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A STUDY ON STRESS ANALYSIS OF FGM STRUCTURES

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ABSTRACT

An FGM typically consists of a composite material with a spatially varying microstructure designed to optimize performance through the corresponding property distribution. Property distributions are found in a variety of common products that must have multiple functions (i.e., multifunctional), such as gears, which must be tough enough inside to withstand the fracture, but must also be hard enough on the outside to prevent wear. Gear teeth are in constant contact, and therefore their surface hardness becomes of primary concern to prevent them from deteriorating during use. However, if this factor were the only considered design criterion, the gear may suffer from fracture under the constant loading since hardness and toughness are mutually exclusive. Similarly, a turbine blade also possesses a property distribution. Again, the blade must be tough to withstand the loading it is subjected to, but it must also have a high melting point to be able to withstand high temperatures on the outer surface. As with hardness and toughness, these different material properties tend to mutually exclude one another.

KEYWORDS: FGM, Residual stresses, Euler-Bernoulli beam theory.

INTRODUCTION

Motivation

A new class of composite materials known as functionally graded materials (FGM) has drawn considerable attention. A typical FGM, with a high bending-stretching coupling effect, is an inhomogeneous composite made from different phases of material constituents (usually ceramic and metal). Within FGM the different micro-structural phases have different functions, and the overall FGM attain the multi-structural status from their property gradation. By gradually varying the volume fraction of constituent materials, their material properties exhibit a smooth and continuous change from one surface to another, thus eliminating interface problems and mitigating thermal stress concentrations.

The term FGM was originated in the mid-1980s by a group of scientists in Japan. Since then, an effort to develop high-resistant materials using FGM had been continued. FGM were initially designed as thermal barrier materials for aerospace structures and fusion reactors. They are now developed for the general use as structural components in high-temperature environments. Potential applications of FGM are both diverse and numerous e.g., FGM sensors and actuators, FGM metal/ceramic armor, FGM photo-detectors and FGM dental implant.

OBJECTIVE

As technology progresses at an ever increasing rate, the need for advanced capability materials becomes a priority in the engineering of more complex and higher performance systems. This need can be seen in many fields in which engineers are exploring the applications of these new engineered materials. Aerospace engineers trying to incorporate new and improved capabilities into air and space systems are pushing the envelope for what current materials can physically handle. Functionally graded materials (FGM) are relatively new technology and are being studied for the use in components exposed to harsh temperature gradients. Before FGM, laminated composite materials were used which provide the design flexibility to achieve desirable stiffness and strength through the choice of lamination scheme. The anisotropic constitution of laminated composite structures often results in stress concentrations near material and geometric discontinuities that can lead to damage in the form of de-lamination, matrix cracking, and adhesive bond separation.

FGM alleviate these problems because they consist of a continuous variation of material properties from one surface to the other. In this work, structural beam member of FGM is considered and thermal and mechanical stress analysis of FGM structure is carried out. It is also attempted to suggest the design parameters for the FGM composite and safe design of FGM layer under residual stresses. Also predict the neutral surface and nature of deflection of FGM composite beam.

Theoretical Background

The fundamental aspects of FGM and their fabrication and applications are presented in this chapter. First, the conceptual idea of FGM and their distinct features in comparison with other engineering materials is introduced. Also, approaches for modeling and calculating the effective properties of FGM are discussed. Some typical engineering applications of FGM are reviewed.

The term functionally graded materials (FGM) refers to solid objects or parts that usually consist of multiple materials or embedded components, that is, they are materially heterogeneous. The term “heterogeneous object” is defined for those having multiple material objects with clear material domains.

A FGM consists of materials whose properties change from one surface to another according to a smooth continuous function based on the position across the thickness of the material. Most often, these materials consist of ceramic and metallic constituents. One surface is generally a pure metal while the opposite surface is usually pure ceramic or a majority ceramic. The metal portion of the material acts in the role of a structural support while the ceramic provides thermal protection when subjected to harsh temperatures.

The continuous change in the microstructure of FGM distinguishes them from the fiber-reinforced laminated composite materials, which have a mismatch of mechanical properties across an interface due to two discrete materials bonded together. As a result the constituents of the fiber-matrix composite are prone to debonding at extremely high thermal loading. Also, the anisotropic constitution of laminated composite structure often results in stress concentrations near material and geometric discontinuities that can lead to damage in the form of delamination, matrix cracking, and adhesive bond separation.

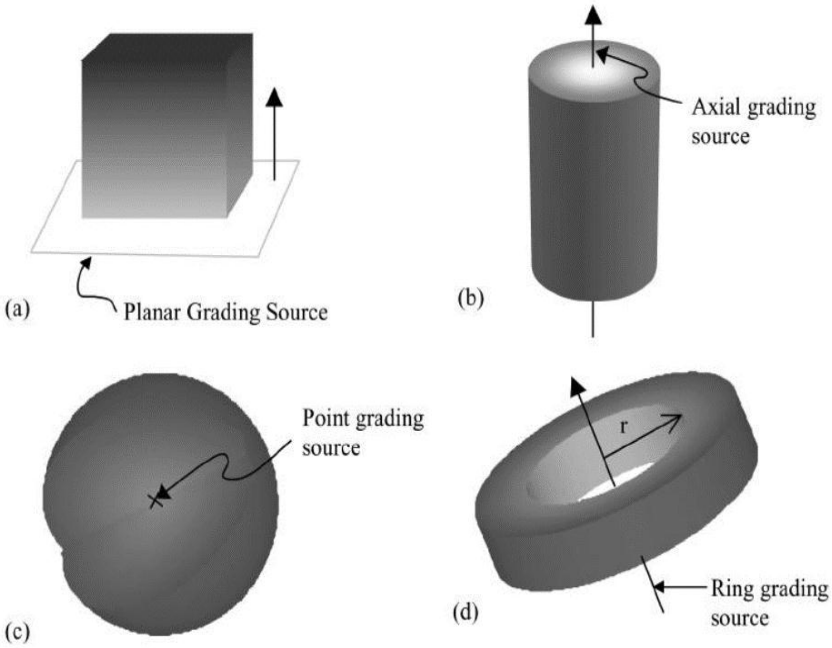

Figure: Examples of Material Grading in Functionally Graded Materials.

Thus FGM consist of a continuous variation of material properties from one surface to the other. The smooth transition through the various material properties reduces both thermal and residual stresses. In most cases the material progresses from a metal on one surface to a ceramic or mostly ceramic on the opposite surface, with a smooth transition throughout the center of the material.

The material transitions from a metal to a ceramic by increasing the percentage of ceramic material present in the metal until the appropriate percentage is reached or a pure ceramic is achieved.

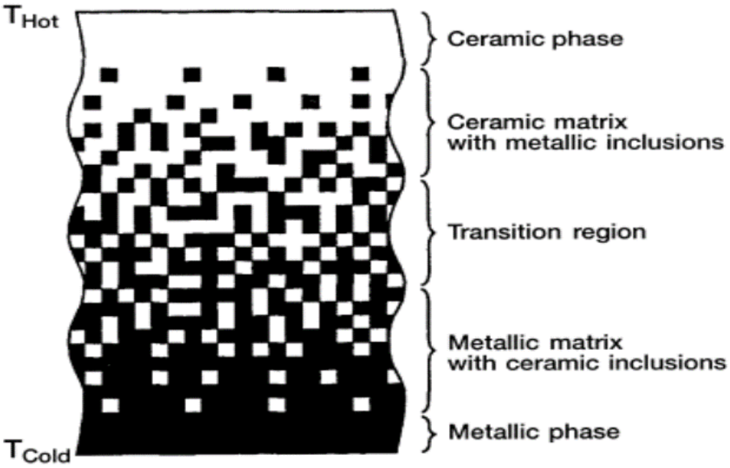

Figure: Graphical FGM Representation of Gradual Transition in the Direction of the Temperature Gradient.

Since the material does not have a dramatic change in material properties at any one point through the thickness, it would not cause a large stress concentration. This material usually exists where an extreme temperature gradient which is designated by Thot and Tcold is. The ceramic face of the material is generally exposed to a high temperature, while the metallic face is usually subjected to a relatively cooler temperature. The smooth transition of material properties allows for a material whose properties provide thermal protection as well as structural integrity reducing the possibilities of failure within the structure. This reduction of failure is of critical importance in space programs where thermal protection tiles are laminated to the metallic structure of the space shuttle to handle the extreme temperature during re-entry into the earth's atmosphere. These tiles are susceptible to cracking and debonding at the superstructure/tile interface due to abrupt transition between thermal expansion coefficients. The smooth transition between material properties reduces the potential cracking and debonding of thermal protection tiles laminated to structural members.

The capabilities of the FGM are quite flexible as one can vary the materials used as well as the function of composition throughout the material at which they transition from surface to surface. A specific metal and ceramic can be chosen for the particular application to capitalize on the positive characteristics of each of the materials. Also, the function between the two outside materials can be mathematically maximized and tailored specifically to meet the needs of the desired applications.

Functionally graded materials (FGM) are new advanced multifunctional composites where the volume fractions of the reinforcements phase vary smoothly. This is achieved by using reinforcements with different properties, sizes, and shapes, as well as by interchanging the functions of the reinforcement and matrix phases in a continuous manner.

Formulation of Governing Equations

The formulation generates axial stresses on functionally graded cantilever beam with and without the temperature consideration.

Beam Theory for Stress Calculations

Mechanical stress distribution has been determined for a three layered composite beam having a middle layer of functionally graded material (FGM), by analytical methods. Beam is subjected to uniformly distributed transverse loading with continuous and smooth grading of metal and ceramics based on P-FGM Law, E-FGM Law and S-FGM Law are considered for study and Poisson ratio is to be held constant across FGM layer. Analytical solution is based on simple Euler-Bernoulli type beam theory.

The dimension of FGM beam of width unity and thickness *h* are considered, where the material property varies continuously in thickness direction (z). FGM beams have their volume fraction of ceramics V_c defined according to the power law function, sigmoid law function and exponential law function and the volume fraction of metal V_m is obtained as

$$V_m(z) = 1 - V_c(z)V_m(z) = 1 - V_c(z)$$

The mathematical modeling for evaluating the properties of functionally graded materials (P (z)), is chosen from any of the three laws expresses as per the Equations 2.2, 2.7, 2.10-2.11.

P (z) =P_m+ (P_c-P_m) V_c (z)

Temperature Profile Modeling

When proposed FGM beam model is subjected to uniform temperature change (ΔT), the total strain under a small strain assumption, can be taken as made up of elastic and thermal part. For a beam under plane strain condition, the only non-zero stress component is $\sigma_x\sigma_x$ [51]:

$$\sigma_x = E(z)[\varepsilon_{x0}^T + z.k^T - \alpha(z)\Delta T]\sigma_x = E(z)[\varepsilon_{x0}^T + z.k^T - \alpha(z)\Delta T]$$

The coefficient of thermal expansion for FGM is obtained by rule of mixture

$$\alpha(z) = (\alpha_c - \alpha_m)(V_c) + \alpha_m$$

Neutral Surface Positioning

Neutral surface of functionally graded beam may not coincide with its geometric mid-surface, because of the material property variation through the thickness. In this section the position of neutral surface for functionally graded beam is obtained.

Consider a functionally graded cantilever beam of length *l*, width *b*, thickness *h*, with co-ordinate system *oxyz* having the origin o as shown in Figure 3.2. The beam is subjected to a uniformly distributed load *q* and the Young’s modulus E varies continuously in the thickness direction.

$$E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^n E(z) = E_m + (E_c - E_m)\left(\frac{2z+h}{2h}\right)^n$$

Neutral surface non-dimensional shift (h_0/h) in the proposed model.

$$\frac{h_0}{h} = \frac{(E_c - E_m)\frac{n}{(n+2)(2n+2)}}{E_m + \frac{(E_c - E_m)}{(n+1)}}\frac{h_0}{h} = \frac{(E_c - E_m)\frac{n}{(n+2)(2n+2)}}{E_m + \frac{(E_c - E_m)}{(n+1)}}$$

Deflection of Functionally Graded Beam

For a functionally graded cantilever beam, bending moment in equilibrium can be expressed as an integral in terms of internal stress:

$$M = \int_{-(\frac{h}{2} - h_0)}^{\frac{h}{2} - h_0} \sigma_{xx} z dAM = \int_{-(\frac{h}{2} - h_0)}^{\frac{h}{2} - h_0} \sigma_{xx} z dA \, w_{max} = -\frac{ql^4}{8D}$$

Thus analytical and theoretical formulation of stress and deflection calculation of proposed model is presented.

Performance Evaluation and Design Analysis of FGM Composite

The analysis of FGM beam under mechanical and thermal loading is performed using formulation presented in the previous chapter and the mathematical tool MATLAB is used for coding. The beam responses are compared with earlier studies. Also a study to determine the influence of neutral surface position on deflection of functionally graded beam under uniformly distributed loading is presented.

Problem Definition

A three layered composite system of Al_2O_3 – FGM – Steel is considered in which middle layer is FGM and other two are ceramic (Al_2O_3) and metal (steel). A cantilever beam of 0.5 m length made of the composite system of thickness 0.02 m is considered. The topmost material is ceramic (Al_2O_3) which has a thickness of 0.005 m and the bottom layer is metal (steel) of same thickness as ceramic. In between these two layers there is a FGM layer of 0.01 m and the beam has unit width. First, fixed-free boundary condition is considered and beam is subjected to a uniformly distributed loading (q = 100 kN/m) and there is no rise in temperature (ΔT = 0). Next, free-free boundary condition is considered and the effect of thermal loading is studied were beam is subjected to a temperature gradient (ΔT = 100^{o}C). Effect of temperature rise/fall is considered by augmenting the thermal strain to the mechanical strain.

CONCLUSIONS AND SUGGESTIONS

Functionally graded materials are good replacement of composite materials because they overcome the debonding type problems. These materials are commonly used in aerospace industries where the harsh temperature is major issue. The basic properties of FGM can be obtained by any of the three function laws, power law (P-FGM), sigmoid law (S-FGM) and exponential law (E-FGM). In the present work the axial stresses in FGM beam under uniformly distributed load and the residual stresses due to temperature drop during curing of the FGM composite or temperature rise in operation are calculated analytically. These results are found to be compared with the previous work. Results of dissertation conclude that the properties evaluation of FGM from exponential low is best than the other two law's and this material is basically used as base materials of thermal protection systems for space craft. Because the residual stresses generated in reentry of space craft in earth atmosphere is moderately reduced.

Nature of neutral surface shift is determined for variation of power law index and also the deflection of FGM beam with changing power law index is examined. When the power law exponent is increased the maximum deflection of the beam is increased.

Future Scope of Work

The following recommendations and future work is suggested.

- A further investigation regarding the techniques for estimating effective material properties of functionally graded materials is desirable. In the graded layer of real FGMs, ceramic and metal particles of arbitrary shapes are mixed up in arbitrary dispersion structures. Hence, the prediction of the thermo-elastic properties is not a simple problem, but complicated due to the shape and orientation of particles, the dispersion structure, and the volume fraction. This situation implies that the reliability of material-property estimations becomes an important key for designing a FGM that meets the required performance.
- The thickness of the middle FGM can be optimized.

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