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THE SPANNING TREE OF THE CORONA PRODUCT GRAPHS IS PRIME

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ABSTRACT

In this paper we discuss a new method of graph labeling which is to be a prime. We consider the spanning of the corona product graph G then we prove that it is vertex labeling, edge labeling and vertex-edge labeling are all prime. Here G is the corona product of different types of graphs. There are number of spanning trees exists for any graph, but in this paper we consider the spanning tree is in one way which contains the largest paths. With the help of program we can find the spanning tree of G and its labeling.

KEYWORDS: Corona Product, Spanning tree, Prime labeling, relatively Prime numbers, vertex labeling, edge labeling.

1. INTRODUCTION

One of the main branches in the graph theory is Graph labeling. In 19th century Arthur Cayley proved that there are n^{n-2} distinct labeled trees of order n called as Cayley's tree formula. Mostly graph labeling traced by Alexander Rosa, he identified three types of labeling, they are α- labeling, β- labeling and ρ – labeling. Later on β- labeling was remained as graceful labeling by Solomon Golomb. Graph labeling has a wide range of applications such as data security, coding theory, and communication networks.

2. Preliminaries

Definition 2.1: (Corona Product)

The corona product of two graphs G and H is defined as the graph obtained by taking one copy of G and |V(G)| copies of H then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H.

Definition 2.2: (Prime Labeling)

Let G= (V, E) be a graph with n vertices, a bijection f: V → {1, 2, 3…..n} is called prime labeling if for each edge e = uv, the gcd (f(u), f(v)) =1 then the graph G is called a prime graph.

Definition 2.3: (Spanning Tree)

Let G be a graph, then the Spanning tree of G is a tree which covers all vertices of the graph G. For any graph G there are more number of spanning trees exists. But in this paper we consider only one type of spanning tree of the corona product graph G. as mentioned in conjecture (1).

Conjecture (1): Let G ʘ H be the corona product of G and H with |V (G)| = n and |V (H)| = m then we consider the spanning tree as first we join the n vertices of G with n-1 edges of G so as to it gives the largest path of G along with this, at each vertex of G we join the largest path of m vertices of H with m-1 edges of H.

Conjecture (2): Let G = (V, E) be a graph of n vertices and e edges, a bijection f: E →{1,2,3,….e} is an edge prime labeling if the labels of the edges incident at each vertex are relatively prime.(i.e., the gcd of all edges incident at a vertex is 1)

Example: G = K_4 ʘ C_3 is the corona product of complete graph and cycle. The graph G is as shown in figure1 and its spanning tree is shown in figure 2.

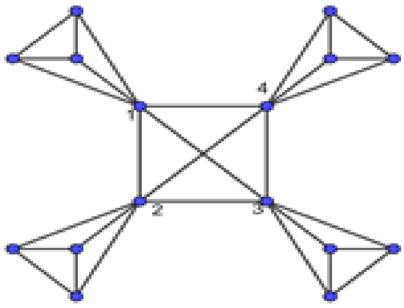

Figure 1

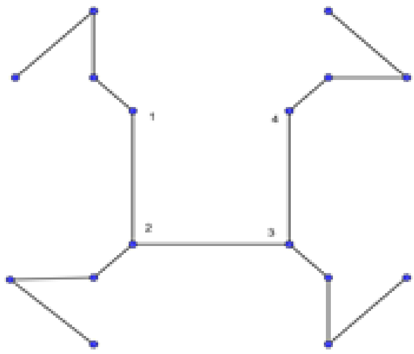

Figure 2

3. Main Results

Lemma 3.1: Let S = {m+2, 2m+3, 3m+4…….. (n-1) m + n} where m, n ∈ N then any two successive numbers in S are relatively prime.

Proof:

Let m ∈ N and n ∈ N.

Consider S = {m+2, 2m+3, 3m+4 ……… (n-1) m + n}.

To prove the successive numbers in S are relatively prime.

Let m+2, 2m+3 be two successive numbers in S then gcd of m+2 and 2m+3 is as follows

(m+2, 2m+3) = (m+2, (m+2) +m+1),

= (m+2, m+1), (by Euclidean formula)

= gcd of consecutive integers is 1,

= 1,

∴ (m+2, 2m+3) = 1.

Consider the next successive numbers 2m+3, 3m+4 then gcd of 2m+3 and 3m+4 is given by

(2 m+3, 3m+4) = (2m+3, (2m+3) +m+1)

= (2m+3, m+1) (by Euclidean formula)

= (2(m+1) +1, m+1)

= (1, m+1)

= 1

∴ (m+2, 2m+3) = 1.

Consider the next successive numbers 3m+4, 4m+5 then gcd of 3m+4 and 4m+5 is given by

(3m+4, 4m+5) = (3m+4, (3m+4) +m+1)

= (3m+4, m+1) (by Euclidean formula)

= (3(m+1) + 1, m+1)

= (1, m+1)

= 1

∴ (3m+4, 4m+5) = 1.

In general consider the gcd of (n-2) m + (n-1) and (n-1) m + n

((n-2) m+ (n-1), (n-1) m + n) = ((n-2) m+ (n-1) , (n-2) m+n-1 +m+1)

= ((n-2) m+ (n-1), m+1)

= ((n-2) m+ (n-2) +1, m+1)

= ((n-2) (m+1) +1, m+1)

= (1, m+1)

= 1

∴ ((n-2) m+ (n-1), (n-1) m + n) = 1

∴ The gcd of all successive numbers in the set S is 1.

Hence any two successive numbers in S are relatively prime.

Lemma 3.2: The spanning trees of the graphs G_1 and G_2 where $G_1= K_n \odot K_m$ and $G_2= C_n \odot P_m$ are isomorphism. (By conjecture (1))

Proof

Let G_1 be the corona product of K_n and K_m & G_2 be the corona product of C_n and P_m then the number of vertices in $G_1= K_n \odot K_m$ and $G_2= C_n \odot P_m$ are same .

i.e., $|V(G_1)| = |V(G_2)| = n(m+1)$.

Now we consider the spanning trees of G_1 and G_2 (as we defined) then G_1 and G_2 contains n (m+1)-1 edges in each respectively.

According to our consideration of spanning tree, the degree sequences of spanning trees of G_1 and G_2 are also same.

Hence we say that the spanning tree of G_1 and G_2 are isomorphism.

Example (1): $G_1 = K_4 \odot K_3$ and $G_2 = C_4 \odot P_3$ are the corona product graphs and S_1 & S_2 are the spanning trees of G_1 and G_2 respectively. [Where P_3 be the path of 3 vertices].

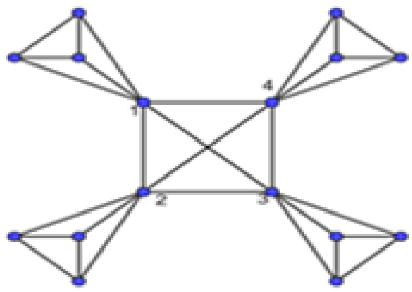

Figure 3: $G_1 = K_4 \odot K_3$

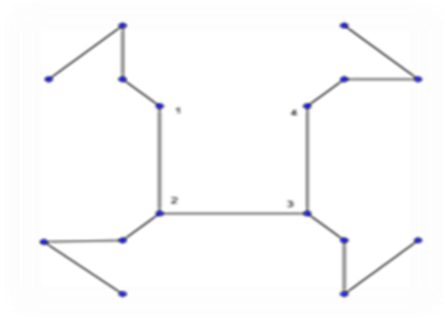

Figure 4: S_1 – spanning tree.

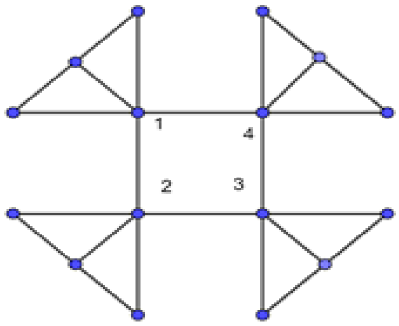

Figure 5: $G_2 = C_4 \odot P_3$

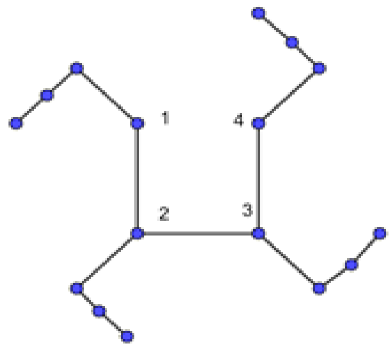

Figure 6: S_2 – spanning tree.

Here we observe that S_1 and S_2 are isomorphism (Figures 4 and 6).

Example (2): $G_1 = K_3 \odot K_{3,2}$ and $G_2 = C_3 \odot P_5$ are the corona product graphs and S_1 & S_2 are the spanning trees of G_1 and G_2 respectively. [Where P_5 be the path of 5 vertices]

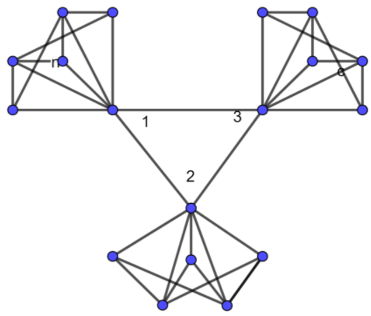

Figure 7: $G_1 = K_3 \odot K_{3,2}$.

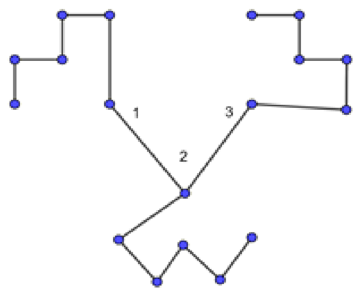

Figure 8: S_1 – spanning tree.

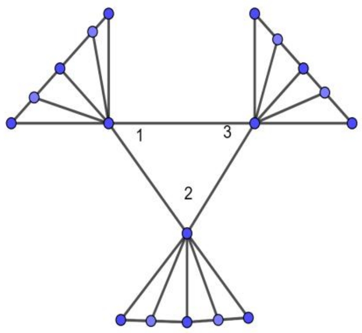

Figure 9: $G_2 = C_3 \Theta P_5$.

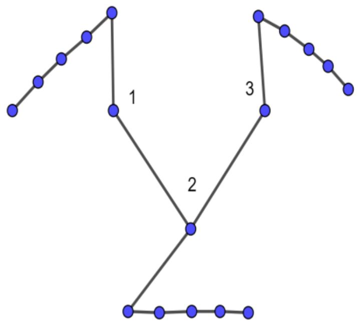

Figure 10: S_2 – spanning tree.

Here we observe that S_1 and S_2 are isomorphism (Figures 8 and 10).

Theorem 3.3: The vertex labeling of spanning tree of the corona product graph is prime.

Proof:

Let C_n be the cycle of n-vertices.

Let P_m be the path of m vertices with (m-1) edges

Consider the corona product graph $G = C_n \Theta P_m$ then G contains (m+1) n vertices as shown in figure 11.

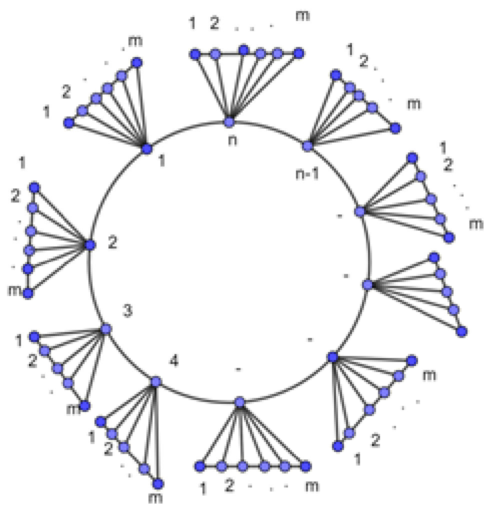

Figure 11: $G = C_n \Theta P_m$

By using our conjecture (1), we can draw the spanning tree of G as shown in figure 12. The total number of vertices in the spanning tree is also n (m+1).

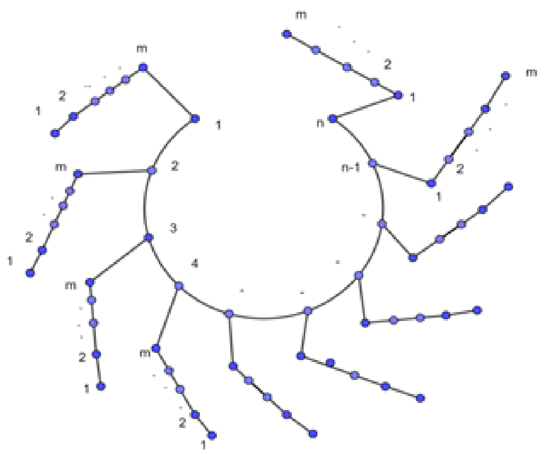

Figure 12.

For vertex labeling of the spanning tree, we consider the mapping.

f : V → { 1,2,3 …….n(m+1) } such that gcd of (f(u), f(v)) =1 where u and v are the adjacent vertices

For that, the vertices of the graph c_n are labeled as the first vertex with m+1 and from second vertex to n^{th} vertices we label as m+2 , 2m+3, 3m+4 ,4m+5…………(n-1) m + n.

At vertex m+1, we label the m vertices as 1, 2, 3, 4 …….. , m.

At vertex m+2, we label the m vertices as m+3, m+4, m+5 ….. 2m+2.

At vertex 2m+3, we label the m vertices as 2m+4, 2m+5, 2m+6 ……. 3m+3.

At vertex 3m+4, we label the m vertices as 3m+5, 3m+6 ,3m+7 …….. 4m+4

On repeating this process

At vertex (n-1) m + n, we label the m vertices as

(n-1) m+n+1, (n-1) m+n+2, (n-1) m+n+3 …….. (n-1) m + n + m = n (m+1).

The labeling of spanning tree is as shown in figure 13.

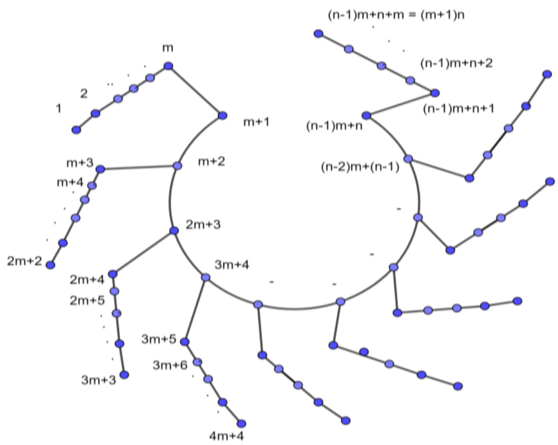

Figure 13.

We know that the consecutive integers are relatively prime and also from lemma (3.1), we observe that.

(m+2, 2m+3) = (2m+3, 3m+4) = ………. = ((n-2) m + (n-1), (n-1) m + n) = 1.

That means for every adjacent vertices the labeling is relatively prime.

Hence the labeling of spanning tree is prime.

Theorem 3.4: The edge labeling of spanning tree of corona product graph is prime. (By conjecture (2)).

Proof

Let C_n be the cycle of n vertices with n-1 edges.

Let P_m be the path of m vertices with m-1 edges.

The corona product graph $G = C_n \odot P_m$ is as shown in fig.(3) , then the spanning tree of G is consider according to conjecture(1) and the graph is shown in fig.(4) this sapping tree contains n(m+1)-1 edges, for edge labeling of the spanning tree.

We consider the mapping f: E → {1, 2, 3 ……..n (m+1)-1} such that the labeling of adjacent edges is relatively prime.

First we assign the cycle of n-1 edges such as m+1, 2m+2, 3m+3, 4m+4……………..…. (n-1) m+ (n-1).

Then at the end vertex of m+1 edge we join the m edges of P_m with the labeling as

1, 2, 3, 4 ……. m then

At common vertex of m+1 and 2m+2 edges we label the m edges as

m+2, m+3, m+4 ……………2m+1.

At common vertex of 2m+2 and 3m+3 edges we label the m edges as

2m+3, 2m+4, 2m+5…. …….3m+2.

At common vertex of 3m+3 and 4m+4 edges we label the m edges as

3m+4, 3m+5, 3m+6……… 4m+3

Repeating this process

At the last vertex of C_n we join m edges as

(n-1)m+(n-1)+1 , (n-1)m+(n-1)+2 ,……..(n-1)m+(n-1)+m= n(m+1)-1.

The edge labeling of spanning tree is as shown in figure.14.

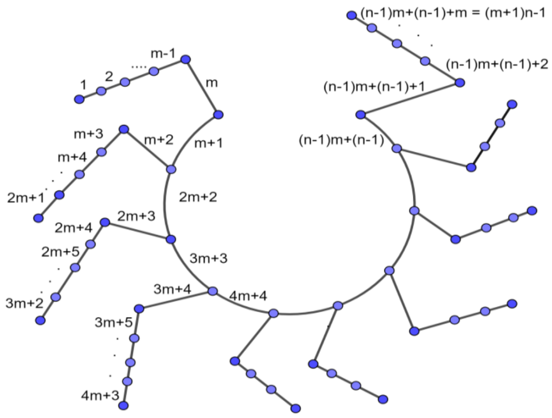

Figure 14.

Here the labeling is prime since at each vertex of C_n we have to calculate the gcd of three numbers, but in that three numbers two are consecutive so its gcd is obviously prime. Hence the gcd of the edges at each vertex is 1. So the spanning tree of corona product is edge prime labeling.

4. CONCLUSIONS

In this paper we briefly discussed about the spanning tree of the corona product Graph is prime and we conclude that

- The spanning tree of corona product graph G is vertex labeling, edge labeling and vertex – edge labeling all are prime.
- Since the spanning tree of some graphs are isomorphism, so their labeling is same.
- Even though the corona product graphs are not an isomorphism but their spanning trees are isomorphism (by conjecture 1).

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