



ON SPECIAL DIO-QUADRUPLE WITH PROPERTY $D(S^2+1)$

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Article Received on 08/07/2016

Article Revised on 29/07/2016

Article Accepted on 19/08/2016

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ABSTRACT

We search for three distinct integers a, b, c such that the product of any two from the set minus s -times their sum and increased by $(s^2 + 1)$ is a perfect square. Also, we show that the triple can be extended to the quadruple with property $D(s^2 + 1)$

KEYWORDS: Diophantine Quadruples, Pell equation 2010

Mathematics subject classification: 11D99.

INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by Diophantus.^[3] A set of m positive integers $\{a_1, a_2, \dots, a_m\}$ is said to have the property $D(n)$, $n \in \mathbb{Z} - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \leq i < j \leq m$ and such a set is called a Diophantine m -tuples with property $D(n)$. Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n and also for any linear polynomials in n . In this context, one may refer^[1,2, 4-18] for an extensive review of various problems on Diophantine quadruples. This paper aims at constructing special dio – quadruple where the product of any two members of the quadruple minus s -times the same members and the addition of $(s^2 + 1)$ satisfies the required property.

Method of analysis

Let $a(k, s) = 2k^2 + 2k + s, b(k, s) = 2k^2 - 2k + s$ be any two distinct integers such that $a(k, s)b(k, s) - s(a(k, s) + b(k, s)) + s^2 + 1$ is a perfect square.

Let $c_N(k, s)$ be any non-zero integer such that

$$(a(k, s) - s)c_N(k, s) - sa(k, s) + s^2 + 1 = p_N^2(k, s) \quad (1)$$

$$(b(k, s) - s)c_N(k, s) - sb(k, s) + s^2 + 1 = q_N^2(k, s) \quad (2)$$

Eliminating $c_N(k, s)$ between (1) and (2), we have

$$(2k^2 - 2k)p_N^2(k, s) - (2k^2 + 2k)q_N^2(k, s) = -4k \quad (3)$$

Introducing the linear transformations

$$\left. \begin{aligned} p_N(k, s) &= X_N(k, s) + (2k^2 + 2k)T_N(k, s) \\ q_N(k, s) &= X_N(k, s) + (2k^2 - 2k)T_N(k, s) \end{aligned} \right\} \quad (4)$$

in (3), we get

$$X_N^2(k, s) = (4k^4 - 4k^2)T_N^2(k, s) + 1 \quad (5)$$

This is a well known Pellian equation whose general solution is given by

$$\left. \begin{aligned} X_N(k, s) &= \frac{1}{2} [(2k^2 - 1 + \sqrt{4k^4 - 4k^2})^{N+1} + (2k^2 - 1 - \sqrt{4k^4 - 4k^2})^{N+1}] \\ T_N(k, s) &= \frac{1}{2\sqrt{4k^4 - 4k^2}} [(2k^2 - 1 + \sqrt{4k^4 - 4k^2})^{N+1} - (2k^2 - 1 - \sqrt{4k^4 - 4k^2})^{N+1}] \end{aligned} \right\} \quad (6)$$

Taking $N=0$ in (6), (4) and using (1), we get

$$c_0(k, s) = s + 8k^2 - 2$$

Note that $(a(k, s), b(k, s), c_0(k, s))$ is the special dio-triple with property $D(s^2 + 1)$

Now, substituting $N=1$ in (6), (4) and using (1), we have

$$c_1(k, s) = s + (8k^2 - 4k - 2)(16k^4 + 8k^3 - 12k^2 - 4k + 2)$$

Thus, we obtain $(a(k, s), b(k, s), c_0(k, s), c_1(k, s))$ as a dio-quadruple with the property

$$D(s^2 + 1).$$

Some numerical examples are presented below.

Dio-quadruple with property $D(s^2 + 1)$

S.No	k	s	$(a(k, s), b(k, s), c_0(k, s), c_1(k, s))$
1	1	1	(5,1,7,21)
2	1	2	(6,2,8,22)
3	2	4	(16,8,34,5856)
4	2	3	(15,7,33,5855)

It is worth to note that, considering the pairs $(a(k, s), c_o(k, s))$, $(a(k, s), c_1(k, s))$, $(c_o(k, s), c_1(k, s))$, $(b(k, s), c_o(k, s))$ and $(b(k, s), c_1(k, s))$ in turn and repeating the above process, one obtains many special dio-quadruples with property $D(s^2 + 1)$.

CONCLUSION

This paper concerns with the construction of special dio - quadruples with property $D(s^2 + 1)$. One may search for special dio-quadruples consisting of special numbers with suitable property.

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