World Journal of Engineering Research and Technology
WJERT
www.wjert.org
SJIF Impact Factor: 4.326

COMPRESSIVE SENSING BASED IMAGE RECONSTRUCTION

Sherin C. Abraham[1]* and Ketki Pathak[2]

[1]Electronics and Communication department, Dr. S and S. S Ghandhy College of Engineering and Technology, Majura Gate, Surat, India.

[2]Electronics and Communication department, Sarvajanik College of Engineering and Technology, Surat, India.

Article Received on 04/03/2017 Article Revised 25/03/2017 Article Accepted on 14/04/2017

***Corresponding Author**
Sherin C. Abraham
Electronics and Communication department, Dr. S and S. S Ghandhy College of Engineering and Technology, Majura Gate, Surat, India.

ABSTRACT

Compressive sensing is a technique of image acquisition and reconstruction from a relatively fewer measurements than what the Nyquist theorem suggests; the sampling rate must be greater than twice the highest frequency in the signal for fidelity of image reconstruction. Compressive sensing is applicable when the signals under consideration are sparse. Two-dimensional discrete wavelet transform is applied for sparse representation of an image in this thesis. When sparsity is more, the performance of compressive sensing image reconstruction algorithm will be better. The sparse level of low frequency sub bands and high frequency sub bands are different. Two different compressive sensing measurement matrixes and recovery algorithms are used for the low-frequency sub bands and high-frequency sub bands for better results. Medical field especially in MRI scanning, compressive sensing can be utilized for less scanning time, thus benefits patients. The reconstructed image will be better in both PSNR and visual quality.

KEYWORDS: Compressive Sensing, Wavelet Transform, Discrete Cosine Transform, Sparsity, MRI.

INTRODUCTION

We are in the midst of a digital revolution that is driving the development and deployment of new kinds of sensing systems with ever-increasing fidelity and resolution. Signals, images, videos, and other data can be exactly recovered from a set of uniformly spaced samples taken at the so-called Nyquist rate. Unfortunately, in many important and emerging applications, the resulting Nyquist rate is so high that will end up with far too many samples. Alternatively, it may simply be too costly, or even physically impossible, to build devices capable of acquiring samples.

A new technology is required in this world where we are using HD videos, video conferencing, online games, etc. much more than ever. Now we have a solution, Compressive Sensing. The signal having a sparse representation can be recovered exactly from a small set of linear, non-adaptive measurements. It means it's possible to sense sparse signals by taking far fewer measurements, lower sampling rate, hence the name compressed sensing.

CS differs from classical sampling in three important respects. First, CS is a mathematical theory focused on measuring finite-dimensional vectors in R^N. Second, CS systems typically acquire measurements in the form of inner products between the signal and more general test functions. Thirdly, the signal recovery is typically achieved using highly nonlinear methods. In short, CS enables a potentially large reduction in the sampling and computation costs for sensing signals that have a sparse or compressible representation.

Nomenclature	
CS	Compressive Sensing
WT	Wavelet Transform
DCT	Discrete Cosine Transform
OMP	Orthogonal Matching Pursuit
DWT	Discrete Wavelet Transform
HD	High Definition
i.i.d	identically and independent dependent
MRI	Magnetic Resonance Imaging
MSE	Mean Square Error
PSNR	Pseudo SNR

In this paper we applied CS in the medical images and analyzed using different sparsity domain, measurement matrix and reconstruction algorithm. WT and DCT are used as sparsity domain. Hadamard matrix and Gaussian matrix are considered as measurement matrix. L1 minimization, Orthogonal Matching Pursuit and pseudo inverse multiplication method are used as recovery algorithm.

Applications

CS has already had notable impact on several applications. One example is medical imaging, where it has enabled speedups by a factor of seven in paediatric MRI while preserving diagnostic quality.

Basic Concept of Medical Imaging

CS is being actively pursued for medical imaging, particularly in MRI. Most MR images, like angiograms, have sparsity properties, in domains such as Fourier or wavelet basis. Generally, MRI is a costly and time consuming process because of its data collection process which is dependent upon physical and physiological constraints. In addition, high gradient amplitudes and rapid switching can produce peripheral nerve stimulation. However, the introduction of CS based techniques has improved the image quality through reduction in the number of collected measurements and by taking advantage of their sparsity, thus benefits patients.

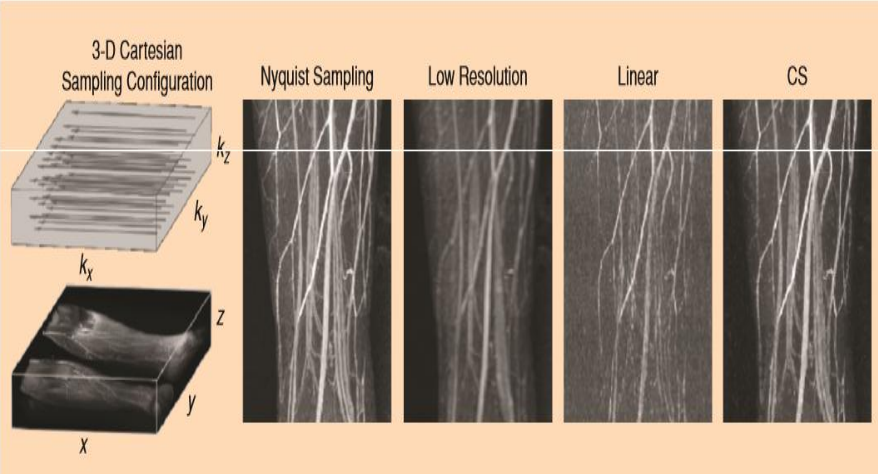

Figure 1: MRI image reconstructed using various techniques.[6]

Compressive Sensing – Basic Introduction

Basic Block Diagram of CS based system

The recent theory of Compressive Sensing introduced by Candes, Romberg, and Tao and Donoho demonstrates that a signal that is K-sparse in one basis (call it the sparsity basis) can be recovered from cK non-adaptive linear projections onto a second basis (call it the measurement basis) that is incoherent with the first, where c is a small over-measuring constant. While the measurement process is linear, the reconstruction process is decidedly nonlinear.

Let x is a real valued, finite length, one dimensional, discrete time signal which is an Nx1 vector in R^N. Thus by $x \in R^N$ can also represent in orthonormal basis $\Psi \in R^{NxN}$.

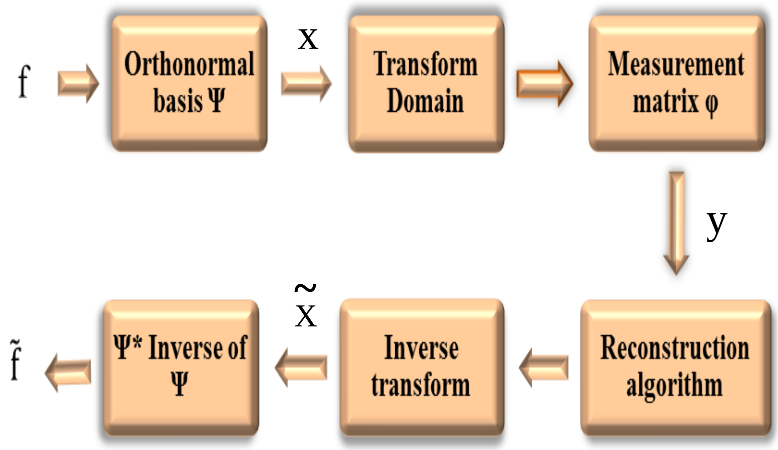

Figure 2: Block diagram of compressive sensing.[1]

Thus signal x can be written as

$x = \Psi f$

- f is Nx1 column vector of weighting coefficients $f = \langle x, \psi \rangle = \Psi^T x$
- f is the sparse representation of signal x in the orthonormal basis Ψ.

The signal x is K-sparse if it is a linear combination of only K nonzero basis vectors. When K << N, the signal x is compressible, means it has just a few large coefficients and many small coefficients.

We measure the signal x by sampling it with respect to a measurement matrix $\Phi \in R^{MxN}$. The measurement MxN matrix Φ must allow the reconstruction of the length-N signal x from M < N measurements. The measurement process is not adaptive, meaning that Φ is fixed and independent of signal x. Thus measurements M×1 vector y can be represented:

$y = \Phi\Psi f = Acsf = \Phi x$

- Acs = ΦΨ is the sensing matrix of M x N.
- Φ is the measurement matrix.
- Ψ and Φ should be incoherent.[1]

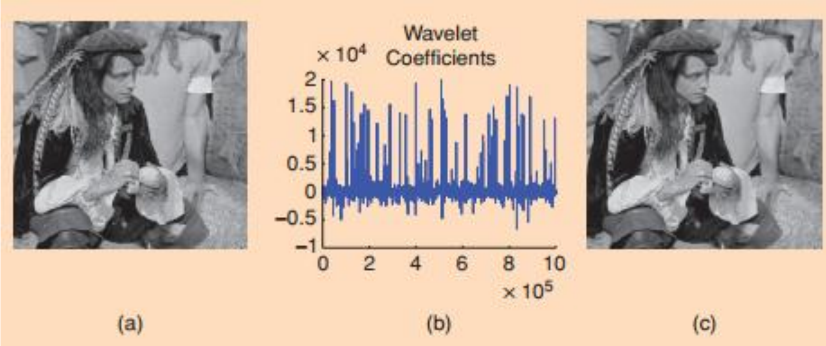

Figure 3: Sparsity and incoherence in wavelet transform enables the megapixel image to its approximation obtained by throwing away 97.5% of the coefficients with negligible perceptual loss. (a) Original Image (b) Wavelet transform coefficients (c) Reconstructed image.[10]

Reconstruction Algorithms

Linear Optimisation

To recover the signal by L1-norm minimization; the reconstruction x* is given by x* = Ψf*, where f* is the solution to the convex optimization program

$$(\|f\|_{l_1} := \textstyle\sum_i |f_i|)\ \min_{\tilde{f}\in R^N} \|\tilde{f}\|_{l_1} \text{ subject to } y_k = \langle \phi k, \Psi\tilde{f}\rangle,\ \forall\, k \in M$$

That is, among all objects $\hat{f} = \Psi^{\tilde{x}}$ consistent with the data, pick only whose coefficient sequence has minimal L1 norm. The CS theory uses L1 norm characteristics which are linear in nature and can be easily computed, thus offering a far simpler and faster way of estimating sparse signals from very limited number of measurements.

Greedy algorithm uses an iterative approach of the coefficient signal to the signal convergence is reached, or get an approximate increase of sparse signal in each of iteration by calculating the measured data mismatch. OMP is one of greedy algorithms.[7]

Proposed Algorithm

The proposed algorithm is given in fig 4. The image is undergone two - dimensional wavelet transform for more sparsity. In the image reconstruction based on the traditional compressive sensing algorithm, the same measurement matrix is used to measure the whole wavelet coefficients. However, since the high-frequency coefficients are sparse while the low frequency coefficients are not sparse,[2] when putting the low frequency coefficients together with the high-frequency coefficients to multiply with the measurement matrix, the coherences among the low-frequency coefficients will be disrupted, which leads to a degraded performance of the reconstructed image.[3] The scaling coefficients of low-frequency component contain most of the image energy.

Due to these DWT features, two random CS sensing matrices are separately used for re-sampling the low-band and high-bands, which can be expressed as follows:

$$\begin{bmatrix} Y_L \\ Y_H \end{bmatrix} = \begin{bmatrix} \varphi_L & \\ & \varphi_H \end{bmatrix} \begin{bmatrix} X_L \\ X_H \end{bmatrix}$$

where Y_L and Y_H denote CS samples measured from low-bands and high-bands, while X_L and X_H represent the scaling and wavelet coefficients, respectively. At the decoder side, two different CS recovery algorithms are developed for the low-frequency subband and high-frequency subbands, respectively.

The required number of CS measurements is much smaller than that of the DWT coefficients, and therefore, CS sampling. The challenge is to get the reconstructed image in better quality by adding more sparsity with maintaining incoherence. For this, we need to choose measurement matrix and recovery algorithm wisely.

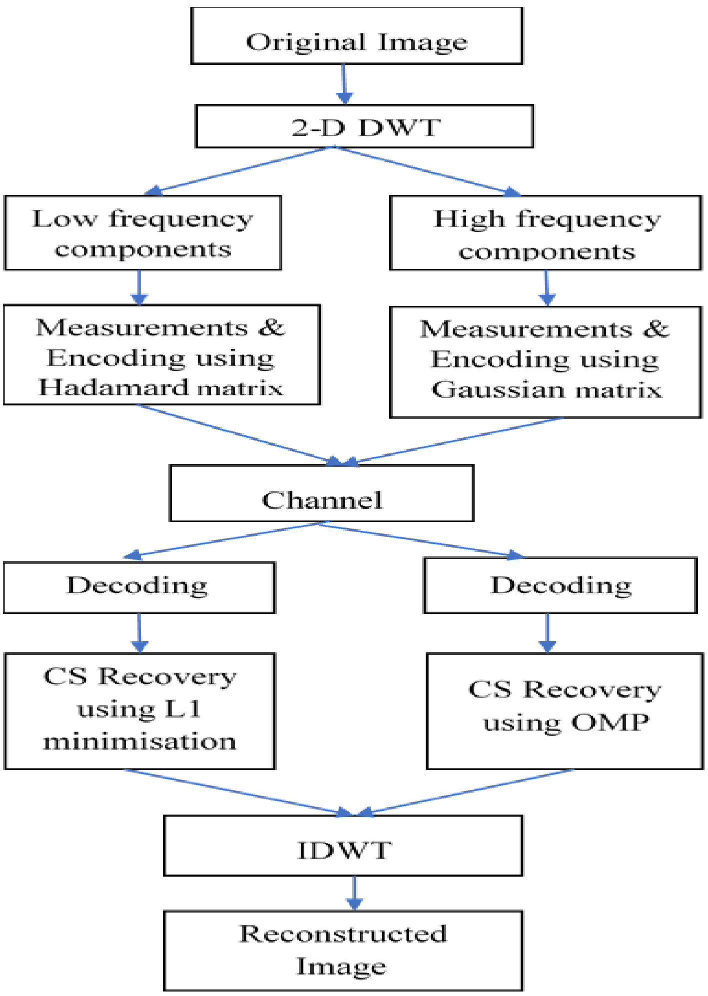

Figure 4: Proposed block diagram for implementation.

Implementation

Algorithm Steps For Implemenation

The steps of the algorithms are as follows:

(1) Perform the wavelet transform of the N*N image, and get the four wavelet sub-band coefficients {LH_1, HL_1, HH_1, LL_1}.

(2) Use the measurement matrix to measure the three high-frequency sub-band coefficients LH_1, HL_1, HH_1 to get the matrices of the measured coefficients while use another measurement matrix for the low-frequency sub-band coefficients LL_1.

(3) Use the reconstruction algorithms to reconstruct the three high-frequency coefficients matrices $\overline{LH}_1$, $\overline{HL}_1$, $\overline{HH}_1$, and low-frequency coefficients matrix LL_1. Then together reconstruct the image.

(4) The image reconstructed will be verified by its PSNR value.

Simulation setup

The scaling coefficients X_L are re-sampled by an i.i.d. random Hadamard matrix Φ_L. The scaling coefficients X_H are re-sampled by an i.i.d. random Gaussian matrix Φ_H. L1 minimization technique, OMP and Pseudo Inverse multiplication method reconstruction techniques are used as reconstruction algorithms.

We have used different size of images like 32x32, 64x64, 128x128 and 512x512. The transform method DCT is also compared with wavelet transform.

Results of implementation of proposed block diagram.

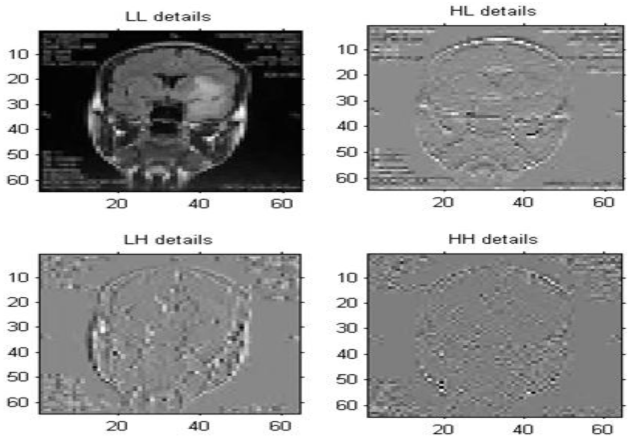

Figure 5: Wavelet Transform Output of 128x128 image.

The low frequency sub band LL has coarse information and details information in high frequency sub bands HL, LH, HH.

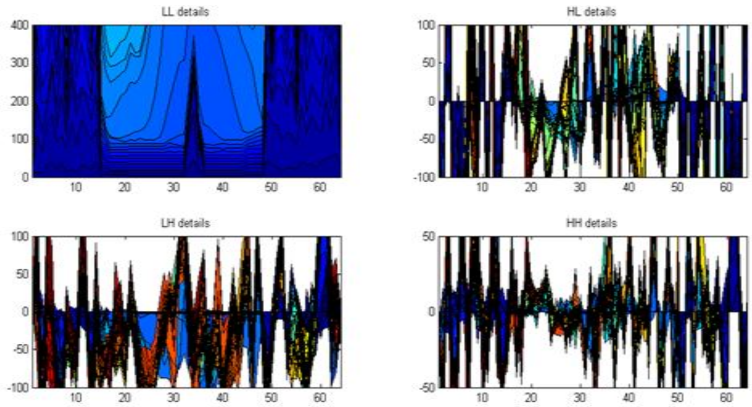

Figure 6: Wavelet coefficients of 128x128 image before measurements taken.

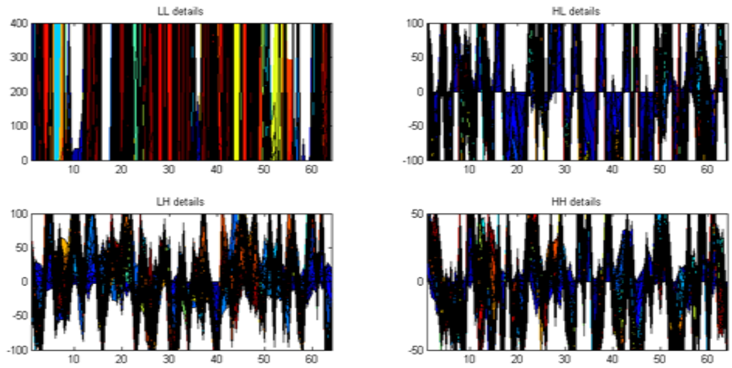

Figure 7: Wavelet coefficients of 128x128 image after measurements taken.

CS makes the signal sparser, which is clear by comparing the Fig.6 and Fig.7. This sparsity leads to the efficient lossless compression. Due to more sparsity, the reconstructed image quality will be more.

The outputs of different images are compared based on PSNR and MSE values with iteration value 50. The images are undergone WT.

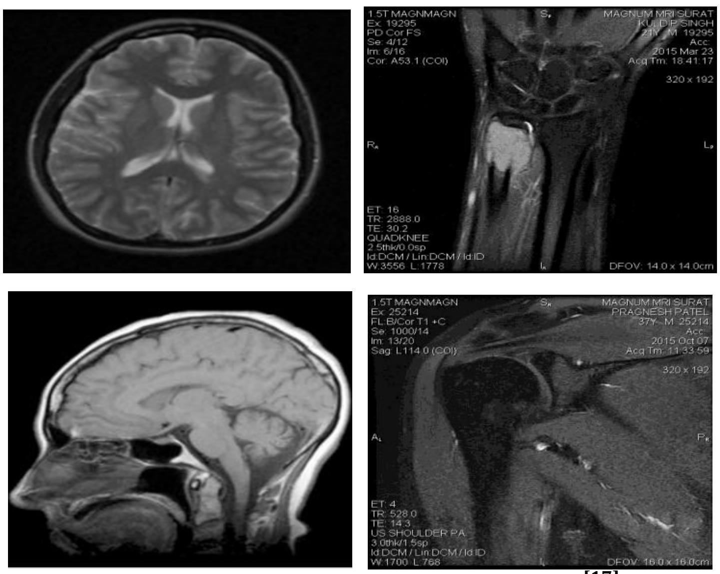

Figure 8: Different medical images.[17]

Table 1: MSE and PSNR values comparison for different medical images for 50 iteration.

		Medical image 1	Medical image 2	Medical image 3	Medical image 4
PSNR	L1 and PINV	43.83	54.85	31.76	30.11
	OMP	15.28	18.21	19.77	14.23
MSE	L1 and PINV	2.699	.212	43.27	63.36
	OMP	1971.3	981	684.86	2455

The outputs of different image sizes are compared based on PSNR and MSE values with iteration value 50. The image has undergone WT.

Table 2: MSE and PSNR values using L1 minimization or OMP with different sizes of images.

		32x32	64x64	128x128	256x256	512x512
PSNR	L1 and PINV	-	50.08	42.74	33.31	31.46
	L1	-	50.08	42.74	-	-
	OMP	-	-	32.38	20.77	16.99
MSE	L1 and PINV	0	0.638	3.45	30.28	46.35
	L1	0	0.638	3.45	-	-
	OMP	0	0	610	542.17	1991.4

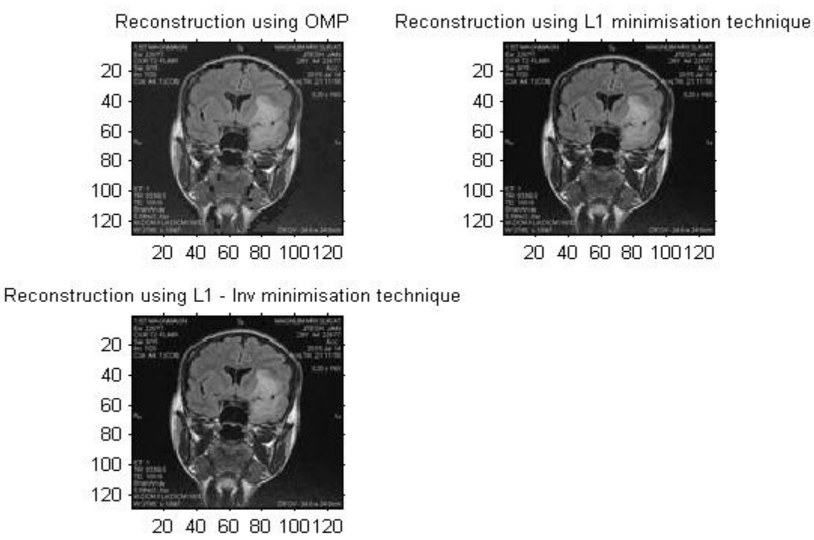

Figure 9: Output of 128x128 image using various techniques.

Table 3: PSNR comparison for various number of iterations.

Iterations	PSNR using 32x32	PSNR using 64x64	PSNR using 128x128	PSNR using 256x256	PSNR using 512x512
5	17.87	15.76	14.53	13.5	12.96
10	23.37	18.57	15.86	14.6	13.66
15	36.67	21.08	17.25	15.34	14.14
25	-	30.39	20.25	16.84	15.1
50	-	-	30.39	20.77	16.99
100	-	-	-	33.88	21.12

The PSNR is compared with different number of iterations using OMP reconstruction algorithm.

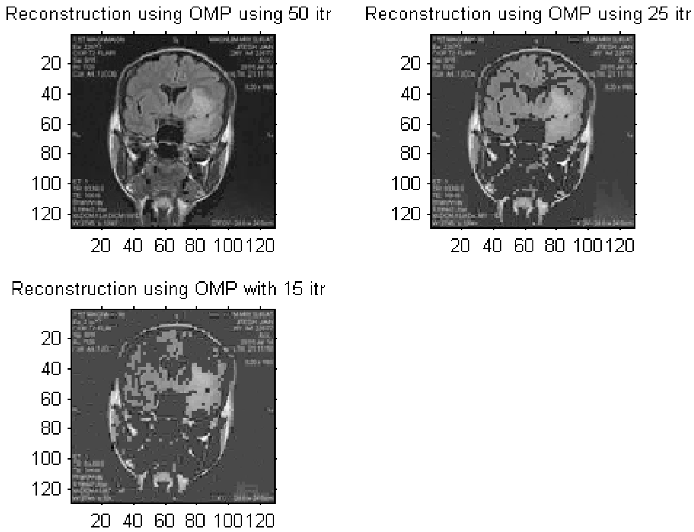

Figure 10: Output of 128x128 image using different iterations of OMP.

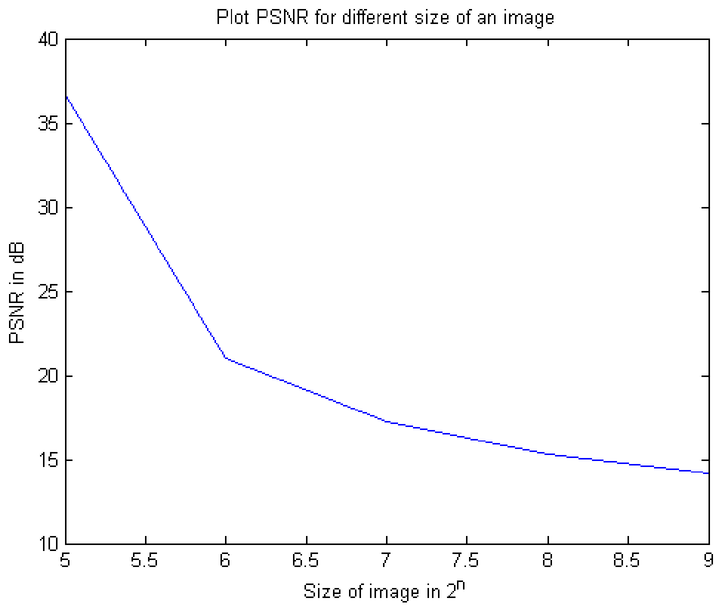

Figure 11: PSNR plot with different size of images.

PSNR values for different size of images respectively are plotted in fig 10. As size increases the PSNR values is getting decreased. But as size decreases the details of the image will get lost. As numbers of iterations are increasing, the image quality is also increasing as shown in fig 11. From table II we can conclude that less size and more iteration will result in better image.

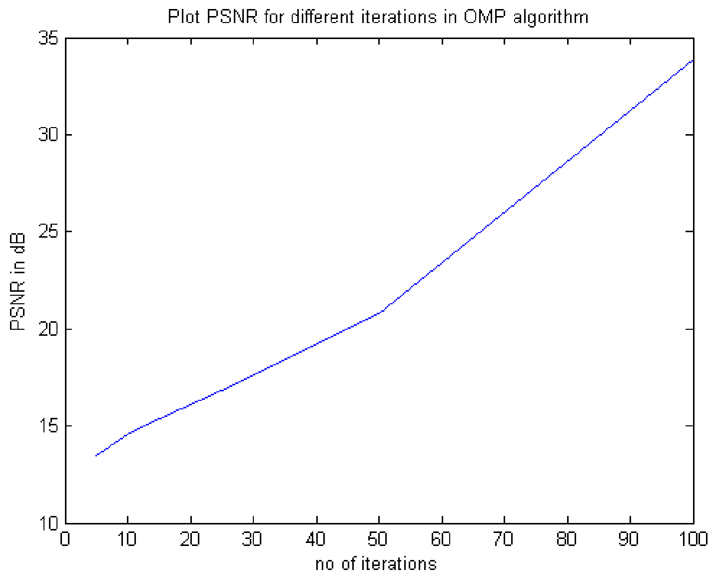

Figure 12: PSNR for different iterations in OMP reconstruction algorithm.

The wavelet transform is compared with DCT. The image undergone DCT, and measurements are taken using Hadamard matrix and without packet loss in the channel. When size increases, the quality will be reducing. OMP with 50 iterations is used.

Table 4: MSE and PSNR comparison using DCT with Hadamard Matrix.

		32x32	**64x64**	**128x128**	**256x256**	**512x512**
PSNR	OMP	-	50.18	38.59	28.55	27.14
MSE	OMP	0	0.546	9.1443	90.73	122.4

Table 5: MSE and PSNR comparison using DCT with Gaussian Matrix.

		32x32	**64x64**	**128x128**	**256x256**	**512x512**
PSNR	PINV	19.9	19.45	19.06	18.04	18.02
	L1	36.13	28.45	26.02	22.24	20.35
	OMP	-	36.14	32.5	25.94	25.37
MSE	PINV	665.4	774.4	805.9	1013.5	1023.5
	L1	15.84	98.85	163.45	388.9	520.3
	OMP	0	15.16	36.56	165.4	188.6

OMP is better than L1 and PINV methods when Gaussian matrix is used as measurement matrix. Gaussian matrix represents random values.

Inferences

- When size of image increases, the PSNR value decreases.
- For small size images, OMP reconstruction is better. And for large size images, L1 minimization output is better.
- PSNR will increase when number of iterations is increased for OMP reconstruction algorithm.
- Hadamard matrix is better than Gaussian matrix when use after DCT.
- Wavelet transform is better when OMP reconstruction algorithm is used.
- Due to the sparsity in the signal, the required compression is achieved.

CONCLUSION

The Compressive sensing measures a relatively small number of "random" linear combinations of the signal value. Sparsity, incoherence and nonlinear reconstruction are three main components of CS. The sparse nature of signals in a particular basis by taking measurements in an 'incoherent' basis is utilized in CS.

Wavelet transform is proven sparsity domain for many signals. The high-frequency coefficients are sparse while the low frequency coefficients are not sparse. So, both should be processed separately for better results. But in DCT transform, we cannot separate the components on basis of frequency. Hadamard measurement matrix is better. L1 minimisation is better with wavelet transform and OMP will be better with DCT.

In Medical field, CS will help in less number of samples, less radiation for patients. The medical images are inherently sparse and incoherence.

Future works possible

- Reduce the processing time using Block-based CS method.
- Add enough sparsity by sparsity tuning and intra-prediction methods.

ACKNOWLEDGEMENT

To all my friends and family who supported me to prepare this paper.

REFERENCE

1. Chenwei Deng, Weisi Lin, Bu-sung Lee, and Chiew Tong Lau, "Robust Image Coding Based Upon Compressive Sensing", IEEE Transactions on Multimedia, April 2014;

14(2).

2. Xiumeni Li and Guoan Bi, “Image reconstruction based on the improved compressive sensing algorithm”, IEEE International Conference on Digital Signal Processing (DSP), 2015.
3. Weisheng Dong, Xiaolin Wu, and Guangming Shi, "Sparsity Fine Tuning in Wavelet Domain with Application to Compressive Image Reconstruction”, IEEE Transactions on Image Processing, December 2014; 23(12).
4. Muhammad Ali Qureshi and M. Deriche, “A new wavelet based efficient image compression algorithm using compressive sensing”, Springer Science Multimedia tools and applications, March 2015.
5. Michael Lustig and et.al, “Compressed sensing MRI”, IEEE signal processing magazine, March, 2008.
6. Sabbisetti Ravindranath and et. al, “Compressive Sensing based Image acquisition and Reconstruction analysis” Green Computing Communication and Electrical Engineering (ICGCCEE), 2014 International Conference.
7. Chen Chen and Junzhou Huang, “Exploiting the wavelet structure in compressed sensing MRI”, Science Direct Magnetic Resonance Imaging, July 2014.
8. Saad Qaisar and et.al, “Compressive sensing: From theory to applications, a survey”, IEEE Journal of communications and networks, October 2013; 15(5).
9. Emmanuel J. Candès and Michael B. Wakin, “An introduction to compressive sensing”, IEEE Signal Processing Magazine, March 2008.
10. S. Spurthi and Parnasree Chakraborty, “Block based compressed sensing algorithm for medical image compression”, International Journal of Engineering and Computer Science, May 2016; 05(5).
11. Sumit Budhiraja and Dipti Bhatnagar, “Wavelet based compressive sensing techniques for image compression”, International journal of Engineering Research and Applications, 2011.
12. Dr. G Vijay Kumar, “Fast compressed sensing based high resolution image reconstruction”, International Journal of New Technology and Research, March 2016; 2(3).
13. Jie Xu and et.al, “Improved total variation minimization method for compressive sensing by intra-prediction”, Science direct, Signal processing, 2012.
14. Wonseok Kang and et.al, “Compressive sensing-based image de-noising using adaptive multiple sampling and optimal error tolerance”, IEEE International Conference on

Acoustics, Speech and Signal Processing, 2013.

15. Quisong Wu and et.al, "Multi-task Bayesian compressive sensing exploiting intra-task dependency", IEEE signal processing letters, 2013.
16. http://dsp.rice.edu/sites/dsp.rice.edu/files/cs/baraniukCSlecture07.pdf
17. http://dsp.rice.edu/cs
18. https://en.wikipedia.org/wiki/Compressed_sensing
19. Civil Hospital, Surat
20. https://www.ncm-c.org/services/diagnostic-imaging/magnetic-resonance-imaging-mri/ http://www.healthhype.com/diagnostic-imaging-of-the-abdomen-x-ray-ultrasound-ct-mri.html