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ESTABLISHING THE EQUATIONS THAT DETERMINE THE VIBRATION MOVEMENT OF THE SPINDLE IN LONGITUDINAL DIRECTION AT CNC LATHE

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ABSTRACT

The paper presents a mathematical model for analysis of vibration movements in case of CNC lathes. It is a known fact that vibrations represent an important component in determining the precision of a CNC machine. The longitudinal direction movement equations are established using Lagrange equations which combined with the transversal movement equations create a complex image of the dynamic response of the system which under limit conditions influences the precision of the CNC lathe. An important aspect is that the proposed model takes into account most of the relevant working hypotheses. The proposed equations describe at the highest possible level the vibration stability of the main spindle of the machine tool. Additionally, the proposed mathematical model has a high generality degree allowing it to be used for dynamic analysis of other types of CNC machine tools as well.

KEYWORDS: CNC, vibrations, Lagrange equations, movement equations, longitudinal direction, lathe.

INTRODUCTION

In case of turning operations, the cutting tool is exposed to dynamic stress because of the workpiece material deformation. The oscillation movements between tool and workpiece material play a decisive role in the processed surface roughness, determining the nature of the given turning operation.[1]

In order to establish the movement equations that describe the vibration of the main spindle of the lathe it is used a work model as presented in figure 1.

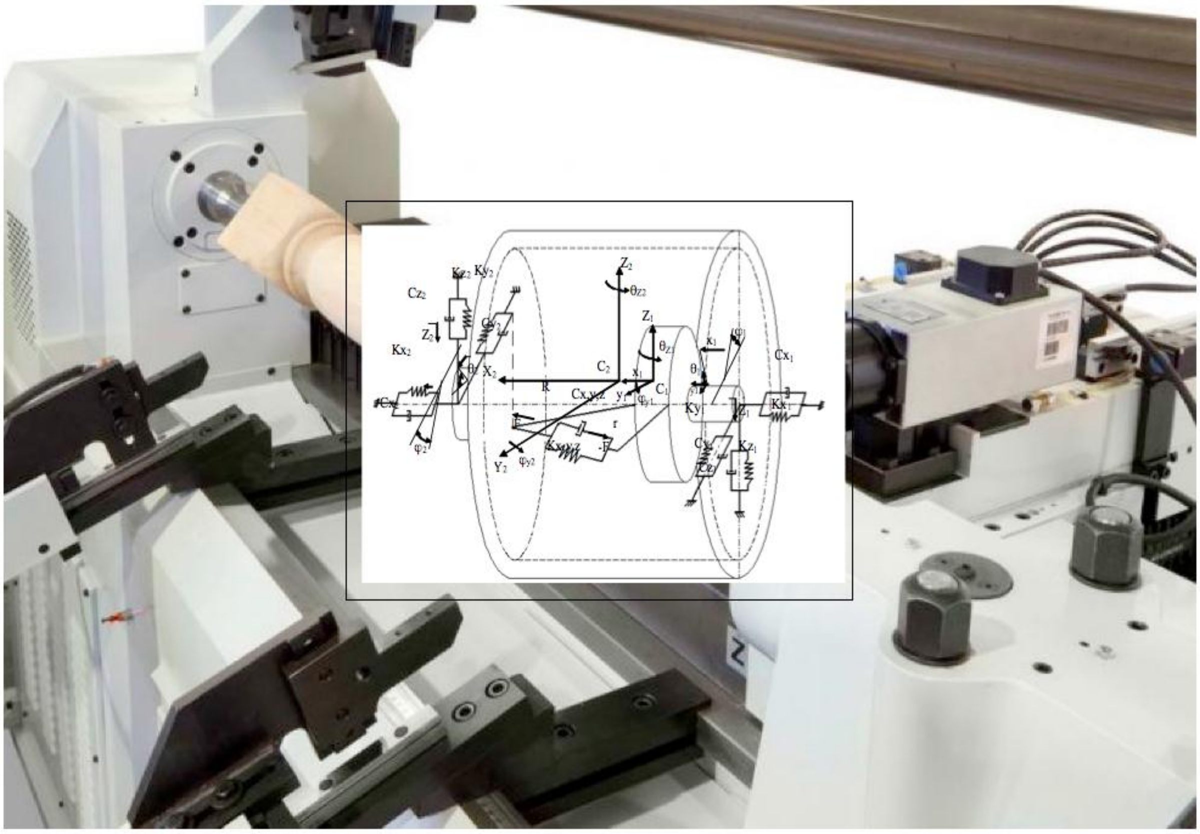

Figure (1): CNC lathe with dynamic model of internal turning operation.

The proposed dynamic model emphasizes the contact between tool and workpiece starting from Lagrange equations.[2] These provide the dependency relations between the components of the cutting force and the relative displacements between the tool and the workpiece.

The initial reference system is positioned with the origin on the tool axis, with X axis along the spindle axis. The Y axis crosses the contact point between tool and workpiece, with the Z axis is perpendicular on the plane defined by another two axes, forming a normal tri-orthogonal system.[3]

The designed model provides a very accurate image of the process. The model is aimed at determining the influence of the cutting speed on the vibration magnitude, and consequently, the influence on the finished surface quality and dimensional precision.[4]

The mathematical model for spindle vibration in stabilized regime is a partial derivative of the constant coefficients equation system.[5]

ESTABLISHING THE MATHEMATICAL MODEL

The model was established based on the following hypotheses:

- there are no distributed superficial forces or torques on the external surface of the spindle;
- during movement there are no additional strains or other links that could produce shocks;
- the initial spindle state is not tensioned (no remnant tension, only elastic strains);
- a plane section perpendicular on the spindle axis before deformation remains plane without remaining perpendicular;
- the transversal contraction effect is not taken into account;
- torsion vibration is ignored, its effect in the spindle rotation is irrelevant in relation with the general rotation;

Since the spindle deformations cannot be measured in its own reference system, the movement equations are projected on the axes of a fixed reference system. These are:

$$F_1 = k_{11}u_1 + k_{12}u_2 + k_{13}u_3 + k_{14}\theta_2 + k_{15}\theta_3$$
$$F_2 = k_{21}u_1 + k_{22}u_2 + k_{23}u_3 + k_{24}\theta_2 + k_{25}\theta_3$$
$$F_3 = k_{31}u_1 + k_{32}u_2 + k_{33}u_3 + k_{34}\theta_2 + k_{35}\theta_3$$
$$M_2 = k_{41}u_1 + k_{42}u_2 + k_{43}u_3 + k_{44}\theta_2 + k_{45}\theta_3$$
$$M_3 = k_{51}u_1 + k_{52}u_2 + k_{53}u_3 + k_{54}\theta_2 + k_{55}\theta_3$$

Where k_{ij} represents the bearing rigidity ($i, j = 1\ldots5$).

Observation: k_{11} represents the longitudinal force in the x_1 direction produced by the δ_1 longitudinal deformation.

Because of the symmetry we have $k_{11} = k_{13}$; $k_{14} = k_{15}$; $k_{22} = k_{13}$; $k_{24} = k_{23}$; ... $k_{21} = k_{31}$; $k_{41} = k_{51}$; ... $k_5 = k_{43}$.

Under null initial conditions $\theta_2 = -u_{3,1}$; $\theta_3 = -u_{2,1}$.

The movement equations become:

$$\rho A u_1 - EAu_{1,11} = 0$$

$$\rho A\ddot{u}_3 - \rho I\ddot{u}_{3,11} + 2\rho I\Omega\dot{u}_{2,11} + \rho I\Omega^2 u_{3,11} + EIu_{3,1111} = 0$$

$$-\rho A\ddot{u}_2 + \rho I\ddot{u}_{2,11} + 2\rho I\Omega\dot{u}_{3,11} - \rho I\Omega^2 u_{2,11} - EIu_{2,1111} = 0$$

$$s^2\rho A\overline{u}_1 - EA\overline{u}_{1,11} = 0$$

$$\overline{u}_1\,(s;x_1) = C_1 s^{s\sqrt{\frac{\rho}{E}}x_1} + C_2 s^{-s\sqrt{\frac{\rho}{E}}x_1}$$

$$\overline{u}_1^{(0)}(s;x_1) = C_1^0 s^{s\sqrt{\frac{\rho}{E}}x_1} + C_2^0 s^{-s\sqrt{\frac{\rho}{E}}x_1}$$

$$\bar{u}_1^{(1)}(s;x_1) = C_1^1 s^{s\sqrt{\frac{\rho}{E}}x_1} + C_2^1 s^{-s\sqrt{\frac{\rho}{E}}x_1}$$

$$\bar{u}_1^{(2)}(s;x_1) = C_1^2 s^{s\sqrt{\frac{\rho}{E}}x_1} + C_2^2 s^{-s\sqrt{\frac{\rho}{E}}x_1}$$

$$EA\bar{u}_{1,1}^{(0)} = 0$$

$$\left[EA\bar{u}_{1,1}\right]_{x_1=l_1^-}^{x_1=l_1^+} = \left[k_{11}\bar{u}_1\right]_{x_1=l_1}$$

$$\bar{u}_1^{(0)}(l_1) = \bar{u}_1^{(1)}(l_1)$$

$$\left[EA\bar{u}_{1,1}\right]_{x_1=l_2^-}^{x_1=l_2^+} = \left[k_{11}\bar{u}_1\right]_{x_1=l_2}$$

$$\bar{u}_1^{(1)}(l_2) = \bar{u}_1^{(2)}(l_2)$$

$$EA\bar{u}_{1,1}^{(2)}(l) + m_p s^2 \bar{u}_1^{(2)}(l) = R_1^f$$

$$C_1^0 - C_2^0 = 0$$

$$\left(-EAs\sqrt{\frac{\rho}{e}}e^{s\sqrt{\frac{\rho}{E}}l_1} - k_{11}e^{s\sqrt{\frac{\rho}{E}}l_1}\right)C_1^0 + \left(EAs\sqrt{\frac{\rho}{e}}e^{-s\sqrt{\frac{\rho}{E}}l_1} - k_{11}e^{-s\sqrt{\frac{\rho}{E}}l_1}\right)C_2^0 + EAs\sqrt{\frac{\rho}{e}}e^{s\sqrt{\frac{\rho}{E}}l_1}C_1^1 - EAs\sqrt{\frac{\rho}{e}}e^{-s\sqrt{\frac{\rho}{E}}l_1}C_2^1 = 0$$

$$e^{s\sqrt{\frac{\rho}{E}}l_1}C_1^0 + e^{-s\sqrt{\frac{\rho}{E}}l_1}C_2^0 - e^{s\sqrt{\frac{\rho}{E}}l_1}C_1^1 - e^{-s\sqrt{\frac{\rho}{E}}l_1}C_2^1 = 0$$

$$\left(-EAs\sqrt{\frac{\rho}{e}}e^{s\sqrt{\frac{\rho}{E}}l_2} - k_{11}e^{s\sqrt{\frac{\rho}{E}}l_2}\right)C_1^1 + EAs\sqrt{\frac{\rho}{e}}e^{s\sqrt{\frac{\rho}{E}}l_2}C_1^2 + \left(EAs\sqrt{\frac{\rho}{e}}e^{-s\sqrt{\frac{\rho}{E}}l_2} - k_{11}e^{-s\sqrt{\frac{\rho}{E}}l_2}\right)C_2^1 - EAs\sqrt{\frac{\rho}{e}}eC_2^2 = 0$$

$$e^{s\sqrt{\frac{\rho}{E}}l_2}C_1^1 + e^{-s\sqrt{\frac{\rho}{E}}l_2}C_2^1 - e^{s\sqrt{\frac{\rho}{E}}l_2}C_1^2 - e^{-s\sqrt{\frac{\rho}{E}}l_2}C_2^2 = 0$$

$$\left(EAs\sqrt{\frac{\rho}{e}}e^{s\sqrt{\frac{\rho}{E}}l} + m_p s^2 e^{s\sqrt{\frac{\rho}{E}}l}\right)C_1^2 + \left(-EAs\sqrt{\frac{\rho}{e}}e^{-s\sqrt{\frac{\rho}{E}}l} + m_p s^2 e^{-s\sqrt{\frac{\rho}{E}}l}\right)C_2^2 = R_1^f$$

$$C_1^0 = \frac{m_1}{m}; C_1^1 = \frac{m_3}{m}; C_2^1 = \frac{m_5}{m}$$

$$C_2^0 = \frac{m_2}{m}; C_1^2 = \frac{m_4}{m}; C_2^2 = \frac{m_6}{m}$$

$$m_1 = 4R_1^f EA^2 \rho s^2$$

$$m_2 = 4R_1^f EA^2 \rho s^2$$

$$m_3 = 2R_1^f As\sqrt{\rho e}\left(k_{11}e^{-2s\sqrt{\frac{\rho}{E}}l_1} + k_{11} + 2EAs\sqrt{\frac{\sigma}{e}}\right)$$

$$m_4 = -2R_1^f As\sqrt{\rho e}\left(k_{11}e^{2s\sqrt{\frac{\rho}{E}}l_1} + k_{11} - 2EAs\sqrt{\frac{\sigma}{e}}\right)$$

$$m_5 = -R_1^f\left[k_{11}^2\left(e^{-2s\sqrt{\frac{\rho}{E}}l_2} - e^{-2s\sqrt{\frac{\rho}{E}}l_1} + e^{2s\sqrt{\frac{\rho}{E}}(l_1-l_2)} - 1\right) - \right.$$

$$\left. - 2k_{11}EAs\sqrt{\frac{\rho}{e}}\left(e^{-2s\sqrt{\frac{\rho}{E}}l_2} + e^{-2s\sqrt{\frac{\rho}{E}}l_2}\right) - 4k_{11}EAs\sqrt{\frac{\rho}{e}} - 4EA^2\rho s^2\right]$$

$$m_6 = -R_1^f\left[k_{11}^2\left(1 + e^{2s\sqrt{\frac{\rho}{E}}l_1} - e^{2s\sqrt{\frac{\rho}{E}}l_2} - e^{2s\sqrt{\frac{\rho}{E}}(l_2-l_1)}\right) - 2k_{11}EAs\sqrt{\frac{\rho}{e}}\left(e^{-2s\sqrt{\frac{\rho}{E}}l_1} + e^{-2s\sqrt{\frac{\rho}{E}}l_2}\right) - 4k_{11}EAs\sqrt{\frac{\rho}{e}}\right]$$

CONCLUSIONS

Establishing the equations that generate the vibration of the main spindle at the CNC lathe is necessary for the stability analysis of the oscillation movement.

These are based on the general equations of the vibration movement, taking into account the impulse derivative axiom and the kinetic moment derivative.[6, 7]

The presented mathematical model provides a general framework which can be used for dynamic analysis in case of other CNC machine tools.[8, 9]

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