wjert, 2017, Vol. 3, Issue 1, 142 -153 Original Article ISSN 2454-695X

World Journal of Engineering Research and Technology
WJERT
www.wjert.org
SJIF Impact Factor: 3.419

THE COMPARISON OF DOMINATION NUMBER OF G AND AN INDUCED SUB GRAPH OF DOMINATION NUMBER OF $\overline{G}$ TOWARDS A NON-SPLIT DOMINATION NUMBER OF G CORRESPONDING TO I.

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Article Received on 05/11/2016 Article Revised on 25/11/2016 Article Accepted on 15/12/2016

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ABSTRACT

Interval graphs have drawn the attention of many researchers for over 30 years. They are extensively studied and revealed their practical relevance for modeling problems arising in the real world. Among the various applications of the theory of domination and the distance, the most often discussed is a communication network. In this paper we investigate the problem of the comparison of domination number of G and an induced sub graph of domination number of $\overline{G}$ towards a non-split domination number of G corresponding to an interval family I.

KEYWORDS: Interval family, interval graph, dominating set, non - split dominating set, domination number, induced sub-graph.

INTRODUCTION

A graph G is an ordered triple (V(G), E(G), ψ_G)) consisting of a non-empty set V(G) of vertices, a set E(G) of edges and an incidence function ψ_G that associates with each edge of G of an undirected pair of vertices of G. Let I = {I_1, I_2,…..I_n} be an interval family, where each I_i is an interval on the real line and I_i = [a_i,b_i], for I = 1,2,3,…..n. Here a_i is called the left end point labeling and b_i is the right end point labeling of I without loss of generality, we assume that all the end points of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to be intersect each other if they have non-empty intersection.

In general an undirected graph G = (V, E) is an interval graph if the vertex set V can be put into one to one correspondence with a set of intervals I on the real line R such that two vertices of G are joined by an edge in E if and only if their corresponding intervals in I have non-empty intersection that is $I = [a_i,b_i]$ and $j = [a_j,b_j]$, then i and j intersect means either $a_j < b_i$ or $a_i < b_j$. The set I is called an interval representation of G and G is referred to as the intersection graph of I. Also we say that the intervals both end points and that no two intervals share a common end point the intervals and vertices of an interval graph are one and the same thing. The graph G is connected and the list of sorted end points is given and the intervals in I are indexed by increasing right end points labeling that is $b_1 < b_2 < \ldots\ldots < b_n$.

The concept of domination in graphs has been extensively researched branch as it has large number of applications in different fields. The theory of domination has been the nucleus of research activities in graph theory in recent times. The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when De Jaenisch [4] studied the problems of dominating the minimum number of queens which are necessary to cover or to dominate $n \times n$ chess board.

The research of the domination in graphs has been an evergreen of the graph theory. Its basic concept is the dominating set and the domination number. The theory of domination in graphs was introduced by O. Ore[1] and C. Berge.[2] A survey on results and applications of dominating sets was presented by E.J. Cockayane and S.T. Hedetniemi.[3] There has been persistent in the algorithmic aspects of interval graphs in past decades spurred much by their numerous applications of interval graphs corresponding to interval families. A set $D \subseteq V(G)$ is said to be a dominating set of G, if every vertex in V-D is adjacent to some vertex in D. The minimum cardinality of vertices in such asset is called the domination number of G and is denoted by $\gamma(G)$. A dominating set $D \subseteq V(G)$is said to be split dominating set of G, if the induced sub graph $\langle V-D \rangle$ is disconnected. A dominating set D of G is called a non-split dominating set if $\langle V-D \rangle$ is connected.

Let G(V,E) be a graph. The neighborhood [5] of a vertex v in G is defined as the set of vertices adjacent with v and is denoted by nbd.[5] A subset S of V in G is called a neighborhood set of G if G = $\bigcup_{v\in S} < nbd[v] >$. Where < nbd[v] > is the vertex induced sub graph of G. The neighborhood number of G is defined as the minimum cardinality of a neighborhood set of G. A neighborhood set with minimum cardinality is called a minimum neighborhood set.

Theorem.1

Let G be a interval graph corresponding to an interval family I, $i \in D$, $j \neq 1$.

Then $\gamma(G) + \gamma(\overline{G}) \leq [\frac{N}{2}] + 2$.

Proof

Let I= $\{I_1, I_2, \ldots\ldots\ldots I_n\}$ be an interval family and let G be an interval graph corresponding to an interval family I. If i and j are any two intervals in I such that $i \in D$, $j \neq 1$ and j is contained in i and if there is at least one interval $k \neq i$ to the right of j that intersects j, then we will show that a non-split domination occurs in G.

Now we have to show that the graph G is a connected graph as well as a non-split domination set of G corresponding to an interval family I. Suppose there is at least one interval $k \neq i$ to the right of j that intersect j. Then it is obvious that j is adjacent to k in < V-D >, so that there will not be any disconnection in < V-D >. Since there is atleast one interval to the left of j that intersect j. There will not be any disconnection in < V-D > to its left thus we get non-split domination number in G denoted by $\gamma(G)$.

Let $\overline{G}$ be a an induced sub graph of G corresponding to an interval family I. In this we have to prove the domination number of $\gamma(\overline{G})$. Suppose D is a domination number of $\gamma(\overline{G})$. Then every vertex j in D, D-{ j } is not a dominating set thus some vertex u in D - D $\cup$ { j } is not a dominated by any vertex in D-{ j }. Now either i = j or $j \in$ V-D. If i = j, then j is an isolated vertex of D. If $i \in$ V –D and i is not dominated by D -{ j } but it is dominated by D, then i is adjacent only to vertex j in D that is N(i) $\cap$ D = {j}. Then $\overline{G}$ is a connected graph. Therefore we will find the domination number of $\gamma(\overline{G})$. Our aim to show that the sum of $\gamma(G) + \gamma(\overline{G})$ is less than or equal to $[\frac{N}{2}]+2$. Where N is a number of intervals in I. Where $\gamma(G)$ is a domination number and $\gamma(\overline{G})$ is also domination number of an induced sub graph of G corresponding to an interval family I. Therefore $\gamma(G)+\ \gamma(\overline{G})\leq[\frac{N}{2}]+2$ at $j\neq 1$, $i\in$ D.

Where N is number of intervals of I. Therefore the theorem is hold from G.

Experimental problem for theorem. 1

I = {1,2,3,………, 13}

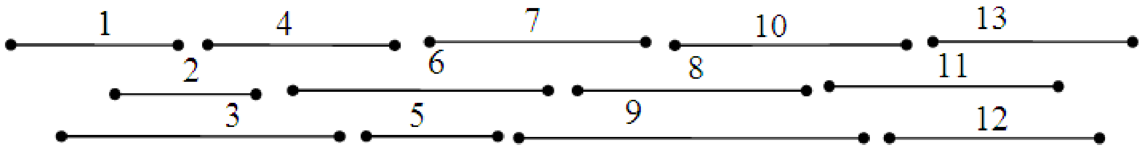

Fig.: Interval family I

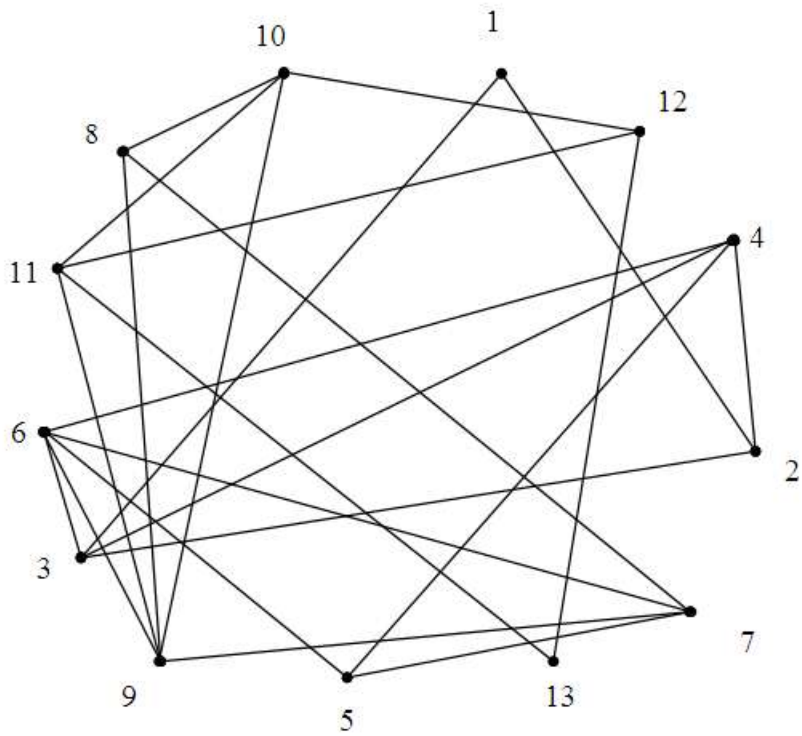

Fig.: Interval graph G

The dominating set of G = {3,7,11} and {3,9,11} .

The domination number γ(G) = 3

The induced sub graph of G is $\overline{G}$ = V – D = {1,2,3,4,5,6,7,8,9,10,11,12,13} – {3,7,11}

= {1,2,4,5,6,8,9,10,11,12,13}

The induced sub graph $\overline{G}$ is

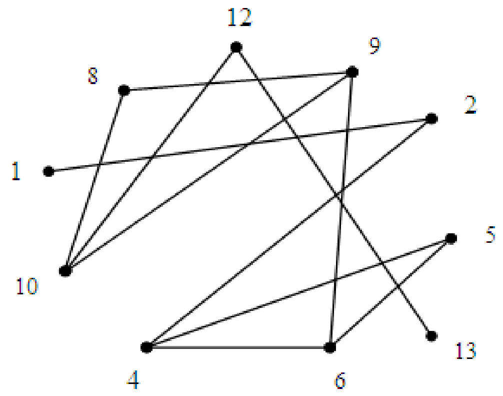

Fig.: Induced sub graph of G

Therefore the dominating set of $\overline{G}$ = {2,6,9,12} and

The domination number of $\overline{G} = \gamma(\overline{G}) = 4$.

Therefore the result is $\gamma(G) + \gamma(\overline{G}) \leq [\frac{N}{2}]+2$

$$3 + 4 \leq \frac{13}{2} + 2$$

$$7 \leq 6.5 + 2$$

$$7 \leq 8.5$$

Therefore the result is true.

An Algorithm for finding a minimum dominating set of an interval graph by verification method

Input: Interval family I = {1,2,3,…., n}.

Step 1: Set MDS = {max(i)}.

Step 2: LI = The largest interval in MDS.

Step 3: Compute NI(LI).

Step 4: If NI(LI) = null then go to step 8

Step 5: Find max(NI(LI)).

Step 6: If max (NI(LI)) does not exist then

Step 6.1: max(NI(LI)) = NI(LI).

Step 7: MDS = MDS U {max(NI(LI))} go to step 2.

Step 8: End.

An Algorithm for finding a minimum dominating set of Theorem.1

We will find nbd[i], max(i) and NI(i), where i = 1,2,3,…..,13 for the above graph.

Nbd[1]= {1,2,3}	max(1)= 3	NI(1)=4
Nbd[2]= {1,2,3,4}	max(2)= 4	NI(2)=5
Nbd[3]= {1,2,3,4,6}	max(3)= 6	NI(3)=7
Nbd[4]= {2,3,4,5,6}	max(4)= 6	NI(4)=7
Nbd[5]= {4,5,6,7}	max(5)= 7	NI(5)=8
Nbd[6]= {3,4,5,6,7,9}	max(6)= 9	NI(6)=10
Nbd[7]= {5,6,7,8,9}	max(7)= 9	NI(7)=10
Nbd[8]= {7,8,9,10}	max(8)= 10	NI(8)=11
Nbd[9]= {6,7,8,9,10,11}	max(9)=11	NI(9)=12
Nbd[10]= {8,9,10,11,12}	max(10)= 12	NI(10)=13
Nbd[11]= {9,10,11,12,13}	max(11)= 13	NI(11)=null
Nbd[12]= {10,11,12,13}	max(12)=13	NI(12)=null
Nbd[13]= {11,12,13}	max(13)= 13	NI(13)=null

Procedure for finding a minimum dominating set

Input: Interval family I = {1,2,3,…..,13}.

Step 1: MDS = 3

Step 2: LI = The largest interval in MDS = 3

Step 3: NI(LI) = NI(3) = 7

Step 4: max(NI(LI)) = max(7) = 9

Step 5: MDS = $\{3\} \cup \{9\}$ = {3,9} go to step 2

Step 6: LI = 9

Step 7: NI(9) = 11

Step 8: MDS = $\{3,9\} \cup \{11\}$ go to step 2

Step 9: LI = 11

Step 10: NI(11) = null

Step 11: End

Output: MDS = {3,9,11} is the minimum dominating set of given interval.

Theorem: 2

Let G be an interval graph corresponding to an interval family I, i $\in$ D, j=1.

Then $\gamma(G) + \gamma(\overline{G}) \leq [\frac{N}{2}]+2$.

Proof: Let G be an interval graph corresponding to an interval family I. We have to prove that $\gamma(G)+\gamma(\overline{G})\leq[\frac{N}{2}]+2$ at $i\in D$, j=1. Where N is a number of intervals in I. First we will discuss the domination numbers of G and $\overline{G}$. Where G and $\overline{G}$ must be connected because G is non split dominating set of an interval graph. First we will prove that G must be non split as well as connected. Let j = 1 be the interval contained in i . Where $i\in D$. Suppose $i\in D$, < V-D > does not contain I. Further in < V-D >, The vertex j is adjacent to the vertex k and hence there will not be any disconnection in < V-D >. Therefore we get a non split domination in G and G must be connected interval graph corresponding to an interval family I. In this fact that we can find easily the domination number $\gamma(G)$ from G. Next will find the domination number $\gamma(\overline{G})$ of an induced sub graph of $\overline{G}$ corresponding to an interval family I. Already we proved in theorem .1.

Then we will get $\gamma(G)+\gamma(\overline{G})\leq[\frac{N}{2}]+2$ at j = 1, $i\in D$.

Experimental problem for Theorem. 2

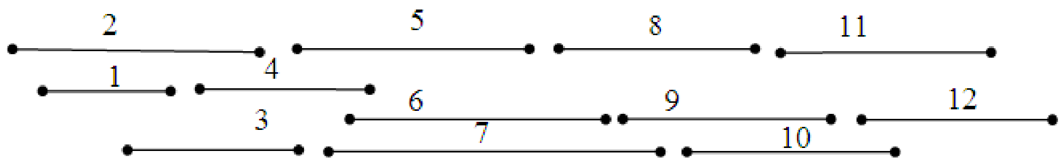

Fig.: Interval family I

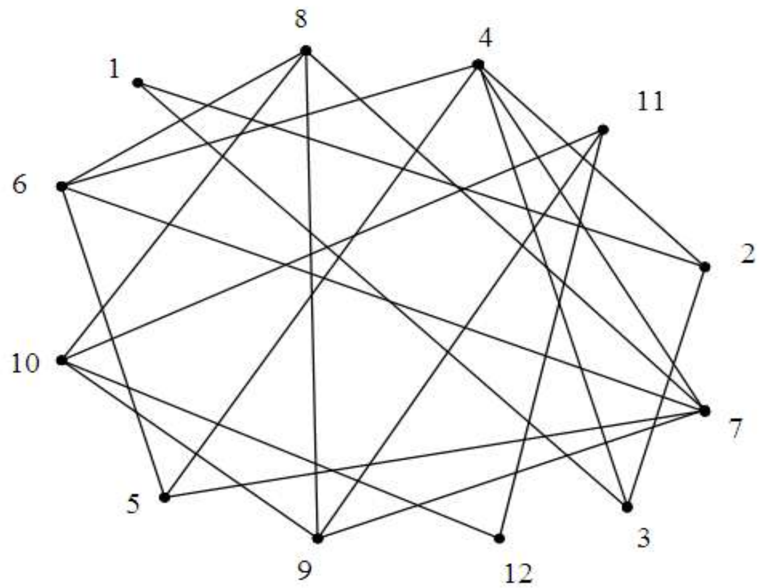

Fig.: Interval graph G

The dominating set of G= {3,7,10}.

The domination number γ(G)= 3

The induced sub graph $\overline{G}$ of G = {1,2,3,4,5,6,7,8,9,10,11,12} – {3,7,10}

= {1,2,4,5,6,8,9,11,12}

The induced sub graph $\overline{G}$ is

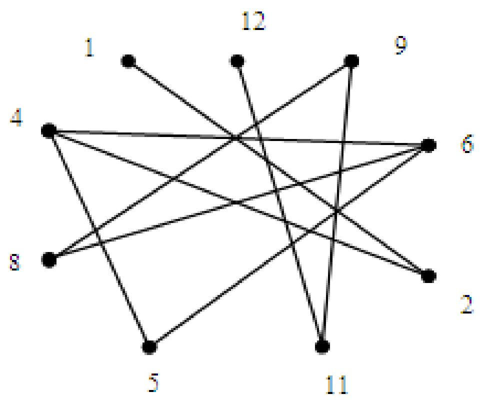

Fig.: Induced sub graph of G

The induced sub graph $\overline{G}$ is connected.

Therefore the dominating set of $\overline{G}$ = {2,6,9,12} and

The domination number of $\overline{G} = \gamma(\overline{G}) = 4$.

Therefore the result is $\gamma(G) + \gamma(\overline{G}) \leq [\frac{N}{2}]+2$

$$3 + 4 \leq \frac{12}{2} + 2$$

$$7 \leq 6 + 2$$

$$7 \leq 8$$

Therefore the result is true.

An Algorithm for finding a minimum dominating set of Theorem.2

We will find nbd[i], max(i) and NI(i), where i = 1,2,3,…..,13 for the above graph.

Nbd[1]= {1,2,3}	max(1)= 3	NI(1)=4
Nbd[2]= {1,2,3,4}	max(2)= 4	NI(2)=5
Nbd[3]= {1,2,3,4}	max(3)= 4	NI(3)=5
Nbd[4]= {2,3,4,5,6,7}	max(4)= 7	NI(4)=8
Nbd[5]= {4,5,6,7}	max(5)= 7	NI(5)=8

Nbd[6]= {4,5,6,7,8}	max(6)= 8	NI(6)=9
Nbd[7]= {4,5,6,7,8,9}	max(7)= 9	NI(7)=10
Nbd[8]= {6,7,8,9,10}	max(8)= 10	NI(8)=11
Nbd[9]= {7,8,9,10,11}	max(9)=11	NI(9)=12
Nbd[10]= {8,9,10,11,12}	max(10)= 12	NI(10)=null
Nbd[11]= {9,10,11,12}	max(11)= 12	NI(11)=null
Nbd[12]= {10,11,12}	max(12)=12	NI(12)=null

Procedure for finding a minimum dominating set

Input: Interval family I = {1,2,3,…..,12}.

Step 1: MDS = 3

Step 2: LI = The largest interval in MDS = 3

Step 3: NI(LI) = NI(3) = 5

Step 4: max(NI(LI)) = max(5) = 7

Step 5: MDS = {3} $\cup$ {7} = {3,7} go to step 2

Step 6: LI = 7

Step 7: NI(7) = 10

Step 8: MDS = {3,7} $\cup$ {10} = {3,7,10} go to step 2

Step 9: LI = 10

Step 10: NI(10) = null

Step 11: End

Output: MDS = {3,7,10} is the minimum dominating set of given interval.

Theorem.3

Let I be three consecutive interval family and G is an interval graph corresponding to I. Then $\gamma(G)+\gamma(\overline{G})\leq[\frac{N}{2}]+2$ at i < j < k and j $\in$ D.

Proof

Let I = {i,j, k,….n} be an interval family and G is an interval graph corresponding to I. If i, j, k are three consecutive intervals such that i < j < k and j $\in$ D, i intersects j, j intersects k and i intersects k then G must be connected and non split domination occurs in G. We have to show that G is a without isolated vertices of G. In this connection we can show that

$\gamma(G)+\gamma(\overline{G})\leq[\frac{N}{2}]+2.$

We consider an interval family and if D is a domination number γ(G) of G then < V-D > is a domination number $\gamma(\overline{G})$ of $\overline{G}$. Suppose < V-D > is not a domination number then there exists a vertex j such that i is not dominated by any vertex in < V-D >. Since an interval graph G has no isolated vertex in D-{i}. Thus D - {i} is a domination number which contradicts the domination of D. Thus every vertex in D is adjacent with atleast one vertex in < V-D >.

Thus G is a non-split domination. We get $\gamma(G)+\ \gamma(\overline{G})\leq[\frac{N}{2}]+2$ towards a non- split dominating set of an interval graph.

Experimental problem for Theorem.3

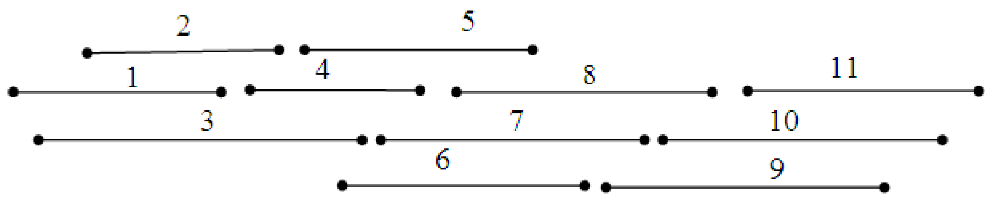

Fig.: Interval family I

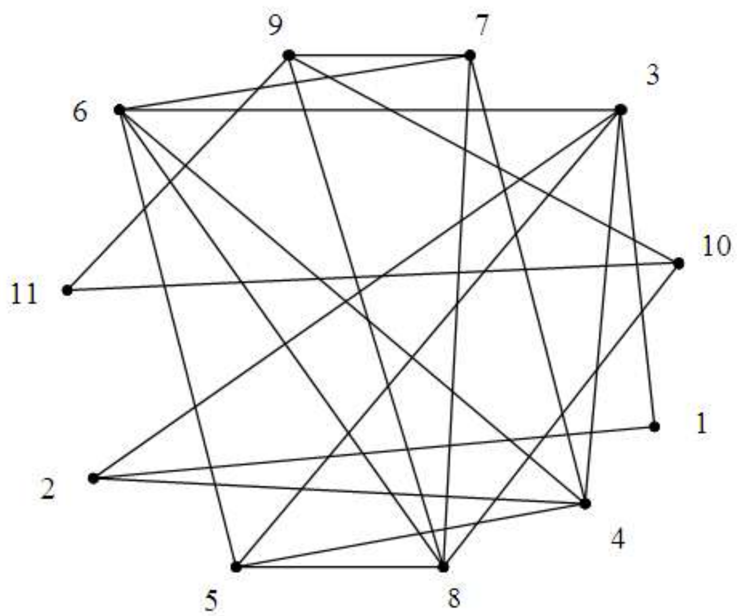

Fig.: Interval graph G

The dominating set of G= {3,9}.

The domination number γ(G)= 2

The induced sub graph $\overline{G}$ of G = {1,2,3,4,5,6,7,8,9,10,11} – {3,9}

= {1,2,4,5,6,7,8,11}

The induced sub graph $\overline{G}$ is

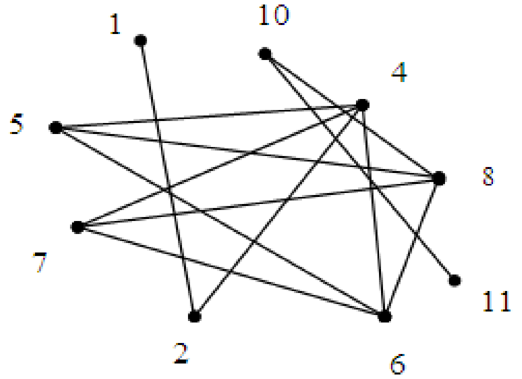

Fig.: Induced sub graph of G

The induced sub graph is connected.

Therefore the dominating set of $\overline{G}$ = {2,6,10} and

The domination number of $\overline{G}$ = $\gamma(\overline{G})$ = 3.

Therefore the result is $\gamma(G)+ \gamma(\overline{G}) \leq [\frac{N}{2}]+2$

$$2 + 3 \leq \frac{11}{2} + 2$$

$$5 \leq 5.5+ 2$$

$$5\leq 7.5$$

Therefore the result is true.

An Algorithm for finding a minimum dominating set of Theorem.3

We will find nbd[i], max(i) and NI(i), where i = 1,2,3,…..,11 for the above graph.

Nbd[1]= {1,2,3}	max(1)= 3	NI(1)=4
Nbd[2]= {1,2,3,4}	max(2)= 4	NI(2)=5
Nbd[3]= {1,2,3,4,5,6}	max(3)= 6	NI(3)=7
Nbd[4]= {2,3,4,5,6,7}	max(4)= 7	NI(4)=8
Nbd[5]= {3,4,5,6,8}	max(5)= 8	NI(5)=9

Nbd[6]= {3,4,5,6,7,8}	max(6)= 8	NI(6)=9
Nbd[7]= {4,6,7,8,9}	max(7)= 9	NI(7)=10
Nbd[8]= {5,6,7,8,9,10}	max(8)= 10	NI(8)=11
Nbd[9]= {7,8,9,10,11}	max(9)=11	NI(9)=null
Nbd[10]= {8,9,10,11}	max(10)= 11	NI(10)=null
Nbd[11]= {9,10,11}	max(11)= 11	NI(11)=null

Procedure for finding a minimum dominating set

Input: Interval family I = {1,2,3,…..,11}.

Step 1: MDS = 3

Step 2: LI = The largest interval in MDS = 3

Step 3: NI(LI) = NI(3) = 7

Step 4: max(NI(LI)) = max(7) = 9

Step 5: MDS = {3} $\cup$ {9} = {3,9} go to step 2

Step 6: LI = 9

Step 7: NI(9) = null

Step 8: End

Output: MDS = {3,9} is the minimum dominating set of given interval.

REFERENCES

1. O.Ore, *Theory of Graph*, Amer, Math. Soc. Colloq. Publ. 38, Providence, 1962; P. 206.
2. C.Berge, *Graphs and Hyperactive graphs*, North Holland, Amsterdram in graphs, Networks, 1980; 10: 211-215.
3. E.J.Cockayne, S.T. Hedetniemi, *Towards a theory of domination in graphs*, Networks, 1977; 7: 247-261.
4. De Jaenisch C.F. De, applications de.1 Analyze mathematique an Jendes Echecs, 1862.
5. E. Sampath Kumar, P.S. Neeralagi, The neighbour of graph, Indian J. Pure. Appl. Math., 1985; 16: 126-13.