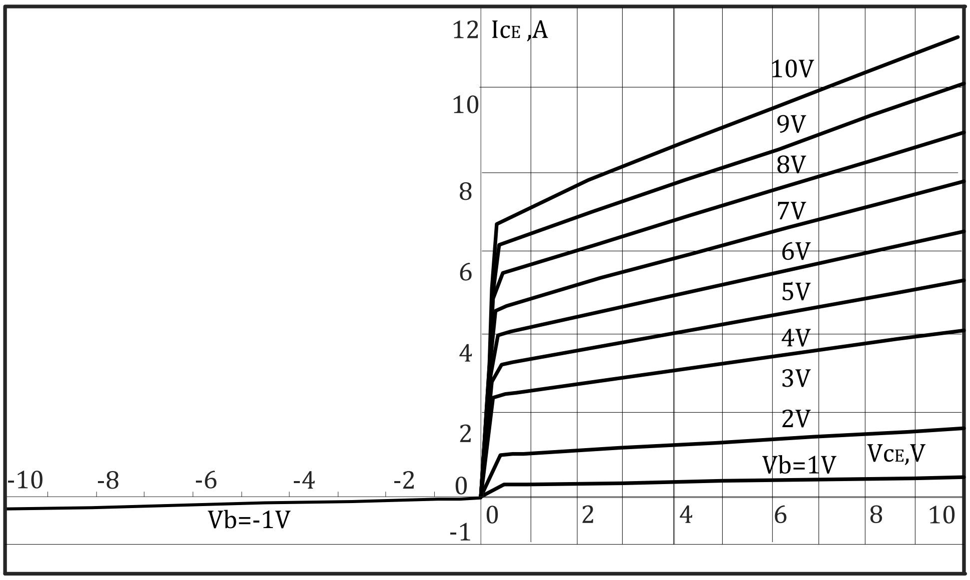


Fig. 4: Full current-voltage characteristics $I_{CE}=f(V_{CE},V_B)$ of the transistor.

Similarly, system of equations (8) and (6), are nonlinear and can be solved using a computer program. Conveniently, to obtain the results in this matter, we used Matlab and its visual Simscape application. The model simulated and introduced in Fig. 5 corresponds to Fig. 1 and reflects the characteristics of the transistor in Fig. 4. For example, the following model parameters were taken: collector voltage $V_{in}=10V$, internal source resistance $R_c=0.1\Omega$, resistance in the base circuit $R_b=100\Omega$, inductance $L_1=L=100\mu H$, coupling coefficient between inductors $k = 0.99$, capacitor value $C=10\mu F$ and load $R=1\Omega$. As a result of the simulation, the curves of voltage across the capacitor and current through the inductors L and L_1 - Fig. 6, a, b, c – respectively, were obtained. It is clear that the curves in Fig. 6 does not coincide, neither in shape nor in magnitude, with the curves in Fig. 3.

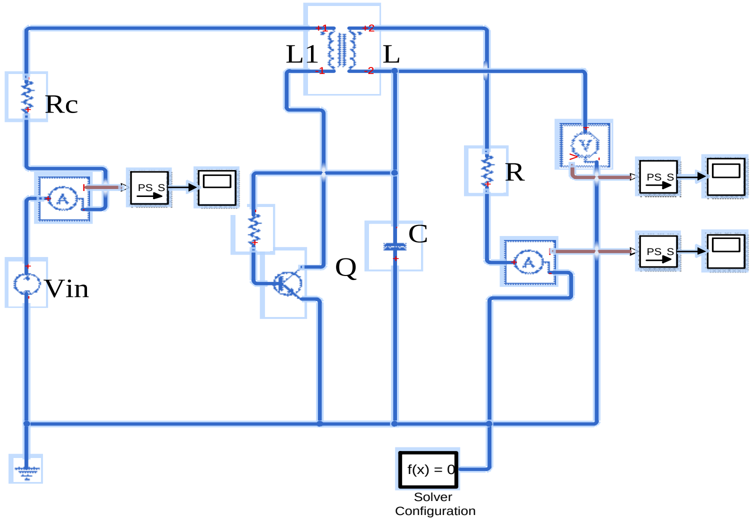


Fig. 5: Diagram of the model in Matlab-Simscape of the Van der Pol transistor oscillator.

This is quite understandable, since the model of the oscillator circuit in Fig. 5 takes into consideration an input voltage source and a current flow in one direction (i_{CE}), i.e. no current flows in the opposite direction, through the transistor (small signal). On the other hand, the oscillation period, obtained through simulation ($T = 249.5 \cdot 10^{-6} s$), is reasonably adjacent to the period obtained by equation (6) ($T = 199.8 \cdot 10^{-6} s$). The maximum and minimum values of the voltage drop across the capacitor *C* is $V_{max}=11.7V$ and $V_{min}=-20V$, respectively. The values of current, through inductance *L*, corresponds to $I_{Lmax}=7.96A$ and $I_{Lmin}=-6.2A$; as for inductance *L1*, the value of the current is - $I_{Lmax}=6.4A$.

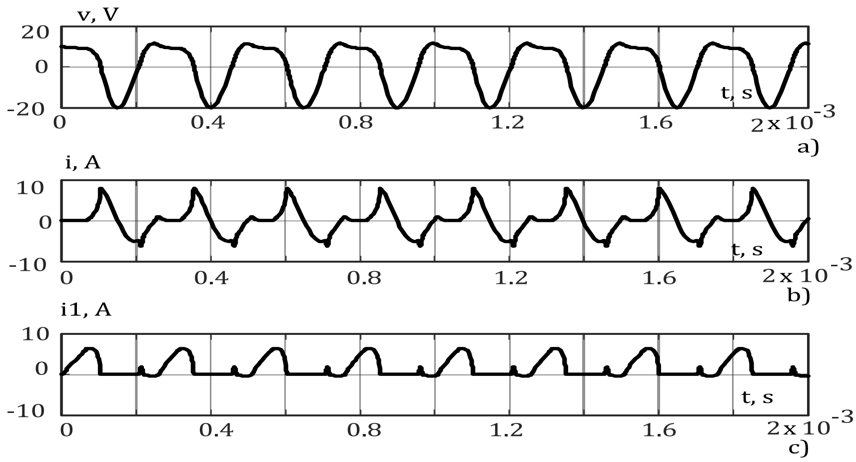


Fig. 6: Diagrams of voltage *v* on a capacitor and currents *i* of inductance $L=100\mu H$ and current i_1 of inductance $L_1=100\mu H$, obtained from simulation results in Matlab-Simscape.

Concurrently, simulating with a different value of inductance $L_1=1000\mu H$, while maintaining the preceding values of all other parameters same, had resulted a greater discrepancy in the value of the oscillation period -$T=369.5\cdot10^{-6}s$ (Fig. 7, a, b, c), compared with the calculated value- $T=219.5\cdot10^{-6}s$, -obtained by equation (6).

Considering the process of transferring active power in the oscillator, at $L=L_1=100\mu H$. Whereas the source of active power is obviously the voltage source V_{in}, a small portion of the input power is dissipated by the resistance R_c and transistor Q, while the main portion is applied to inductance L_1 and then transferred to inductance L.

The power of the inductor L is then transferred to the load R, while a small portion of it is dissipated by the resistance R_b and also transferred to the base-emitter junction of the transistor Q.

We shell also dwell on the important issue of the reactive power flow. For this purpose, we begin with considering the current-voltage characteristic of the voltage and current through inductance L_1, at steady state –as presented in Fig. 8, for two cases: $L_1=100\mu H$ - Fig. 8,a and $L_1=1000\mu H$ - Fig. 8,b. As presented in Figs. 6, 7, the current i_1 consists of a constant and variable component i.e. ripple, that when added with the voltage across this inductance, forms some reactive (non-active) power, which also circulates through the input voltage source.

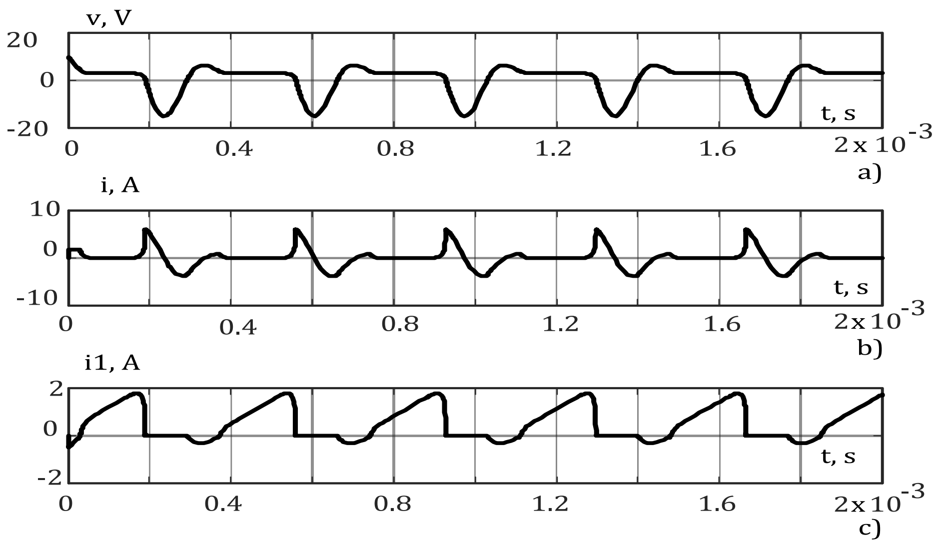


Fig. 7: Diagrams of voltage *v* on a capacitor and currents *i* of inductance $L=100\mu H$ and current i_1 of inductance $L_1=1000\mu H$, obtained from simulation results in Matlab-Simscape.

An area of the closed curve, representing a limit cycle in the plane i_1 - v_{L1} (Fig. 8) and divided by 2π, gives the value of the reactive power, which can be calculated by the formula as follows:

$$Q = \sum_{n=1}^{\infty} n \cdot V_n \cdot \tilde{I}_n \cdot \sin\varphi_n \qquad (9)$$

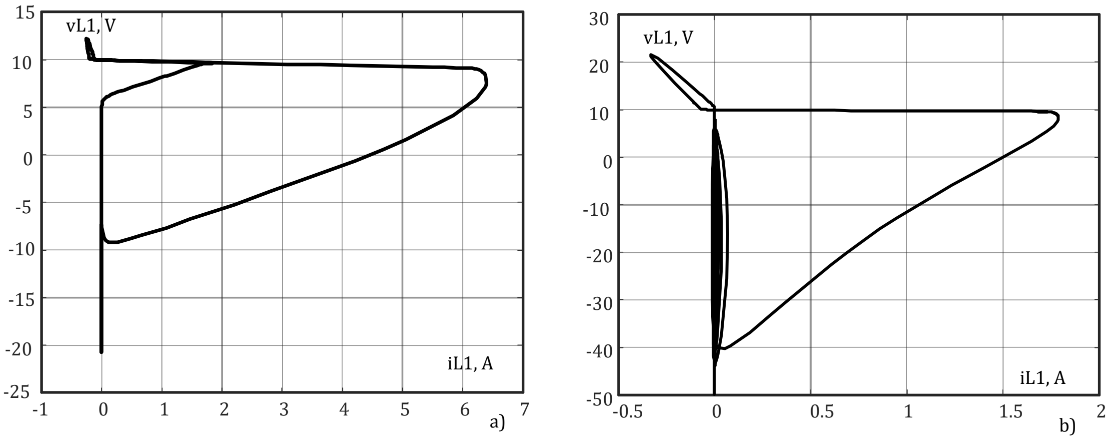


Fig. 8: Volt-ampere characteristics for voltage and current across inductance L_1 in the steady-state mode of the circuit Fig. 5, a) L_1=100μH, b) L_1=1000μH.

where V_n, I_n are the effective values of the voltage and current harmonics of the ripple, φ_n is the phase difference between them and n is the total number of the harmonics for these quantities (Emde, 1921, Mayevsky, 1978). In the particular example, referring to Fig. 8, the reactive power calculated from the area is, respectively, Q = *10.775VA* - Fig. 8,a and Q = *7.6367VA* - Fig. 8,b. Whereas calculating this power, by formula (9), through the expansion of the Fourier series, yields to Q = *11.77VA* and Q = *8.0266VA*, respectively. As will be clarify from the subsequent presentation, it is the effect on the current ripple and reactive power which impact the self-oscillation mode.

4. Synchronization of oscillations in the oscillator using a source in the secondary circuit.

According to the solutions of the oscillator equation (6) and the results of the electrical model simulation, the circuit's oscillation frequency is described by parameter μ in equation (6), i.e., in particular, with the change of the mutual inductance value *M*. In the model, those variations were carried out by altering the inductance L_1. Despite the variations with the value of μ (corresponding L_1 in the model), we will aim to remain the frequency unchanged and equal to the *L-C-R* oscillatory circuit frequency (considering the example noted, *f=5 kHz*).

Below we will consider the well-known and also proposed in this paper methods of synchronization (Fig. 9, a, b).

To this end, in a known method an additional sinusoidal voltage source is connected to the oscillator circuit (Anishchenko, et al., 2007), with a frequency value of *5 kHz*, as noted in the example above. It is expressed by the appearance at the right-hand side term of equation (6), $A\,sin(2\pi ft+\varphi)$, where A is the voltage amplitude, $f=5kHz$ and φ is the phase angle. As a result, for certain parameters of these quantities, the oscillation frequency remains constant, regardless to the variations of parameter. Regarding the case to the electric model, an additional source is connected to the *L-C-R* circuit, as introduced in Fig. 9,a – V_s. The analysis shows that when the value at the specified sinusoidal voltage source is reached i.e., $A=5V, \varphi=0$, the oscillation frequency actually remains equal, even though the inductance L_1 varies from $100\mu H$ to $1000\mu H$. However, the functioning of the oscillator takes on a completely different nature, which is explained from the flow of active and reactive power. This did not follow from equation (6), particularly, because the input source was not reflected in it.

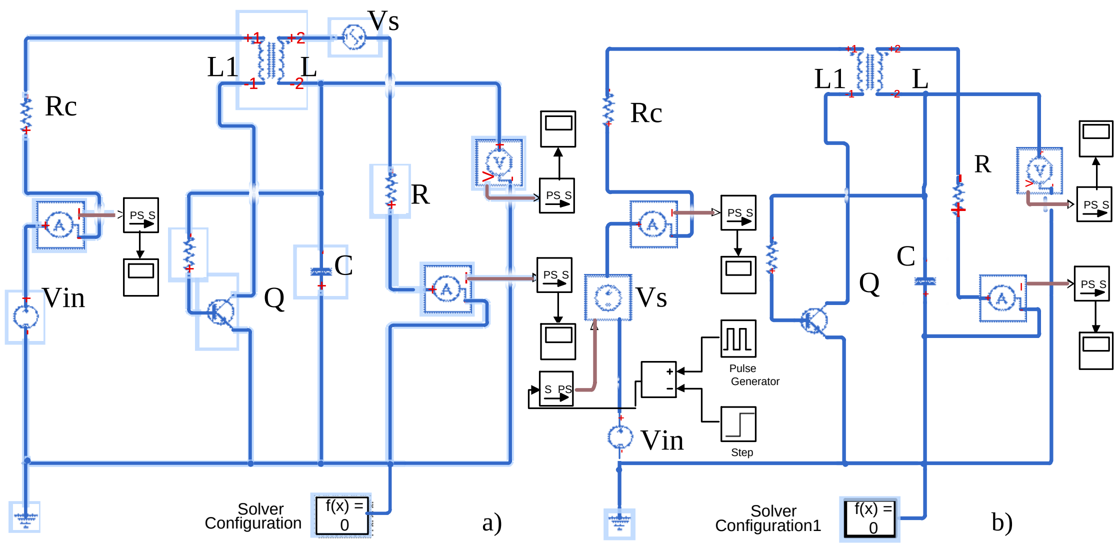


Fig. 9: Diagram of the model in Matlab-Simscape of a Van der Pol transistor oscillator, a) with a sinusoidal synchronization source V_s in the secondary circuit and b) with a square-wave synchronization source V_s in the primary circuit.

Indeed, an analysis of such model with inductors value of $L=L_1=100\mu H$, while keeping other parameters as indicated above, shows that the main power of the load *R* (*12.1W*) is consumed from a sinusoidal source, while the *Vin* source supplies merely *2.69W* to the load. It is noteworthy to mention, that a portion of this power is dissipated by the control on the base of the transistor, summing it up to a total of *14.18W* of power delivered to the load *R*.

This means that the *L-C-R* circuit with a sinusoidal voltage source is virtually operating, and all other components of the circuit do not affect the processes and all the more so no oscillations were obtained. Moreover, at $L_1=1000\mu H$, a portion of the power is delivered to the V_{in} source, i.e., the entire circuit begins to function as a charger. Actually in this case, the source V_s produces only active power, since the *L-C-R* circuit operates close to resonant.

5. Synchronization of oscillations in the oscillator using a source in the primary circuit.

In this part we propose a method of synchronizing oscillations in the oscillator while maintaining the nature of its functioning. For the purpose, a source of periodic oscillations with a lower amplitude, compared to V_{in}, is suggested, for example, a square-wave signal is connected in series with the input source V_{in} (Fig. 9b, V_s). In the example, under consideration, at $V_{in}=7V$ the source amplitude V_s is assumed to be *6V* and the frequency is *5kHz*. Testing the model, results that the oscillation frequency remains unchanged at these values, even though L_1 varies from $100\mu H$ to $7000\mu H$. Naturally, all of the power is drawn from the two sources of the input circuit, specifically the majority power is delivered by V_{in}. Note that with an increase of inductance L_1, the power dissipation at the load decreases, although it can be increased back by a proportional increase in both input sources. Back to the considered example, when determining $L_1 =100\mu H$, we obtain that the active power, between the sources are distributed as follows: $P_{Vin}=7.89W$, $P_{Vs}= 4.45W$, $P_R=12.22W$, while at $L_1=1000\mu H$ we obtain that- $P_{Vin}=0.96W$, $P_{Vs}=0.44W$, $P_R=1.23W$.

In the input circuit, reactive power is accumulated in inductance L_1 and is also produced by the source V_s. The current-voltage characteristics of inductance L_1, in its two modes of operation is presented in Fig. 10: mode I- $L=L_1=100\mu H$, mode II- $L=100\mu H$, and $L_1=1000\mu H$.

The reactive power reflected by them, accumulated by the inductance L_1 is, respectively, $Q_{L1}=11.69VA$ and $Q_{L1}= 1.63VA$. The reactive power produced by the source *Vs* is respectively $Q_{Vs} =10.77VA$ and $Q_{Vs}= 1.13VA$. Table I shows the case for the circuits in Fig. 5

and Fig. 9b, concluding calculation results of the reactive power, developed by inductance L_1, when setting the values respectively, to $L_1=100\mu H$ and $L_1=1000\mu H$.

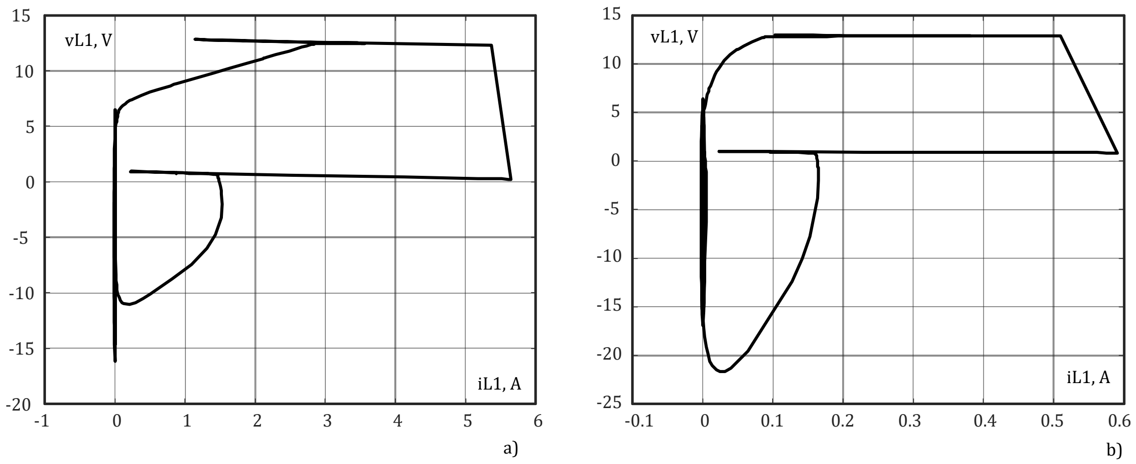


Fig. 10: Volt-ampere characteristics for voltage and current on inductance L_1 in the steady-state mode of the circuit Fig. 9b, a) $L_1=100\mu H$, b) $L_1=1000\mu H$.

These reactive powers are reduced to the unit of active power consumption: $Q^*=Q/Pin$. It is also feasible to obtain the values of the reactive power, of the alternating voltage source Vs, from the diagram in Fig. 9b.

Previously described, the order of the active power transfer delivered from the energy of the input source was traced. Consumption of only active power is not sufficient to an oscillatory process, in the oscillator circuit, it also requires power of a different nature: namely reactive (non-active) power. This reactive power was consumed from the source and will return, circulating, without consuming active power. Only the circulation of reactive energy (power) can generate oscillations and determine their frequency. We shell determine, using the example with data from Table I, the relation between the nature of the oscillations and the values of reactive power of the primary circuit connected with the power source.

Table I.

<table>
<tr><th>Scheme</th><th colspan="3">$L_1=100\mu H$</th><th colspan="3">$L_1=1000\mu H$</th></tr>
<tr><td>Fig. 5</td><td>$P_{in}=16.08$</td><td>$Q_{L1}=10.95$</td><td>$Q^*_{L1}=0.68$</td><td>$P_{in}=5.13$</td><td>$Q_{L1}=7.64$</td><td>$Q^*_{L1}=1.49$</td></tr>
<tr><td rowspan="2">Fig. 9b</td><td colspan="2">$P_{in}=P_{Vin}+P_{Vs}=12.15$</td><td>$Q_{L1}=11.70$</td><td colspan="2">$P_{in}=P_{Vin}+P_{Vs}=1.21$</td><td>$Q_{L1}=1.63$</td></tr>
<tr><td>$Q_{Vs}=10.77$</td><td>$Q=Q_{L1}+Q_{Vs}==22.47$</td><td>$Q^*=1.85$</td><td>$Q_{Vs}=1.13$</td><td>$Q=Q_{L1}+Q_{Vs}==2.76$</td><td>$Q^*=2.28$</td></tr>
</table>

Table I shows, for the scheme in Fig. 5, that in order to maintain oscillations, already at low frequency, when switching from a lower inductance to a higher value a large value of reactive power is required to transfer a unit of active power i.e. *1.49* against *0.68*. Similar constrain is obtained for scheme in Fig. 9b: *2.28* against *1.85*. In addition , for transition of oscillations from lower to higher frequency, equal to the frequency of the oscillating circuit, an increase of reactive power is also required: from *0.68* to *1.85* at $L_1=100\mu H$ and from *1.49* to *2.28* for $L_1=1000\mu H$.

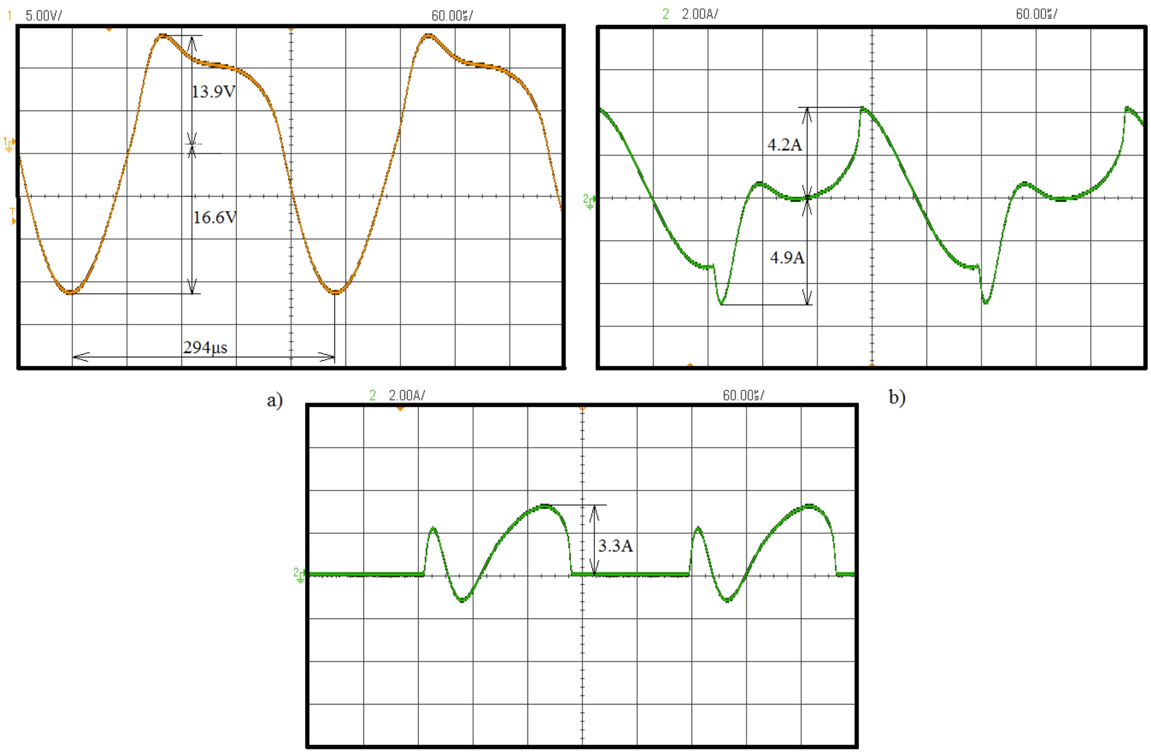
Therefore, in its final form, a certain factor N, which determines the magnitude of the oscillations, must take into account, in addition to the magnitude of reactive power, also the frequency of its circulation in the primary circuit, namely in the form of $N=\frac{Q^*}{T}$, where T is the period of oscillations. As a result, it can be argued that the synchronization of oscillations, achieved in this case, is based on the factor of increasing the circulation of reactive power in the circuit of the main source V_{in}.

6. Experimental results

The operation of Van der Pol transistor oscillator was experimentally tested on a laboratory prototype, with similar parameters previously used for modeling the circuit in Fig. 5: $V_{in}=10V$, $R_c=0.1\Omega$, $R_b=100\Omega$, $L_1=L=100\mu H$, $C=10\mu F$, $R=1\Omega$. Coefficient between the windings is k = *0.96-0.97*. An *MJE3055T* transistor was selected for the prototype circuit. Inverse coupled inductors L-L_1 assembled, using 105/36 (strands/awg) litz wire.

The measurements results of the voltage across the capacitor and current through the inductors are given in the oscillograms (Fig. 11). It is clear, from the oscillograms, that the nature of change in the voltage across the capacitor V, the currents through the inductors L/ L_1 and the duration of the time intervals do confirm adequacy with the Simscape program model functionality. Indeed, the shapes of the curves in Fig. 11 and Fig. 5 are completely identical.

Nevertheless, there is an understandable deviation in the experimental values, compared with the values obtained from simulation. The maximum and minimum capacitor voltage value is 18% lower than the simulated model, and the oscillation period value is 18% higher. Moreover, the current values, obtained from experiment, though with identical shapes, are significantly less than those obtained through simulation. This occurs due to number of reasons, firstly, the increased value of the leakage inductance, between L and L_1 and their active resistance in the prototype compared with simulation. Secondly, a discrepancy between the real *MJE3055T* transistor and the digital model, which is obviously a generalized component of the Simscape program. In summary, another important note is that the experiment, of the transistor oscillator, confirms a significant discrepancy to the curves obtained from the solution of the classical Van der Pol equation.



a) b) c)

Fig. 11: Results of experimental verification of the Van der Pol transistor oscillator, (a) voltage across the capacitor, (b) inductance current *i*, (c) inductance current i_1.

7. CONCLUSIONS

1. The well-known mathematical Van der Pol equation, illustrating the possibility of self-oscillations occurrence, does not reflect electrical processes in the oscillator, particularly when taking into account the use of modern transistors, rather than vacuum tube.
2. A mathematical model of the physical circuit of the Van der Pol oscillator, with the use of a transistor, is proposed and studied .The model reflects the process of self-oscillations occurrence, also makes it possible to establish the processes of the active and reactive power transfer in the oscillator.
3. The method of oscillation synchronization, considered in literature, regarding the connection of a sinusoidal voltage source into the oscillator circuit, changes the character of the circuit's functioning and turning it into an elementary alternating current circuit.
4. A new method of synchronization, preserving the principle of oscillator operation, is proposed.

5. The establishment of synchronized oscillations, associated with an increase of the reactive power circulation, is shown in the paper.

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