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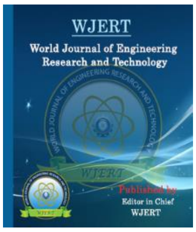

NEW DIFFUSION-MOBILITY-VISCOSITY-ACTIVATION ENERGY-FERMI ENERGY RELATIONS, INVESTIGATED IN N(P)-TYPE DEGENERATE (OR VISCOUS) GaSb(1-x) Te(x)-CRYSTALLINE ALLOY, SUGGESTING A DEGENERACY-AND-VISCOSITY EQUIVALENCE CONCEPT (XXI)

H. Van Cong*, O. Coquand, V. A. T. Nguyen, S. Dumont, P. Blaise, M. Cayrol, V. L'Henaff, N. Tahri-Roe

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS), EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

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***Corresponding Author**

H. Van Cong

Université de Perpignan Via Domitia, Laboratoire de Mathématiques et Physique (LAMPS), EA 4217, Département de Physique, 52, Avenue Paul Alduy, F-66 860 Perpignan, France.

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ABSTRACT

In degenerate $\mathbf{n^{+}(p^{+}) - p(n) - X(x) \equiv GaSb(1-x)Te(x)}$ -crystalline alloy, $0 \leq x \leq 1$, various optical and electrical laws and Stokes-Einstein-Sutherland-Reynolds-Van Cong relations, enhanced by: the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP), our static dielectric constant law given in Equations (1a, 1b), our accurate Fermi energy expression given in Eq. (11), and finally our conductivity model given in Eq. (18), are now investigated, by basing on the same physical model and the mathematical treatment method, as those used in our recent works [1-5]. It should be noted that, for x=0 (1), this crystalline alloy is reduced to the n(p)-type degenerate **GaSb (GaTe)-crystals**. For the physical conditions, as those given in Eq. (15), one remarks that the optical conductivity, σ_O, obtained from the O-EP, has a same form with that of the electrical conductivity, given from the E-OP, σ_E, as those determined in Eq. (20a), but $\sigma_O > \sigma_E$ since $m_r(x) < m_{c(v)}(x)$, $m_{c(v)}$

and m_r, being the unperturbed reduced effective electron (hole) mass in conduction (valence) bands and the relative carrier mass, respectively. Then, by basing on our optical [electrical] conductivity models, $\sigma_{O[E]}$, given in Eq. (18), the diffusion-mobility-viscosity-activation energy-Fermi energy relations are determined, and their numerical results are reported in Tables 3n(p), 4n(p), 5n(p) and 6n(p), suggesting an equivalence between the degeneracy-and-viscosity concept in this X(x)-degenerate (viscous) crystalline alloy.

KEYWORDS: Conductivity, Mobility, Viscosity coefficient, Diffusion coefficient, Activation energy, Fermi energy.

INTRODUCTION

In the $\mathbf{n^+(p^+) - X(x) \equiv GaSb_{1-x}Te_x}$ -crystalline alloy, $0 \le x \le 1$, x being the concentration, various optical and electrical laws and Stokes-Einstein-Sutherland-Reynolds-Van Cong relations, enhanced by : (i) the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP), (ii) our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b), (iii) our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq. (11), accurate with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$ [9], affecting strongly all the expressions of optical and electrical coefficients, and (iv) our optical-and-electrical conductivity models, given in Eq. (18, 20a), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as those given in our recent works.[1-5] It should be noted here that for x=0 (=1), the present obtained numerical results are reduced to those given in the n(p)-type degenerate $\mathbf{GaSb}$ $(\mathbf{GaTe})$-**crystals.**[1, 6-18]

Then, some important remarks can be summarized as follows.

(1) As observed in Equations (3, 5, 6a, 6b), the critical impurity density $N_{CDn(CDp)}$, defined by the generalized Mott criterium in the metal-insulator transition (**MIT**), is just the density of electrons (holes), localized in the exponential conduction (valence)-band tail (**EBT**), $N^{EBT}_{CDn(CDp)}$, being obtained with a precision of the order of $\mathbf{2.87 \times 10^{-7}}$, as given in our recent work.[3] Therefore, the effective electron (hole)-density can be defined as: $N^* \equiv N - N_{CDn(CDp)} \simeq N - N^{EBT}_{CDn(CDp)}$, N being the total impurity density, as that observed in the compensated crystals.

(2) The ratio of the inverse effective screening length $k_{sn(sp)}$ to Fermi wave number $k_{Fn(kp)}$ at 0 K, $R_{sn(sp)}(N^*)$, defined in Eq. (7), is valid at any N^*.

(3) For given $[N, r_{d(a)}, x, T]$, the coefficients: $\sigma_{O[E]}(E)$, $\kappa_{O[E]}(E)$, $\varepsilon_{2\ O[E]}(E)$, and $\propto_{O[E]}(E)$, are determined in Equations (18, 19b-19d), as functions of the photon energy E, and then their numerical results are reported in Tables 3-8, being new ones.

(4) Finally, for particular physical conditions, as those given in Eq. (15), one observes that the optical conductivity σ_O has a same form with that of the electrical conductivity, σ_E, as those given in Eq. (20a), but $\sigma_O > \sigma_E$ since $m_r(x) < m_{c(v)}(x)$, $m_{c(v)}$ and m_r, being the unperturbed reduced effective electron (hole) mass in conduction (valence) bands and the relative carrier mass, respectively. Then, by basing on those $\sigma_{O[E]}$-expressions, the diffusion-mobility-viscosity-activation energy-Fermi energy relations are determined, and their numerical results are reported in Tables 3n(p), 4n(p), 5n(p) and 6n(p), suggesting an equivalence between the degeneracy-and-viscosity concept in this X(x)-degenerate (viscous) crystalline alloy.

In the following, various Sections are presented in order to investigate the conductivity, the mobility, the viscosity coefficient, and the activation energy, expressed as functions of the Fermi energy, given in the degenerate (viscous) $n^+(p^+) - \mathbf{X}(\mathbf{x})$- crystalline alloy.

OUR STATIC DIELECTRIC CONSTANT LAW AND GENERALIZED MOTT CRITERIUM IN THE METAL-INSULATOR TRANSITION

First of all, in the degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$- crystalline alloy, at T=0 K[1-5], we denote : the donor (acceptor) d(a)-radius by $r_{d(a)}$, the corresponding intrinsic one by: $\mathbf{r_{do(ao)}} = \mathbf{r_{Sb(Ga)}}$, respectively, the effective averaged numbers of equivalent conduction (valence)-bands by: $g_{c(v)}$, the unperturbed reduced effective electron (hole) mass in conduction (valence) bands by $m_{c(v)}(x)/m_o$, m_o being the free electron mass, the relative carrier mass by: $m_r(x) \equiv \frac{m_c(x)\times m_v(x)}{m_c(x)+m_v(x)} < m_{c(v)}(x)$ for a given x , the unperturbed static dielectric constant by: $\varepsilon_o(x)$, and the intrinsic band gap by: $E_{go}(x)$, as those given in **Table 1, reported in Appendix 1**.

Here, the effective carrier mass $m^*_{n(p)}(x)$ is equal to $m_{c(v)}(x)$. Therefore, we can define the effective donor (acceptor)-ionization energy in absolute values as:

$E_{do(ao)}(x) = \frac{13600\times[m_{c(v)}(x)/m_o]}{[\varepsilon_o(x)]^2}$ meV , and then, the isothermal bulk modulus, by : $B_{do(ao)}(x) \equiv \frac{E_{do(ao)}(x)}{\left(\frac{4\pi}{3}\right)\times(r_{do(ao)})^3}$.

Our Static Dielectric Constant Law $\left[m^*_{n(p)}(x) \equiv m_{c(v)}(x)\right]$

Here, the changes in all the energy-band-structure parameters, expressed in terms of the effective dielectric constant $\varepsilon(r_{d(a)}, x)$, are developed as follows.

At $r_{d(a)} = r_{do(ao)}$, the needed boundary conditions are found to be, for the impurity-atom volumes : $V = (4\pi/3)\times\left(r_{d(a)}\right)^3$ and $V_{do(ao)} = (4\pi/3)\times\left(r_{do(ao)}\right)^3$, corresponding to the pressures : p, $p_o = 0$, and to the deformation potential energies (or the strain energies) : α, $\alpha_o = 0$. Further, the two important equations, used to determine the α -variation, $\Delta\,\alpha \equiv \alpha - \alpha_o = \alpha$, are defined by : $\frac{dp}{dV} = -\frac{B_{do(ao)}(x)}{V}$ and $p = -\frac{d\alpha}{dV}$, which lead to : $\frac{d}{dV}\left(\frac{d\alpha}{dV}\right) = \frac{B_{do(ao)}(x)}{V}$. After integration, one obtains :

$$\left[\Delta\alpha(r_{d(a)}, x)\right]_{n(p)} = B_{do(ao)}(x)\times(V - V_{do(ao)})\times\ \ln\left(\frac{V}{V_{do(ao)}}\right) = E_{do(ao)}(x)\times\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right]\times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0.$$

Furthermore, we also showed that, as $r_{d(a)} > r_{do(ao)}$ ($r_{d(a)} < r_{do(ao)}$), the compression (dilatation) leads to an increase (decrease) in the energy gap $E_{gn(gp)}\left(r_{d(a)}, x\right)$, and the effective donor (acceptor)-ionization energy $E_{d(a)}\left(r_{d(a)}, x\right)$ in absolute values, obtained in the effective Bohr model. Those are represented respectively by : $\pm\left[\Delta\alpha(r_{d(a)}, x)\right]_{n(p)}$, as :

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x)\times\left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = + \left[\Delta\alpha(r_{d(a)}, x)\right]_{n(p)},$$

for $r_{d(a)} \geq r_{do(ao)}$, and for $r_{d(a)} \leq r_{do(ao)}$,

$$E_{gn(gp)}(r_{d(a)}, x) - E_{go}(x) = E_{d(a)}(r_{d(a)}, x) - E_{do(ao)}(x) = E_{do(ao)}(x)\times\left[\left(\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)})}\right)^2 - 1\right] = - \left[\Delta\alpha(r_{d(a)}, x)\right]_{n(p)}.$$

Therefore, the relative dielectric constant $\varepsilon(r_{d(a)}, x)$ and the energy band gap $E_{gn(gp)}\left(r_{d(a)}, x\right)$ are given as fiollows.

First, for $r_{d(a)} \geq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x)=\frac{\varepsilon_o(x)}{\sqrt{1+\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \leq \varepsilon_o(x)$, being a **new** **$\varepsilon(r_{d(a)},x)$-law**,

$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \geq 0,$ (1a)

according to the increase of both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with increasing $r_{d(a)}$ and for a given x. Secondly, for $r_{d(a)} \leq r_{do(ao)}$, since $\varepsilon(r_{d(a)},x)=\frac{\varepsilon_o(x)}{\sqrt{1-\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3-1\right]\times\ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3}} \geq \varepsilon_o(x)$, with a condition, given by: $\left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 < 1$, being also a **new $\varepsilon(r_{d(a)},x)$-law**,

$E_{gn(gp)}(r_{d(a)},x) - E_{go}(x) = E_{d(a)}(r_{d(a)},x) - E_{do(ao)}(x) = -E_{do(ao)}(x) \times \left[\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 - 1\right] \times \ln\left(\frac{r_{d(a)}}{r_{do(ao)}}\right)^3 \leq 0,$ (1b)

corresponding to the decrease of both $E_{gn(gp)}(r_{d(a)},x)$ and $E_{d(a)}(r_{d(a)},x)$, with decreasing $r_{d(a)}$ and for a given x.

It should be noted that, in the following, all the optical and electrical properties strongly depend on this **new $\varepsilon(r_{d(a)},x)$-law.**

Furthermore, the effective Bohr radius $a_{Bn(Bp)}(r_{d(a)},x)$ is defined by:

$$a_{Bn(Bp)}(r_{d(a)},x) \equiv \frac{\varepsilon(r_{d(a)},x)\times\hbar^2}{m^*_{n(p)}(x)\times m_o\times q^2} = 0.53 \times 10^{-8}\ \text{cm} \times \frac{\varepsilon(r_{d(a)},x)}{m^*_{n(p)}(x)}, \qquad (2)$$

Where q=e, according to an electron charge equal to : -e.

Generalized Mott Criterium in the MIT $\left[m^*_{n(p)}(x) \equiv m_{c(v)}(x)\right]$

Now, it is interesting to remark that the critical total donor (acceptor)-density in the MIT at T=0 K, $N_{CDn(NDp)}(r_{d(a)},x)$, was given by the Mott's criterium, with an empirical parameter, $M_{n(p)}$, as[3] :

$$N_{CDn(CDp)}(r_{d(a)},x)^{1/3} \times a_{Bn(Bp)}(r_{d(a)},x) = M_{n(p)},\ \ M_{n(p)} = 0.25, \qquad (3)$$

Depending thus on our **new $\varepsilon(r_{d(a)},x)$-law**.

This result can be explained by using the definition of the reduced effective Wigner-Seitz (**WS**) radius $\mathrm{r}_{\mathrm{sn(sp),M}}$, in the Mott's criterium, being characteristic of interactions, by:

$$\mathrm{r}_{\mathrm{sn(sp),M}}\left(\mathrm{N}=\mathrm{N}_{\mathrm{CDn(CDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x}),\mathrm{r}_{\mathrm{d(a)}},\mathrm{x}\right)\equiv\left(\frac{3}{4\pi\mathrm{N}_{\mathrm{CDn(CDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})}\right)^{\frac{1}{3}}\times\frac{1}{\mathrm{a}_{\mathrm{Bn(Bp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})}=\mathbf{2.4813963},\qquad(4)$$

for any $(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})$-values. Then, from Eq. (4), one also has :

$$\mathrm{N}_{\mathrm{CDn(CDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})^{1/3}\times\mathrm{a}_{\mathrm{Bn(Bp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})=\left(\frac{3}{4\pi}\right)^{\frac{1}{3}}\times\frac{1}{2.4813963}=\mathbf{0.25}=\mathbf{M}_{\mathbf{n(p)}},\qquad(5)$$

Explaining thus the existence of the Mott's criterium.

Furthermore, by using $\mathrm{M}_{\mathrm{n(p)}}=0.25$, according to the empirical Heisenberg parameter $\boldsymbol{\mathcal{H}}_{\mathbf{n(p)}}=\mathbf{0.47137}$, as those given in our previous work[3], we have also showed that $\mathrm{N}_{\mathrm{CDn(CDp)}}$ is just **the density of electrons (holes) localized in the exponential conduction (valence)-band tail** , $\mathbf{N}_{\mathbf{CDn(CDp)}}^{\mathbf{EBT}}$, with a precision of the order of $\mathbf{2.87}\times\mathbf{10^{-7}}$, respectively.[3] So,

$$\mathrm{N}_{\mathrm{CDn(NDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})\cong\mathrm{N}_{\mathrm{CDn(CDp)}}^{\mathrm{EBT}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x}).\qquad(6a)$$

It shoud be noted that the values of $\mathrm{M}_{\mathrm{n(p)}}$ and $\mathcal{H}_{\mathrm{n(p)}}$ could be chosen such that those of $\mathrm{N}_{\mathrm{CDn(CDp)}}$ and $\mathrm{N}_{\mathrm{CDn(CDp)}}^{\mathrm{EBT}}$ are found to be in good agreement with their experimental results.

Therefore, the effective density of electrons (holes) given in parabolic conduction (valence) bands, N^*, can be defined, as that given in compensated materials:

$$\mathrm{N}^*(\mathrm{N},\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})\equiv\mathrm{N}-\mathrm{N}_{\mathrm{CDn(NDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})\cong\mathrm{N}-\mathrm{N}_{\mathrm{CDn(CDp)}}^{\mathrm{EBT}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})\geq0.\qquad(6b)$$

One notes here that, with increasing $\mathrm{r}_{\mathrm{d(a)}}$ and for given x and N, $\mathrm{N}_{\mathrm{CDn(NDp)}}(\mathrm{r}_{\mathrm{d(a)}})$ increases, as observed in Ref.[3], and therefore, $\mathrm{N}^*(\mathrm{r}_{\mathrm{d(a)}})$ decreases.

In summary, as observed in our previous paper[3], for a given x and an increasing $\mathrm{r}_{\mathrm{d(a)}}$, $\varepsilon(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})$ decreases, while $\mathrm{E}_{\mathrm{gno(gpo)}}(\mathrm{r}_{\mathrm{d(a)}},x)$, $\mathrm{N}_{\mathrm{CDn(NDp)}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})$ and $\mathrm{N}_{\mathrm{CDn(CDp)}}^{\mathrm{EBT}}(\mathrm{r}_{\mathrm{d(a)}},\mathrm{x})$ increase, affecting strongly all the optical, electrical, and thermoelectric coefficients, as those observed in following Sections.

PHYSICAL MODEL

In the degenerate $\mathrm{n^+(p^+)} - \mathbf{X}(\mathbf{x})$ -crystalline alloy, the reduced effective Wigner-Seitz (**WS**) radius $\mathrm{r_{sn(sp)}}$, characteristic of interactions, being given in Eq. (4), in which N is replaced by $\mathrm{N^*}$, is now defined by:

$$\gamma \times \mathrm{r_{sn(sp)}}\left(\mathrm{N^*}, \mathrm{r_{d(a)}}, \mathrm{x}\right) \equiv \frac{\mathrm{k^{-1}_{Fn(Fp)}}}{a_{\mathrm{Bn(Bp)}}} < 1 \quad , \quad \mathrm{r_{sn(sp)}}\left(\mathrm{N^*}, \mathrm{r_{d(a)}}, \mathrm{x}\right) \equiv \left(\frac{3\mathrm{g_{c(v)}}}{4\pi\mathrm{N^*}}\right)^{1/3} \times \frac{1}{\mathrm{a_{Bn(Bp)}(r_{d(a)},x)}} \quad ,$$

being proportional to $\mathrm{N^*}^{-1/3}$. Here, $\gamma = (4/9\pi)^{1/3}$, $\mathrm{k_{Fn(Fp)}(N^*)} \equiv \left(\frac{3\pi^2\mathrm{N^*}}{\mathrm{g_{c(v)}}}\right)^{\frac{1}{3}}$ is the Fermi wave.

Then, the ratio of the inverse effective screening length $\mathrm{k_{sn(sp)}}$ to Fermi wave number $\mathrm{k_{Fn(kp)}}$ is defined by:

$$\mathrm{R_{sn(sp)}(N^*)} \equiv \frac{\mathrm{k_{sn(sp)}}}{\mathrm{k_{Fn(Fp)}}} = \frac{\mathrm{k^{-1}_{Fn(Fp)}}}{\mathrm{k^{-1}_{sn(sp)}}} = \mathrm{R_{snWS(spWS)}} + \left[\mathrm{R_{snTF(spTF)}} - \mathrm{R_{snWS(spWS)}}\right] e^{-\mathrm{r_{sn(sp)}}} <$$

1, (7)

being valid at any $\mathrm{N^*}$.

Here, these ratios, $\mathrm{R_{snTF(spTF)}}$ and $\mathrm{R_{snWS(spWS)}}$, can be determined as follows.

First, for $\mathrm{N \gg N_{CDn(NDp)}(r_{d(a)}, x)}$, according to the **Thomas-Fermi (TF)-approximation**, the ratio $\mathrm{R_{snTF(spTF)}(N^*)}$ is reduced to

$$\mathrm{R_{snTF(spTF)}(N^*)} \equiv \frac{\mathrm{k_{snTF(spTF)}}}{\mathrm{k_{Fn(Fp)}}} = \frac{\mathrm{k^{-1}_{Fn(Fp)}}}{\mathrm{k^{-1}_{snTF(spTF)}}} = \sqrt{\frac{4\gamma\mathrm{r_{sn(sp)}}}{\pi}} \ll 1, \qquad (8)$$

being proportional to $\mathrm{N^*}^{-1/6}$.

Secondly, for $\mathrm{N \ll N_{CDn(NDp)}(r_{d(a)})}$, according to the **Wigner-Seitz (WS)-approximation**, the ratio $\mathrm{R_{snWS(snWS)}}$ is respectively reduced to

$$\mathrm{R_{sn(sp)WS}(N^*)} \equiv \frac{\mathrm{k_{sn(sp)WS}}}{\mathrm{k_{Fn}}} = 0.5 \times \left(\frac{3}{2\pi} - \gamma \frac{\mathrm{d}\left[r^2_{\mathrm{sn(sp)}} \times E_{\mathrm{CE}}(\mathrm{N^*})\right]}{\mathrm{d}r_{\mathrm{sn(sp)}}}\right), \qquad (9a)$$

Where $\mathrm{E_{CE}(N^*)}$ is the majority-carrier correlation energy (CE), being determined by:

$$\mathrm{E_{CE}(N^*)} = \frac{-0.87553}{0.0908+\mathrm{r_{sn(sp)}}} + \frac{\frac{0.87553}{0.0908+\mathrm{r_{sn(sp)}}} + \left(\frac{2[1-\ln(2)]}{\pi^2}\right) \times \ln(\mathrm{r_{sn(sp)}}) - 0.093288}{1 + 0.03847728 \times \mathrm{r^{1.67378876}_{sn(sp)}}} .$$

Furthermore, in the highly degenerate case, the physical conditions are found to be given by:

$\frac{k_{Fn(Fp)}^{-1}}{a_{Bn(Bp)}} < \frac{\mathbb{U}_{n(p)}}{E_{Fno(Fpo)}} \equiv \frac{1}{A_{n(p)}} < \frac{k_{Fn(Fp)}^{-1}}{k_{sn(sp)}^{-1}} \equiv R_{sn(sp)} < 1$, $\mathbb{U}_{n(p)}\left(N^*, r_{d(a)}, x\right) \equiv \frac{\sqrt{2\pi \times (\frac{N^*}{g_{c(v)}})}}{\varepsilon(r_{d(a)}, x)} \times q^2 k_{sn(sp)}^{-1/2}$, (9b)

which gives: $A_{n(p)}(N^*, r_{d(a)}, x) = \frac{E_{Fno(Fpo)}(N^*)}{\mathbb{U}_{n(p)}(N^*, r_{d(a)}, x)}$, $E_{Fno(Fpo)}(N^*, r_{d(a)}, x) \equiv \frac{\hbar^2 \times k_{Fn(Fp)}^2(N^*)}{2 \times m_{n(p)}^*(x) \times m_o}$.

Here, one remarks that: (i) the generalized Thomas-Fermi energy $\mathbb{U}_{n(p)}\left(N^*, r_{d(a)}, x\right)$ can thus be approximately expressed as: $C \times \frac{\sqrt{N^*}}{\varepsilon(r_{d(a)}, x)}$, C being a constant, and (ii) $\mathbb{U}_{n(p)}(r_{d(a)})$ increases with increasing $r_{d(a)}$ and for given x and N, since $\varepsilon(r_{d(a)})$ decreases, as given in Ref.[3]

BAND GAP NARROWING (BGN)

First, the BGN is found to be given by[1]:

$$\Delta E_{gn(gp)}\left(N^*, r_{d(a)}, x\right) \simeq$$

$$a_1 + \frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)} \times N_r^{\frac{1}{3}} + a_2 \times \frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)} \times N_r^{\frac{1}{3}} \times \left(2.503 \times \left[-E_{CE}\left(r_{sn(sp)}\right)\right] \times r_{sn(sp)}\right) + a_3 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{5}{4}} \times \sqrt{\frac{m_{v(c)}}{m_{n(p)}^*(x)}} \times N_r^{\frac{1}{4}} + 2a_4 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{1}{2}} \times N_r^{\frac{1}{2}} + 2a_5 \times \left[\frac{\varepsilon_o(x)}{\varepsilon(r_{d(a)}, x)}\right]^{\frac{3}{2}} \times N_r^{\frac{1}{6}},\ N_r = \frac{N^*}{9.999 \times 10^{17}\,cm^{-3}}, \quad (10a)$$

Here, for $\Delta E_{gn;N}(N^*, r_d, x)$, one has: $a_1 = 3.8 \times 10^{-3}(eV)$, $a_2 = 6.5 \times 10^{-4}(eV)$, $a_3 = 2.8 \times 10^{-3}(eV)$, $a_4 = 5.597 \times 10^{-3}(eV)$, and $a_5 = 8.1 \times 10^{-4}(eV)$, and for $\Delta E_{gp;N}(N^*, r_a, x)$, one has: $a_1 = 3.15 \times 10^{-3}(eV)$, $a_2 = 5.41 \times 10^{-4}(eV)$, $a_3 = 2.32 \times 10^{-3}(eV)$, $a_4 = 4.12 \times 10^{-3}(eV)$, and $a_5 = 9.8 \times 10^{-5}(eV)$.

Therefore, at T=0 K and $N^* = 0$, and for any x and $r_{d(a)}$, one obtains: $\Delta E_{gn(gp)} = 0$, in good agreement with the metal-insulator transition (MIT).

Secondly, the temperature dependent BGN is proposed by[1]:

$$\Delta E_{gn(gp)}(T, x) = 10^{-4} T^2 \times \left[\frac{5.405 \times x}{T+204} + \frac{7.205 \times (1-x)}{T+94}\right] (eV). \quad (10b)$$

FERMI ENERGY AND FERMI-DIRAC DISTRIBUTION FUNCTION

Fermi Energy

Here, for presentation simplicity, we change the sign of all various parameters, given in the degenerate $p^+ - \mathbf{X(x)}$-crystalline alloy, in order to obtain the same one, as given in the degenerate $n^+ - \mathbf{X(x)}$ - crystalline alloy, according to the reduced Fermi energy $E_{Fn(Fp)}$, $\xi_{n(p)}(N^*, r_{d(a)}, x, T) \equiv \frac{E_{Fn(Fp)}(N^*, r_{d(a)}, x, T)}{k_B T} > 0 (< 0)$, obtained respectively in the degenerate (non-degenerate) case.

For any $(N^*, r_{d(a)}, x, T)$, the reduced Fermi energy $\xi_{n(p)}(N^*, r_{d(a)}, x, T)$ or the Fermi energy $E_{Fn(Fp)}(N^*, r_{d(a)}, x, T)$, obtained in our previous paper[9], obtained with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$, is found to be given by:

$$\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u) + Au^B F(u)}{1 + Au^B} \equiv \frac{V(u)}{W(u)},\ A = 0.0005372 \text{ and } B = 4.82842262, \quad (11)$$

Where u is the reduced electron density, $u(N^*, r_{d(a)}, x, T) \equiv \frac{N^*}{N_{c(v)}(T,x)}$, $N_{c(v)}(T, x) = 2g_{c(v)} \times \left(\frac{m^*_{n(p)}(x) \times m_o \times k_B T}{2\pi\hbar^2}\right)^{\frac{3}{2}}$ (cm^{-3}), $F(u) = au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}}$, $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$, $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$, and $G(u) \simeq \mathrm{Ln}(u) + 2^{-\frac{3}{2}} \times u \times e^{-du}$; $d = 2^{3/2}\left[\frac{1}{\sqrt{27}} - \frac{3}{16}\right] > 0$.

So, in the non-degenerate case $(u \ll 1)$, one has: $E_{Fn(Fp)}(u) = k_B T \times G(u) \simeq k_B T \times \mathrm{Ln}(u)$ as $\mathbf{u \longrightarrow 0}$**, the limiting non-degenerate condition**, and in the very degenerate case $(u \gg 1)$, one gets: $E_{Fn(Fp)}(u \gg 1) = k_B T \times F(u) = k_B T \times au^{\frac{2}{3}}\left(1 + bu^{-\frac{4}{3}} + cu^{-\frac{8}{3}}\right)^{-\frac{2}{3}} \simeq \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m^*_{n(p)}(x) \times m_o}$ as $\mathbf{u \longrightarrow \infty}$**, the limiting degenerate condition**. In other words, $\xi_{n(p)} \equiv \frac{E_{Fn(Fp)}}{k_B T}$ is accurate, and it also verifies the correct limiting conditions.

In particular, as $T \to 0\,K$, since $u^{-1} \to 0$, Eq. (11) is reduced to: $E_{Fno(Fpo)}(N^*) \equiv \frac{\hbar^2 \times k^2_{Fn(Fp)}(N^*)}{2 \times m^*_{n(p)}(x) \times m_o}$, proportional to $(N^*)^{2/3}$, noting that, for a given N^*, $E_{Fno(Fpo)}\left(m^*_{n(p)}(x) = m_r(x)\right) > E_{Fno(Fpo)}\left(m^*_{n(p)}(x) = m_{c(v)}(x)\right)$ since $m_r(x) < m_{c(v)}(x)$ for given x. Further, at T=0 K and $N^* = 0$, being the physical conditions, given for the metal-insulator transition (MIT).

In the following, it should be noted that all the optical and electrical-and-thermoelectric properties strongly depend on such an accurate expression of $\xi_{n(p)}\left(N^*, r_{d(a)}, x, T\right)$.[9]

Fermi-Dirac Distribution Function (FDDF)

The Fermi-Dirac distribution function (FDDF) is given by: $f(E) \equiv (1+e^{\gamma})^{-1}$, $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$.

Thus, the average of E^p, calculated using the FDDF-method, as developed in our previous works[1, 6] is found to be given by:

$\langle E^p \rangle_{FDDF} \equiv G_p(E_{Fn(Fp)}) \times E^p_{Fn(Fp)} \equiv \int_{-\infty}^{\infty} E^p \times \left(-\frac{\partial f}{\partial E}\right) dE$, $\quad -\frac{\partial f}{\partial E} = \frac{1}{k_B T} \times \frac{e^{\gamma}}{(1+e^{\gamma})^2}$.

Further, it is noted that, at 0 K, $-\frac{\partial f}{\partial E} = \delta\left(E - E_{Fno(Fpo)}\right)$, $\delta\left(E - E_{Fno(Fpo)}\right)$ being the Dirac delta (δ)-function. Therefore, $G_p\left(E_{Fno(Fpo)}\right) = 1$.

Then, at low T, by a variable change $\gamma \equiv (E - E_{Fn(Fp)})/(k_B T)$, one has:

$G_p\left(E_{Fn(Fp)}\right) \equiv 1 + E^{-p}_{Fn(Fp)} \times \int_{-\infty}^{\infty} \frac{e^{\gamma}}{(1+e^{\gamma})^2} \times \left(k_B T\gamma + E_{Fn(Fp)}\right)^p d\gamma = 1 + \sum_{\mu=1,2,\ldots}^{p} C_p^{\beta} \times (k_B T)^{\beta} \times E^{-\beta}_{Fn(Fp)} \times I_{\beta}$, where $C_p^{\beta} \equiv p(p-1)\ldots(p-\beta+1)/\beta!$ and the integral I_{β} is given by:

$I_{\beta} = \int_{-\infty}^{\infty} \frac{\gamma^{\beta} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma = \int_{-\infty}^{\infty} \frac{\gamma^{\beta}}{\left(e^{\gamma/2}+e^{-\gamma/2}\right)^2} d\gamma$, vanishing for old values of β. Then, for even values of $\beta = 2n$, with n=1, 2, …, one obtains:

$I_{2n} = 2\int_0^{\infty} \frac{\gamma^{2n} \times e^{\gamma}}{(1+e^{\gamma})^2} d\gamma$.

Now, using the identity$(1+e^{\gamma})^{-2} \equiv \sum_{s=1}^{\infty} (-1)^{s+1} s \times e^{\gamma(s-1)}$, a variable change: $s\gamma = -t$, the Gamma function: $\int_0^{\infty} t^{2n} e^{-t}\, dt \equiv \Gamma(2n+1) = (2n)!$, and also the definition of the Riemann's zeta function: $\zeta(2n) \equiv 2^{2n-1}\pi^{2n}|B_{2n}|/(2n)!$, B_{2n} being the Bernoulli numbers, one finally gets: $I_{2n} = (2^{2n}-2) \times \pi^{2n} \times |B_{2n}|$. From the above Eq. of $\langle E^p \rangle_{FDDF}$, we get in the degenerate case the following ratio:

$$G_{p>1}\left(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}}\right) \equiv \frac{\langle E^p \rangle_{FDDF}}{E^p_{Fn(Fp)}} = 1 + \sum_{n=1}^{p} \frac{p(p-1)\ldots(p-2n+1)}{(2n)!} \times (2^{2n}-2) \times |B_{2n}| \times y^{2n}, \quad (12)$$

Noting that $G_{p>1}(y=0) = 1$, as T= 0 K and for a given value of $E_{Fn(Fp)}$.

Then, some usual results of $G_{p\geq 1}(y)$ are given in the **Table 2, reported in Appendix 1**, suggesting that, with increasing T (or decreasing T) and for given (N, r_d, x), since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases).

OPTICAL-AND-ELECTRICAL PROPERTIES

Optico-Electrical Phenomenon (O-EP) and Electro-Optical Phenomenon (E-OP)

In the degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$-crystalline alloy, the following relations hold:

(i) in the **E-OP**, the reduced band gap is defined by:

$$E_{gn2(gp2)} \equiv E_{gn(gp)} - \Delta E_{gn(gp)}\left(N^*, r_{d(a)}, x\right) - \Delta E_{gn(gp)}(T, x), \qquad (13)$$

Where the intrinsic band gap $E_{gn(gp)}$ is defined in Equations (1a, 1b), $\Delta E_{gn(gp)}\left(N^*, r_{d(a)}, x\right)$ and $\Delta E_{gn(gp)}(T, x)$ are respectively determined in Equations (10a, 10b), and

(ii) in the (**O-EP**), the photon energy is defined by: $E \equiv \hbar\omega$, and the optical band gap, as: $E_{gn1(gp1)} \equiv E_{gn2(gp2)} + E_{Fn(Fp)}$.

Therefore, for $E \geq E_{gn1(gp1)}(E_{gn2(gp2)})$, the effective photon energy E^* is found to be given by:

$$E^* \equiv E - E_{gn1(gp1)}(E_{gn2(gp2)}) \geq 0. \qquad (14)$$

From the above Equations, one notes that: $E^* \equiv [E - E_{gn1(gp1)}] = E_{Fn(Fp)}$, given in the O-EP, if $E = \left[E_{gn1(gp1)} + E_{Fn(Fp)}\right] \equiv E_{gn(gp)O}$ and $m^*_{n(p)}(x) = m_r(x)$, and $E^* \equiv E - E_{gn2(gp2)} = E_{Fn(Fp)}$, given in the E-OP, if $E = [E_{gn2(gp2)} + E_{Fn(Fp)}] \equiv E_{gn(gp)E}$ and $m^*_{n(p)}(x) = m_{c(v)}(x)$, noting that $E_{Fn(Fp)}\left(m_r(x)\right) > E_{Fn(Fp)}\left(m_{c(v)}(x)\right)$, since $m_r(x) < m_{c(v)}(x)$, for a given x. (15)

Eq. (15) thus shows that, in both O-EP and E-OP, the Fermi energy-level penetrations into conduction (valence)-bands, observed in the $n^+(p^+) -$ type degenerate $n^+(p^+) - \mathbf{X}(\mathbf{x})$-crystalline alloy, $E_{Fn(Fp)}$, are well defined.

Optical Coefficients

The optical properties for any medium, defined in the O-EP and E-OP, respectively, according to: $[m^*_{n(p)} \equiv m_r(x)[m_{c(v)}(x)]]$, can be described by the complex refraction: $\mathbb{N}_{O[E]} \equiv n_{O[E]} - i\kappa_{O[E]}$, $n_{O[E]}$ and $\kappa_{O[E]}$ being the refraction index and the extinction coefficient, the complex dielectric function: $\mathcal{E}_{O[E]} = \varepsilon_{1\,O[E]} - i\varepsilon_{2\,O[E]}$, where

$i^2 = -1$, and $\mathcal{E}_{O[E]} = \mathbb{N}_{O[E]}{}^2$. Further, if denoting the normal-incidence reflectance and the optical absorption by $R_{O[E]}$ and $\propto_{O[E]}$, and the effective joint parabolic conduction (parabolic valence)-band density of states by:

$$\mathrm{JDOS}_{n(p)\ O[E]}\left(E, N^*, r_{d(a)}, x, T\right) \equiv$$

$$\frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{3/2} \times \sqrt{E_{Fno(Fpo)}(N^*)} \times \left[\frac{E-E_{gn1(gp1)}(E_{gn2(gp2)})}{E-[E_{gn1(gp1)}(E_{gn2(gp2)})+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \quad , \quad \text{and}$$

$F_{O[E]}(E) \equiv \frac{\hbar q^2 \times |v(E)|^2}{n_{O[E]}(E)\times cE\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}}$, one gets[2]:

$$\propto_{O[E]}(E) = \mathrm{JDOS}_{n(p)\ O[E]}(E) \times F_{O[E]}(E) = \frac{E\times\varepsilon_{2\ O[E]}(E)}{\hbar c\ n_{O[E]}(E)} = \frac{2E\times\kappa_{O[E]}(E)}{\hbar c} =$$

$$\frac{4\pi\sigma_{O[E]}(E)}{cn_{O[E]}(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}},$$

$$\varepsilon_{1\ O[E]}(E) \equiv n_{O[E]}{}^2 - \kappa_{O[E]}{}^2,\ \varepsilon_{2\ O[E]}(E) \equiv 2\kappa_{O[E]}n_{O[E]},\ \text{and}\ R_{O[E]}(E) \equiv \frac{[n_{O[E]}-1]^2+\kappa_{O[E]}{}^2}{[n_{O[E]}+1]^2+\kappa_{O[E]}{}^2}. \quad (16a)$$

One notes here that, at the MIT-conditions : $N^* = 0$, both $E_{gn1(gp1)}(E_{gn2(gp2)}) = E_{gn(gp)}$, according to :

$$\left[\frac{E-E_{gn1(gp1)}(E_{gn2(gp2)})}{E-[E_{gn1(gp1)}(E_{gn2(gp2)})+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 = \frac{0}{0}\ \text{for}\ E=E_{gn(gp)},$$

$$\left[\frac{E-E_{gn1(gp1)}(E_{gn2(gp2)})}{E-[E_{gn1(gp1)}(E_{gn2(gp2)})+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 = 1\ \text{for}\ E \gtrsim E_{gn(gp)},$$ so that, in such the MIT,

$$\mathrm{JDOS}_{n(p)\ O[E]}\left(E, N^*, r_{d(a)}, x, T\right) \equiv \frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{\frac{3}{2}} \times \sqrt{E_{Fno(Fpo)}(N^* = 0)} = 0, \quad \text{for} \quad E \gtrsim$$

$E_{gn(gp)}$, which is largely verified since $N_{CDn(NDp)}(r_{d(a)}, x) \cong N^{EBT}_{CDn(CDp)}(r_{d(a)}, x)$ or $E_{gn2(gp2)}(N_{CDn(NDp)}, T = 0K) \cong E_{gn2(gp2)}(N^{EBT}_{CDn(CDp)}, T = 0K) \cong E_{gn(gp)}$, as those given in Equations (6a, 6b). In other words, the critical photon energy can be defined by: $E \cong E_{gn(gp)}$. Then, Eq. (6a) states that $N_{CDn(CDp)}$, given in parabolic conduction (parabolic valence)-band density of states, is just the density of electrons (holes) localized in the exponential conduction (valence)-band tail, $N^{EBT}_{CDn(CDp)}$, with a precision of the order of $\mathbf{2.87 \times 10^{-7}}$.[3]

Therefore, for $E \cong E_{gn(gp)}$, the exponential conduction (valence)-band tail states can be approximated with the same precision as:

$$\mathrm{JDOS}^{EBT}_{n(p)O[E]}\left(E, N^*, r_{d(a)}, x, T\right) \equiv \frac{1}{2\pi^2} \times \left(\frac{2m^*_{n(p)}(x)}{\hbar^2}\right)^{\frac{3}{2}} \times \sqrt{E_{Fno(Fpo)}\left(N^* = N_{CDn(NDp)}\right)}. \ 16b)$$

Here, $\varepsilon_{\text{free space}} = 8.854187817 \times 10^{-12} \left(\frac{\mathrm{c}^2}{\mathrm{N} \times \mathrm{m}^2}\right)$ is the permittivity of free space, -q (<0) is the charge of the electron, $\left| v_{\mathbf{O[E]}}(\mathrm{E}) \right|$ denotes the matrix elements of the velocity operator between valence (conduction)-and-conduction (valence) bands, and our approximate expression for the refraction index $n_{\mathbf{O[E]}}$ is found to be defined by:

$$n_{O[E]}\left(E, N^*, r_{d(a)}, x, T\right) = n_\infty(r_{d(a)}, x) + \sum_{i=1}^{4} \frac{X_i(E_{gn1(gp1)}) \times E + Y_i(E_{gn1(gp1)})}{E^2 - B_i E + C_i}, \tag{17}$$

Going to a constant as $E \to \infty$, since $n(E \to \infty, r_{d(a)}, x) \to n_\infty(r_{d(a)}, x) = \sqrt{\varepsilon(r_{d(a)}, x)} \times \frac{\omega_T}{\omega_L}$, given in the well-known Lyddane-Sachs-Teller relation, in which $\omega_T \simeq 5.1 \times 10^{13}\ \mathrm{s}^{-1}$ and $\omega_L \simeq 8.9755 \times 10^{13}\ \mathrm{s}^{-1}$ are the transverse (longitudinal) optical phonon frequencies, giving rise to: $n_\infty(r_{d(a)}, x) \simeq \sqrt{\varepsilon\left(r_{d(a)}, x\right)} \times 0.568$.

Here, the other parameters are determined by: $X_i\left(E_{gn1(gp1)}\right) = \frac{A_i}{Q_i} \times \left[-\frac{B_i^2}{2} + E_{gn1(gp1)} B_i - E_{gn1(gp1)}^2 + C_i\right]$, $Y_i\left(E_{gn1(gp1)}\right) = \frac{A_i}{Q_i} \times \left[\frac{B_i \times (E_{gn1(gp1)}^2 + C_i)}{2} - 2E_{gn1(gp1)} C_i\right]$, $Q_i = \frac{\sqrt{4C_i - B_i^2}}{2}$, where, for i=(1, 2, 3, and 4),

$A_i = 4.7314 \times 10^{-4},\ 0.2313655, 0.1117995, 0.0116323$, $B_i = 5.871, 6.154, 9.679$, 13.232, and $C_i = 8.619, 9.784, 23.803, 44.119$.

Now, the optical [electrical] conductivity $\sigma_{O[E]}$ can be defined and expressed in terms of the kinetic energy of the electron (hole), $E_k \equiv \frac{\hbar^2 \times k^2}{2 \times m^*_{n(p)}(x) \times m_o}$, k being the wave number, as:

$\sigma_{O[E]}(k) \equiv \frac{q^2 \times k}{\pi \times \hbar} \times \frac{k}{k_{sn(sp)}} \times \left[k \times a_{Bn(Bp)}\right] \times \left(\frac{E_k}{\mathbb{U}_{n(p)}}\right)^{\frac{1}{2}} \left(\frac{1}{\Omega \times \mathrm{cm}}\right)$, which is thus proportional to ${E_k}^2$,

Where $\frac{q^2}{\pi \times \hbar} = 7.7480735 \times 10^{-5}\ \mathrm{ohm}^{-1}$ and $\mathbb{U}_{n(p)}\left(N^*, r_{d(a)}, x\right)$ is determined in Eq. Eq. (9b).

Then, we obtain: $\langle E^2 \rangle_{FDDF} \equiv G_2\left(y = \frac{\pi k_B T}{E_{Fn(Fp)}}\right) \times E_{Fn(Fp)}^2$, and $G_2(y) = \left(1 + \frac{y^2}{3}\right) \equiv G_2(N^*, r_{d(a)}, x, T)$, with $y \equiv \frac{\pi}{\xi_{\mathbf{n(p)}}}$, $\xi_{n(p)} = \xi_{n(p)}\left(N^*, r_{d(a)}, x, T\right)$ for simplicity.

Therefore, from above equations (16, 17), if denoting the function $H\left(N^*, r_{d(a)}, x, T\right)$ by:

$\mathrm{H}\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) =$

$\left[\frac{\mathrm{k}_{\mathrm{Fn(Fp)}}(\mathrm{N}^*)}{\mathrm{R}_{\mathrm{sn(sp)}}(\mathrm{N}^*)} \times \left[\mathrm{k}_{\mathrm{Fn(Fp)}}(\mathrm{N}^*) \times \mathrm{a}_{\mathrm{Bn(Bp)}}\left(\mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}\right)\right] \times \sqrt{\mathrm{A}_{\mathrm{n(p)}}(\mathrm{N}^*) = \frac{\mathrm{E}_{\mathrm{Fno(Fpo)}}(\mathrm{N}^*)}{\mathbb{U}_{n(p)}(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, x)}}\right] \times$ $\mathrm{G}_2\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right)$, which can be approximately expressed in terms of: $\mathrm{E}^2_{\mathrm{Fno(Fpo)}}(\mathrm{N}^*) \times \mathrm{G}_2\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) \times \frac{\sqrt{\varepsilon(\mathrm{r}_{\mathrm{d(a)}}, \mathrm{x})}}{(\mathrm{N}^*)^{\frac{1}{4}}}$, since as noted in Eq. (9b), $\mathbb{U}_{n(p)}\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, x\right)$ is approximately expressed as: $\mathrm{C} \times \frac{\sqrt{\mathrm{N}^*}}{\varepsilon(\mathrm{r}_{\mathrm{d(a)}}, \mathrm{x})}$, C being a constant. Thus, with increasing $\mathrm{r}_{\mathrm{d(a)}}$ and for given x, T and N, the function $\mathrm{H}\left(\mathrm{r}_{\mathrm{d(a)}}\right)$ therefore decreases since $\varepsilon\left(\mathrm{r}_{\mathrm{d(a)}}\right)$ decreases, as noted in Ref.[3]

Then, our optical [electrical] conductivity models, defined in the O-EP and E-OP, respectively, for a simple representation, can thus be assumed to be as:

$\sigma_{\mathrm{O}}\left(\mathrm{E}, \mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) =$

$\frac{\mathrm{q}^2}{\pi \times \hbar} \times \mathrm{H}\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) \times \left[\frac{\mathrm{E} - \mathrm{E}_{\mathrm{gn1(gp1)}}}{\mathrm{E} - \left[\mathrm{E}_{\mathrm{gn1(gp1)}} + \mathrm{E}_{\mathrm{Fn(Fp)}} - \mathrm{E}_{\mathrm{Fno(Fpo)}}\right]}\right]^2 \left(\frac{1}{\Omega \times \mathrm{cm}}\right)$, and

$\sigma_{\mathrm{E}}\left(\mathrm{E}, \mathrm{N}, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) =$

$\frac{\mathrm{q}^2}{\pi \times \hbar} \times \mathrm{H}\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) \times \left[\frac{\mathrm{E} - \mathrm{E}_{\mathrm{gn2(gp2)}}}{\mathrm{E} - \left[(\mathrm{E}_{\mathrm{gn2(gp2)}} + \mathrm{E}_{\mathrm{Fn(Fp)}} - \mathrm{E}_{\mathrm{Fno(Fpo)}}\right]}\right]^2 \left(\frac{1}{\Omega \times \mathrm{cm}}\right)$. (18)

It should be noted here that:

(i) $\sigma_{\mathrm{O[E]}}\left(\mathrm{E} = \mathrm{E}_{\mathrm{gn1(gp1)}}[\mathrm{E}_{\mathrm{gn2(gp2)}}]\right) = 0$, and $\sigma_{\mathrm{O[E]}}(\mathrm{E} \to \infty) \to \text{Constant}$ for given $\left(\mathrm{N}, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right)$ −physical conditions, and

(ii) as T→ 0 K and $\mathrm{N}^* = 0$ [or $\mathrm{E}_{\mathrm{Fno(Fpo)}}(\mathrm{N}^*) = 0$], according to: $\mathrm{H}\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right) = 0$, and for a given E, $\left[\mathrm{E} - \mathrm{E}_{\mathrm{gn1(gp1)}}\right] = \left[\mathrm{E} - \mathrm{E}_{\mathrm{gn(gp)}}\right]$=Constant, then from Equations (16-18), $\mathrm{n}_{\mathrm{O[E]}}$ (E)= Constant, $\sigma_{\mathrm{O[E]}}(\mathrm{E}) = 0$, $\kappa_{\mathrm{O-EP[E-OP]}}(\mathrm{E}) = 0$, $\varepsilon_{1\,\mathrm{O[E]}}(\mathrm{E}) = (\mathrm{n}_\infty)^2 = \text{Constant}$, $\varepsilon_2(\mathrm{E}) = 0$, and $\propto_{\mathrm{O[E]}}$ (E) = 0.

This result (18) should be new, in comparison with that, obtained from an improved Forouhi-Bloomer parameterization, as given in our previous work.[2]

Using Equations (16-18), one obtains all the analytical results as:

$\frac{|\mathrm{v(E)}|^2}{\mathrm{E}} = \frac{8\pi^2\hbar}{(2\mathrm{m_r})^{\frac{3}{2}} \times \sqrt{\mathbb{U}_{n(p)}(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, x)}} \times \left[\frac{\mathrm{k}_{\mathrm{Fn(Fp)}}(\mathrm{N}^*)}{\mathrm{R}_{\mathrm{sn(sp)}}(\mathrm{N}^*)} \times \left[\mathrm{k}_{\mathrm{Fn(Fp)}}(\mathrm{N}^*) \times \mathrm{a}_{\mathrm{Bn(Bp)}}\left(\mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}\right)\right]\right] \times$

$\mathrm{G}_2\left(\mathrm{N}^*, \mathrm{r}_{\mathrm{d(a)}}, \mathrm{x}, \mathrm{T}\right)$, (19a)

$$\kappa_{O}(E) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times E}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times$$

$$\left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \quad \text{and}$$

$$\kappa_{E}(E) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times E}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \quad ,$$

(19b)

which gives: $\kappa_{O[E]}\left(E = E_{gn1(gp1)}[E_{gn2(gp2)}]\right) = 0$, and $\kappa_{O[E]}(E \to \infty) \to 0$, as those given in Ref.[2],

$$\varepsilon_{2\,O}(E) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times E}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \quad \text{and}$$

$$\varepsilon_{2\,E}(E) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times E}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2,$$

(19c)

which gives: $\varepsilon_{2O-EP[2E-OP]}\left(E = E_{gn1(gp1)}[E_{gn2(gp2)}]\right) = 0$, and $\varepsilon_{2O-EP[2E-OP]}(E \to \infty) \to 0$, as those given in Ref.[2],

$$\propto_{O}\ (E) =$$

$$\frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left[\frac{E-E_{gn1(gp1)}}{E-[E_{gn1(gp1)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \ \left(\frac{1}{cm}\right) \quad \text{and}$$

$$\propto_{E}\ (E) =$$

$$\frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times$$

$$\left[\frac{E-E_{gn2(gp2)}}{E-[(E_{gn2(gp2)}+E_{Fn(Fp)}-E_{Fno(Fpo)}]}\right]^2 \left(\frac{1}{cm}\right), \qquad (19d)$$

Which gives: $\propto_{O[E]}\ \left(E = E_{gn1(gp1)}[E_{gn2(gp2)}]\right) = 0$, and $\propto_{O[E]}\ (E \to \infty) \to$ Constant.

Furthermore, from Equations (16, 17, 19b), we can also determine $\varepsilon_{1\,O[E]}(E)$ and $R_{O[E]}(E)$.

Now, from Equations (18, 19b, 19c, 19d), using Eq. (15) as $E \equiv E_{gn(gp)O[E]}$, one obtains respectively, as:

$$\sigma_{O}\left(N^{*}, r_{d(a)}, x, T\right) = \frac{q^2}{\pi\times\hbar}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \ \left(\frac{1}{\Omega\times cm}\right),$$

having the same form as that of $\sigma_{E}\left(N, r_{d(a)}, x, T\right)$ [1], as:

$$\sigma_{E}\left(N^{*}, r_{d(a)}, x, T\right) = \frac{q^2}{\pi\times\hbar}\times H\left(N^{*}, r_{d(a)}, x, T\right)\times\left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \ \left(\frac{1}{\Omega\times cm}\right), \qquad (20a)$$

noting here that for given physical conditions $\left(N^*, r_{d(a)}, x, T\right)$ we obtain: $\sigma_O > \sigma_E$ since $m_r(x) < m_{c(v)}(x)$,

$\kappa_O\left(N^*, r_{d(a)}, x, T\right) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn1(gp1)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$ and

$\kappa_E\left(N^*, r_{d(a)}, x, T\right) = \frac{2q^2}{n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn2(gp2)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$, (20b)

$\varepsilon_{2\,O}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn1(gp1)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$

and

$\varepsilon_{2\,E}\left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}\times(E_{gn2(gp2)}+E_{Fn(Fp)})} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2$,

(20c) $\propto_O \left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{cm}\right)$ and

$\propto_E \left(N^*, r_{d(a)}, x, T\right) = \frac{4q^2}{\hbar c n(E)\times\varepsilon(r_{d(a)},x)\times\varepsilon_{\text{free space}}} \times H\left(N^*, r_{d(a)}, x, T\right) \times \left(\frac{E_{Fn(Fp)}}{E_{Fno(Fpo)}}\right)^2 \left(\frac{1}{cm}\right)$. (20d)

Further, from Equations (16, 17, 20b), we can also determine $\varepsilon_{1\,O[E]}(E)$ and $R_{O[E]}(E)$.

By returning to **Eq. (20a)**, one remarks that, as noted above for the function $H\left(N^*, r_{d(a)}, x, T\right)$, the function $\sigma_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$ can thus be approximately expressed in terms of $E^2_{Fn(Fp)}\left(N^*, r_{d(a)}, x, T\right) \times G_2\left(N^*, r_{d(a)}, x, T\right) \times \frac{\sqrt{\varepsilon(r_{d(a)},x)}}{(N^*)^{\frac{1}{4}}}$, being proportional to: $G_2\left(N^*, r_{d(a)}, x, T\right) \times \sqrt{\varepsilon\left(r_{d(a)},\ x\right)} \times (N^*)^{\frac{13}{12}}$ at low T and high N, giving raise to some concluding remarks as follows.

(1) With **increasing** $r_{d(a)}$ **and for given x, T and** N, since as observed in Ref.[3], $\varepsilon\left(r_{d(a)}\right)$ **decreases**, thus $\sigma_{O[E]}\left(r_{d(a)}\right)$ **decreases**, as observed in Tables 9n and 9p given in Ref.[1],[2] With decreasing T and for given x, N and $r_{d(a)}$, since $G_2(T)$ decreases as noted in Table 2, thus $\sigma_{O[E]}(\,T)$ **decreases**, as observed in Tables 9n and 9p given in Ref.[1]

(3) With increasing N and for given x, T and $r_{d(a)}$, as noted above, $\sigma_{O[E]}(N)$ **increases**, as observed in Tables 9n and 9p given in Ref.[1]

OPTICAL [ELECTRICAL] PROPERTIES $\left[m^*_{n(p)} \equiv m_r(x)[m_{c(v)}(x)]\right]$

Here, if denoting, for majority electrons (holes), the thermal conductivity by: $\sigma_{Th.\ O[E]}(N^*, r_{d(a)}, x, T)$ in $\frac{W}{cm\times K}$, and the Lorenz number L by: $L = \frac{\pi^2}{3} \times \left(\frac{k_B}{q}\right)^2 = 2.4429637 \left(\frac{W\times ohm}{K^2}\right) = 2.4429637 \times 10^{-8}\ (V^2 \times K^{-2})$, then the well-known Wiedemann-Frank law states that the ratio, $\frac{\sigma_{Th.O[E]}}{\sigma_{O[E]]}}$, due to the O-EP [E-OP], is proportional to the temperature T(K), as:

$$\frac{\sigma_{Th\,.O[E]}(N^*, r_{d(a)}, x, T)}{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)} = L \times T. \qquad (21)$$

Further, the resistivity is found to be given by: $\rho_{O[E]}(N^*, r_{d(a)}, x, T) \equiv 1/\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$, noting again that $N^* \equiv N - N_{CDn(NDp)}(r_{d(a)}, x)$.

In Eq. (20a), one notes that at T= 0 K, $\sigma_{O[E]}(N^*, r_{d(a)}, x, T)$ is proportional to $E^2_{Fno(Fpo)}$, or to $(N^*)^{\frac{4}{3}}$. Thus , from Eq. (21), one has: $\sigma_{O[E]}(N^* = 0, r_{d(a)}, x, T = 0K) = 0$ and also $\sigma_{Th.\ O[E]}(N^* = 0, r_{d(a)}, x, T = 0K) = 0$ at $N^* = 0$, at which the MIT occurs.

New Optical [Electrical] Coefficients

The relaxation time $\tau_{O[E]}$ is related to $\sigma_{O[E]}$ by[1]:

$\tau_{O[E]}(N^*, r_{d(a)}, x, T) \equiv \sigma_{O[E]}(N^*, r_{d(a)}, x, T) \times \frac{m^*_{n(p)}(x)\times m_o}{q^2\times(N^*/g_{c(v)})}$. Therefore, the mobility $\mu_{O[E]}$ is given by:

$$\mu_{O[E]]}\left(N^*, r_{d(a)}, x, T\right) = \frac{q\times\tau_{O[E]}(N^*, r_{d(a)}, x, T)}{m^*_{n(p)}(x)\times m_o} = \frac{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)}{q\times(N^*/g_{c(v)})}\ \left(\frac{cm^2}{V\times s}\right), \qquad (22a)$$

Being expressed in terms of $\frac{\sigma_{O[E]}(N^*, r_{d(a)}, x, T)}{N^*}$. Further, as noted in above Eq. (20a) for $\sigma_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$, $\mu_{O[E]]}\left(N^*, r_{d(a)}, x, T\right)$ can thus be expressed in terms of:

$$E^2_{Fn(Fp)}\left(N^*, r_{d(a)}, x, T\right) \times G_2\left(N^*, r_{d(a)}, x, T\right) \times \frac{\sqrt{\varepsilon(r_{d(a)}, x)}}{(N^*)^{\frac{5}{4}}}.$$

Then, from the well-known idea of Stokes, Einstein, Sutherland and Reynolds, we can define our viscosity coefficient, $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, and its reduced one, $R\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, by:

$\frac{\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T)}{q} \equiv \frac{1}{6\pi\times\ \mu_{O[E]]}(N^*,r_{d(a)},x,T)\times R_{WS}(N^*,x)}\left(\frac{V}{cm}\times\frac{s}{cm^2}\right)$, $R\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T) \equiv \frac{\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T)}{\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T=0K)}$, (22b)

Where $R_{WS}(N^*,x) \equiv \left(\frac{3g_{c(v)}(x)}{4\pi N^*}\right)^{1/3}$ is the effective Wigner-Seitz radius, decreasing with increasing N.

Further, as noted above for $\mu_{O[E]]}(N^*,r_{d(a)},x,T)$, $\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T)$ can thus be expressed in terms of $\frac{(N^*)^{\frac{19}{12}}}{E^2_{Fn(Fp)}(N^*,r_{d(a)},x,T)\times G_2(N^*,r_{d(a)},x,T)\times\sqrt{\varepsilon(r_{d(a)},x)}}$, being proportional to: $\frac{N^{\frac{1}{4}}}{G_2(N^*,r_{d(a)},x,T)\times\sqrt{\varepsilon(r_{d(a)},x)}}$ at lowest T and highest N, giving raise to some concluding remarks as follows.

(1) **With increasing $r_{d(a)}$** and for given x, T and N, since as observed in Ref.[3], $\varepsilon(r_{d(a)})$ **decreases**, thus $\mathbb{V}_{O[E]}(r_{d(a)})$ **increases**, as observed in next Tables 4n and 4p in Appendix 1.

(2) **With increasing x** and for given $r_{d(a)}$, T and N, since as observed in Ref.[3], $\varepsilon(r_{d(a)})$ **decreases**, thus $\mathbb{V}_{O[E]}(r_{d(a)})$ **increases**, as observed in next Tables 4n and 4p in Appendix 1.

(3) **With decreasing T** and for given x, N and $r_{d(a)}$, since $G_2(T)$ decreases as noted in Table 2, thus $\mathbb{V}_{O[E]}(r_{d(a)})$ **increases**, as observed in next Tables 4n and 4p in Appendix 1.

(4) **With increasing N** and for given x, T and $r_{d(a)}$, as noted above: $\mathbb{V}_{O[E]}(N)$ is proportional to: $(N^*)^{\frac{1}{4}}$, thus $\mathbb{V}_{O[E]}(N)$ **increases**, as observed in next Tables 4n and 4p in Appendix 1.

We now define the activation energy, $AE_{O[E]}(N^*,r_{d(a)},x,T)$, as[17] by:

$$AE_{O[E]}(N^*,r_{d(a)},x,T) \equiv k_BT\times Ln\left(R\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T)\right) \leq 0\ eV, \qquad (22c)$$

According to the reduced activation energy, $RAE_{O[E]}(N^*,r_{d(a)},x,T)$, given by:

$$RAE_{O[E]}(N^*,r_{d(a)},x,T) \equiv \frac{AE_{O[E]}(N^*,r_{d(a)},x,T)}{k_BT} \equiv Ln\left(R\mathbb{V}_{O[E]}(N^*,r_{d(a)},x,T)\right) \leq 0.$$

Furthermore, the Hall factor is defined by:

$r_{HO[HE]}(N^*,r_{d(a)},x,T) \equiv \frac{\langle\tau_{O[E]}{}^2\rangle_{FDDF}}{\left[\langle\tau_{O[E]}\rangle_{FDDF}\right]^2} = \frac{G_4(y)}{[G_2(y)]^2}$, $y \equiv \frac{\pi}{\xi_{n(p)}(N^*,r_{d(a)},x,T)} = \frac{\pi k_BT}{E_{Fn(Fp)}(N^*,r_{d(a)},x,T)}$, and the Hall mobility is found to be given by:

$$\mu_{HO[HE]}(N^*,r_{d(a)},x,T) \equiv\ \mu_{O[E]}(N^*,r_{d(a)},x,T)\times r_{HO[HE]}(N^*,r_{d(a)},x,T)\ \left(\frac{cm^2}{V\times s}\right). \qquad (23)$$

It should be noted that, as T= 0 K and for a given value of $E_{Fn(Fp)}$, since $G_{p>1}(y=0)=1$ as given in Eq. (12) and Table 2, one obtains: $r_{HE[HO]}(N^*, r_{d(a)}, x, T) = 1$, and therefore: $\mu_{HO[HE]}(N^*, r_{d(a)}, x, T) \equiv \mu_{O[E]}(N^*, r_{d(a)}, x, T)$.

New relation between diffusion, mobility, and viscosity

By taking into account Equations (22a, 22b), our relation is found to be defined by[1]:

$$\mathbb{R}_{E[O]}\left(N^*, r_{d(a)}, x, T\right) \equiv \frac{D_{O[E]}\left(N^*, r_{d(a)}, x, T\right)}{\mu_{O[E]}\left(N^*, r_{d(a)}, x, T\right)} \equiv D_{O[E]}\left(N^*, r_{d(a)}, x, T\right) \times \frac{\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)}{q} \times 6\pi \times R_{WS}(N^*, x) \equiv \frac{N^*}{q} \times \frac{dE_{Fn(Fp)}}{dN^*} \equiv \frac{k_B \times T}{q} \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right) = \sqrt{\frac{3\times L}{\pi^2}} \times T \times \left(u\frac{d\xi_{n(p)}(u)}{du}\right), \quad \frac{k_B}{q} = \sqrt{\frac{3\times L}{\pi^2}}, \qquad (24)$$

Where $D_{E[O]}\left(N^*, r_{d(a)}, x, T\right)$ is the diffusion coefficient, $\xi_{n(p)}(u)$ is defined in Eq. (11), the mobility $\mu_{O[E]}\left(N^*, r_{d(a)}, x, T\right)$ is determined in Eq. (22a), and the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$ is defined in Eq. (22b). By differentiating this function $\xi_{n(p)}(u) \equiv \frac{E_{Fn(Fp)}(u)}{k_B T} = \frac{G(u)+Au^B F(u)}{1+Au^B} \equiv \frac{V(u)}{W(u)}$, with respect to u, being defined in Eq. (11), one obtains $\frac{d\xi_{n(p)}(u)}{du}$. Therefore, Eq. (24) can also be rewritten as: $\mathbb{R}_{E[O]}(u) = \frac{k_B \times T}{q} \times u\frac{V'(u)\times W(u) - V(u)\times W'(u)}{W^2(u)}$ where $W'(u) = ABu^{B-1}$ and $V'(u) = u^{-1} + 2^{-\frac{3}{2}}e^{-du}(1-du) + \frac{2}{3}Au^{B-1}F(u)\left[\left(1+\frac{3B}{2}\right) + \frac{4}{3}\times\frac{bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}}{1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}}\right]$. One remarks that: (i) as $u \to 0$, one has: $W^2 \simeq 1$ and $u[V' \times W - V \times W'] \simeq 1$, and therefore: $\mathbb{R}_{E[O]}(u \to 0) \simeq \frac{k_B \times T}{q}$, being a well-known relation given by Stokes, Einstein, Sutherland and Reynolds, and (ii) as $u \to \infty$, one has: $W^2 \approx A^2 u^{2B}$ and $u[V' \times W - V \times W'] \approx \frac{2}{3}au^{2/3}A^2u^{2B}$, and therefore, in this **highly degenerate case** and at T=0K, our relation (24) is reduced to: $\mathbb{R}_{E[O]}\left(N^*, r_{d(a)}, x, T=0K\right) \approx \frac{2}{3}E_{Fno(Fpo)}(N^*)/q$. In other words, **Eq. (24) verifies all the correct limiting conditions**.

Furthermore, in the present degenerate case ($u \gg 1$), Eq. (24) can be rewritten as:

$$\mathbb{R}_{E[O](VC)}\left(N^*, r_{d(a)}, x, T=0K\right) \simeq \frac{2}{3} \times \frac{E_{Fno(Fpo)}(u)}{q} \times \left[1 + \frac{4}{3} \times \frac{\left(bu^{-\frac{4}{3}}+2cu^{-\frac{8}{3}}\right)}{\left(1+bu^{-\frac{4}{3}}+cu^{-\frac{8}{3}}\right)}\right],$$

where $a = \left[3\sqrt{\pi}/4\right]^{2/3}$, $b = \frac{1}{8}\left(\frac{\pi}{a}\right)^2$ and $c = \frac{62.3739855}{1920}\left(\frac{\pi}{a}\right)^4$.

Then, in Tables 3n and 3p, reported in Appendix 1, for given ($r_d, x \text{ and } T$), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed as functions of N, are obtained by using Equations (22b, 22a and 24).

In Tables 4n and 4p, reported in Appendix 1, the numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$ are obtained by using Eq. (22b).

In Tables 5n and 5p, reported in Appendix 1, the numerical results of reduced Fermi energy $\xi_{nO[E]}(N^*, r_{d(a)}, x, T)$, mobility $\mu_{O[E]}(N^*, r_{d(a)}, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_{d(a)}, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, and activation energy $AE_{O[E]}(N^*, r_{d(a)}, x, T)$ are obtained by using Equations (11, 22a, 24, 22b, 22c), respectively.

Finally, in Tables 6n and 6p, For given x, r_a, T and N, the numerical results of reduced Fermi energy $\xi_{pO[E]}(N^*, r_{d(a)}, x, T)$, mobility $\mu_{O[E]}(N^*, r_{d(a)}, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_{d(a)}, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_{d(a)}, x, T)$, and activation energy $AE_{O[E]}(N^*, r_{d(a)}, x, T)$ are obtained by using Equations (11, 22a, 24, 22b, 22c), respectively.

CONCLUDING REMARKS

In the $\mathbf{n}^+(\mathbf{p}^+) - \mathbf{X}(\mathbf{x})$-crystalline alloy, $0 \leq x \leq 1$, x being the concentration, the diffusion-mobility-activation energy-Fermi energy relations, enhanced by : **(i)** the optico-electrical phenomenon (O-EP) and the electro-optical phenomenon (E-OP), **(ii)** our static dielectric constant law, $\varepsilon(r_{d(a)}, x)$, $r_{d(a)}$ being the donor (acceptor) d(a)-radius, given in Equations (1a, 1b), **(iii)** our accurate reduced Fermi energy, $\xi_{n(p)}$, given in Eq. (11), accurate with a precision of the order of $\mathbf{2.11 \times 10^{-4}}$ [9], and finally **(iv)** our optical-and-electrical conductivity models, given in Eq. (18, 20a), are now investigated by basing on our physical model and Fermi-Dirac distribution function, as presented in our recent works.[1-5]

Some important concluding remarks can be given and discussed as follows.

(I)- Then, in Tables 3n and 3p, reported in Appendix 1, for given x, $r_{d(a)}$, and T=(4.2 K and 77 K), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed as functions of N, are obtained by using Equations (22b, 22a and 24). In particular, for given (x, $r_{d(a)}$ and N), those of $\mu_{O[E]}(T)$ decrease with decreasing T, due to the increasing reduced Fermi energy $\xi_{nO[E]}$. Further, for given (x, $r_{d(a)}$ and N) the numerical results of $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and for given (x, T and $\mathrm{r_{d(a)}}$) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (or with decreasing T and increasing N), both the reduced Fermi energy $\xi_{\mathrm{nO[E]}}$ and the viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

In Tables 4n and 4p, the numerical results of the viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, expressed in $\left(\frac{\mathrm{eV}}{\mathrm{cm}} \times \frac{\mathrm{s}}{\mathrm{cm^2}}\right)$, are obtained by using Eq. (22b), suggesting that: (i) for given (x, T and N), they increase with increasing $\mathrm{r_{d(a)}}$, (ii) for given (x, $\mathrm{r_{d(a)}}$ and N) the numerical results of $\mathbb{V}_{\mathrm{O[E]}}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and (iii) for given (x, T and $\mathrm{r_{d(a)}}$) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (or with decreasing T and increasing N), both the reduced Fermi energy $\xi_{\mathrm{n(p)O[E]}}$ and the viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**. In Tables 5n and 5p, reported in Appendix 1, the numerical results of reduced Fermi energy $\xi_{\mathrm{nO[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, mobility $\mu_{\mathrm{O[E]}}(\mathrm{N^*, r_{r_{d(a)}}, x, T})$, diffusion coefficient $\mathrm{D_{O[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}\left(\mathrm{N^*, r_{d(a)}, x, T}\right)$, and activation energy $\mathrm{AE_{O[E]}}\left(\mathrm{N^*, r_{r_{d(a)}}, x, T}\right)$, are obtained by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{\mathrm{O[E]}}$, one notes that, for given (x, $\mathrm{r_{d(a)}}$ and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] In other words, with increasing degeneracy, both the reduced Fermi energy $\xi_{\mathrm{nO[E]}}$ and the viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Finally, in Tables 6n and 6p, for given x, $\mathrm{r_a}$, T and N, the numerical results of reduced Fermi energy $\xi_{\mathrm{pO[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, mobility $\mu_{\mathrm{O[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, diffusion coefficient $\mathrm{D_{O[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, viscosity coefficient $\mathbb{V}_{\mathrm{O[E]}}\left(\mathrm{N^*, r_{d(a)}, x, T}\right)$, and activation energy $\mathrm{AE_{O[E]}}\left(\mathrm{N^*, r_{d(a)}, x, T}\right)$, are obtained by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{\mathrm{O[E]}}(\mathrm{N^*, r_{d(a)}, x, T})$, one observes that, for given (x, $\mathrm{r_{d(a)}}$ and T), they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy, both the reduced

Fermi energy $\xi_{n(p)O[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

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APPENDIX 1

Table 1: In the $X(x) \equiv GaSb_{1-x}Te_x$-crystalline alloy, the different values of energy-band-structure parameters, for a given x, are given in the following.[3]

In the $\mathbf{X(x)}$**-crystalline alloy**, in which $r_{do(ao)} = r_{\mathbf{Sb(Ga)}}$ =0.136 nm (0.126 nm), we have [3]: $g_{c(v)}(x) = 1 \times x + 1 \times (1-x) = 1$, $m_{c(v)}(x)/m_o = 0.209\ (0.4) \times x + 0.047\ (0.3) \times (1-x)$, $\varepsilon_o(x) = 12.3 \times x + 15.69 \times (1-x)$, $E_{go}(x) = 1.796 \times x + 0.81 \times (1-x)$.

Table 2: Expressions for $G_{p>1}(y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}} = \frac{\pi}{\xi_{n(p)}})$, due to the Fermi-Dirac distribution function, are used to determine the electrical-and-thermoelectric coefficients, suggesting that, with increasing T (or decreasing T) and for given (N, r_d, x), since $\xi_{n(p)}(T)$ decreases (or increases), the function $G_{p>1}(T)$ increases (or decreases). It should be noted that, since $y \equiv \frac{\pi k_B T}{E_{Fn(Fp)}}$, $G_{p>1}(y=0) = 1$, as T= 0 K and for a given value of $E_{Fn(Fp)}$.

$G_{3/2}(y)$	$G_2(y)$	$G_{5/2}(y)$	$G_3(y)$	$G_{7/2}(y)$	$G_4(y)$	$G_{9/2}(y)$
$\left(1+\frac{y^2}{8}+\frac{7y^4}{640}\right)$	$\left(1+\frac{y^2}{3}\right)$	$\left(1+\frac{5y^2}{8}-\frac{7y^4}{384}\right)$	$(1+y^2)$	$\left(1+\frac{35y^2}{24}+\frac{49y^4}{384}\right)$	$\left(1+2y^2+\frac{7y^4}{15}\right)$	$\left(1+\frac{21y^2}{8}+\frac{147y^4}{128}\right)$

Table 3n: For given x, r_d, and T=(4.2 K and 77 K), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed respectively in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22b, 22a and 24). In particular, for given (x, r_d and N), those of $\mu_{O[E]}(T)$ decrease with decreasing T, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), and therefore, those of the viscosity coefficient $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] Further, for given (x, T and r_d), those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{nO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (or with decreasing T and increasing N), both $\xi_{nO[E]}$ and $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Donor	**As**	Sb
r_d (nm) ↗	**0.118**	0.136
For x=0 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	5.839 [7.634], 45.50 [34.81], 26.28 [17.38]	6.766 [8.840], 39.27 [30.06], 22.68 [15.01]
7	8.079 [10.58], 43.63 [33.31], 44.32 [29.26]	9.369 [12.26], 37.62 [28.74], 38.21 [25.24]
10	9.254 [12.13], 42.89 [32.73], 55.27 [36.47]	10.73 [14.06], 36.97 [28.23], 47.64 [31.45]
For x=0 and at T=77 K		

$N\ (10^{19}\ \mathrm{cm}^{-3})$		
3	1.543 [0.395], 172.2 [672.6], 99.45 [335.8]	1.787 [0.457], 148.6 [580.8], 85.83 [289.9]
7	4.252 [1.529], 82.88 [230.5], 84.20 [202.5]	4.932 [1.772], 71.46 [198.7], 72.60 [174.6]
10	5.935 [2.591], 66.88 [153.2], 86.18 [170.6]	6.885 [3.005], 57.65 [132.1], 74.29 [147.1]
For x=0.5 and at T=4.2 K		
$N\ (10^{19}\ \mathrm{cm}^{-3})$		
3	32.74 [56.56], 8.110 [4.694], 2.028 [0.859]	37.71 [64.92], 7.041 [4.090], 1.760 [0.748]
7	46.11 [80.50], 7.641 [4.377], 3.363 [1.411]	53.24 [92.69], 6.618 [3.801], 2.912 [1.225]
10	53.14 [93.10], 7.468 [4.263], 4.171 [1.743]	31.40 [107.3], 6.463 [3.697], 3.609 [1.512]
For x=0.5 and at T=77 K		
$N\ (10^{19}\ \mathrm{cm}^{-3})$		
3	8.627 [2.916], 30.78 [91.06], 7.692 [16.66]	9.930 [3.344], 26.73 [79.38], 6.679 [14.52]
7	24.25 [11.62], 14.53 [30.33], 6.394 [9.774]	28.00 [13.37], 12.58 [26.34], 5.537 [8.488]
10	34.06 [19.88], 11.65 [19.96], 6.505 [8.162]	39.36 [22.91], 10.08 [17.32], 5.629 [7.081]
For x=1 and at T=4.2 K		
$N\ (10^{19}\ \mathrm{cm}^{-3})$		
3	79.22 [158.9], 3.339 [1.665], 0.566 [0.185]	90.54 [180.2], 2.918 [1.466], 0.493 [0.163]
7	114.0 [234.0], 3.085 [1.503], 0.924 [0.296]	130.9 [267.1], 2.685 [1.316], 0.803 [0.259]
10	132.3 [273.9], 2.996 [1.447], 1.140 [0.362]	152.2 [313.3], 2.604 [1.265], 0.990 [0.316]
For x=1 and at T=77 K		
$N\ (10^{19}\ \mathrm{cm}^{-3})$		
3	20.62 [8.054], 12.83 [32.84], 2.171 [3.647]	23.48 [9.092], 11.25 [29.06], 1.899 [3.218]
7	59.76 [33.56], 5.886 [10.48], 1.763 [2.061]	68.56 [38.24], 5.128 [9.194], 1.534 [1.806]
10	84.67 [58.24], 4.681 [6.807], 1.780 [1.700]	97.33 [66.54], 4.071 [5.955], 1.547 [1.486]

Table 3p: For given x, r_a, and T=(4.2 K and 77 K), the numerical results of $\mathbb{V}_{O[E]}$, $\mu_{O[E]}$ and $D_{O[E]}$, expressed respectively in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}, \frac{10^3 \times cm^2}{V \times s}, \frac{10^3 \times cm^2}{s}\right)$, as functions of N, are obtained by using Equations (22b, 22a and 24). In particular, for given (x, r_a and N), those of $\mu_{O[E]}(T)$ decrease with decreasing T, due to the increasing reduced Fermi energy $\xi_{pO[E]}$ (or with increasing degeneracy), and therefore, those of the viscosity coefficient $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] Further, for given (x, T and r_a), those of $\mathbb{V}_{O[E]}$ increase with increasing N, due to the increasing reduced Fermi energy $\xi_{pO[E]}$ (or with increasing degeneracy), in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, with increasing degeneracy (or with decreasing T and increasing N), both $\xi_{pO[E]}$ and $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Acceptor	Mg	In
r_a (nm) ↗	0.140	0.144
For x=0 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	7.407 [235.0], 35.52 [1.119], 20.12 [0.086]	7.883 [247.4], 33.35 [1.062], 18.84 [0.081]
7	10.33 [353.9], 33.97 [0.991], 34.23 [0.135]	11.01 [374.0], 31.87 [0.938], 32.08 [0.128]
10	11.86 [417.3], 33.38 [0.948], 42.76 [0.164]	12.64 [441.4], 31.31 [0.896], 40.09 [0.155]
For x=0 and at T=77 K		
N (10^{19} cm^{-3})		
3	7.405 [230.6], 35.54 [1.141], 20.12 [0.087]	7.880 [242.7], 33.36 [1.083], 18.85 [0.082]
7	10.33 [351.8], 33.97 [0.997], 34.23 [0.136]	11.01 [371.7], 31.87 [0.944], 32.08 [0.128]
10	11.86 [415.8], 33.38 [0.952], 42.76 [0.165]	12.64 [439.8], 31.31 [0.900], 40.09 [0.156]
For x=0.5 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	40.48 [344.8], 6.420 [0.754], 1.537 [0.048]	42.87 [361.1], 6.047 [0.718], 1.441 [0.046]
7	58.16 [537.1], 6.004 [0.650], 2.595 [0.075]	61.78 [565.5], 5.646 [0.617], 2.435 [0.071]
10	67.34 [640.0], 5.856 [0.616], 3.229 [0.091]	71.60 [674.9], 5.504 [0.584], 3.031 [0.086]
For x=0.5 and at T=77 K		
N (10^{19} cm^{-3})		
3	40.40 [335.6], 6.433 [0.754], 1.540 [0.049]	42.78 [351.3], 6.059 [0.738], 1.443 [0.047]
7	58.12 [532.6], 6.007 [0.655], 2.596 [0.076]	61.74 [560.8], 5.649 [0.622], 2.437 [0.072]
10	67.32 [636.7], 5.858 [0.619], 3.230 [0.091]	71.57 [671.5], 5.506 [0.587], 3.032 [0.086]
For x=1 and at T=4.2 K		
N (10^{19} cm^{-3})		
3	93.53 [477.2], 2.699 [0.529], 0.417 [0.028]	98.24 [495.2], 2.554 [0.507], 0.389 [0.026]
7	140.7 [783.3], 2.453 [0.441], 0.707 [0.044]	148.8 [820.7], 2.313 [0.419], 0.664 [0.041]
10	164.9 [947.6], 2.372 [0.413], 0.878 [0.052]	174.8 [995.3], 2.234 [0.392], 0.825 [0.050]

For x=1 and at T=77 K		
N (10^{19} cm^{-3})		
3	93.09 [458.7], 2.712 [0.550], 0.418 [0.029]	97.77 [475.6], 2.567 [0.528], 0.391 [0.027]
7	140.5 [774.4], 2.456 [0.446], 0.708 [0.044]	148.6 [811.3], 2.316 [0.424], 0.664 [0.042]
10	164.8 [941.1], 2.373 [0.416], 0.879 [0.053]	174.6 [988.4], 2.236 [0.395], 0.826 [0.050]

Table 4n: The numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, expressed in $\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$, are obtained as functions of N, by using Eq. (22b), suggesting that: (i) for given (x, T and N), they increase with increasing r_d, (ii) for given (x, r_d and N) the numerical results of $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and (iii) for given (x, T and r_d) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3n, with increasing degeneracy (or with decreasing T and increasing N), both the reduced Fermi energy $\xi_{nO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Donor	P	**As**	Sb	Sn
r_d (nm) [4] ↗	0.110	**0.118**	0.136	0.140
For x=0 and at T=4.2 K				
N (10^{19} cm^{-3})				
3 ↗	4.870 [6.372]	5.839 [7.634]	6.766 [8.841]	6.815 [8.905]
7 ↗	6.731 [8.820]	8.079 [10.58]	9.369 [12.26]	9.438 [12.35]
10 ↗	7.708 [10.10]	9.254 [12.13]	10.73 [14.06]	10.81 [14.16]
For x=0 and at T=77 K				
N (10^{19} cm^{-3})				
3 ↗	1.287 [0.330]	1.543 [0.395]	1.787 [0.457]	1.800 [0.461]
7 ↗	3.543 [1.274]	4.252 [1.529]	4.932 [1.772]	4.968 [1.785]
10 ↗	4.944 [2.159]	5.935 [2.591]	6.885 [3.005]	6.936 [3.027]
For x=0.5 and at T=4.2 K				
N (10^{19} cm^{-3})				
3 ↗	27.49 [47.69]	32.74 [56.56]	37.71 [64.92]	37.97 [65.36]
7 ↗	38.61 [67.61]	46.11 [80.50]	53.24 [92.69]	53.61 [93.34]
10 ↗	44.46 [78.10]	53.14 [93.10]	61.40 [107.3]	61.84 [108.1]
For x=0.5 and at T=77 K				
N (10^{19} cm^{-3})				
3 ↗	7.249 [2.460]	8.627 [2.916]	9.930 [3.344]	9.999 [3.367]
7 ↗	20.31 [9.760]	24.25 [11.62]	28.00 [13.37]	28.19 [13.46]

10	↗	28.50 [16.68]	34.06 [19.88]	39.36 [22.91]	39.64 [23.07]
For x=1 and at T=4.2 K					
N (10^{19} cm^{-3})					
3	↗	67.09 [135.7]	79.22 [158.9]	90.54 [180.2]	91.13 [181.3]
7	↗	96.05 [198.5]	114.0 [234.0]	130.9 [267.1]	131.8 [268.9]
10	↗	111.3 [231.7]	132.3 [273.9]	152.2 [313.3]	153.2 [315.4]
For x=1 and at T=77 K					
N (10^{19} cm^{-3})					
3	↗	17.52 [6.910]	20.62 [8.054]	23.48 [9.092]	23.63 [9.146]
7	↗	50.39 [28.51]	59.76 [33.56]	68.56 [38.24]	69.02 [38.48]
10	↗	71.25 [49.33]	84.67 [58.24]	97.33 [66.54]	98.00 [66.98]

Table 4p: The numerical results of the viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, expressed in $\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$, are obtained as functions of N, by using Eq. (22b), suggesting that: (i) for given (x, T and N), they increase with increasing r_a, (ii) for given (x, r_a and N) the numerical results of $\mathbb{V}_{O[E]}$ increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao[18], and (iii) for given (x, T and r_a) they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3p, with increasing degeneracy (or with decreasing T and increasing N), both the reduced Fermi energy $\xi_{pO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Acceptor		**Ga**	Mg	In	Cd
r_a (nm)	↗	**0.126**	0.140	0.144	0.148
For x=0 and at T=4.2 K					
N (10^{19} cm^{-3})					
3	↗	6.700 [216.1]	7.407 [235.0]	7.883 [247.4]	8.484 [262.8]
7	↗	9.328 [323.7]	10.33 [354.0]	11.01 [374.0]	11.86 [399.0]
10	↗	10.70 [381.0]	11.86 [417.3]	12.64 [441.4]	13.63 [471.7]
For x=0 and at T=77 K					
N (10^{19} cm^{-3})					
3	↗	6.697 [212.1]	7.405 [230.6]	7.880 [242.7]	8.481 [257.8]
7	↗	9.327 [321.8]	10.33 [351.8]	11.01 [371.7]	11.86 [396.6]
10	↗	10.70 [379.6]	11.86 [415.8]	12.63 [439.8]	13.62 [469.9]
For x=0.5 and at T=4.2 K					
N (10^{19} cm^{-3})					
3	↗	36.87 [**240.1**]	40.48 [260.7]	42.87 [274.1]	45.85 [290.7]
7	↗	52.74 [362.2]	58.16 [395.6]	61.78 [417.6]	66.35 [445.1]

10	↗	61.00 [427.2]	67.34 [467.5]	71.60 [494.2]	76.97 [527.6]
For x=0.5 and at T=77 K					
N (10^{19} cm^{-3})					
3	↗	36.80 [235.8]	40.40 [255.9]	42.78 [269.0]	45.76 [285.3]
7	↗	52.71 [360.1]	58.12 [393.3]	61.74 [415.2]	66.31 [442.5]
10	↗	60.97 [425.7]	67.32 [465.8]	71.57 [492.4]	76.94 [525.7]
For x=1 and at T=4.2 K					
N (10^{19} cm^{-3})					
3	↗	15.41 [86.20]	17.00 [93.53]	18.05 [98.24]	19.38 [103.9]
7	↗	21.68 [128.4]	23.97 [140.7]	25.51 [148.8]	27.46 [159.0]
10	↗	24.95 [150.1]	27.61 [164.9]	29.39 [174.8]	31.66 [187.1]
For x=1 and at T=77 K					
N (10^{19} cm^{-3})					
3	↗	15.40 [85.81]	16.98 [93.09]	18.04 [97.77]	19.37 [103.4]
7	↗	21.68 [128.2]	23.97 [140.5]	25.51 [148.6]	27.46 [158.8]
10	↗	24.95 [150.0]	27.60 [164.8]	29.39 [174.6]	31.66 [187.0]

Table 5n: For given x, r_d and N, the numerical results of reduced Fermi energy $\xi_{nO[E]}(N^*, r_d, x, T)$, mobility $\mu_{O[E]}(N^*, r_d, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_d, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, and activation energy $AE_{O[E]}(N^*, r_d, x, T)$, are obtained, as functions of T=[4.2K, 77K, 150K], by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, one observes that, for such given (x, r_d and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] In other words, as discussed in above Table 3n, with increasing degeneracy, both the reduced Fermi energy $\xi_{nO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Donor		As	Sb	Sn
For **x=0** and **N=3** × $\mathbf{10^{19}}$ cm^{-3}**,**				
$\xi_{nO[E](T=4.2K)}$	↘	**2393.249 [2069.091]**	**2393.217 [2069.064]**	**2393.215 [2069.062]**
$\xi_{nO[E](T=77K)}$	↘	130.550 [112.870]	130.54852 [112.869]	130.54842 [112.869]
$\xi_{nO[E](T=150K)}$	↘	67.0294 [57.95585]	67.0285 [57.95508]	67.0284 [57.95503]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.4551 [0.3481]	0.3927 [0.3006]	0.3899 [0.2984]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	1.722 [6.7263]	1.4865 [5.8084]	1.4758 [5.7666]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	18.281 [91.9175]	15.777 [79.3738]	15.663 [78.803]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	2.6278 [1.7379]	2.2678 [1.5007]	2.2514 [1.4899]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	9.9455 [33.577]	8.5832 [28.995]	8.5212 [28.786]

$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	105.537 [458.721]	91.081 [396.12]	90.424 [393.27]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**5.8391 [7.6333]**	**6.7659 [8.8398]**	**6.8151 [8.9038]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	1.5427 [0.3950]	1.7875 [0.4575]	1.8005 [0.4608]
$\mathbb{V}_{O[E]\,(150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	0.1453 [0.0289]	0.1684 [0.0335]	0.1696 [0.0337]
$-AE_{O[E](4.2K)}$ (meV× 10^{-6})	↗	16.6325 [66.7896]	16.6329 [66.7914]	16.6330 [66.7915]
$-AE_{O[E](77K)}$ (meV)	↗	8.8322 [19.65031]	8.83237 [19.65048]	8.83238 [19.65049]
$-AE_{O[E](150K)}$ (meV)	↗	47.7387 [72.07950]	47.73907 [72.07985]	47.73909 [72.07987]

For **x=0.5** and **N=7** × $\mathbf{10^{19}}$ $\boldsymbol{cm^{-3}}$, one has:

$\xi_{nO[E](T=4.2K)}$	↘	**1824.338 [1335.813]**	**1824.040 [1335.595]**	**1824.024 [1335.582]**
$\xi_{nO[E](T=77K)}$	↘	99.5217 [72.8794]	99.5055 [72.8675]	99.5046 [72.8668]
$\xi_{nO[E](T=150K)}$	↘	51.1056 [37.4358]	51.0973 [37.4297]	51.0968 [37.4293]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0764 [0.0438]	0.0662 [0.0380]	0.0657 [0.0377]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.1453 [0.3033]	0.1258 [0.2634]	0.1249 [0.2616]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	1.0451 [3.7725]	0.9054 [3.2770]	0.8991 [3.2544]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.3363 [0.1411]	0.2912 [0.1225]	0.2892 [0.1216]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.6394 [0.9774]	0.5537 [0.8488]	0.5498 [0.8430]
$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	4.5983 [12.148]	3.9830 [10.551]	3.9550 [10.478]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**46.111 [80.499]**	**53.237 [92.694]**	**53.614 [93.338]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	24.253 [11.616]	27.997 [13.372]	28.196 [13.465]
$\mathbb{V}_{O[E]\,(150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	3.3713 [0.9340]	3.8911 [1.0751]	3.9186 [1.0825]
$-AE_{O[E](4.2K)}$ (meV× 10^{-6})	↗	5.5982 [22.0393]	5.6000 [22.0465]	5.6001 [22.0469]
$-AE_{O[E](77K)}$ (meV)	↗	4.2632 [12.8452]	4.26426 [12.8471]	4.26432 [12.8472]
$-AE_{O[E](150K)}$ (meV)	↗	33.8116 [57.6065]	33.8155 [57.6104]	33.8157 [57.6107]

For **x=1** and **N=**$\mathbf{10^{20}}$ $\boldsymbol{cm^{-3}}$, one has:

$\xi_{nO[E](T=4.2K)}$	↘	**1576.596 [1035.532]**	**1575.438 [1034.771]**	**1575.373 [1034.729]**
$\xi_{nO[E](T=77K)}$	↘	86.010 [56.5053]	85.947 [56.464]	85.944 [56.461]
$\xi_{nO[E](T=150K)}$	↘	44.173 [29.037]	44.140 [29.016]	44.138 [29.015]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0299 [0.0145]	0.0260 [0.0126]	0.0258 [0.0125]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0468 [0.0681]	0.0407 [0.0595]	0.0404 [0.0592]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.2671 [0.7850]	0.2324 [0.6870]	0.2309 [0.6825]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.1140 [0.0362]	0.0990 [0.0316]	0.0983 [0.0314]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.1780 [0.1700]	0.1547 [0.1486]	0.1537 [0.1476]

$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	1.0154 [1.9585]	0.8831 [1.7127]	0.8770 [1.7015]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	**132.32 [273.88]**	**152.19 [313.29]**	**153.24 [315.36]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	84.675 [58.238]	97.335 [66.542]	98.003 [66.977]
$\mathbb{V}_{O[E]\,(150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	14.841 [5.0496]	17.047 [5.7680]	17.163 [5.8056]
$-AE_{O[E](4.2K)}$ (meV× 10^{-6}) ↗	3.6860 [14.2765]	3.6914 [14.2975]	3.6917 [14.2987]
$-AE_{O[E](77K)}$ (meV) ↗	2.9623 [10.2727]	2.9658 [10.2804]	2.9660 [10.2809]
$-AE_{O[E](150K)}$ (meV) ↗	28.2804 [51.6188]	28.2972 [51.6375]	28.2982 [51.6386]

Table 5p: For given x, r_a and N, the numerical results of reduced Fermi energy $\xi_{pO[E]}(N^*, r_a, x, T)$, mobility $\mu_{O[E]}(N^*, r_a, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_a, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, and activation energy $AE_{O[E]}(N^*, r_a, x, T)$, are obtained, as functions of T=[4.2K, 77K, 150K], by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, one observes that, for given (x, r_a and N), they increase with decreasing T, in good agreement with those, obtained in liquids by Ewell and Eyring[17] and complex fluids by Wenhao.[18] In other words, as discussed in above Table 3p, with increasing degeneracy, both the reduced Fermi energy $\xi_{pO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Acceptor	Ga	Mg	In
For **x=0** and **N=3** × $\mathbf{10^{19}}\ \boldsymbol{cm^{-3}}$,			
$\xi_{pO[E](T=4.2K)}$ ↘	**2354.186 [318.870]**	**2347.046 [317.903]**	**2341.965 [317.215]**
$\xi_{pO[E](T=77K)}$ ↘	128.42 [17.4642]	128.03 [17.41]	127.75 [17.37]
$\xi_{pO[E](T=150K)}$ ↘	65.936 [9.070]	65.736 [9.044]	65.594 [9.025]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3934 [0.0122]	0.3552 [0.0112]	0.3335 [0.0106]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3935 [0.0124]	0.3553 [0.0114]	0.3336 [0.0108]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3939 [0.0131]	0.3557 [0.0120]	0.3339 [0.0114]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	2.2344 [0.0094]	2.0117 [0.0086]	1.8843 [0.0081]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	2.2350 [0.0095]	2.0123 [0.0087]	1.8848 [0.0082]
$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	2.2367 [0.0099]	2.0138 [0.0091]	1.8863 [0.0086]
$\mathbb{V}_{O[E]\,(4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	**6.6997 [216.15]**	**7.4076 [235.04]**	**7.8827 [247.42]**
$\mathbb{V}_{O[E]\,(77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	6.6974 [212.11]	7.4050 [230.61]	7.8799 [242.75]
$\mathbb{V}_{O[E]\,(150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	6.6909 [201.40]	7.3977 [218.90]	7.8722 [230.37]
$-AE_{O[E](4.2K)}$ (eV× 10^{-9}) ↗	0.3760 [20.49]	0.3783 [20.62]	0.3799 [20.71]
$-AE_{O[E](77K)}$ (eV× 10^{-6}) ↗	2.3165 [125.63]	2.3307 [126.39]	2.3408 [126.94]
$-AE_{O[E](150K)}$ (eV× 10^{-6}) ↗	17.1211 [914.60]	17.2254 [920.05]	17.3002 [923.95]

For **x=0.5** and **N=**$\mathbf{7\times10^{19}}\ \boldsymbol{cm^{-3}}$, one has:

$\xi_{po[E](T=4.2K)}$	↘	**1796.78 [481.15]**	**1791.56 [479.75]**	**1787.85 [478.76]**
$\xi_{po[E](T=77K)}$	↘	98.019 [26.291]	97.734 [26.215]	97.532 [26.161]
$\xi_{po[E](T=150K)}$	↘	50.334 [13.565]	50.188 [13.526]	50.084 [13.498]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0663 [0.00708]	0.0600 [0.00650]	0.0564 [0.00618]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0663 [0.00714]	0.0601 [0.00655]	0.0565 [0.00622]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0664 [0.00731]	0.0602 [0.00671]	0.0566 [0.00637]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.2874 [0.00822]	0.2595 [0.00752]	0.2435 [0.00712]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.2876 [0.00827]	0.2596 [0.00757]	0.2437 [0.00717]
$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.2879 [0.00842]	0.2600 [0.00771]	0.2440 [0.00730]
$\mathbb{V}_{O[E]\ (4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**52.739 [493.78]**	**58.157 [537.08]**	**61.781 [565.51]**
$\mathbb{V}_{O[E]\ (77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	52.707 [489.69]	58.122 [532.61]	61.744 [560.78]
$\mathbb{V}_{O[E]\ (150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	52.619 [478.50]	58.024 [520.37]	61.640 [547.85]
$-AE_{O[E](4.2K)}\ (eV\times 10^{-9})$	↗	0.6454 [9.001]	0.6492 [9.053]	0.6519 [9.091]
$-AE_{O[E](77K)}\ (eV\times 10^{-6})$	↗	3.9765 [55.34]	3.9997 [55.66]	4.0164 [55.89]
$-AE_{O[E](150K)}\ (eV\times 10^{-6})$	↗	29.384 [406.463]	29.555 [408.81]	29.678 [410.50]

For **x=1** and **N=**$\mathbf{10^{20}}\ \boldsymbol{cm^{-3}}$, one has:

$\xi_{po[E](T=4.2K)}$	↘	**1542.59 [529.40]**	**1535.60 [527.00]**	**1530.62 [525.29]**
$\xi_{po[E](T=77K)}$	↘	84.156 [28.919]	83.775 [28.788]	83.503 [28.695]
$\xi_{po[E](T=150K)}$	↘	43.221 [14.907]	43.026 [14.840]	42.886 [14.793]
$\mu_{O[E](4.2K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0261 [0.00448]	0.02370 [0.00413]	0.02234 [0.00392]
$\mu_{O[E](77K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.02614 [0.00451]	0.02373 [0.00416]	0.02236 [0.00395]
$\mu_{O[E](150K)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.02619 [0.00460]	0.02379 [0.00424]	0.02241 [0.00403]
$D_{O[E](4.2K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0972 [0.00573]	0.0878 [0.00525]	0.0825 [0.00497]
$D_{O[E](77K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0973 [0.00576]	0.0879 [0.00528]	0.0826 [0.00500]
$D_{O[E](150K)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0974 [0.00584]	0.0881 [0.00536]	0.0827 [0.00508]
$\mathbb{V}_{O[E]\ (4.2K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**150.15 [874.40]**	**164.96 [947.61]**	**174.80 [995.34]**
$\mathbb{V}_{O[E]\ (77K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	150.02 [868.41]	164.82 [941.06]	174.66 [988.42]
$\mathbb{V}_{O[E]\ (150K)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	149.68 [851.96]	164.44 [923.08]	174.25 [969.41]
$-AE_{O[E](4.2K)}\ (eV\times 10^{-9})$	↗	0.876 [7.43]	0.884 [7.50]	0.889 [7.55]
$-AE_{O[E](77K)}\ (eV\times 10^{-6})$	↗	5.395 [45.73]	5.444 [46.15]	5.479 [46.45]
$-AE_{O[E](150K)}\ (eV\times 10^{-6})$	↗	39.857 [336.3]	40.221 [339.3]	40.482 [341.5]

Table 6n: For given x, r_d, T=(4.2K, 77K, 150K), the numerical results of reduced Fermi energy $\xi_{nO[E]}(N^*, r_d, x, T)$, mobility $\mu_{O[E]}(N^*, r_d, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_d, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, and activation energy $AE_{O[E]}(N^*, r_d, x, T)$, are obtained, as functions of **N=[N1=** $\mathbf{3 \times 10^{19}\ cm^{-3}}$**, N2=**$\mathbf{7 \times 10^{19}\ cm^{-3}}$**, N3=**$\mathbf{10^{20}\ cm^{-3}}$], by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(N^*, r_d, x, T)$, one observes that, for given (x, r_d and T), they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3n, with increasing degeneracy, both the reduced Fermi energy $\xi_{nO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Donor	As	Sb	Sn
For **x=0** and **T=4.2 K**			
$\xi_{nO[E](N1)}$ $\searrow$	2393.25 [2069.09]	2393.22 [2069.064]	2393.21 [2069.062]
$\xi_{nO[E](N2)}$ $\searrow$	4210.35 [3640.07]	4210.323 [3640.05]	4210.321 [3640.04]
$\xi_{nO[E](N3)}$ $\searrow$	**5340.58 [4617.22]**	**5340.56 [4617.20]**	**5340.55 [4617.19]**
$\mu_{O[E](N1)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.4550 [0.3481]	0.3927 [0.3006]	0.3899 [0.2984]
$\mu_{O[E](N2)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.4363 [0.3331]	0.3762 [0.2874]	0.3734 [0.2853]
$\mu_{O[E](N3)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.4289 [0.3273]	0.3697 [0.2823]	0.3670 [0.2802]
$D_{O[E](N1)}\left(\frac{10^4 \times cm^2}{s}\right)$ $\searrow$	2.6278 [1.7379]	2.2678 [1.5007]	2.2514 [1.4899]
$D_{O[E](N2)}\left(\frac{10^4 \times cm^2}{s}\right)$ $\searrow$	4.4320 [2.9260]	3.8215 [2.5240]	3.7937 [2.5057]
$D_{O[E](N3)}\left(\frac{10^4 \times cm^2}{s}\right)$ $\searrow$	5.5273 [3.6470]	4.7645 [3.1448]	4.7298 [3.1220]
$\mathbb{V}_{O[E]\ (N1)}\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$ $\nearrow$	5.8391 [7.6333]	6.7659 [8.8398]	6.8151 [8.9038]
$\mathbb{V}_{O[E]\ (N2)}\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$ $\nearrow$	8.0786 [10.579]	9.3691 [12.264]	9.4377 [12.354]
$\mathbb{V}_{O[E]\ (N3)}\left(\frac{eV}{cm} \times \frac{s}{cm^2}\right)$ $\nearrow$	**9.2540 [12.126]**	**10.735 [14.062]**	**10.814 [14.164]**
$-AE_{O[E](N1)}\ (eV \times 10^{-9})$ $\nearrow$	16.632552 [66.7896]	16.632997 [66.7914]	16.633022 [66.7915]
$-AE_{O[E](N2)}\ (eV \times 10^{-9})$ $\nearrow$	5.3740949 [21.5812]	5.3741564 [21.5814]	5.3741599 [21.5815]
$-AE_{O[E](N3)}\ (eV \times 10^{-9})$ $\nearrow$	3.3401431 [13.4134144]	3.3401699 [13.413522]	3.3401714 [4.482246]
For **x=0.5** and **T=77 K**, one has:			
$\xi_{nO[E](N1)}$ $\searrow$	56.5412 [41.4143]	56.5196 [41.3985]	56.5184 [41.3976]
$\xi_{nO[E](N2)}$ $\searrow$	99.5217 [72.8794]	99.5055 [72.8675]	99.5046 [72.8668]
$\xi_{nO[E](N3)}$ $\searrow$	**126.2536 [92.4513]**	**126.2392 [92.4407]**	**126.2384 [92.4401]**
$\mu_{O[E](N1)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.3078 [0.9106]	0.2673 [0.7938]	0.2655 [0.7885]
$\mu_{O[E](N2)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.1453 [0.3033]	0.1258 [0.2634]	0.1249 [0.2616]
$\mu_{O[E](N3)}\left(\frac{10^5 \times cm^2}{V \times s}\right)$ $\searrow$	0.1165 [0.1996]	0.1008 [0.1732]	0.1001 [0.1720]

$D_{O[E](N1)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.7692 [1.6657]	0.6679 [1.4516]	0.6633 [1.4418]
$D_{O[E](N2)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.6394 [0.9774]	0.5537 [0.8488]	0.5498 [0.8430]
$D_{O[E](N3)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.6505 [0.8162]	0.5629 [0.7081]	0.5589 [0.7031]
$\mathbb{V}_{O[E]\,(N1)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	8.6275 [2.9162]	9.9303 [3.3445]	9.9991 [3.3670]
$\mathbb{V}_{O[E]\,(N2)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	24.253 [11.616]	27.997 [13.372]	28.196 [13.465]
$\mathbb{V}_{O[E]\,(N3)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**34.065 [19.878]**	**39.360 [22.911]**	**39.640 [23.071]**
$-AE_{O[E](N1)}$ (meV)	↗	8.8497 [19.6757]	8.8534 [19.6805]	8.8536 [19.6807]
$-AE_{O[E](N2)}$ (meV)	↗	4.26324 [12.8452]	4.26426 [12.8470]	4.26432 [12.8472]
$-AE_{O[E](N3)}$ (meV)	↗	2.95019 [10.2457]	2.95073 [10.2468]	2.95076 [10.2469]

For **x=1** and **T=150 K**, one has:

$\xi_{nO[E](N1)}$	↘	19.7208 [13.0083]	19.6723 [12.9766]	19.6696 [12.9748]
$\xi_{nO[E](N2)}$	↘	34.7976 [22.8865]	34.7611 [22.8625]	34.7590 [22.8612]
$\xi_{nO[E](N3)}$	↘	**44.1726 [29.0375]**	**44.1402 [29.0163]**	**44.1384 [29.0151]**
$\mu_{O[E](N1)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	1.3732 [4.5395]	1.2059 [4.0172]	1.1983 [3.9934]
$\mu_{O[E](N2)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.4250 [1.3077]	0.3706 [1.1474]	0.3682 [1.1401]
$\mu_{O[E](N3)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.2671 [0.7850]	0.2324 [0.6870]	0.2309 [0.6825]
$D_{O[E](N1)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	2.3186 [5.0118]	2.0310 [4.4241]	2.0179 [4.3973]
$D_{O[E](N2)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	1.2719 [2.5668]	1.1080 [2.2498]	1.1005 [2.2353]
$D_{O[E](N3)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	1.0154 [1.9585]	0.8831 [1.7127]	0.8770 [1.7015]
$\mathbb{V}_{O[E]\,(N1)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	1.9263 [0.5827]	2.1909 [0.6576]	2.2046 [0.6615]
$\mathbb{V}_{O[E]\,(N2)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	8.2761 [2.6899]	9.4853 [3.0641]	9.5488 [3.0836]
$\mathbb{V}_{O[E]\,(N3)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**14.841 [5.0496]**	**17.047 [5.7680]**	**17.1628 [5.8056]**
$-AE_{O[E](N1)}$ (meV)	↗	48.0418 [72.4954]	48.1047 [72.5602]	48.1082 [72.5638]
$-AE_{O[E](N2)}$ (meV)	↗	33.9041 [57.7283]	33.9294 [57.7553]	33.9308 [57.7568]
$-AE_{O[E](N3)}$ (meV)	↗	28.2803 [51.6188]	28.2972 [51.6375]	28.2982 [51.6386]

Table 6p: For given x, r_a and T=(4.2K, 77K, 150K), the numerical results of reduced Fermi energy $\xi_{pO[E]}(N^*, r_a, x, T)$, mobility $\mu_{O[E]}(N^*, r_a, x, T)$, diffusion coefficient $D_{O[E]}(N^*, r_a, x, T)$, viscosity coefficient $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, and activation energy $AE_{O[E]}(N^*, r_a, x, T)$, are obtained as functions of **N=[N1=** $\mathbf{3 \times 10^{19}\ cm^{-3}}$**, N2=**$\mathbf{7 \times 10^{19}\ cm^{-3}}$**, N3=**$\mathbf{10^{20}\ cm^{-3}}$], by using Equations (11, 22a, 24, 22b, 22c), respectively. In particular, from the numerical results of $\mathbb{V}_{O[E]}(N^*, r_a, x, T)$, one observes that, for given (x, r_a and T), they increase with increasing N, in good agreement with those, obtained in complex fluids by Wenhao.[18] In other words, as discussed in above Table 3p, with increasing degeneracy, both the reduced Fermi energy $\xi_{pO[E]}$ and the viscosity coefficient $\mathbb{V}_{O[E]}$ increase, according to **an equivalence between the degeneracy-and-viscosity concept**.

Acceptor	Ga	Mg	In
For **x=0** and **T=4.2 K**			
$\xi_{pO[E](N1)}$ ↘	2354.19 [318.870]	2347.05 [317.903]	2341.96 [317.215]
$\xi_{pO[E](N2)}$ ↘	4181.0 [566.300]	4175.6 [565.575]	4171.8 [565.059]
$\xi_{pO[E](N3)}$ ↘	**5314.5 [719.834]**	**5309.7 [719.191]**	**5306.4 [718.733]**
$\mu_{O[E](N1)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3934 [0.0122]	0.3552 [0.0112]	0.3335 [0.0106]
$\mu_{O[E](N2)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3765 [0.0108]	0.3397 [0.0099]	0.3187 [0.0094]
$\mu_{O[E](N3)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.3700 [0.0104]	0.3338 [0.0095]	0.3131 [0.0090]
$D_{O[E](N1)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	2.2344 [0.0094]	2.0117 [0.0086]	1.8843 [0.0081]
$D_{O[E](N2)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	3.7981 [0.0148]	3.4227 [0.0135]	3.2082 [0.0128]
$D_{O[E](N3)}\left(\frac{10^4\times cm^2}{s}\right)$ ↘	4.7446 [0.0180]	4.2762 [0.0164]	4.0087 [0.0155]
$\mathbb{V}_{O[E]\ (N1)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	6.6997 [216.15]	7.4076 [235.04]	7.8827 [247.42]
$\mathbb{V}_{O[E]\ (N2)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	9.3285 [323.76]	10.332 [353.98]	11.007 [373.99]
$\mathbb{V}_{O[E]\ (N3)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$ ↗	**10.702 [380.99]**	**11.858 [417.31]**	**12.637 [441.43]**
$-AE_{O[E](N1)}\ (eV\times 10^{-9})$ ↗	0.3760 [20.4934]	0.3783 [20.6183]	0.3799 [20.7078]
$-AE_{O[E](N2)}\ (eV\times 10^{-9})$ ↗	0.1192 [6.4975]	0.1195 [6.5142]	0.1197 [6.5261]
$-AE_{O[E](N3)}\ (eV\times 10^{-9})$ ↗	0.0738 [4.0214]	0.0739 [4.0286]	0.0740 [4.0337]
For **x=0.5** and **T=77 K**, one has:			
$\xi_{pO[E](N1)}$ ↘	54.537 [**14.68**]	54.155 [14.58]	53.883 [14.51]
$\xi_{pO[E](N2)}$ ↘	98.019 [26.29]	97.734 [26.21]	97.532 [26.16]
$\xi_{pO[E](N3)}$ ↘	**124.92 [33.48]**	**124.67 [33.42]**	**124.49 [33.37]**
$\mu_{O[E](N1)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.0708 [0.00838]	0.0643 [0.00774]	0.0606 [0.0074]
$\mu_{O[E](N2)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.0663 [0.00714]	0.0601 [0.00655]	0.0565 [0.0062]
$\mu_{O[E](N3)}\left(\frac{10^5\times cm^2}{V\times s}\right)$ ↘	0.0647 [0.00676]	0.0586 [0.00619]	0.0551 [0.0059]

$D_{O[E](N1)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.1708 [0.00538]	0.1540 [0.00493]	0.1443 [0.0047]
$D_{O[E](N2)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.2876 [0.00827]	0.2596 [0.00757]	0.2437 [0.0072]
$D_{O[E](N3)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.3577 [0.00999]	0.3230 [0.00914]	0.3032 [0.0086]
$\mathbb{V}_{O[E]\ (N1)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	36.800 [311.18]	40.399 [335.61]	42.783 [351.36]
$\mathbb{V}_{O[E]\ (N2)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	52.707 [489.69]	58.122 [532.61]	61.744 [560.78]
$\mathbb{V}_{O[E]\ (N3)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**60.975 [583.89]**	**67.318 [636.68]**	**71.570 [671.49]**
$-AE_{O[E](N1)}\ (eV\times 10^{-6})$	↗	12.85 [177.940]	13.03 [180.442]	13.16 [182.26]
$-AE_{O[E](N2)}\ (eV\times 10^{-6})$	↗	3.976 [55.3396]	4.000 [55.6618]	4.016 [55.89]
$-AE_{O[E](N3)}\ (eV\times 10^{-6})$	↗	2.448 [34.0973]	2.458 [34.2352]	2.465 [34.33]
For **x=1** and **T=150 K**, one has:				
$\xi_{pO[E](N1)}$	↘	17.979 [12.28]	17.763 [12.07]	17.474 [11.93]
$\xi_{pO[E](N2)}$	↘	33.502 [22.57]	33.344 [22.43]	33.133 [22.32]
$\xi_{pO[E](N3)}$	↘	**43.026 [28.92]**	**42.886 [28.79]**	**42.701 [28.69]**
$\mu_{O[E](N1)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0275 [0.0059]	0.0260 [0.0055]	0.0244 [0.0053]
$\mu_{O[E](N2)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0246 [0.0048]	0.0232 [0.0044]	0.0217 [0.0042]
$\mu_{O[E](N3)}\left(\frac{10^5\times cm^2}{V\times s}\right)$	↘	0.0238 [0.0045]	0.0224 [0.0041]	0.0209 [0.0039]
$D_{O[E](N1)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0422 [0.0031]	0.0395 [0.0029]	0.0364 [0.0027]
$D_{O[E](N2)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0710 [0.0048]	0.0666 [0.0044]	0.0618 [0.0042]
$D_{O[E](N3)}\left(\frac{10^4\times cm^2}{s}\right)$	↘	0.0881 [0.0057]	0.0827 [0.0053]	0.0768 [0.0050]
$\mathbb{V}_{O[E]\ (N1)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	91.880 [431.24]	96.462 [458.68]	102.00 [475.57]
$\mathbb{V}_{O[E]\ (N2)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	139.99 [717.40]	148.09 [774.43]	158.19 [811.30]
$\mathbb{V}_{O[E]\ (N3)}\left(\frac{eV}{cm}\times\frac{s}{cm^2}\right)$	↗	**164.44 [868.41]**	**174.25 [941.06]**	**186.55 [988.41]**
$-AE_{O[E](N1)}\ (eV\times 10^{-6})$	↗	230.89 [254.951]	236.55 [263.534]	244.45 [269.95]
$-AE_{O[E](N2)}\ (eV\times 10^{-6})$	↗	66.36 [75.0895]	66.99 [76.085]	67.84 [76.807]
$-AE_{O[E](N3)}\ (eV\times 10^{-6})$	↗	40.22 [45.7300]	40.48 [46.146]	40.83 [46.45]