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PRIMAL-DUAL INTERIOR-POINT TECHNIQUE FOR OPTIMISATION OF 330KV POWER SYSTEM FOR TWO VARIABLES

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ABSTRACT

The work handles a method of optimisation of 330KV power system load flow per excellence. This method is called, PRIMAL-DUAL INTERIOR-POINT TECHNIQUE and it is used in solving optimal load flow problems. As load-sheddings, power outages and system losses have been cause for worries, especially among the developing nations such as Nigeria, hence a need for a more functional load flow solution technique, which, this work addresses. Optimisation is achieving maximum of required and minimum of un-required and it is obtained mathematically by differentiating the objective function with respect to the control variable(s) and equating the resulting expression(s) to zero. This work developed a mathematical model that solves load flow problems by engaging non-negative PRIMAL variables, "S" and "z" into the inequality constraint of the load flow problems in other to transform it to equality constraint(s). Another non-negative DUAL variables "π" and "v" are incorporated together with Lagrangian multiplier "λ" to solve optimisation. While solving optimization, Barrier Parameter "μ" which ensures feasible point(s) exist(s) within the feasible region (INTERIOR POINT). Damping factor or step length parameter "α", in conjunction with Safety factor "γ" (which improves convergence and keeps the non-negative variables strictly positive) are employed to achieve result. The key-words which are capitalized joined to give this work its name, the PRIMAL-DUAL INTERIOR-POINT. The initial feasible point(s) is/are tested for convergence and where it/they fail(s), iteration starts. Variables are updated by using the computed step size ΔY and the step length parameter "α",

which thereafter, undergo another convergence test. This technique usually converges after first iteration. Primarily, this technique excels the existing methods as; it solves load flow problems with equality and inequality constraints simultaneously, it often converges after first iteration as against six or more iterations of the existing methods for one variable objective, for two variables, the iteration number is very few compared to existing method. Its solution provides higher power generations from available capacity and minimum system loss as example, Geregu Power Station on Bus 12 where, result shows 89.3% generation as against 60% of existing methods. Generation loss is 1.8% as against 80.3% of existing methods and availability loss of 12.5% as against 88.2% of existing. Therefore this method ensures very high system stability.

INTRODUCTION

1.1 Background to the Study

The first known interior-point (i.p) method is attributed to frisch of 1955, which is a logarithmic barrier method of wright, of 1957 that was later in 1960s extensively studied by fiacco and mc cormick, of 1968 to solve non-linear inequality constrained problems (irisari et al, 1984 torren and quintina, 2001 granville,2007). The greatest break-through in ip research took place in1984, when karmarka came up with a new ip method, reporting solution times of up to 50 times faster than the simplex method. karmarka's algorithm is based on non-linear projective transformations. After 1984, several variants of karmarka's ip method have been proposed and implemented with primal-dual method being proposed, developed and waiting for implementation since its algorithm proved to perform better than earlier ip algorithms.

1.2 Problem statement since problems that occur in 330kv power system if not handled with dispatch and swiftness, result in total blackout with often catastrophic effects. As the existing conventional methods to solution of Optimal Load Flow problems are time consuming, giving rooms for blackouts.

1.2 AIM of this work is to obtain optimisation of 330KV power system by using **Primal Dual Interior Point** technique.

The **Objectives** include;

- To obtain solution for optimisation of Load Flow problems on 330KV power system that converges at first iteration.

- To be able to solve load flow problems with equality and inequality constraints simultaneously by engaging non-negative PRIMAL variables into inequality constraint in other to become equality constraints.
- To develop a technique whose solution results in generation of about 90% of power station's available capacity.
- To obtain solution that provides minimum running cost of system.
- To articulate all the technique's variables, such as Primal variables, Dual variables and the independent variable(s), parameters, such as Barrier parameter and Step length parameters and constants, such as Centering parameter, Safety factor and others to achieve the above mentioned objectives.
- To sensitise power industries the need to adopt the technique in their operations as reliability,security, stability and efficiency are guaranteed.

1.4 Scope of The Research Work Since the thesis is universal, its utility transcends Nigerian boundaries hence Nigeria 330 KV Network system study is adopted. Useful data are obtained internally and externally and from the existing conventional methods.

1.5 Justification For Study As Engineering research works are aimed at advancement of technology and moving the system over to the next level, the new technique has faster solution time, fewer iterations and handles both equality and inequality constraints problems simultaneously.

1.6 Limitations Of The Research The technique, the Primal-Dual Interior-Point Technique is globally utilisable, but inability of accessing enough foreign materials affected the work's 100% success.

1.7 Motivation For The Research Study analysis of the data collected from November 2012 to October 2018 from Discos,TCN and Gencos on 330kV, 52 bus power system network reveal disturbing facts, hence the research to develop best operational method to achieve optimum, continuous, higher system stability and reliable service to the consumers hence this technique.

Relasted Work

2.1 Optimisation Based On Economic Operation of Power System

Consideration is made so that power system is operated as to supply all the (complex) loads at minimum cost. Often total load is less than the available generation capacity (Fliscounakis et al, 2013) and so there are many possible generation assignment,(Hiyama, 1982), but when there is peak load/demand for power, it means, all the available generation capacity is used resulting in no option. During options, power generation (PGi) is picked to minimise cost of production while satisfying load and the losses in the transmission system (Capitanescu et al, 2012) min $C(PG)= \alpha+\beta PG+\gamma PG^2$. Optimal economic dispatch may require that all the power be imported from neighboring utility through a single transmission system (Street et al, 2014). Also, it is noted that, small variations in demand are taken care of by adjusting the generations already on line, while large variations are accommodated basically by starting up generator units when the loads are on the upswing and shutting down when the loads decrease (Mao and Iravani, 2014). Although the problem is complicated by considering the long lead time required (6-8 hours) for preparing a "cold thermal unit for service", (Colombo and Grothey, 2013). To avoid the cost of start-up or shut-down, there is a requirement that enough spare generation capacity (spinning reserve) be available on-line in the event of a random generator failure (Awosope, 2003).

2.2 Optimisation Based On Minimum Mismatch Method

Generally, load flow equation of an N-bus network can be expressed as:

$S = P + jQ = V^T I^* = V^T (YV)^*$ (Kamel et al, 2013) (2.1)

Where:- "S" is the power injection vector

"I" is the current injection vector

"V" is the bus voltage vector and;

"Y" = G + jB is the system admittance matrix.

All the above quantities are complex, except P and Q which are real and imaginary parts of S. Because of non-linearity of load flow equations, several mathematical solutions exist, giving rise to non-uniqueness in the load flow calculations, with only one of the solutions with the minimum system losses and acceptable high voltages, as low voltage may correspond to unstable operation, is taken (Wu, et al 2010).

2.3 Optimisation Based On Fast Decoupled Load Flow Method

This is a modification of the Newton-Raphson (NR) technique which takes advantage of the weak coupling between the real and reactive power (Bhowmick et al, 2008) with two constant matrices used to approximate and decouple the Jacobian Matrix(Song and Cai, 2013).

2.4 Optimisation Based On Second Order Load Flow (Solf) Method

Load flow equation, with variables defined in rectangular form for nodal real and reactive power mismatches. (Ferreira and Dacosta, 2005)

$$P_i = \sum_{i=j}^{N} (ei\ ej\ Gij - ei\ ef\ Bij + fifj\ Gij + fi\ ej\ Bij),$$

$$Q_i = \sum_{i=j}^{N} (fi\ fj\ Gij - fi\ fj\ Bij - eifj\ Gij - ei\ ej\ Bij), \quad (2.2)$$

$Ei^2 = ei^2 + fi^2$ Where Ei is modulus of ith bus Voltage. (2.3)

2.5. Optimisation Based On Mathematical Model of Primal-Dual Interior-Point Technique

Min f(x)

Such that g(x) = 0 (2.4)

$\underline{H} \leq h(x) \leq \hat{h}$

$x \in IR^n$ is a vector of decision variable including control and non-functional dependent variable,

f: IR^n –IR is a scalar function representing the power system operation optimisation goal.

g: IR^n –IR^m is a vector function representing the conventional power flow equation and other equality constraints.

h: IR^n –IR^p is a vector of functional variables with lower bound $\underline{h}$ and upper bound $\hat{h}$ representing the operating limits on the system.

It is assumed that f(x), g(x) and h(x) are twice continuously differentiable. Since the above problem minimises f(x) subject to h(x) $\geq$ 0. The objective is to obtain a feasible point X. that attains the desired (Chiang and Grothey, 2014. Farivar and Low, 2013, Gan, et al 2015)

2.5.1 Greek Alphabets Used and Their Meanings in Primal-Dual Interior-Point Technique.

"s" and "z" (small and big zeta) **PRIMAL VARIABLES** are non-negative slack vectors**,** for transforming inequality constraint(s) to equality constraint(s) **"slack"** means loosely attached, **"Primal"** means **basic.**

"π" "v" and (small letter pi, and nu) are non-negative Lagrangian vector called **Dual Variables**. They are vector multipliers incorporated with **"λ" (lambda)** the **lagrangian multiplier** to help Primal Variables solve the emerged equality constraints for optimisation.**"Dual"** means joint action.

"μ"(small letter mu) is a Barrier parameter or Complimentary Gap which is incorporated to ensure.

that the feasible point(s) exist(s) within the Feasible region (**Interior-Point**).

"Ω"(Omega) is centering parameter used with **"ρ" (Rho),** the confining parameter in computing **'μ"**

"γ"(gamma) is **safety factor** that ensures, next point satisfies positivity condition, used in computing step lengths (**damping factor) "α"** that improve convergence and keep non-negative variables strictly positive. The constants **"γ" and "Ω"** stand for personnelemolument in the system

s, z, π and v are variables for static var compensators and **FACTS** (flexible ac transmission system).

METHODOLOGY

3.1 Transforming Inequality Constraint to Equality Constraints

Transformation of (2.4) is done (Yang, et al 2016) by incorporating non- negative slack vectors 's' and 'z' into the inequality constraint $\underline{h} \leq h(x) \leq \hat{h}$, imposing strict positivity conditions on those slacks by incorporating them into logarithmic barrier terms as follows;

Min f(x)

Subject to g(x) = 0

$-s - z + \hat{h} - h = 0$

$-H(x) - z + \hat{h} = 0$

Into logarithmic barrier term as

$$\text{Min } f(x) - \mu^k \textstyle\sum_{i=0}^{p} (insi + inzi) \tag{3.1}$$

Subject to g(x) = 0

$-S - z + \hat{h} - h = 0$

$-H(x) - z + \hat{h} = 0$

"S" $\geq$ 0; "z" $\geq$ 0

Where, k is the iteration count or number and p the number of interconnected systems. Solving these equality constraints (Wu, et al 2012, and Ling, 2007), we apply vectors of lagrangian multipliers called Dual-Variables "λ" "$\hat{\pi}$" and "v" together with the Newton method

$L\mu(y) = f(x) - \mu^k\, \Sigma(\text{in } si + \text{in } zi) - \lambda^T gx - \pi^T (s - z + h - \underline{h}) - V^T (-h(x) - z + h)$. **(3.2)**

3.2 Optimality Conditions

A local minimiser of (3.1) is expressed in terms of stationary point of Lx (y) satisfying the Karush- Kuhn Tucker (KKT) optimality conditions for the NLP problem (2.10) (Torren and Quintana, 2001) as

$$\nabla y1(y) = \left\{ \begin{array}{l} s\,\hat{\pi} \\ zv \\ s + z - \hat{h} + \underline{h} \\ h(x) + z - \hat{h} \\ \nabla x f(x) - Jg(x)^T \lambda + Jh(x)^T v \\ -\, g(x) \end{array} \right\} = 0 \qquad \textbf{(3.3)}$$

$\hat{V} = v + \pi$ for simplification

Where l or L is local minimiser

Strict feasibility starting point is not mandatory for Primal Dual Interior Point technique but the condition (s, z)>0 and (π, v) >0 must be satisfied at every point in order to define the barrier term (Capitanescu and Wehenkel, 2012). The algorithm terminates when the Primal and Dual infeasibilities and the complementary gap fall below pre-determined tolerance otherwise, with (s, z)>0 and (π, v) > 0 a new estimate y^k is computed using one step of Newton method to find zeroes (the roots) of the NL functions.

3.3 Estimating New Point (Y^K)

3.3.1 Computing Newton Direction or Step Size ΔY

The Newton direction is obtained by solving. Newton method (Tinney and Hart, 2007) with large sparse coefficient matrix (Geletu et al, 2011), with step size column matrix as below (Molzahn et al, 2013):-

$$\begin{Bmatrix} \pi & 0 & s & 0 & 0 & 0 \\ 0 & v & z & z & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & Jh & 0 \\ 0 & 0 & 0 & Jh^T & \nabla^2_x l\mu & -Jg^T \\ 0 & 0 & 0 & 0 & -Jg & 0 \end{Bmatrix} \begin{Bmatrix} \Delta s \\ \Delta z \\ \Delta\pi \\ \Delta v \\ \Delta x \\ \Delta\lambda \end{Bmatrix} = \begin{Bmatrix} rs \\ rz \\ r\pi \\ rv \\ rx \\ r\lambda \end{Bmatrix} \quad \textbf{(3.5)}$$

(Tinney and Hart, 2007).

$$\left.\begin{aligned} rs &= -s\pi + \mu^k e \\ rz &= -z\hat{v} + \mu^k e \\ r\pi &= -s - z + \hat{h} - \underline{h} \\ rv &= -h(x) - z + \hat{h} \end{aligned}\right\} \quad \textbf{(3.6)}$$

$$rx = -\nabla x\, f(x) + Jg\,(x)^T\lambda - Jh\,(x)^T v$$

$$r\lambda = g(x)$$

Where, $\nabla^2_x l\mu$ is the combination of Hessians of objective and constraints functions. $\nabla^2_x l\mu(y) = \nabla^2_x f(x) - \nabla^2_x gj(x)\lambda j + \nabla^2_x hj\,(x)\, vj$ (3.7)

Where "l" is local minimiser a function of differentiation, $\nabla^2_x f(x)$ is the Hessian or Second differentiation of objective function w.r.t.x, $\nabla^2_x g(x)$ is the Hessian or Second differentiation of equality constraint function w.r.t.x, $\nabla^2_x h\,(x)$ is the Hessian or Second differentiation of inequality constraint function w.r.t. x, $\nabla_x f(x)$ is the first differentiation of objective function w.r.t.x, Jg(x) is the first differentiation or Jacobian value of equality constraint w.r.t.x. Jh(x) is the first differentiation or Jacobian value of inequality constraint w.r.t.x. In the computation of ΔY, factorisation of the coefficient matrix (3.5) is much more expensive than the forward and backward solutions that follow factorisation.

3.3.2 Computing Step Length Parameter (α)

The **scalars $\alpha^k{}_P$€ (0,1) and $\alpha^k{}_D$€ (0,1) are step length parameters** called **damping factor.** They improve convergence and keep non-negative variables strictly positive. k is the iteration counts.

$$\left.\begin{aligned} \alpha^k{}_P &= \min [1, \gamma \min\{-si^k/\Delta si/\Delta si<0, -zi^k/\Delta zi/\Delta zi<0\}] \\ \alpha^k{}_D &= \min[1, \gamma \min \{-\pi i^k/\Delta\pi i/\Delta\pi i<0, -vi^k/\Delta vi/\Delta Vi<0\}] \end{aligned}\right\} \quad \textbf{(3.8)}$$

The **scalar γ (0,1) is a safety factor** which ensures that the next point will satisfy the strict positivity conditions; typical constant values, γ^O = 0.25. γ^k = 0.99995.

3.3.3 Updating Variables

3.3.3.1Updating control variable(s) and primal variables

$$\left.\begin{aligned} X_1{}^k &= X_1{}^{k-1} + \alpha_p{}^k \Delta X_1{}^{k-1} \\ X_2{}^k &= X_2{}^{k-1} + \alpha_p{}^k \Delta X_2{}^{k-1} \end{aligned}\right\} \text{2 control variables}$$

$$S^k = S^{k-1} + \alpha_p{}^k \Delta S^{k-1}$$

$$Z^k = Z^{k-1} + \alpha^k{}_p \Delta Z^{k-1} \quad \textbf{(3.9)}$$

3.3.3.2 Updating dual variables and lagrange multiplier

$$\pi^k = \pi^{k-1} + \alpha^k{}_D\Delta\pi^{k-1}$$

$$V^k = V^{k-1} + \alpha^k{}_D V^{k-1}$$

$$\lambda^k = \lambda^{k-1} + \alpha^k{}_D\Delta\lambda^{k-1}$$

3.4 Reducing the Barrier Parameter (μ^k)

The **scalar μ^k** is the **barrier parameter** or **complementary gap** which ensures the feasible point X exist within the feasible region (Lage et al, 2009) and it is obtained by

$$\mu^{k+1} = \Omega^k \rho^k \quad \textbf{(3.10)}$$

Where $\mathbf{\Omega^k}$ is chosen = max ($0.99\Omega^{k-1}/2$; 0.1) and it is called the **Centering Parameters**

With Ω^0 = (0.2 fixed) and μ^O = (0.1 fixed)

$$\rho^k = (S^k)^T\pi^k + (Z^k)^T V^k \quad \textbf{(3.11)}$$

μ^k is computed first, only if iteration (1) fails, then μ^1 and Y^1 is used to form iteration (2) as Y^0 and μ^0 (given) are used to form iteration (1)

3.5 Testing For Convergence

Interior-Point (IP) Iterations Are Considered Terminated Whenever

$$V_1{}^k = \max [\max\{h-h(x); h(x) - \hat{h} \}, \|g(x)\|\infty],$$

$V_2^k = \frac{\|\nabla x f(x) - Jg(x)^T \lambda + Jh(x)^T V\|_\infty}{1+\|x\|2+\|\lambda\|2+\|V\|2}$

Since$\|\lambda\|_2$ & $\|V\|_2$ are vectors of lagrangian multipliers,

they have no vector addition and so denominator reduces to 1 + $\|x\|_2$

(3.12)

$V_3^k = \frac{\rho^k}{1+\|x\|2}$

$V_4^k = \frac{|f(x^k) - f(x^{k-1})|}{1-|f(x^k)|}$

Typically, V_1^k and $V_2^k \le \xi_1 = 10^{-4}$, or $\|g(X^k)\|:< \xi_1$

V_3^k and $V_4^k \le \xi_2 = 10^{-2}\xi_1$ (i.e. 10^{-6}) , or $\|\Delta X\|:< \xi_2$,

$\mu^k \le \xi\mu$ or $\xi_x = 10^{-12}$, **is satisfied**

Generally, $\xi_1 = 10^{-8}$ is chosen for quadratic functions with 2 variables.
If V^k_1, V^k_2 and V^k_3 are satisfied, then primal feasibility, scaled dual feasibility and complementary condition are satisfied which means that iterate K is a Karush Khun Turker (KKT) point of accuracy.

When numerical problems prevent verifying this condition, the algorithm stops as soon as feasibility of the equality constraint is achieved along with a very small fractional change in the objective value and negligible changes in the variables. The typical tolerances are **$\xi_1 = 10^{-4}$, $\xi_2 = 10^{-2}\xi_1$ and $\xi\mu = 10^{-12}$.**

3.6 Primal-Dual Interior-Point Technique Numerical Algorithms

Step 0: (Initialisation)

Set K = 0, define μ^0 and choose a starting point Y^0 that satisfies the strict positivity conditions.

Step 1: (Compute Newton Direction)

Form the Newton System at the current point and solve for the Newton Direction.

Step 2: (Update Variables)

Compute the step lengths in the Newton direction and update the primal and dual variables.

Step 3: (Test for Convergence)

If the new point satisfies the convergence criteria, stop. Otherwise, set K = K + 1, update the barrier parameter μ^k and return to step 1.

3.6.1 Implementation of the Algorithms of Primal-Dual Interior–Point Technique

3.6.1.1 Step Zero (0), Choosing an initial point

Starting point needs only to meet the strict positivity conditions, IP method performs better if some initial heuristics are used, for instance, X^0 is between the upper and the lower limits of the bounded variables.

3.6.1.1.1 Initial point for two variables with quadratic inequality constraint $\underline{h} \leq h(x) \leq \hat{h}$)

$$X^0 = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \hat{h} - \underline{h}$$

X^0_i are tested by substituting them into h (x) without considering constant term.

Example, if h (x) is, $1 \leq X_1^2 + X_2^2 - 6x_1 - 2x_2 + 10 \leq 4$,

Heuristically picking X_1 as 5 and X_2 as 4 and substituting them into above inequality ignoring constants gives 3 equals the range (4 - 1) of the inequality.

$$X^0 = \begin{Bmatrix} 5 \\ 4 \end{Bmatrix} = 3$$

3.6.1.1.2 Initialising primal slack variables (S^0 and Z^0)

$S^0 = \min [\max \{\gamma^o h^\Delta, h(X^0) - h \min\}, (1-\gamma^0) h^\Delta]$

$S^0 = \min [\max \{0.25h^\Delta, h(X^0) - h \min\}; 0.75h^\Delta]$

Where: $h^\Delta = h \max - h \min$

$\gamma^0 = 0.25$ **(3.13)**

$1 - \gamma^0 = 0.75$

H (X^0) = values of X^0 including constant

$Z^0 = h^\Delta - S^0$

3.6.1.1.3 Initialising dual variables (π^0, V^0)

$\pi^0 = \mu^0 (S^0)^{-1}$ e(e is diagonal I of matrix)

$\pi^0 = 0.1(S^0)^{-1}$ **(3.14)**

$V^0 = \mu^0 (Z^0)^{-1}$ e - π^0

$\lambda^0 = 0$ (since the power balance of steady state system is passive).

Convergence of the initial point is tested and if it fails then:

3.6.1.2 Step one (1), Computing Newton direction ΔY

With μ^o defined and initial point Y^o obtained; Newton method (3.5), is formed and Newton direction computed with (3.6) and (3.7) of (3.5)

3.6.1.2 .1 For two variables with quadratic constraint

With μ^o defined and initial point Y^o obtained; Newton method (3.5), formed and Newton direction computed with (3.6) and (3.7) of (3.5).Factorisation starts from row 3 of (3.5) where Δs is substituted for -Δz and applied to row 1, then row 2, row 4 and with row 4, row 5 and row 6 are factorised and finally row 6 and row 7 are simultaneously factorised to obtain ΔX_1 and ΔX_2 before backward substitutions.

3.6.1.3 Step two (2), Updating variables (Y^k) with step length parameter "α" (3.8). $Y^{1} = Y^{O} + \alpha \Delta Y^o$

Newton direction ΔY is computed from (3.5) and variables are updated from (3.9)

3.6.1.4 Step three (3), Testing for convergence

If the new point satisfies the convergence criteria, stop. Otherwise, set K= K + 1, update the barrier parameter μ^k and return to step 1.

Result and Analysis

4.1 min $x_1^2 + x_2^2 - 4x_1 - 8x_2 + 20$ (Nagrath and Kothari, 2010) subject to $x_1^2 + x_2^2 - 2x_1 - 2x_2 - 2 = 0$ $1 \leq x_1^2 + x_2^2 - 6x_1 - 2x_2 + 10 \leq 4$

Step 0: Initialisation

Heuristically X_1 and X_2 are chosen as follows:

$\hat{h} - \underline{h} = h(x)$

$4 - 1 = 3 = h(x)$

To choose (X_1, X_2) to give 3 we have

$X^0 = \begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

s^0 = 1min [max {0.25 x 3, 13-1}, 0.75 x 3]

= min [12, 2.25]

s^0 = 2.25

Note after obtaining x^0 from h(x) as 3, subsequent h(x) includes the constant term in the inequality, which is 0 to give 13.

$z^0 = h^\Delta - s^o$, i.e 3-2.25=0.75

$\pi = \mu(S^0)^{-1}e$ *where e = diagonal 1*

$= 0.1\ (2.25)^{-1}$

= 0.0444

$v^0 = \mu(Z^0)^{-1}e - \pi^0$

v^0 = 0.1333 – 0.0444

= 0.0889

$\mu(Z^0)^{-1}e = 0.1333$

λ^0 = 0 (for passive power balance) i.e steady state condition.

$\mu^0 = 0.1$, $\Omega^0 = 0.2$ (fixed), $\gamma^0 = 0.25$ and other $\gamma^k = 0.99995$, $E = 10^{-8}$ other $\Omega^k = 0.1$

$\nabla_x f(x) = \begin{matrix} 2x_1 - 4 \\ 2x_2 - 8 \end{matrix}$; $\nabla_x^2 f(x) = \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$

$Jg(x) = \{2x_1 - 2; 2x_2 - 2\}$; $\nabla_x^2 g(x) = \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$

$Jh(x) = \{2x_1 - 6; 2x_2 - 2\}$; $\nabla_x^2 h(x) = \begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$

$\nabla^2_x l\mu^0 = \begin{matrix} 2.17778 & 0 \\ 0 & 2.17778 \end{matrix}$

Where $\nabla^2_x l\mu = \nabla^2_x f(x) - \sum_{j=1}^{m} \lambda_j \nabla^2 xgj(x) + \sum V_j \nabla^2 xhj(x)$

Therefore, Y^0 is initialised as

$$Y^0 = \begin{pmatrix} s^0 \\ z^0 \\ \pi^0 \\ v^0 \\ x_1^{\,0} \\ x_2^{\,0} \\ \lambda^0 \end{pmatrix} = \begin{pmatrix} 2.2500 \\ 0.7500 \\ 0.0444 \\ 0.0889 \\ 5.0000 \\ 4.0000 \\ 0.0000 \end{pmatrix} \qquad \begin{matrix} f(x^0) = 9 \\ g(x^0) = 21 \\ h(x^0) = 13 \end{matrix}$$

Testing for convergence,

$V_1^{\,0} = 21 > 10^{-8}$

$V_2^{\,0} = 0.8585 > 10^{-8}$

$V_3^{\,0} = 0.0205 > 10^{-8}$

Not converged. (Convergence failed)

Iteration I: With Y^0 and μ^0 known, Newton System is formed and solved as follows

$$\begin{pmatrix} 0.0444, & 0 & 2.2500 & 0 & 0 & 0 & 0 \\ 0 & 0.0889 & 0.7500 & 0.7500 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 4 & 6 & 0 \\ 0 & 0 & 0 & 4 & 2.1778 & 0 & -8 \\ 0 & 0 & 0 & 6 & 0 & 2.1778 & -6 \\ 0 & 0 & 0 & 0 & -8 & -6 & 0 \end{pmatrix} \begin{pmatrix} \Delta s^o \\ \Delta z^o \\ \Delta \pi^o \\ \Delta v^o \\ \Delta x_1^{\,o} \\ \Delta x_2^{\,o} \\ \Delta \lambda^o \end{pmatrix} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0001 \\ -9.7500 \\ -6.3550 \\ 0.5334 \\ 21 \end{pmatrix}$$

From (row 3) $\Delta s^O = -\Delta z^O$

From (row 1) $-0.0444\Delta z^{\,O} + 2.25\Delta\pi^O = 0.0001$

$$\Delta\pi^0 = \frac{0.0001 + 0.0444\Delta z^0}{2.25}$$

$\Delta\pi^O = 0.01973\Delta z^O + 0.00004$

From (row 2) $0.1037\ \Delta z + 0.75\Delta v^o = -0.00003$

$$\Delta z^0 = \frac{-0.00003 - 0.75\Delta v^0}{0.1037}$$

$\Delta z^O = -7.2324\Delta v^o - 0.00029$

in row (4) $\Delta v^o = \dfrac{9.7497 + 4\Delta x_1^{\,o} + 6\Delta x_2^{\,o}}{7.2324}$

$\Delta v^o = 1.3484 + 0.5532\Delta x_1^{\,o} + 0.8298\Delta x_2^{\,o}$

in row (5) $4.3901\Delta x_1^{\,o} + 3.3184\Delta x_2 - 8\Delta\lambda^o = -11.7418$

in row (6) $3.3184\Delta x_1^{\,o} + 7.1554\Delta x_2^{\,o} - 6\Delta\lambda^{\,o} = -8.6218$

from row (5) $\Delta\lambda^o = \dfrac{11.7478 + 4.3901\Delta x_1^{\,o} + 3.3184\Delta x_2^{\,o}}{8}$

Substituting $\Delta\lambda^o$ into row (6) gives $0.0262\Delta x_1^o + 4.6672\Delta x_2^o = 0.1881$ row (7) is $8\Delta x_1^o - 6\Delta x_2^o = 21$

Then $\Delta x_1^o = -2.6665$, $\Delta x_2^o = 0.0553$

$\Delta\lambda^o$ from, row (5) = 0.02827

$\Delta\lambda^o$ from, row (6) = 0.02815

$\Delta\lambda^o$ mean is 0.0282

$\Delta v^o = -0.0808$

$\Delta z^o = 0.5841$

$\Delta s^o = -0.5841$

$\Delta\pi^o = 0.0115$

$\alpha_p^1 = \alpha_D^1 = 1$

$$\Delta Y^0 = \begin{pmatrix} \Delta s^0 \\ \Delta z^0 \\ \Delta\pi^0 \\ \Delta v^0 \\ \Delta x_1^0 \\ \Delta x_2^0 \\ \Delta\lambda^0 \end{pmatrix} = \begin{pmatrix} -0.5841 \\ 0.5841 \\ 0.0115 \\ -0.0808 \\ -2.6665 \\ 0.0553 \\ 0.0282 \end{pmatrix}$$

$$Y^1 \quad \begin{pmatrix} 1.6659 \\ 1.3341 \\ 0.0560 \\ 0.0081 \\ 2.3335 \\ 4.0553 \\ 0.0282 \end{pmatrix}$$

$f(x^1) = 0.1143$
$g(x^1) = 7.1131$
$h(x^1) = 9.7290$

Testing for convergence:

$V_1^1 = 7.1131 > 10^{-8}$

$V_2^1 = 0.1023 > 10^{-8}$

$V_3^1 = 0.0183 > 10^{-8}$

$V_4^1 = 7.9742 > 10^{-8}$

Convergence fail

$\mu^1 = \Omega^1\rho^1$ from (3.10), while ρ^1 is from (3.11),

$= 0.1 \times 0.1041 = 0.0104$

Iteration 2

With Y^1 and μ^1 the next Newton System is formed, from where Newton directions are computed, and variables updated with convergence tested as

$V_1^2 = 3.1720 > 10^{-8}$

$V_2^2 = 0.0900 > 10^{-8}$

$V_3^2 = 2.2640 \text{ x } 10^{-6} > 10^{-8}$

$V_4^2 = 0.1198 > 10^{-8}$,

Resulting in failure of the system to converge after iteration 2. Although this process continues until at 8 IP iteration, the system converges with $x_1 = 2.0000$ and $x_2 = 2.7721$

Where $V_1^8 = 2.8499 \text{ x } 10^{-10} < 10^{-8}$

5.1 DISCUSSION OF RESULTS

Generally, the work reveals that Primal-Dual IP load flow technique optimisation excels others as it solves two variables with quadratic constraints function of equality and inequality and obtains solutions at a very fast rate as it converges in few number of iteration..

5.2 Summary of Findings

1. Other techniques solve load flow separately on Equality constraint and Inequality constraint, while the PD-IP technique solves load flow problems containing both constraints at the same time.
2. Number of Iterations to convergence (solution) to load flow problems of two variables are always so many for the other techniques while PD-IP technique often converges to solution wthin eight iterations
3. PD-IP technique convergence save time and so better optimisation technique.

5.3 Contributions to Knowledge

PD-IP technique has many advantages compared to the others, as it solves load flow problems containing both equality and inequality constraints simultaneously, it has few iterations, it obtains solutions at a short period. It is imperative that students and Electric Utility Industries have to study and operate with it.

5.4 Recommendations

For the technique's merits over others, intense efforts is needed to study deeper into the technique by making it accessible to various Institutions of learning and to Electricity Industries.

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