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SELECTION OF OPTIMAL BASE WAVELET FOR IMAGE COMPRESSION

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ABSTRACT

Signal compression is the process of converting data files into smaller files for efficiency of storage and transmission. Image compression is a method through which we can reduce the storage space of images which is helpful to increase storage and transmission process's performance. The image compression is implemented in software using MATLAB's Wavelet Toolbox.[8,9] The study is carried out on .jpg format images. We describe the comparison of performance of discrete wavelets like Symlet, Daubechies, Coiflet and Haar wavelets. This paper includes the discussion on the basics of wavelet, discrete wavelet transforms, principles of image compression, image compression methodology, the selection of discrete wavelet transform with results and conclusion.

KEYWORDS: Wavelet, Discrete Wavelet Transforms, Global Thresholding, Image Compression.

INTRODUCTION

Wavelet is a new development in the emerging field of data analysis for Physicists, Engineers, and Environmentalists.[5,6] It represents an efficient computational algorithm under the interest of a broad community. Fourier sine‟s extracts only frequency information from a time signal, thus losing time information, while wavelet extracts both time evolution and frequency composition of a signal. Wavelet is a special kind of the functions which exhibits oscillatory behaviour for a short time interval and then dies out. In wavelet we use a single function and its dilation and translation to generate a set of orthonormal basis functions to represent a signal. Number of such functions is infinite and we choose one that suits to our application. The range of interval over which scaling function and wavelet function are defined is known as support of wavelet. Beyond this interval (support) the functions should be identically zero. There is an interesting relation between length of support and number of coefficients in the refinement relation. For orthogonal wavelet system, the length of support is always less than no. of coefficients in the refinement relation. It is also very helpful to require that the mother function have a certain number of zero moments, according to.

$$\int_{-\infty}^{\infty} \psi(t)dt = 0$$

The mother function can be used to generate a whole family of wavelets by translating and scaling the mother wavelet.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) = T_b D_a \psi$$

Here b is the translation parameter and a is the dilation or scaling parameter. Provided that ψ (t) is real-valued, this collection of wavelets can be used as an orthonormal basis. A critical sampling of the continuous wavelet transform is

$$W_{a,b} = \int f(t)\frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right)dt$$

is obtained via $a = 2^{-j}$, where j and k are integers representing the set of discrete translations and discrete dilations. Upon this substitution, we can write discrete wavelet transform as;

$$W_{j,k} = \int f(t)2^{j/2}\psi\left(2^j t - k\right)dt$$

Wavelet coefficients for every (a, b) combination whereas in discrete wavelet transform, we find wavelet coefficients only at very few points by the dots and the wavelets that follow these values are given by;

$$\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right)$$

These wavelet coefficient for all j and k produce an orthonormal basis. We call $\psi_{0,0}(t) = \psi(t)$ as mother wavelet. Other wavelets are produced by translation and dilation of mothere wavelet. The wavelet transform of a signal captures the localized time frequency information of the signal. Suppose we are given a signal or sequences of data $S = \{S_n\}_{n\in\mathbb{Z}}$ sampled at regular time interval $\Delta t = \tau$. S is split into a "blurred" version a1 at the coarser interval $\Delta t = 2\tau$ and "detail" d1 at scale $\Delta t = \tau$. This process is repeated and gives a sequence S_n , a_1, a_2, a_3, a_4,.....of more and more blurred versions together with the details d_1, d_2, d_3, d_4, ...removed at every scale ($\Delta t = 2^m\tau$ in a_m and d_m). Here a_ms and d_ms are approximation and details of original signal. After N iteration the original signal S can be reconstructed as

$$S = a_N + d_1 + d_2 + d_3 + \cdots \ldots . d_N$$

DISCRETE WAVELET TRANSFORMS

The primary and most important work in the spectral analysis of any signal using wavelet transforms is the selection of suitable wavelet according to the signal. Suitable wavelet is selected on the basis of compatibility with signal characteristics. Accurate wavelet selection retains the original signal and also enhances the frequency spectrum of denoised signal. Wavelet extracts both time evolution and frequency composition of a signal.

A multiresolution analysis for $L^2(\mathbb{R})$ introduced by Mallat [9, 10] and extended by other researchers [11, 12] consists of a Sequence V_j, $j\epsilon\mathbb{Z}$ of closed subspaces of $L^2(\mathbb{R})$. Let f (x) be a function in $L^2(\mathbb{R})$. We can write $f(x)$ in V_{j+1} space, i.e.,

$$f(x) = \sum c_{j+1,k}\, \phi_{j+1,k}(x)$$

Since

$$V_{j+1} = V_j \oplus W_j$$

where

$$V_{j+1} = span\ \overline{\left(\phi_{j+1,k}(x)\right)}$$

$$V_j = span\ \overline{\left(\phi_{j,k}(x)\right)}$$

$$W_j = span\ \overline{\left(\psi_{j,k}(x)\right)}$$

Therefore

$$f(x) = \sum_k c_{j,k}\, \phi_{j,k}\,(x) + \sum_{j_0}^{j} \sum_k d_{j,k}\, \psi_{j,k}\,(x)$$

where

$$\begin{aligned} c_{j,k} &= \langle f,\ \phi_{j,k} \rangle \\ &= \int f(x)\, \phi_{j,k}(x)\, dx, \quad \forall\, k \in \mathbb{Z} \end{aligned}$$

and

$$\begin{aligned} d_{j,k} &= \langle f,\ \psi_{j,k} \rangle \\ &= \int f(x)\, \psi_{j,k}(x)\, dx \end{aligned}$$

are collectively known as approximation and detailed coefficients.

Thus given signal takes place a new version such as.[1,2]

$$f(1) = a_1 + d_1$$

$$f(2) = a_2 + d_2 + d_1$$

$$f(3) = a_3 + d_{3+} d_2 + d_1$$

$$f(4) = a_4 + d_4 + d_{3+} d_2 + d_1$$

$$f(5) = a_5 + d_5 + d_4 + d_{3+} d_2 + d_1$$

$$f(6) = a_6 + d_6 + d_5 + d_4 + d_{3+} d_2 + d_1$$

$$f(7) = a_7 + d_7 + d_6 + d_5 + d_4 + d_{3+} d_2 + d_1$$

$$f(8) = a_8 + d_8 + d_7 + d_6 + d_5 + d_4 + d_{3+} d_2 + d_1$$

Here a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 are approximations of signal and d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8 are details of signal at various scale or time frames. Since maximum 8 level are possible therefore, f(8) will be combined into a zero frequency component a_8 as well as 8 frequency components $d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8$. We have

$$f(1) = \sum_k c_{1,k}\, \phi_{1,k}(x) + \sum_k d_{1,k}\, \psi_{1,k}(x)$$

$$f(2) = \sum_k c_{2,k}\, \phi_{2,k}(x) + \sum_k d_{2,k}\, \psi_{2,k}(x) + \sum_k d_{1,k}\, \psi_{1,k}(x)$$

$$\begin{aligned} f(8) = {} & \sum_k c_{8,k}\, \phi_{8,k}(x) + \sum_k d_{8,k}\, \psi_{8,k}(x) + \sum_k d_{7,k}\, \psi_{7,k}(x) + \sum_k d_{6,k}\, \psi_{6,k}(x) + \\ & \sum_k d_{5,k}\, \psi_{5,k}(x) + \sum_k d_{4,k}\, \psi_{4,k}(x) + \sum_k d_{3,k}\, \psi_{3,k}(x) + \sum_k d_{2,k}\, \psi_{2,k}(x) \\ & + \sum_k d_{1,k}\, \psi_{1,k}(x) \end{aligned}$$

METHODOLOGY FOR IMAGE COMPRESSION

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space. It also reduces the time required for images to be sent over the Internet or downloaded from Web pages. Compression ratio[7] is defined as ratio of the size of original data set to the size of the size of compressed data set.

$$\text{Compression Percentage} = \frac{C-D}{C} \text{x} 100$$

Where C = Number of bytes in the original data set

D= Number of bytes in the compressed data set.

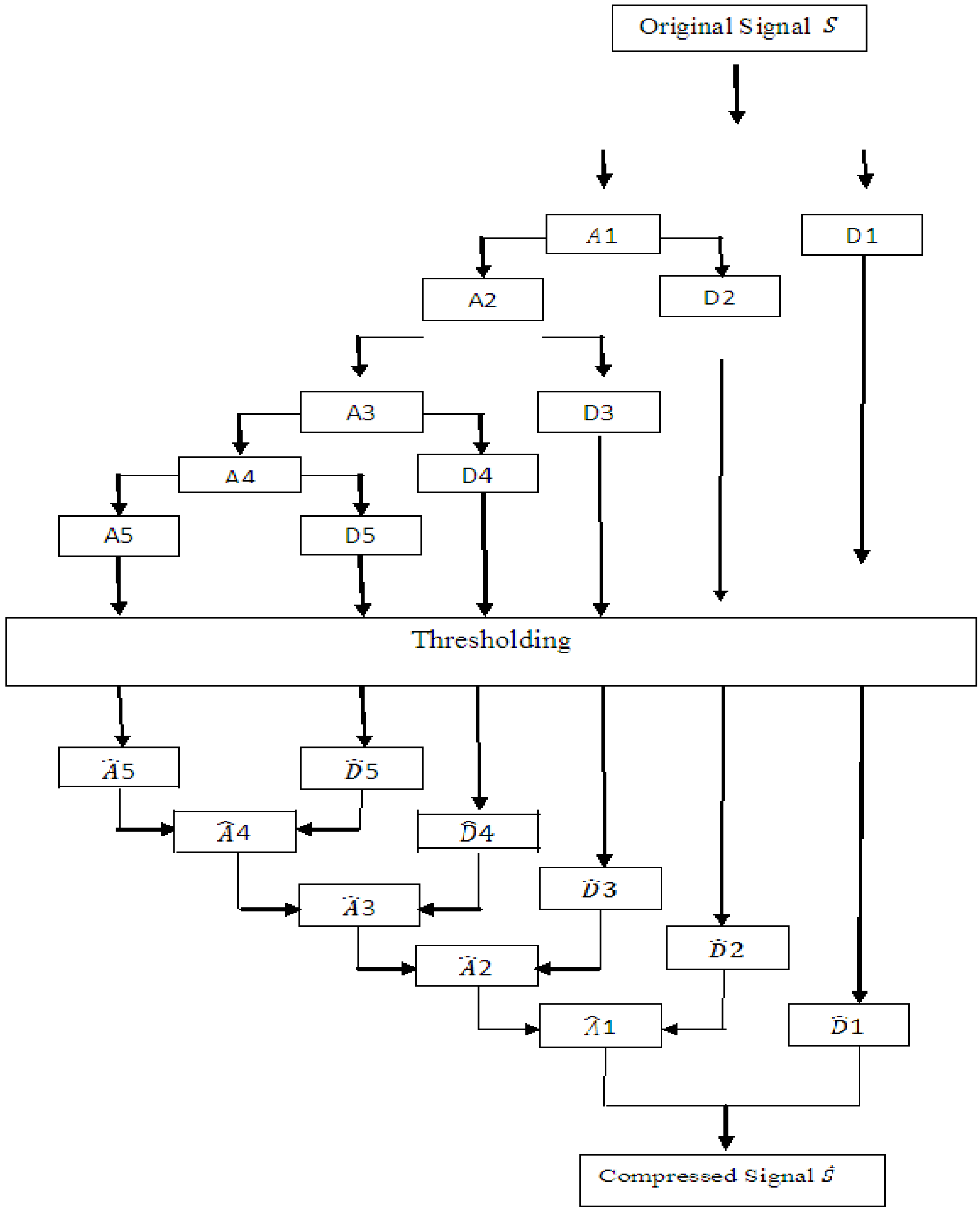
Global Thresholding

Thresholding[12] is a process of converting a gray scale input image to a bi-level image by using an optimal threshold. In global thresholding, a single threshold for all the image pixels is used. When the pixel values of the components and that of background are fairly consistent in their respective values over the entire image, global thresholding could be used. Global thresholding consists of setting an intensity value (threshold) such that all voxels having intensity value below the threshold belong to one phase; the remainder belong to the other. Global thresholding is as good as the degree of intensity separation between the two peaks in the image. It is an unsophisticated segmentation choice. In thresholding we select the maximum value in array and calculate the threshold value by:

$T_o = |X_{max}| / 2$

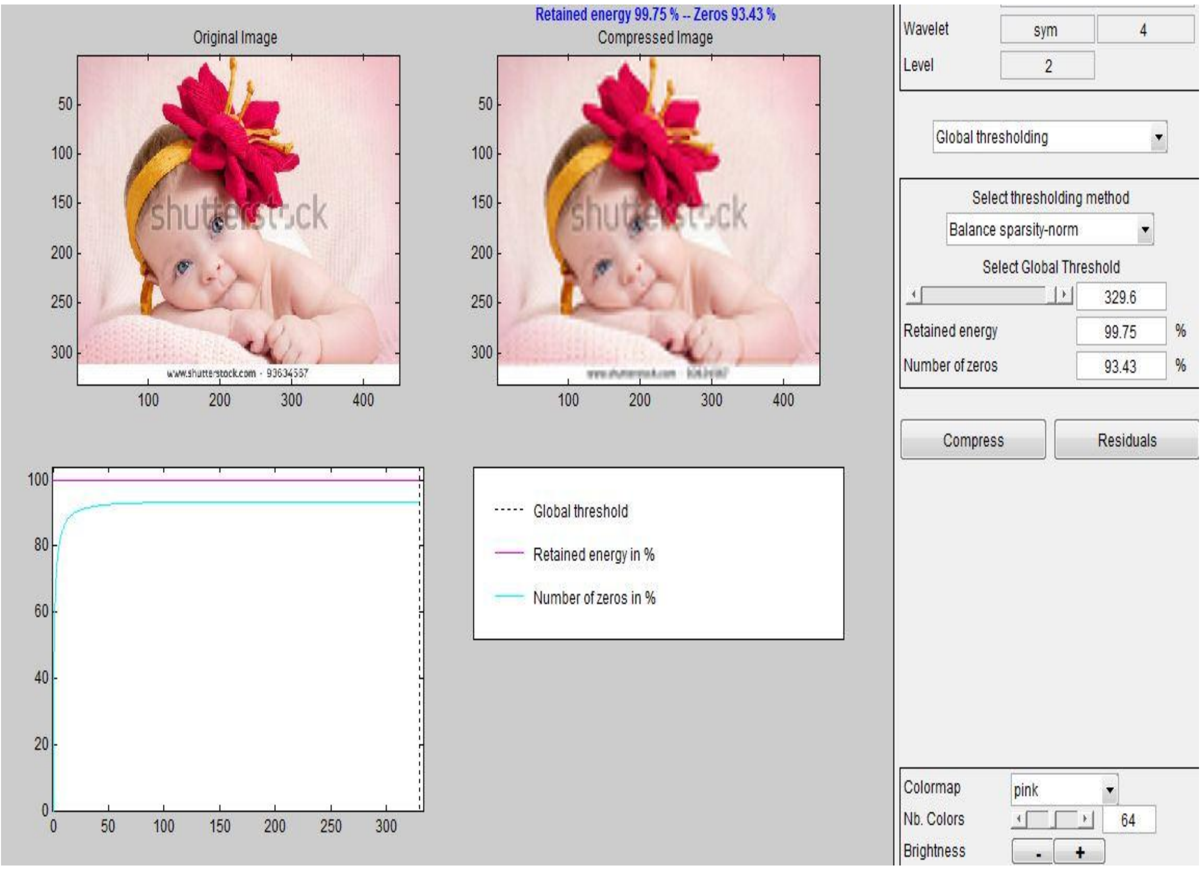
The value which are greater than T_o is significant and others are insignificant form zero tree root.[6] By choosing a particular intensity value as "threshold", images can be segmented by setting those pixels whose original intensity is above the threshold as "white pixels", and setting the other pixels as "black pixels." Thres holding is one of the easiest methods to automatically segment an image using a computer.

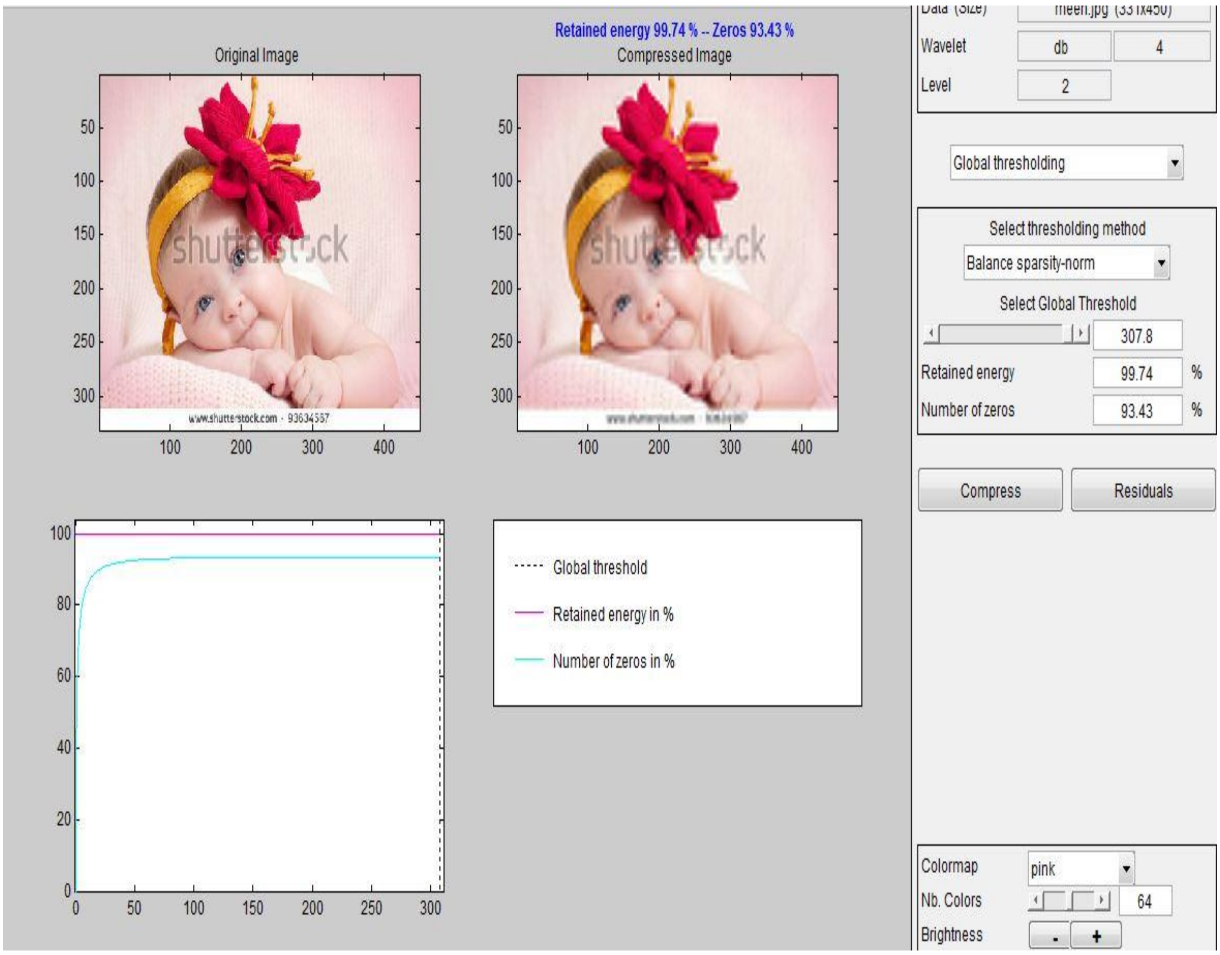
A wavelet transform method of spectral analysis is applied to data to extract fluctuations without long time averaging characteristics.

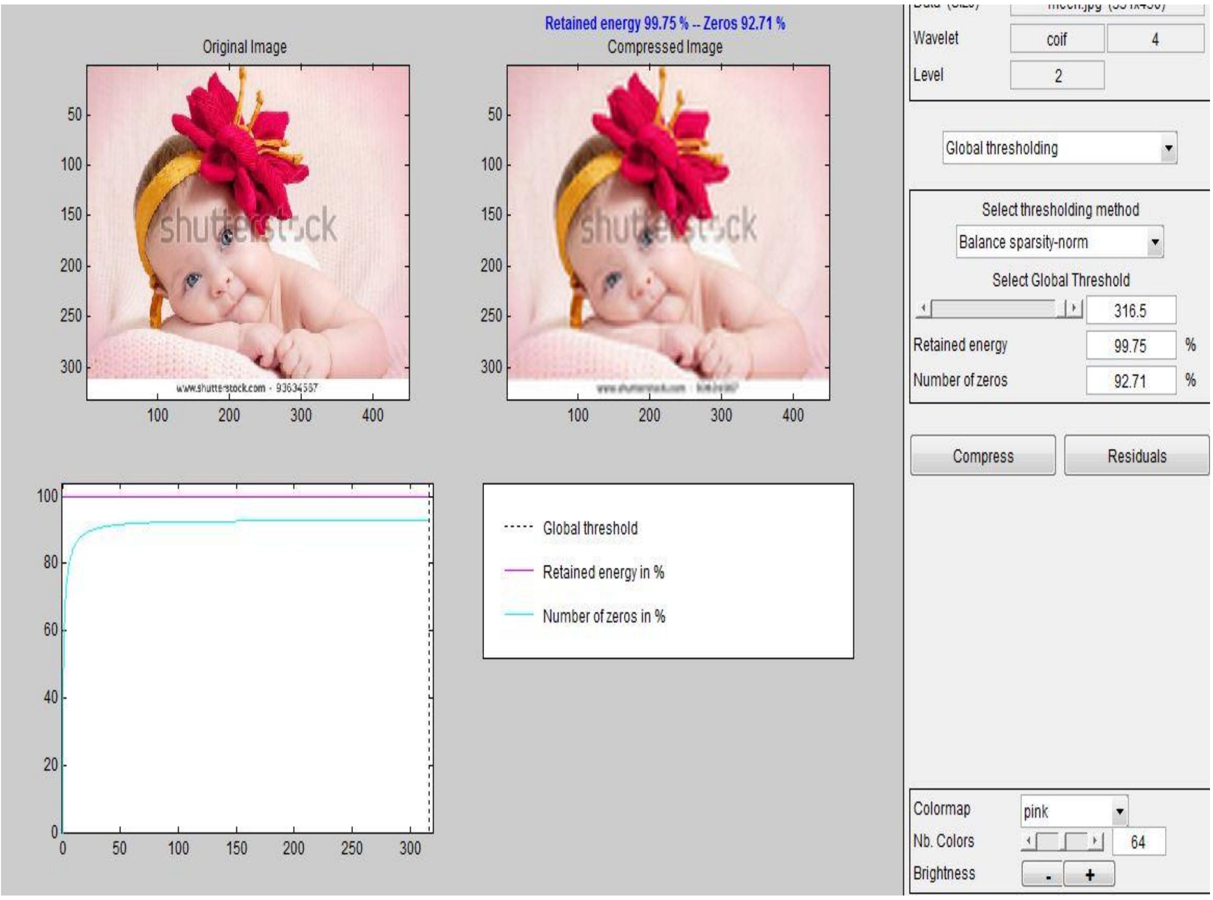


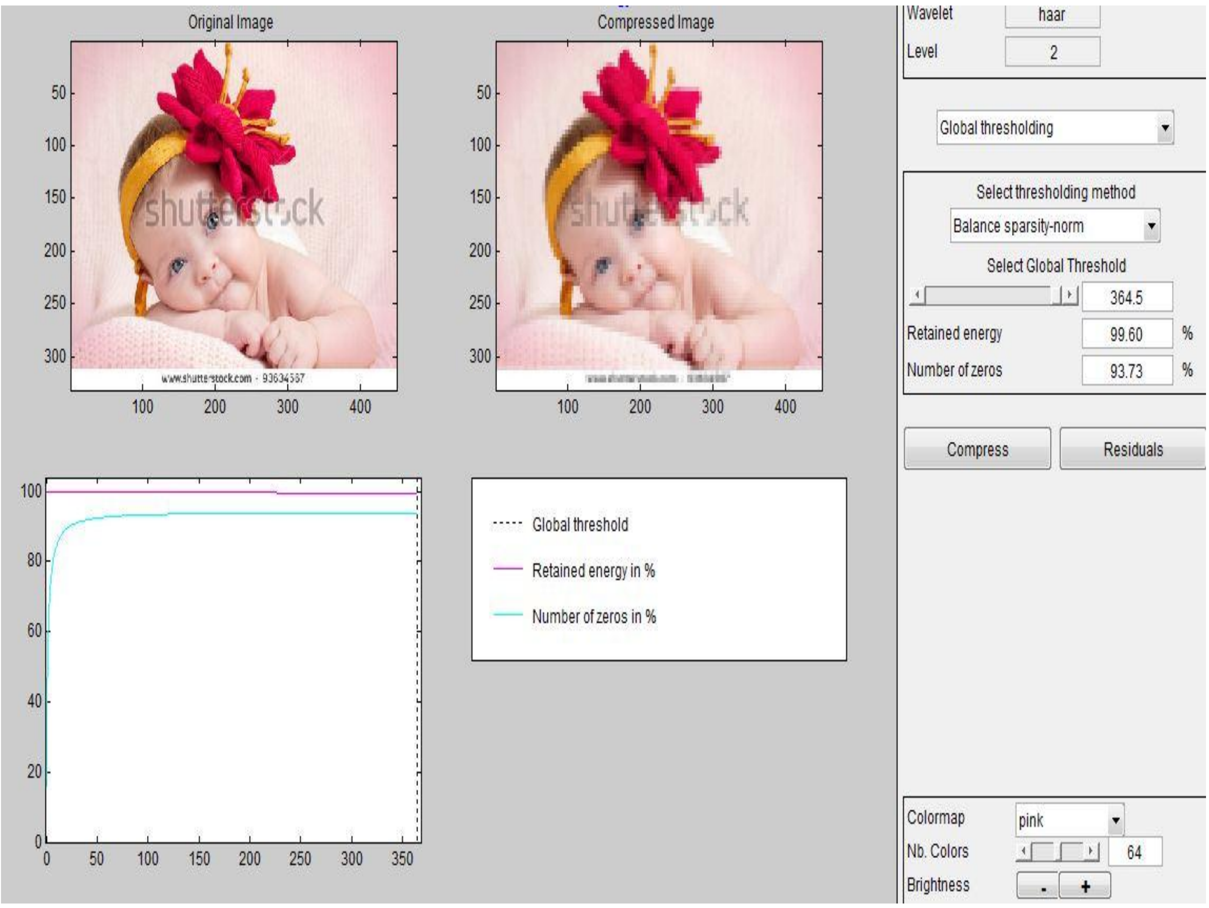
RESULTS

Discrete wavelet transform (DWT) of given image is done using Haar wavelet, Daubechies wavelet, Symlet and Coiflet. Thereafter using global thresholding inverse wavelet transform is applied.


Retained energy 99.75 % -- Zeros 93.43 %
Original Image
Compressed Image
Wavelet sym 4
Level 2
Global thresholding
Select thresholding method
Balance sparsity-norm
Select Global Threshold
329.6
Retained energy 99.75 %
Number of zeros 93.43 %
Compress
Residuals
Global threshold
Retained energy in %
Number of zeros in %
Colormap pink
Nb. Colors 64
Brightness


Retained energy 99.74 % -- Zeros 93.43 %
Original Image
Compressed Image
Wavelet db 4
Level 2
Global thresholding
Select thresholding method
Balance sparsity-norm
Select Global Threshold
307.8
Retained energy 99.74 %
Number of zeros 93.43 %
Compress
Residuals
Global threshold
Retained energy in %
Number of zeros in %
Colormap pink
Nb. Colors 64
Brightness


Retained energy 99.75 % -- Zeros 92.71 %
Original Image
Compressed Image
Wavelet
coif
4
Level
2
Global thresholding
Select thresholding method
Balance sparsity-norm
Select Global Threshold
316.5
Retained energy
99.75
%
Number of zeros
92.71
%
Compress
Residuals
Global threshold
Retained energy in %
Number of zeros in %
Colormap
pink
Nb. Colors
64
Brightness


Original Image
Compressed Image
Wavelet
haar
Level
2
Global thresholding
Select thresholding method
Balance sparsity-norm
Select Global Threshold
364.5
Retained energy
99.60
%
Number of zeros
93.73
%
Compress
Residuals
Global threshold
Retained energy in %
Number of zeros in %
Colormap
pink
Nb. Colors
64
Brightness

In above figures original and compressed image by wavelet transform and their residual are shown. Retain energy and number of zeroes in % using differenr wavelets are also shown.

S.No.	Wavelet	Retain Energy %	No. of zeros %
1.	Symlet4	99.75	93.43
2.	Daubechies4	99.74	93.43
3.	Coiflet4	99.75	92.71
4.	Haar	99.60	93.73

CONCLUSION

As in above discussion more percentage of zeros are responsible for more compression, and high value of the energy retained shows the less loss of the information. The results stated above indicate that for the given image Symlet4 and Coiflet4 are optimal base wavelets. But number of zeros is slightly greater for Sym4 and Hence Sym4 transform is better for the given image.

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