World Journal of Engineering Research and Technology

www.wjert.org

Impact Factor: 7.029 Coden USA: WJERA4

AUTONOMOUS HUMAN BODY CONTROL, PART XI: SERUM PHOSPHATE CONCENTRATION CONTROL USING A PD-I CONTROLLER and AN I-FIRST ORDER COMPENSATOR COMPARED WITH A PI CONTROLLER

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Article Received on 07/10/2025 **Article Revised on 27/10/2025** **Article Published on 01/11/2025**

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https://doi.org/10.5281/zenodo.17490525

How to cite this Article: Galal Ali Hassaan*. (2025). Autonomous Human Body Control, Part Xi: Serum Phosphate Concentration Control Using A Pd-I Controller And An I-First Order Compensator Compared With A Pi Controller. World Journal of Engineering Research and Technology, 11(11), 208–219.

ABSTRACT

This is the 11th research paper concerned with human body control. It handles the control of serum phosphate concentration through using the dialysate flow rate of the hemodialysis machine. The paper proposes a PD-I controller and an I-first order compensator to control the serum phosphate concentration with comparison with a PI controller. The proposed controller and compensator are tuned using non-traditional technique and the time-based characteristics of the resulting control systems are compared graphically and numerically for sake of the assignment of the best controller/compensator for serum phosphate concentration control.

KEYWORDS: Human serum hypophosphatemia control, PD-I controller, I-first order compensator, PI controller, Controller/compensator tuning.

INTRODUCTION

Serum phosphate is one of the human health parameters that indicate the present condition of his health. The serum phosphate has normal levels and other levels may take place either below or above those normal levels. Too much phosphate in the blood is called ‘hyperphosphatemia’ and it can remove calcium from bones and blood causing

'hypocalcemia'.[1] In such a case, hyperphosphatemia results in a number of symptoms including muscle cramps, brittle nails, dry skin, coarser hair, memory problems, irritability, seizures and abnormal heart rhythms.[1] This is why serum phosphate control is required to keep human health in good conditions. The purpose of this research paper is to apply efficient automatic control techniques capable of decreasing the high serum phosphate levels in reduced time span with perfect characteristics using the dialysate flow rate. We start with some efforts of researchers in this aspect since 2002.

Spalding, Chamney and Farrington (2002) measured phosphate concentration in 29hemodialysis patients during short and long dialysis and one hour after. They developed a phosphate kinetics model and implemented it through simulation. They presented graphical output of the model for time up to 250 min during dialysis and for 50 min after dialysis showing the phosphate concentration in mmol/L.[2] Misheda et al. (2006) examined an acute effect of oral inorganic phosphate (Pi) loading on serum fibroblast growth factor 23 (FGF23) levels. They outlined that serum Pi levels significantly increased within 1 h after P400 and P200 intake. They presented graphically the variation of serum Pi with time for Pi loading P400, P800 and P1200 for time up to 8h.[3] Shaikha, Berndt and Kumar (2008) stated that a number of peptides known as 'phosphonins' have been identified for various diseases associated with hypophosphatemia. They concluded that 'phosphonins' played a vital role in the pathogenesis of wide array of disorders.[4]

Yu et al. (2013) evaluated the effect of some medical factors on the serum phosphate clearance in 80 hemodialysis patients. Those factors included predialysis serum phosphate level, continual renal replacement, membrane surface area, effective blood flow rate, blood chamber volume and hematocrit. They concluded that the measured serum phosphate referred to inorganic phosphate.[5] Debowska et al. (2015) outlined that the control of serum phosphate concentration is a considerable clinical problem. They applied a pseudo-one-compartment model to describe the phosphate profile in the blood serum. The concluded that the model parameters strongly correlated with the adequacy parameters of the dialytic removal of phosphate. They presented the serum phosphate profile (model output and experimental) for three hemodialysis patients for a time span from 0 to 5 hours.[6] Leypoldt et al. (2017) proposed a pseudo-one compartment model to describe phosphate kinetics during hemodialysis and post-dialysis periods. They evaluated the ability of the model to describe phosphate kinetics during two hemodialysis treatments separated by a 60 min inter-treatment

period without dialysis. They presented graphically the serum phosphate concentration against time from 0 to 120 min, from 120 to 180 min and from 180 to 300 min.[7] Daugirdas (2018] modified a conventional two-pool urea model predicting measured 1 h intra-dialysis, end dialysis and 30 min post dialysis serum phosphate dialysis. He concluded that his model can be used to predict serum phosphate levels for patients following three times/week dialysis. He presented graphically the predicted and measured serum phosphate in mg/dL for time from 0 to 200 min during dialysis and after 30 min after dialysis for 4 months and 36 months of follow up.[8]

Barreto et al. (2019) outlined that experimental data supported the role of phosphate in the development of bone and cardiovascular diseases. Thy discussed currently available treatment approaches for controlling phosphate removal by dialysis and management of renal osteodystrophy with focus on challenges and limitations, benefits and harms. They tabulated nine phosphate binders with dose, advantages and disadvantages.[9] Anderson et al. (2023) derived analytical expressions for phosphate clearance during dialysis including single-pass and multi-pass models. They calibrated the two models to clinical data and estimated the kinetic parameters. They derived a model for the rebound effect immediately after dialysis for single-pass and multi-pass dialysis. They presented graphically the analytical solution of the single-pass and multi-pass models in terms of phosphate concentration in mmol/L against time from 0 to 4 hours followed by a rebound for another 2 hours.[10] Rosolowska et al. (2024) analyzed laboratory data from 3 months before dialysate flow reduction from 500 to 300 mL/min, in the month with dialysate flow rate of 300 mL/L and from 3 months thereafter with 500 mL/L. They concluded that reducing the dialysate flow rate from 500 to 300 did not cause any short-term negative effects.[11] Waniewski et al. (2025) constructed three mathematical models for phosphate kinetics in patients on maintenance hemodialysis describing the kinetics of phosphate during one-week cycle of 3 dialysis patients. They concluded that the kinetics of the total phosphate levels in plasma predicted by the 3 models agreed with the clinical data. They presented graphically simulated total plasma phosphate and ion phosphate in mmol/L for the 3 models against time in the range of 0 to 52 h.[12]

Serum Phosphate Concentration as a Process

In the work of Daugirdas, he modeled a hemodialysis operation to predict serum phosphate concentration in patients following three times/week dialysis.[8] I have used his graphical simulations for 36 weeks of follow-up and extracted values for serum phosphate

concentrations in mg/dL against time in hours during dialysis. Then I referred to the initial value to define an array for the change in serum phosphate concentration and used MATLAB to fit a reasonable transfer function model. The best model was an un-delayed first-order model, $G_p(s)$ in the form:

$$G_p(s) = K/(Ts+1) \quad (1)$$

Where: K is the process gain in (mg/dL)/(dL/min), T is the time constant in min of the phosphate concentration process.

The process gain K is defined as:

$$K = (P_i - P_{ss})/Q_d \quad (2)$$

Where P_i is the initial serum phosphate concentration (origin), P_{ss} is the steady-state value (referred to the initial value) and Q_d is the dialysate flow rate in the beginning of the dialysis session (5 dL/min[11]). This gives the phosphorate concentration process gain as:

$$K = -0.62274 \quad (3)$$

The model fitted to the normalized data by MATLAB assigned the process time constant T and the multiple correlation coefficient R^2 as:

$$T = 41.249756 \text{ min} \; and \; R^2 = 0.99981 \quad (4)$$

The step time response of the phosphate concentration process for the 5dL/min dialysate flow rate input is shown in **Figure 1** using the process transfer function in Eq.1 with parameters in Eqs.4 and 5 compared with the simulated parameters of Daugirdas[8] as generated by the MATLAB step response.[13]

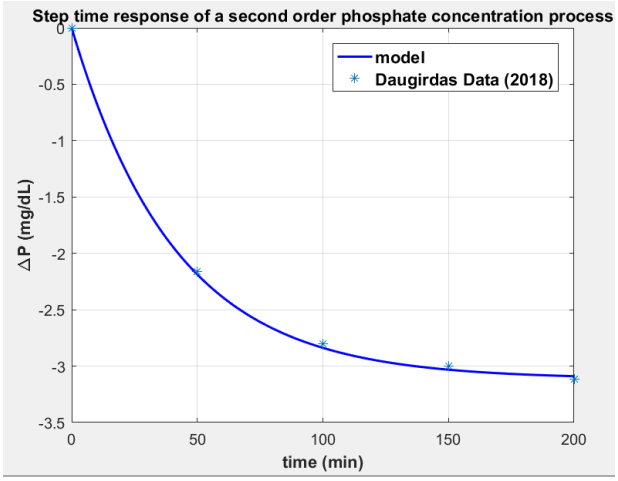

Figure 1: Step time response of the phosphate concentration process.

3. Controlling the Serum Phosphate using a PI Controller

The PI controller is one of the first generation of PID controllers. It is used in regulating some medications related to the control of some human body functions such as blood pressure and glucose concentration.[14], [15] The PI controller is proposed here for sake of comparison only with the proposed controller and compensator from the second generation of control compensators and PID controllers.

A PI controller has a transfer function, $G_{PI}(s)$ given by:

$$G_{PI}(s) = K_{pc1} + (K_{i1}/s) = (K_{pc1}s + K_{i1})/s \quad (5)$$

The PI controller and the hemodialysis operation with phosphate concentration output are set in one control loop. The closed-loop transfer function of the control system, $M_1(s)$ is derived from this single-loop block diagram using Eq.1 for the process and Eq.5 for the PI controller revealing:

$$M_1(s) = (b_0 s + b_1)/(s^2 + a_1 s + b_1) \quad (6)$$

Where:

$$b_0 = KK_{pc1}/T \quad , b_1 = KK_{i1}/T \quad , a_1 = (1 + KK_{pc1})/T \quad (7)$$

For the optimal performance of the control system, the controller parameters K_{pc1} and K_{i1} have to be tuned. They are tuned as follows:

- The transfer function in Eq.6 is set in a standard form as follows:

$$M_1(s) = \omega_n^2 (T_z s + 1)/(s^2 + 2\varsigma\omega_n s + \omega_n^2) \quad (8)$$

Where ω_n is the natural frequency of the second order denominator of Eq.8, ζ is its damping ratio and T_z is the time constant of the simple zero of Eq.8.

- The parameters K_{pc1} and K_{i1} are tuned such that there is no maximum overshoot through forcing the damping ratio to have a unit value (critically damped dynamic system) and the settling time of the step time response of the control system for ± 2 % band to be 10 min.

- Applying those conditions and using the step time response derived from Eq.8 gives the tuned PI controller parameters as:

$$K_{pc1} = -24.245177 \quad , K_{i1} = -6.061294 \quad (9)$$

Using the PI controller parameters in Eq.9, the transfer function in Eq.6 and MATLAB step command[13], the step time response of the serum phosphate concentration change is generated and shown in **Figure 2.** The figure depicts also lower phosphate level change (corresponding to 3.5 mg/dL[9]) and upper phosphate limit (corresponding to 5.5 mg/dL[9]).

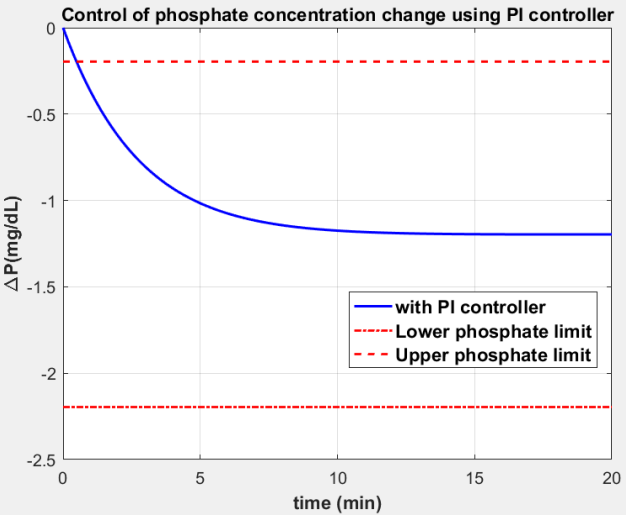

Figure 2: PI controlled phosphate concentration.

The time-based characteristics of the control system time response using a PI controller are:

- There is no overshoot (due to the high efficiency of the used tuning technique).
- Settling time with respect to ± 2 % tolerance: 10 min
- Settling time with respect to phosphate concentration limits: 0.485 min
- Rise time: 5.71 min
- Steady-state error: zero

4. Controlling the Serum Phosphate using a PD-I Controller

The PD-I controller is one of the second generation PID controllers introduced by the author since 2014. The author used a PD-I controller in March 2018 to control undamped second-order-like processes.[16] A PD-I controller consists of a proportional-derivative and integral control modes cascaded in the forward path of a single-loop block diagram of a control system. It has the transfer function $G_{PDI}(s)$ given by[16]:

$$G_{PDI}(s) = (K_{pc2} + K_{d2}s)(K_{i2}/s) \quad (10)$$

Eq.10 is written in another form suitable for the tuning operation of the controller as follows:

$$G_{PDI}(s) = (K_{pc2}K_{i2}/s)[(K_{d2}/K_{pc2})s + 1) \quad (11)$$

Eq.10 and Eq.11 are used to provide the forward path transfer function $G_{PDI}(s)G_p(s)$. The PD-I controller parameters K_{pc2}, K_{d2} and K_{i2} are tuned as follows:

- The zero/pole cancellation technique[17] is used to cancel simple zero of Eq.11 with the simple pole of Eq.1. This step reveals:

$$K_{d2} = TK_{pc2} \tag{12}$$

- Now, with unit feedback single-loop block diagram of the proposed control system, the closed-loop transfer function $M_2(s)$ will be given by:

$$M_2(s) = 1/[(KK_{pc2}K_{i2})^{-1}s + 1] = 1/(T_2 s + 1) \tag{13}$$

- The control system now is a standard first-order dynamic system characterized by its time constant T_2.

- The settling time of a first-order control system, T_{s2} for ± 2 % tolerance is related to its time constant T_2 through.[18]

$$T_{s2} = 3.912T_2 \tag{14}$$

- For a desired settling time of 10 min and a unit proportional gain (K_{pc2} = 1), Eq.12, Eq.13 and Eq.14 give the tuned PD-I controller gain parameters as:

$$K_{pc2} = 1,\ K_{d2} = 41.249756,\ K_{i2} = -0.6281915 \tag{15}$$

Using the PD-I controller parameters in Eq.15, the transfer function in Eq.13 and MATLAB step command[13], the step time response of the serum phosphate concentration change is generated and shown in **Figure 3.**

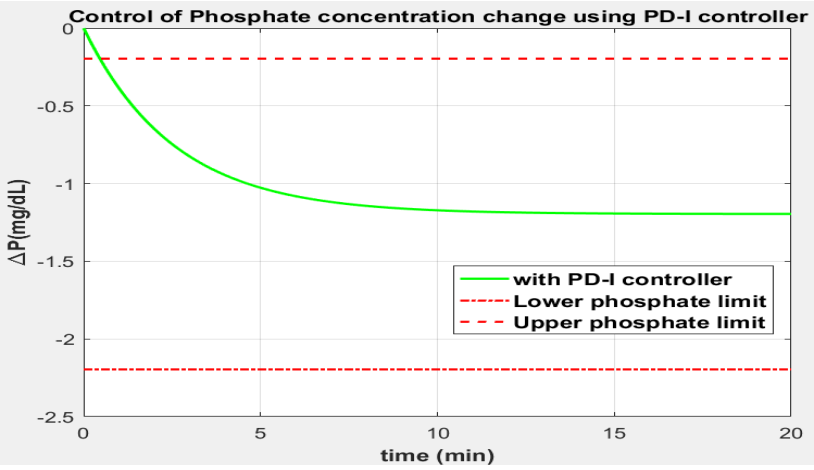

Figure 3: PD-I controlled phosphate concentration.

The time-based characteristics of the control system time response using a PD-I controller are:

- Maximum percentage overshoot: zero
- Settling time with respect to ± 2 % tolerance: 10 min
- Settling time with respect to phosphate concentration limits: 0.462 min
- Rise time: 5.663 min
- Steady-state error: zero

5. Controlling the Serum Phosphate using an I-First Order Compensator

The I-first order compensator is one of the second generation control compensators introduced by the author since 2014. The author used an I-first order compensator in September 2024 to control car longitudinal velocity.[18] The proposed I-first order compensator is composed of two cascaded control elements: An integrator wit gain K_{i3} and first-order compensator with time constants T_{z3} and T_{p3}. It has a transfer function $G_{I1st}(s)$ given by[18]:

$$G_{I1st}(s) = (K_{i3}/s)(T_{z3}s + 1)/(T_{p3}s + 1) \quad (16)$$

The I-first order compensator has three gain parameters (K_{i3}, T_{z3} and T_{p3}) that have to be tuned to produce optimal performance of the control system proposed to control the serum phosphate concentration. It is tuned as follows:

- The zero/pole cancellation technique[17] is used to cancel simple zero of Eq.16 with the simple pole of Eq.1. This step reveals:

$$T_{z3} = T \quad (17)$$

- Now, with unit feedback single-loop block diagram of the proposed control system, the closed-loop transfer function $M_3(s)$ will be given by:

$$M_3(s) = \omega_{n3}^2/(s^2 + 2\varsigma_3\omega_{n3}s + \omega_{n3}^2) \quad (18)$$

Where:

$$\omega_{n3}^2 = KK_{i3}/T_{p3}\ ,\ 2\varsigma_3\omega_{n3} = 1/T_{p3}$$

- In Eq.18, ς_3 is the damping ratio of the closed-loop control system incorporating the I-first order compensator and the phosphate concentration process and ω_{n3} is its natural frequency.

- The closed-loop transfer function in Eq.18 depicts a standard second-order dynamic system. The rest of its compensator parameters (K_{i3} and T_{p3}) have to be tuned.

- For a dynamic second order control system without maximum overshoot, it has to be critically damped ($\varsigma_3 = 1$).

- The relationship for the settling time against the control system natural frequency for critically damped dynamic systems is not available in the literature. I investigated this matter in details and fitted this relationship using private MATLAB scripts designed by the author and the following relationship between the settling time, T_{s3} and the system natural frequency ω_{n3} through:

$$T_{s3} = 5.83356 / \omega_{n3} \quad (19)$$

- Now, with a desired settling time of 10 min, Eq.19, Eq.17 and Eq.18 give the tuned compensator parameters as:

$$T_{z3} = 41.24975, T_{p3} = 0.85749, K_{i3} = -0.468378 \quad (20)$$

Using the I-first order compensator with parameters in Eq.20, the transfer function in Eq.18 with unit damping ratio and MATLAB step command[13], the step time response of the serum phosphate concentration change is generated and shown in **Figure 4.**

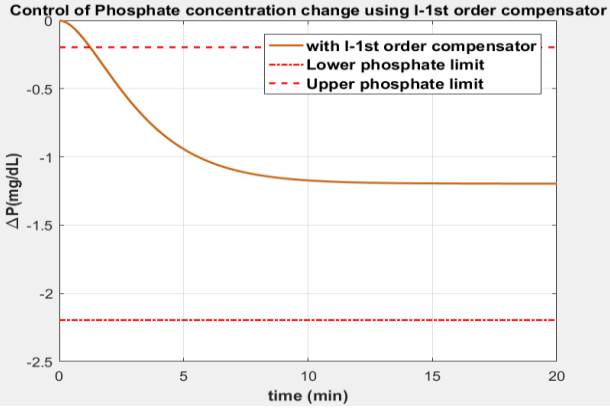

Figure 4: I-first order compensator controlled phosphate concentration.

The time-based characteristics of the control system time response using an I-first order compensator are:

- Maximum percentage overshoot: zero
- Settling time with respect to ± 2 % tolerance: 10 min

- Settling time with respect to phosphate concentration limits: 1.250 min
- Rise time: 5.760 min
- Steady-state error: zero

6. Controllers/Compensator-based Characteristics Comparison

The characteristics of the control systems comprising the controllers/compensator proposed in this research work are compared graphically in **Figure 5** and quantitively in **Table 1.**

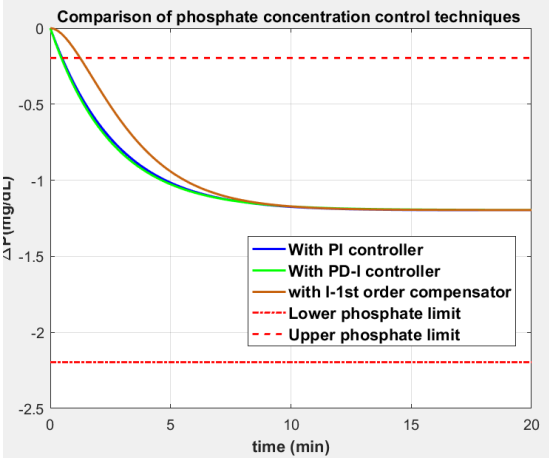

Figure 5: Step time response comparison.

Table 1: Characteristics comparison for serum phosphate control.

Time-based Characteristics	Controllers/Compensator		
	PI controller	PD-I Controller	I-first Order Compensator
OS_{max} (%)	0	0	0
$T_{s2\%}$ (min)	10	10	10
$T_{slimits}$ (min)	0.485	0.462	1.250
T_r (min)	5.710	5.663	5.760
e_{ss} (mg/dL)	0	0	0

OS_{max}: Maximum percentage overshoot.

$T_{s2\%}$: Settling time for ± 2 % tolerance.

$T_{slimits}$: Settling time for phosphate concentrate limits.

T_r: Rise time.

e_{ss}: Steady-state error.

7. CONCLUSIONS

- One controller from the second generation of PID controllers and one compensator from the second generation of control compensators were proposed to control the serum

phosphate concentration with comparison with a PI controller from the first generation of PID controllers.

- The serum phosphate concentration as a process was modeled for its transfer function using simulation date from previous work and its parameters were identified with high mathematical correlation.
- Simple tuning technique of the three proposed controllers/compensator was adopted based on zero/pole cancellation technique and fulfilling specific time-based specifications for the designed control systems.
- All the proposed controllers/compensator succeeded to achieve exactly the desired zero maximum overshoot, 10 min settling time to 2% tolerance and zero steady-state error.
- 10 min settling time is a great achievement from the proposed controllers/compensator compared with 183.3 min without control meaning saving time and costs of the hemodialysis operation.
- New relationships for the settling time of overdamped second-order systems were used in the tuning operation of the I-first order compensator.
- The PD-I controller provided a closed-loop control system for the serum phosphate concentration with the least settling time with respect to the limits of the phosphate concentration and the rise time. Therefore, it is recommended as the best controller for the control of serum phosphate concentration.

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