wjert, 2016, Vol. 2, Issue 6, 12 -19 Research Article ISSN 2454-695X

World Journal of Engineering Research and Technology
WJERT
www.wjert.org
SJIF Impact Factor: 3.419

A BRIEF NOTE ON THE CLASSES OF ABSORPTION SEMIRING

M. Amala*

Dept. of Applied Mathematics, Yogi Vemana University, Kadapa, Andhra Pradesh, India.

Article Received on 13/09/2016 **Article Revised on 03/10/2016** **Article Accepted on 24/10/2016**

***Corresponding Author**
Dr. M. Amala
Dept. of Applied Mathematics, Yogi Vemana University, Kadapa, Andhra Pradesh, India.

ABSTRACT

In this paper we have proved that Suppose S is a totally ordered Absorption semiring. If (S, +) and (S, •) are positively totally ordered, then (S, +) is a band.

AMS Mathematics Subject Classification (2010): 20M10, 16Y60.
Index terms: Rectangular band, Square regular, Positive totally ordered, Zeroid.

1. INTRODUCTION

Algebraic systems are artistic with a partially or fully ordered met within several disciplines of Mathematics. In recent years interest in the study of partially ordered and fully ordered semigroups, groups, semirings, semi modules, rings and fields has been increasing enormously. The theory of semigroups had essentially two origins. One was an attempt to generalize both group theory and ring theory to the algebraic system consisting of a single associated operation which from the group theoretical point of view omits the axioms of the existence of the identities and inverses and from the ring theoretical point of view omits the additive structure of the ring.

Semirings are used for modeling Economics, social network analysis, queuing theory, computation of bio-polymers, penalty theory in artificial intelligence, computation theory, modern control theory of psychological phenomenon. Semirings are used for physical theory on cognitive processes. S.Gosh studied the class of idempotent semiring. He revealed that S is distributive lattice if S is an idempotent commutative semiring satisfying the absorption equality a + ax = a for all a, x in S. This paper is concerned with structures of Absorption semiring and its ordering. In first section, structures of Absorption semiring are given. In last section we study structures of totally ordered Absorption semiring.

A semiring S is Absorption if it satisfies the condition a + ax = a for all a, x in S.

2. PRELIMINARIES

Definition 2.1

The zeroid of a semiring S is $\{x \in S \mid z + x = z \text{ or } x + z = z \text{ for some } z \in R\}$.

Definition 2.2

A semiring S is viterbi if S is additively idempotent and multiplicatively subidempotent.

i.e., a + a = a and $a + a^2 = a$ for all 'a' in S.

Definition 2.3

A semigroup (S, •) is rectangular band if a = axa for all a, x in S.

Definition 2.4

In a semigroup (S, •) an element *a* is left (right) square regular if there exists an element x such that $a^2x = a$ ($xa^2 = a$). A semigroup (S, •) is left (right) square regular if each element in S is left (right) square regular.

Definition 2.5

A semigroup (S, •) is rectangular band if a = axa for all a, x in S.

Definition 2.6

An element 'a' of a semigroup (S, •) is E-inverse if there exists an element x

in S such that $(ax)^2 = ax$ i.e ax € E(S), Where E(S) is the set of all idempotent elements of S.

A semigroup (S, •) is E-Inverse if every element of S is E-inverse.

Definition 2.7

A semigroup (S, +) is a band if a + a = a for all a in S.

Definition 2.8

A semiring S is zerosumfree if a + a = 0, for all a in S. Here zero is an additive identity.

Definition 2.9

In a totally ordered semiring $(S, +, \bullet, \leq)$

(i) $(S, +, \leq)$ is positively totally ordered (p.t.o), if $a + x \geq a, x$ for all a, x in S

(ii) $(S, \bullet, \leq)$ is positively totally ordered (p.t.o), if $ax \geq a, x$ for all a, x in S.

(iii) $(S, +, \leq)$ is negatively totally ordered (n.t.o), if $a + x \leq a, x$ for all a, x in S

(iv) $(S, \bullet, \leq)$ is positively totally ordered (n.t.o), if $ax \leq a, x$ for all a, x in S.

3. CLASSES OF ABSORPTION SEMIRING

Theorem 3.1: If S is an Absorption semiring and (S, +) is left cancellative, then $ax^n = ax$ for $n \geq 1$.

Proof: By hypothesis S is Absorption semiring then $a + ax = a$ $\rightarrow$ **(1)**

On multiplication of 'a' on both sides we get $ax + ax^2 = a$

On addition of 'a' on both sides we obtain $a + ax + ax^2 = a + ax$

$\Rightarrow a + ax^2 = a$ $\rightarrow$ **(2)**

$\Rightarrow ax + ax^3 = ax$

$\Rightarrow a + ax + ax^3 = a + ax$

$\Rightarrow a + ax^3 = a$ $\rightarrow$ **(3)**

From equations (1), (2) and (3) we can conclude that $a + ax^n = a$ $\rightarrow$ **(4)**

Comparing equations (1) and (4) we get $a + ax^n = a + ax$

Using (S, +) left cancellation law in above we obtain $ax^n = ax$ for $n \geq 1$

Theorem 3.2: Let S be an Absorption Semiring and (S, +) be zeroid. Then S is a viterbi semiring.

Proof: Given (S, +) is zeroid then $x + a = x$ or $a + x = x$ for some x in S

Also we have $a + a.a = a$

$\Rightarrow a + a^2 = a$ for all a in S

Thus S is multiplicatively subidempotent semiring

Since (S, +) is zeroid $a + a = a$ for all a in S

Therefore S is viterbi semiring

Proposition 3.3: If S is an Absorption semiring and $(S, \bullet)$ is rectangular band, then $a^2 + a = a^2$ for all a in S.

Proof: By hypothesis S is Absorption then $a + ax = a$

$\Rightarrow a^2 + axa = a^2 \rightarrow$ **(1)**

Since (S, •) is rectangular band then axa = a for all a, x in S

Then equation (1) reduces to $a^2 + a = a^2$ for all a in S

Lemma 3.4: Suppose S is a semiring which contains an additive identity 'e' also multiplicative identity. Then a ($\neq$ e) in S is an Absorption element if and only if ax = a for all x in S.

Proof: By hypothesis S contains an additive identity 'e' also multiplicative identity then e + a = a + e = a and e.a = a.e = a for all a, e in S

Since a in S is an Absorption element, a + ax = a for all x in S

$\Rightarrow$ a (e + x) = a

$\Rightarrow$ ax = a

Therefore ax = a for all x in S

To prove the converse part let us consider ax = a

This implies a + ax = a + a

Which again leads to the form a + ax = a (e + e)

Thus a + ax = a for all x in S

Theorem 3.5: Let S be a semiring which contains an additive identity 'e' also multiplicative identity. If a ($\neq$ e) in S is an Absorption element, then a = a + x for all x in S.

Proof: Given that S contains an additive identity 'e' also multiplicative identity then e + a = a + e = a and e.a = a.e = a for all a, e in S

By hypothesis a in S is an Absorption element, then a + ax for all x in S $\rightarrow$ **(1)**

This implies a + ax + x = a + x

$\Rightarrow$ a + (a + e) x = a + x

$\Rightarrow$ a = a + x $\rightarrow$ **(2)**

Therefore a = a + x for all x in S

Example 3.6: Example for above Lemma 3.4 and theorem 3.6 with S = {e, a, x}. Here 'a' is an Absorption element.

+	e	a	x
e	e	a	x
a	a	a	a
x	x	a	x

•	e	a	x
e	e	a	x
a	a	a	a
x	x	a	x

Theorem 3.7: Let a in S be an Absorption element in a semiring S. If 'a' is regular and (S, +) is commutative, then $a + x^n = a$ for $n > 1$ where 'x' depends on 'a'.

Proof: By hypothesis 'a' is regular then $a + x + a = a \quad \rightarrow$ **(1)**

Also 'a' in S is an Absorption element then $a + ax = a$ for all x in S $\rightarrow$ **(2)**

This implies $a + (a + x + a)\, x = a$

$\Rightarrow a + ax + x^2 + ax = a$

$\Rightarrow a + x^2 + ax = a$

Using (S, +) commutative in above we get $a + ax + x^2 = a$

$\Rightarrow a + x^2 = a \quad \rightarrow$ **(3)**

$\Rightarrow ax + x^2x = ax$

$\Rightarrow ax + x^3 = ax$

$\Rightarrow a + ax + x^3 = a + ax$

Using equation (2) in above we obtain $a + x^3 = a \quad \rightarrow$ **(4)**

Continuing like this as equations (3) and (4) we get

$a + x^n = a$ for $n > 1$ where 'x' depends on 'a'

Theorem 3.8: Suppose S is a semiring which contains multiplicative identity 1. Then S is an Absorption semiring if and only if $1 + x = 1$.

Proof: First assume $1 + x = 1$ for all x in S

If $a, x \in S$, then $a = a.1 = a\,(1 + x)$

$= a + ax$

Thus $a + ax = a$ for all a, x in S

Therefore S is an Absorption semiring

Conversely assume that $a + ax = a$ for all a, x in S $\rightarrow$ **(1)**

Consider $1 + x = 1 + 1.x = 1$

Hence $1 + x = 1$ for all x in S

Proposition 3.9: If S is an Absorption semiring and (S, •) is right square regular semigroup, then $a + x^n = a$.

Proof: Given that S is an Absorption semiring $a + ax = a$ for all a, x in S

Consider $ax = (a + ax)\, x = ax + ax^2$

Using (S, •) right square regular, $ax^2 = x$ then above equation becomes

$ax = ax + x$

This implies $a + ax = a + ax + x$

$\Rightarrow a + x = a \rightarrow$ **(1)**

$\Rightarrow ax + x^2 = ax$

$\Rightarrow a + ax + x^2 = a + ax$

$\Rightarrow a + x^2 = a \rightarrow$ **(2)**

Proceeding in a similar manner as above we get $a + x^n = a$ for $n \geq 1$

Proposition 3.10: Let S be an Absorption semiring and $a + ax + x = ax$, for all a, x in S, then S is a mono semiring and (S, •) is E-inverse semigroup.

Proof: By hypothesis $a + ax = a$ for all a, x in S $\rightarrow$ **(1)**

Let us consider $a + ax + x = ax \Rightarrow a + x = ax$ for all a, x in S

Thus S is a mono semiring

Again $a + ax + x = ax$

$\Rightarrow a + x + ax = ax$

$\Rightarrow ax + ax = ax$

Since S is a mono semiring it implies $ax\,(ax) = ax \Rightarrow (ax)^2 = ax$

Hence (S, •) is an E-inverse semigroup

4. CLASSES OF TOTALLY ORDERED ABSORPTION SEMIRING

Proposition 4.1: Let S be a totally ordered Absorption semiring. If (S, +) and (S, •) are positively totally ordered, then (S, +) is a band.

Proof: Let us consider $a + ax = a$ for all a, x in S $\rightarrow$ **(1)**

Since (S, +) is positively totally ordered $a = a + ax \geq ax$

This implies $a \geq ax \rightarrow$ **(2)**

Also (S, •) is positively totally ordered then $ax \geq a \rightarrow$ **(3)**

Therefore from equations (2) and (3) $a = ax$ for all a, x in S

By hypothesis $a + ax = a$ for all a, x in S

Since $ax = a$ then equation (1) becomes $a + a = a$ for all a in S

Therefore (S, +) is a band

Theorem 4.2: If S is a totally ordered Absorption semiring satisfying the condition $a + ax + x = ax$ and (S, +) is positively totally ordered, then (S, •) is positively totally ordered.

Proof: By hypothesis $a + ax + x = ax \rightarrow$ **(1)**

Also S is absorption semiring then $a + ax = a$ for all a, x in S

Then equation (1) becomes a + x = ax

Using (S, +) positively totally ordered in above we get $ax = a + x \geq a, x$

$\Rightarrow ax \geq a, x$ for all a, x in S

Therefore (S, •) is positively totally ordered

Proposition 4.3: If S is a totally ordered zerosumfree and Absorption semiring and (S, •) is positively totally ordered, then 0 is the maximum element.

Proof: By hypothesis S is zerosumfree then a + a = 0 for all a in S

This implies a + a + ax = ax

It leads to the form a + a = ax

$\Rightarrow 0 = ax$

Since (S, •) is positively totally ordered $0 = ax \geq a, x$ for all a, x in S

Hence 0 is the maximum element

Theorem 4.4: If S is a totally ordered Absorption semiring which contains multiplicative identity '1' and (S, +) is positively totally ordered, then 1 is the maximum element.

Proof: Consider a + ax = a for all a, x in S

Since S contains multiplicative identity '1' then 1 + 1.x = 1

Using (S, +) positively totally ordered in above then $1 = 1 + x \geq x$

$\Rightarrow 1 \geq x$

Therefore 1 is the maximum element

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