wjert, 2020, Vol. 6, Issue 2, 452-471. *Original Article* ISSN 2454-695X

World Journal of Engineering Research and Technology

WJERT

www.wjert.org SJIF Impact Factor: 5.924

15OPTIMISATION OF 330KV NIGERIA ELECTRIC POWER SYSTEM FOR IMPROVED PERFORMANCE BY PRIMAL-DUAL INTERIOR-POINT TECHNIQUE

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Article Received on 11/02/2020 **Article Revised on 01/03/2020** **Article Accepted on 22/03/2020**

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ABSTRACT

The paper developed a technique that made much impact recently in optimization of 330 kV and other Extra High Voltage Networks as it solves load flows which are non-linear with both equality and inequality constraints at the same time thereby saving time and also the system from encountering problems due to delays in fault clearings. The existing solves one constraint after the other and has more than 6 iterations before converging, while the developed method has few iterations and often converge after first iteration. The developed technique guarantees higher system power generation and consequently, larger loading with high system stability. With these advantages over the other methods the technique stands the best for optimisation. This technique is realised by applying the non-negative Primal Variables ,"S" and "z" into the problem formulation to transform the Inequality constraint part to Equality constraints and subsequently apply another non-negative Dual Variables, "π" and "v" together with Lagrange multiplier "λ" to solve optimisation. Optimisation is solved by incorporating, Barrier Parameter "μ" which ensures feasible point(s) exist(s) within the feasible region (INTERIOR POINT), Damping Factor or Step length parameter "α", Step Size ΔY, in conjunction with Safety Factor "γ" (which improves convergence and keeps the non-negative variables strictly positive) are used for updating variables ($Y^1=Y^0+\alpha\Delta Y^0$). If initialised variables fail convergence test, iteration starts with the updated variables. The problem formulation is done economically through minimisation of cost of power generation; min $C(PG)= \alpha+\beta PG+\gamma PG^2$, $g(x)=0$, stands for conventional power flow equation and other equality constraints, which is represented as; PG−PD−loss=0 and $\underline{h}$ £ h(x) £ $\hat{h}$, stands for operating limits on the system, which is represented as PGmin £ PG) £PGmax. The numerical algorithms of the method runs; Step Zero (Initialisation), Step One (Compute Newton Direction ΔY), Step Two (Update Variables), Step Three (Test for Convergence). Studies with results and analysis of improved perforformance by using PD-IP technique on the 330KV Bus Power Stations using Shiroro HydroPower Station of Nigeria of Bus number 1 as example and from table shows that percentage improvement to the existing methods is 22% on power generation, 15% on powere demand and 64% on power loss. Therefore, this method ensures and guarantees1 high system stability. Finally PD-IP technique proved to stands most desired and so should be introduced to Institutions and Utility Companies.

INTRODUCTION

Though Fast Decoupled Load Flow (FDLF) (Vincovic and Mihalic, 2008) method was widely accepted by the industry because of its fast, simple to implement and with reduced computer storage requirements, several refinements were later made such as the Carpentiers Implicit Coupling (CrIC) modification, Carpenter J.L Active reactive decoupling for improved convergence characteristics of the reactive model (Zhang and Tolbert, 2007) and hybrid model (Gomez-Exposito et al, 2015).

The first known Interior-Point (I.P) method is usually attributed to Frisch, which is a logarithmic barrier method that was later in 1960s extensively studied by Fiacco and Mc Cormick to solve non-linear inequality constrained problems (Torren and Quintina, 2001, Granville, 2007). The greatest break-through in IP research took place in 1984, when Karmarka came up with a new IP method for Linear Programming LP reporting solution times of up to 50 times faster than the simplex method. Then Karmarka's algorithm is based on non-linear projective transformations. Later, several variants of Karmarka's IP method have been proposed and implemented.Finally, the Primal-Dual methods show that its algorithm (Shyamasundar, 2010) proved to perform better than earlier IP algorithms.

One of the drawbacks of IP methods is their difficulty in detecting infeasibility. The computational efforts of each iteration of an IP algorithm is dominated by the solution of large, sparce linear systems (Geletu et al, 2011). Therefore the performance of any IP code is highly dependent on the linear algebra kernel (Alamaniotis et al, 2012). Although in the last decade IP methods have achieved significant development, there are still many open questions that need more research to further improve their performance. This work addresses some of these issues (Qui and Deconinck, 2009).

Optimal load flow methods are essentially static optimisation procedures in which the optimal generation schedule that satisfies the load flow equations and minimises production cost C(X, U) is sought. The problem for a system of A interconnected areas may be stated as follows:

$$\text{Min } C(X, U) = \sum_{k=1}^{A} C_k (X_k, U_k)$$

Subject to the constraints that:

$$F (X, U, D^0) = 0$$

$$\underline{X} \leq X \leq \bar{X}$$

$$\underline{U} \leq U \leq \bar{U} \qquad (1.1)$$

Where X and U are vectors of control variables,

D^0 is constant introduced to facilitate solutions to the problems.

This formulation is appropriate for areas operating in a pool arrangement (Jabr et al, 2016) whereby generation schedules are determined to minimise the production cost for the entire system. The optimum generation schedule for a separate single system is determined by minimising C_k (X_k, U_k) subject to the constraints that F_k (X_k, U_k, D_k^0) = 0 etc. Since the problems are mathematically identical, it is valid restricting the work to a single area, for a system of A interconnected areas.

From the above equation, it is noted that basically three (3) constraints must be satisfied by X and U. The first is equality constraint that disallows any value of X and U that does not satisfy the load flow equations. The other two constraints are inequality constraints on X and U within the defined ranges.

A constrained minimisation problem like the above is solved by transforming it into an unconstrained minimisation problem.

The merit of this new technique is; solving the above Load Flow problems faster, that is having faster algorithm (Xin-She and Xingshi, 2013) and in effect protect the 33OkV System from incessant blackouts(Capitanescu et al, 2012). As one of the shortcomings of the existing methods of load flow analysis (Momo et al, 2017) on 330kV power system is, time consumption as it has more than six iterations for one variable and this gives room for unwarranted blackouts and islanding conditions (Wu et al, 2010), as the existing methods solve equality and inequality constraints in Non-linear load flow problems separately, contributing to time consumption (Lui and Wu, 2017 and Yuan et al, 2016). Data collected from TCN, GenCos, DisCos (Odiah, 2011, Ebewele, 2014 and Awosope, 2003), show that the existing methods have poor generation-assignment resulting in poor power generation and system stability.

Overview Of The **Works Cited**

There are no sources in the current document.

2.1 Optimisation Based On Economic Operation of Power System

Consideration is made so that power system is operated as to supply all the (complex) loads (Moradi, et al 2011) at minimum cost (Wang and Murillo, 2007). Often total load is less than the available generation capacity (Fliscounakis et al, 2013 in developed world but not always so in Nigeria (Alawode and Jubril, 2010). Where the total load is less than, there are many possible generation assignment (Bakare et al, 2005 and Orike and Corne, 2013), but when there is peak load/demand for power, it means, all the available generation capacity is used resulting in no option. During options, (Capitanescu et al, 2011) power generation in system (PGi) is picked to minimise cost of production while satisfying load and the losses in the transmission system (Kimbark, 1966 and Arya, 1990), min $C(PG) = \propto + \beta\ PGi + \gamma(PGi^2)$.

Optimal economic dispatch (Yuan and Hesamzadel, 2017 and Xia and Elaiw, 2010) may require that all the power be imported from neighboring utility through a single transmission system (Street et al, 2014). Also, it is noted that, small variations in demand are taken care of by adjusting the generations already on line, while large variations_are accommodated basically by starting up generator units when the loads are on the upswing and shutting down when the loads decrease (Mao and Iravani, 2014). Although the problem is complicated by considering the long lead time required (6-8 hours) for preparing a “cold thermal unit for service”, (Colombo and Grothey, 2013). To avoid the cost of start-up or shut-down, there is a requirement that enough spare generation capacity (spinning reserve) (Arul et al, 2013) be available on-line in the event of a random generator failure (Awosope, 2003).

Mathematically, Economic Operation of Power System run thus,:-

Fi (PGi) = PGi . Hi (PGi) also, Fi (PGi) = $a^i + b^i$ PGi + y^i PGi^2

(2.1)

Cost, Ci (PGi) = K. (PGi Hi (PGi) in (₦/hr), ie K. (Fi (PGi) in (₦/hr)

(2.2)

6.1.4 Energy Balance of gas fired Power Plants

A further indication of the given situation in the Nigerian power business is shown in **Table 6-3** by the energy balance for gas fired power plants in the year 2015 as example.

Table 6-3 Energy Balance of Gas fired Power Plants in 2015

	Quantity of Gas Consumed		Energy Sent Out	Net Energy Efficiency	Net Heat Rate	Average operation
	[SCF]	[GJ]	[MWhe]	[kWhe/kWhth]	[MJ/kWhe]	[h/year]
EGBIN	53,965,156,491	59,612,264	5,192,951	0.31	11.479	3934
DELTA	35,372,927,009	39,074,477	2,761,016	0.25	14.152	3140
SAPELE	7,632,762,897	8,431,483	550,937	0.24	15.304	1043
SAPELE NIPP	8,448,997,178	9,333,131	919,606	0.35	10.149	2026
AFAM VI (GTCC)	23,935,446,666	26,440,138	2,991,284	0.41	8.839	4109
OMOTOSHO GAS	16,588,767,738	18,324,676	1,448,663	0.28	12.649	4325
OMOTOSHO NIPP	15,160,500,000	16,746,949	1,089,840	0.23	15.366	2156
GEREGU GAS	11,748,628,000	12,978,047	1,165,646	0.32	11.134	2816
GEREGU NIPP	1,246,166,000	13,546,557	1,165,646	0.31	11.622	2625
IBOM	6,223,319,000	6,874,549	510,565	0.27	13.465	2606
OKPAI (GTCC)	19,868,522,319	21,947,636	2,604,661	0.43	8.426	5788
IHOVBOR NIPP	12,461,912,480	13,765,972	1,106,267	0.29	12.444	2447
OLORUNSOGO GAS	31,984,785,036	35,331,788	1,522,245	0.16	23.210	4544
OLORUNSOGO NIPP (GTCC)	13,414,042,652	14,817,736	1,113,488	0.27	13.307	1469

The above table shows that none of the plants is operating in continuous base load over the year. The average utilization factor is about 0.35.

Under ideal operating conditions - without overhaul or other planned / unplanned outages - a utilization factor (load factor) of up to 0.95 is possible for modern open / combined cycle gas turbine plants.

An annual load factor of about 0.85 is assumed to be achievable as average over the whole lifetime under consideration of frequent and regular maintenance stops according to manufacturers' recommendations and under consideration of unrestricted fuel availability (no vandalism or other reason for shortages of gas supply).

Under ideal operating conditions (ISO), modern Open Cycle Gas Turbine Plants have an electrical efficiency of up to 35 - 39%. As this is highly dependent on the local ambient air temperature, about 33 - 36% should be considered for given climate conditions in Nigeria.

Modern Combined Cycle Gas Turbine Plants can achieve energy efficiencies of up to 60% (ISO conditions), which result in efficiencies of 50 - 55% under climate conditions in Nigeria.

The Sapele NIPP plant (GTOC) has high energy efficiency although operating only 2000 hours in 2015 only. It can be seen that it was mainly used in base load during this period.

The low energy efficiency of the Olorunsogo NIPP plant (GTCC) is resulting from short operating periods at part load only.

Economically, the problem is formulated by having the objective of ith generator power generation PGi and voltage magnitude $|Vi|$ from; the ith generator of m generators as to minimise the total cost (CT). min CT $\triangleq$ ∑Ci (PGi) (m generators are committed) (Oluseyi, 2010) such that ∑PG $\triangleq$ ∑PDi + $\sum P_{los}$ (Sambo, 2011), (2.3)

Where, PDi is power demand from ith generator. Subject to the satisfaction of power flow equation above and inequality constraints on generator power, line flow and voltage-magnitude respectively below (Bai and Wei, 2009)

1. $PGi_{min} \leq PGi \leq PGi_{max}$ i = (1, 2,…m)
2. $|Pij| \leq Pij_{max}$ all linesp
3. $|Vi|_{min} \leq |Vi| \leq |Vi|_{max}$ i = (1, 2, …m…)

Where, Pij is line flow between ith generator and jth generator.

The above inequalities show that:-

1. Upper limits on PGi, is set by thermal limits (Nlu and Wei, 2013) on the generator unit, while the lower limits is set by other thermodynamics.
2. Constraints on the transmission-line power, relate to thermal and stability limits. Constraints on $|V|$ (Lesientre et al, 2011) keep the system voltage from varying too far from their rated or nominal values. The objective is to help maintain consumers' voltage which should neither be too high nor too low.
3. Formulation of the problem is consistent with the availability of injected active power and the bus-voltage magnitude, as control variables can be extended to such, as phase angle across phase-shifting transformer, the turn-ratios of tap-changing transformers and the admittance of variable (controllable) shunts and series inductors and capacitors (Adebayo et al, 2012 and Jabr et al, 2016).
4. The minimisation of cost function subject to equality and inequality constraints is a problem of optimization.

By power flow analysis, if PGi and transmission-line power angles obtained satisfy the inequality constraint, the choice of set of $|Vi|$ and the power generated in ith system reduced by 1 (PGi -1) within the constraint set is feasible and so the total cost CT is calculated.

Under normal operating condition, there is relatively time-coupling between active power flows and the power angles on the one hand, and reactive power flows and the voltage-magnitudes on the other hand. The results is formulated in terms of active power flow, setting the $|Vi|$ at their nominal values. While performing calculations, some approximations made are;-

1. Neglecting line-power flow and line losses.

2. No generator limits and no line losses.
3. Line losses considered

For approximation (1), there is no power flow equation.

For approximation (2), there is no inequality constraint and so it is optimal to operate every generator at equal incremental costs IC.

IC $=\frac{dCi\,(PGi)}{dPGi}$ (which is slope of fuel cost curve), Ci is cost in ith generator (2.4)

IC unit is ₦/MWH. IC is the increase in cost-rate per unit increase in MW Output power; or in crease in cost per unit increase in MWH (Afi, 2012).

2.2 Optimisation Based On Minimum Mismatch Method

Generally, load flow equation of an N-bus network can be expressed as:

$S = P + jQ = V^T I^* = V^T (YV)^*$ (Kamel, et al 2013) (2.5)

All the above quantities are complex, except P and Q which are real and imaginary parts of S. Because of non-linearity of load flow equations, several mathematical solutions may exist and this gives rise to non-uniqueness in the load flow calculations (Wu, et al 2010), hence, for practical purpose, only one of the solutions is acceptable and this is the solution with the minimum system losses and acceptable high voltages.

2.3 Optimisation Based On Fast Decoupled Load Flow Method

This is a modification of the Newton-Raphson (NR) technique which takes advantage of the weak coupling between the real and reactive power (Bhowmick, et al 2008). Two constant matrices are used to approximate and decouple the Jacobian Matrix under the following assumptions,

i. $\cos \theta_{kj} \cong 1$

ii. $G_{kj} \sin \theta_{kj} << B_{kj}$

iii. $Q_k << B_{kk} V_k^2$ (because for most transmission lines X/R << 1) (Lin and Teng 2000). Where X and R are shunt reactance and series resistance respectively, with shunt reactance being very small compared to series resistance of most transmission lines.

2.4 Optimisation Based On Second Order Load Flow (Solf) Method

Load flow equation, with variables defined in rectangular form for nodal real and reactive power mismatches.

N

$P_i = \sum$ (ei ej Gij – ei ef Bij +fifj Gij+ fi ej Bij),

i= j

N

$Q_i = \sum$ (fi fj Gij – fi fj Bij - eifj Gij- ei ej Bij),

i= j

$Ei^2 = ei^2 + fi^2$ Where Ei is modulus of ith bus Voltage. (2.6)

Where Vi = ei +jfi (nodal or bus voltage),

2.6. Optimisation Based On Mathematical Model Of Primal-Dual Interior-Point Technique

Min f(x)

Such that g(x) = 0 (2.7)

$\underline{h}$ £ h(x) £ $\hat{h}$

$x \in IR^n$ is a vector of decision variable with control/ non-functional dependent variable.

f: IR^n –IR is a scalar function standing for power system operation optimisation goal.

g: IR^n –IR^m is a vector function representing the conventional power flow equation and other equality constraints(Chiang and Grothey, 2014 and Chiang,2013).

h: IR^n –IR^p is a vector of functional variables with lower bound $\underline{h}$ and upper bound $\hat{h}$ representing the operating limits on the system.

It is assumed that f(x), g(x) and h(x) are twice continuously differentiable. Since the above problem minimises f(x) subject to h(x) $\geq$ 0. The objective is obtaining feasible point X. that attains the desired minimum

III Methodology

3.1 Transforming Inequality Constraint To Equality Transformation of (2.7) is done (Yang, et al 2016) by incorporating non- negative slack vectors 's' and 'z' into the inequality constraint $\underline{h} \leq h(x) \leq \hat{h}$, imposing strict positivity conditions on those slacks (Yang, et al 2016) by incorporating them into logarithmic barrier terms (Babu and Harini, 2016):-

Min f(x)

Subject to g(x) = 0

$-s - z + \hat{h} - h = 0$

$-h(x) - z + \hat{h} = 0$

Into logarithmic barrier term as

(3.1)

Min f(x)- $\mu^k \sum_{i=0}^{p}(insi + inzi)$

Subject to g(x) = 0

-s – z + ĥ – h = 0

-h(x) – z + ĥ = 0

"s" ≥ 0; "z" ≥ 0

Where, k is the iteration count or number and p the number of interconnected systems. Solving these equality constraints, ^we apply vectors of lagrangian multipliers called Dual-Variables "λ" "π" and "v" together with the Newton method. Where $\lambda \in IR^m$, $\pi \in IR^p$ and $v \in IR^{p.}$ The point "Y" becomes (s, z, π, v, x, λ) [Wu, et al 2012 and Ling, 2007) lagrangian function Lμ(y),

$L\mu(y) = f(x) - \mu^k \Sigma(\text{in } si + \text{in } zi) - \lambda^T gx - \pi^T(s - z + h - \underline{h}) - V^T(-h(x) - z + h).$ **(3.2)**

3.3 Optimality Conditions

A local minimiser of (3.1) is expressed in terms of stationary point of Lx(y) satisfying the Karush-Kuhn Tucker (KKT) optimality conditions for the NLP (2.7)(Torren and Quintana, 2001 and Wu, et al 2012) as

$$\nabla y1(y) = \begin{Bmatrix} s\pi \\ zv \\ s + z - \hat{h} + \underline{h} \\ h(x) + z - \hat{h} \\ \nabla xf(x) - Jg(x)^T \lambda + Jh(x)^T v \\ -g(x) \end{Bmatrix} = 0 \quad \textbf{(3.3)}$$

^

V= v + π for simplification

$$\nabla ylm(y) = \begin{Bmatrix} s\pi - m^k e \\ zv - m^k e \\ s + z - \hat{h} + \underline{h} \\ h(x) + z - \hat{h} \\ \nabla xf(x) - Jg(x)^T \lambda + Jh(x)^T v \\ -g(x) \end{Bmatrix} = 0 \quad \textbf{(3.4)}$$

Where l or L is local minimiseStrict feasibility starting point is not mandatory for Primal Dual Interior Point technique but the condition (s, z)>0 and (p, v) >0 must be satisfied at every point in order to define the barrier term (Sivasubramani and Swarup, 2011 and Lage et al 2009). So, IP starts from a point y^0 that satisfies $(s^0, z^0)>0$ and $(p^o v^o) > 0$. Primal Dual (IP) iterates (Capitanescu and Wehenkel, 2008) by one step of Newton method for NL equation to solve the KKT system (3.4). A step size is computed in Newton Direction and variables updated and m^k values reduced. The algorithm terminates when the Primal and Dual infeasibilities and the complementary gap fall below pre-determined tolerance otherwise, with (s, z)>0 and (p, v) > 0 a new estimate y^k is computed using one step of Newton method to find zeroes (the roots of the NL functions applied to (3.3).

3.4 Estimating New Point (Y^K)

3.4.1 Computing Newton Direction or Step Size ΔY

The Newton direction is obtained by solving. Newton method (Tinney and Hart, 2007) with large sparse coefficient matrix (Geletu, et al 2011), with step size column matrix as shown below (Flicousnakis, et al 2013 and Molzahn, et al 2013):-

$$\begin{Bmatrix} p & 0 & s & 0 & 0 & 0 \\ 0 & v & z & z & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & Jh & 0 \\ 0 & 0 & 0 & Jh^T & \nabla^2_x lm & -Jg^T \\ 0 & 0 & 0 & 0 & -Jg & 0 \end{Bmatrix} \begin{Bmatrix} Ds \\ Dz \\ Dp \\ Dv \\ Dx \\ Dl \end{Bmatrix} = \begin{Bmatrix} rs \\ rz \\ rp \\ rv \\ rx \\ rl \end{Bmatrix} \qquad \textbf{(3.5)}$$

(Minot and Li, 2015 andMolzahn, et al 2013).

Where:

$$\left.\begin{aligned} rs &= -sp + m^k e \\ rz &= -zv + m^k e \\ rp &= -s - z + \hat{h} - \underline{h} \\ rv &= -h(x) - z + \hat{h} \\ rx &= -\nabla x\, f(x) + Jg(x)^T l - Jh(x)^T v \\ rl &= g(x) \end{aligned}\right\} \qquad \textbf{(3.6)}$$

Where, $\nabla^2_x lm$ is the combination of Hessians of objective and constraints functions.

$$\nabla^2_x lm(y) = \nabla^2_x f(x) - \nabla^2_x gj(x)lj + \nabla^2_x hj(x)\, vj \qquad (3.7)$$

Where "l" is local minimiser a function of differentiation, $\nabla^2_x f(x)$ is the Hessian or 2nd differentiation of objective function w.r.t.x, $\nabla^2_x g(x)$ is the Hessian or 2nd differentiation of equality constraint function w.r.t.x, $\nabla^2_x h(x)$ is the Hessian or 2nd differentiation of inequality constraint function w.r.t. x, $\nabla_x f(x)$ is the 1st differentiation of objective function w.r.t.x, Jg(x) is the 1st differentiation or Jacobian value of equality constraint w.r.t.x. Jh(x) is the 1st differentiation or Jacobian value of inequality constraint w.r.t.x.

Evaluation of the Newton directions is usually the computationally most expensive task in single iteration of PD-IP algorithm. In the computation of DY, factorisation of the coefficient matrix (3.5) is much more expensive than the forward and backward solutions that follow factorisation.

$$\Delta \mathbf{Y} = \begin{Bmatrix} Ds \\ Dz \\ Dp \\ Dv \\ Dx \\ Dl \end{Bmatrix}$$

Where, **the scalars α^k_P€ (0,1) and α^k_D€ (0,1) are step length parameters** called **damping factors** which improve convergence and keep non-negative variables strictly positive, k is the iteration counts.

$$a^k_P = \min [1, \gamma \min\{-si^k/Dsi/Dsi<0, -zi^k/Dzi/Dzi<0\}]$$
$$a^k_D = \min[1, \gamma \min \{-pi^k/Dpi/Dpi<0, -vi^k/Dvi/DVi<0\}] \quad \textbf{(3.8)}$$

The **scalar γ (0,1) is a safety factor** which ensures that the next point will satisfy the strict positivity conditions; typical constant values, $\gamma^O = 0.25$. $\gamma^k = 0.99995$.

3.4.3 Updating Variables

3.4.3.1Updating control variable(s) and primal variables

$X_1^k = X_1^{k-1} + a_p^k DX_1^{k-1}$ 2 control

$X_2^k = X_2^{k-1} + a_p^k DX_2^{k-1}$ variables

$S^k = S^{k-1} + a_p^k DS^{k-1}$

$Z^k = Z^{k-1} + a^k_p DZ^{k-1}$

(3.9)

3.4.3.2 Updating dual variables and lagrange multiplier

$p^k = p^{k-1} + a^k_D Dp^{k-1}$

$V^k = V^{k-1} + a^k_D V^{k-1}$

$l^k = l^{k-1} + a^k_D Dl^{k-1}$

3.5 Reducing The Barrier Parameter (μ^k)

The **scalar μ^k** is the **barrier parameter** or **complementary gap** (Lage, et al 2009) which ensures the feasible point X exist within the feasible region and it is obtained by

$\mu^{k+1} = W^k r^k$ **(3.10)**

Where Ω^k is chosen = max0.99W^{k-1}/2; 0.1) and it is called the **Centering Parameters**

With W^0 = (0.2 fixed) and μ^O = (0.1 fixed)

$r^k = (S^k)^T p^k + (Z^k)^T V^k$ **(3.11)**

μ^k is computed first, only if iteration (1) fails, then μ^1 and Y^1 is used to form iteration (2) as Y^0 and μ^0 (given) are used to form iteration (1)

3.6 Testing For Convergence

Interior-Point (IP) Iterations Are Considered Terminated Whenever

$V_1^k \leq \xi_1$, $m^k \leq \xi m$,

$V_2^k \leq \xi_1$, $\|\Delta X\|$:< ξ_2,

$V_3^k \leq \xi_2$, $\|g(X^k)\|$:< ξ_1,

$V_4^k \leq \xi_2$, $V_4^k \leq \xi_2$

is satisfied, where

V_1^k = max [max{h-h(x); h(x) – $\hat{h}$ },$\|g(x)\|$¥],

$V_2^k = \frac{\|\nabla x f(x) - Jg(x)^T \lambda + Jh(x)^T V\|_\infty}{1+\|x\|2+\|\lambda\|2+\|V\|2}$

Since$\|\lambda\|_2$ & $\|V\|_2$ are vectors of lagrangian multipliers,

they have no vector addition and so denominator reduces to 1 + $\|x\|_2$ **(3.12)**

$V_3^k = \frac{\rho^k}{1+\|x\|2}$

$V_4^k = \frac{|f(x^k) - f(x^{k-1})|}{1-|f(x^k)|}$

Typically, $\xi_1 = 10^{-4}$,

$\xi_2 = 10^{-2} E_1$ (i.e. 10^{-6}) ,

$\xi_x = 10^{-12}$.

Generally, $\xi_1 = 10^{-8}$ is chosen for quadratic functions with 2 variables.

If $V^k{}_1$, $V^k{}_2$ and $V^k{}_3$ are satisfied, then primal feasibility, scaled dual feasibility and complementary condition are satisfied which means that iterate K is a Karush Khun Turker (KKT) point of accuracy.

When numerical problems prevent verifying this condition, the algorithm stops as soon as feasibility of the equality constraint is achieved along with a very small fractional change in the objective value and negligible changes in the variables. The typical tolerances are $\xi_1 = 10^{-4}$, $\xi_2 = 10^{-2}\,\xi_1$ **and** $\xi\mu = 10$ (Lavaei and Low, 2012)

3.7 Primal-Dual Interior-Point Technique Numerical Algorithms

Step 0: (Initialisation)

Set K = 0, define μ^0 and choose a starting point Y^0 that satisfies the strict positivity conditions.

Step 1: (Compute Newton Direction)

Form the Newton System at the current point and solve for the Newton Direction.

Step 2: (Update Variables)

Compute the step lengths in the Newton direction and update the primal and dual variables.

Step 3: (Test for Convergence)

If the new point satisfies the convergence criteria, stop. Otherwise, set K = K + 1, update the barrier parameter μ^k and return to step 1.

3.7.1 Representation of the Algorithms in Flow Chart (Xin-She andXingShi-He, 2013)

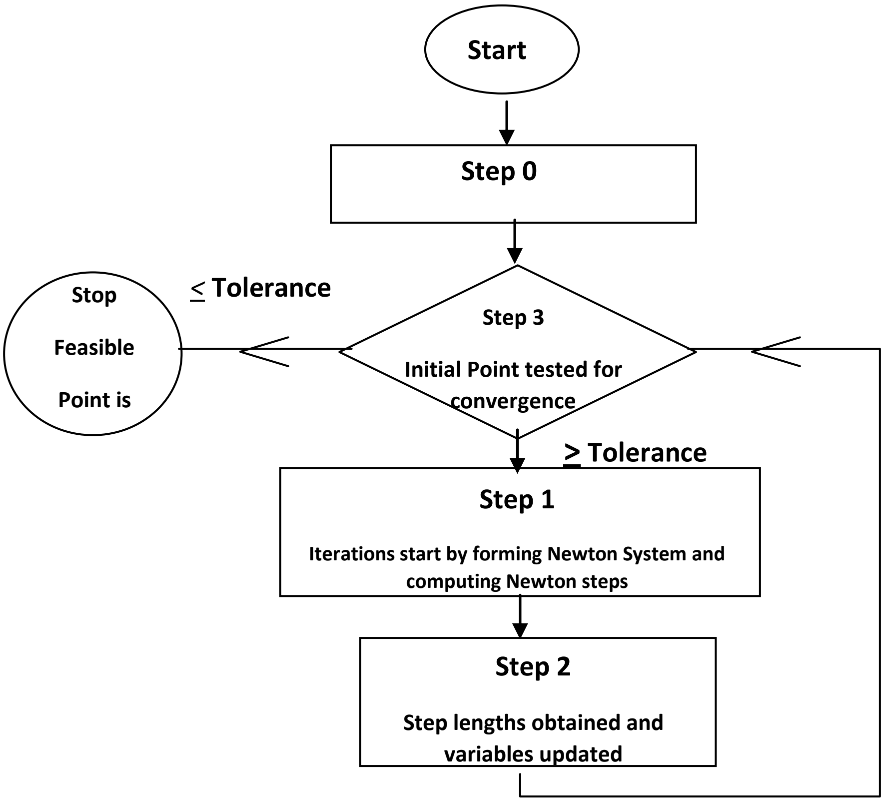

Fig. 3.1: PD-IP Technique's Flow Chart of Optimal Load Flow.

3.7.2 Implementation of the Algorithms of Primal-Dual Interior–Point Technique

3.7.2.1 Step Zero (0), Choosing an initial point

Although the starting point needs only to meet the strict positivity conditions (Cao, et al 2016), IP method performs better if some initial heuristics (Niu, et al 2014) are used, for instance, X^0 is middle point between the upper and the lower limits of the bounded variables.

3.7.2.1.1Initial point for one variable with linear inequality constraint, pick X^o a little less than $\hat{h}$. E.g $100 \leq X \leq 300$

Pick $X^0 = 250$

3.7.2.1.2 Initialising primal slack variables (S^0 and Z^0)

$$\left.\begin{aligned} &S^0 = \min\,[\mathrm{Max}\{\gamma^o h^\Delta, h(X^0) - h\ min\}, (1-\gamma^0)\, h^\Delta] \\ &S^0 = \min\,[\max\,\{0.25h^\Delta, h(X^0) - h\ min\}; 0.75h^\Delta] \\ &\text{Where:}\ h^\Delta = h\ max - h\ min \\ &\gamma^0 = 0.25 \\ &1 - \gamma^0 = 0.75 \end{aligned}\right\} \quad (3.13)$$

$H\,(X^0)$ = values of X^0 including constant

$$Z^0 = h^\Delta - S^0$$

3.7.2.1.3 Initialising dual variables (π^0, V^0)

$$\left.\begin{aligned} p^0 &= m^0 (S^0)^{-1} e \quad \text{(e is diagonal I of matrix)} \\ p^0 &= 0.1(S^0)^{-1} \\ V^0 &= m^0 (Z^0)^{-1} e - p^0 \end{aligned}\right\} \quad (3.14)$$

l^0 = 0 (since the power balance of steady state system is passive)

s^0

z^0

$$Y^o = \begin{Bmatrix} p^0 \\ v^0 \\ x^0 \\ l^0 \end{Bmatrix}$$

Convergence of the initial point is tested and if it fails then:

3.7.2.2 Step one (1), Computing Newton direction ΔY

With m^o defined and initial point Y^o obtained; Newton method (3.5), of page 22 is formed and Newton direction computed with (3.6) and (3.7) of (3.5)

3.7.2.2.1 Newton direction for one variable with linear constraint

After iteration one, rs^o, rz^o, $rp^{0,}$ rv^o and $\nabla^2{}_x lm^0$ of (3.5) are zeros and convergence often occur.

From, row 6 of equation (3.5), where value of Dx^o is obtained. Dx value is substituted into row 4 to obtain Dz which in turn is substituted into row 3 where Ds = -Dz to obtain Ds. Ds value is substituted into row 1 to obtain Dp which in turn is substituted into row 2 to obtain Dv and finally Dv with Dx of row 6 are substituted into row 5 to obtain Dl.

3.7.2.3 Step two (2), Updating variables (Y^k) with step length parameter "α" (3.8). $Y^1 = Y^O + aDY^o$

Newton direction DY is computed from (3.5) and variables are updated from (3.9)

3.7.2.4 Step three (3), Testing for convergence If the new point satisfies the convergence criteria, stop. Otherwise, set K= K + 1, update the barrier parameter m^k and return to step 1 (Lavaei and Low, 2012).

3.8 Work Example of One Variable:

Min C (PG) = 20 + 4.1 PG + 0.0035 P^2G (Nagrath and Kothari, 2010 and Gill, et al 2014)

S. t PG – 240 – 0.02PG = 0

1. $100 \leq PG \leq 300$
2. $QG_{min} \leq QG \leq QG_{max}$
3. $|V|_{min} \leq |V| \leq |V|_{max}$

Where, C is the cost

Step 0: Initialisation

$PG^0 = 250$, $hD = PG_{max} – PG_{min} = 200$

$Jg(PG) = 0.98$, $J^2g(PG) = 0$

$Jh(PG) = 1$, $J^2h(PG) = 0$

$C^1(PG) = 4.1 + 0.007PG$, $\nabla^2C(PG) = 0.007$

$m^0 = 0.1$, $\Omega^0 = 0.2$ (fixed), $\gamma^0 = 0.25$, $\gamma^k = 0.99995$

$\xi = 10^{-4}$, Ω^k always 0.

Choosing heuristically,

1. PG^0, is picked little less than PG_{max} and here, 250
2. P_D, is picked little less than PG^o and here, 240
3. P_{loss}, is picked as hundredth of PG^o and here 0.02 PG^o

s^0, must be greater than z^0

$s^0 = 150$, $z^0 = 50$, $PG^0 = 250$ from (3.13) of page 26.

$p^0 = 0.0007$, $v^0 = 0.0013$, $l^0 = 0$, $V^{O\wedge} = \pi^o + v^0 =$ **(0.**0020) from (3.14) of page 26.

150 s^0

50 z^0

Therefore, y^0 = 0.0007 = p^0 g(PG) = 5

0.0013 v^0 h(PG) = 250 from (2.21)

250 PG^0

0 l^0

Test for Convergence: from (3.12)

$V_1^0 = 5 \geq 10^{-4}$

$V_2^0 = 0.02 \geq 10^{-4}$

$V_3^0 = 0.00067 \geq 10^{-4} \quad = \frac{150*0.0007+50*0.0013}{1+250}$

Iteration 1 With y^0 and with $m^0 = 0.1$, Newton System is formed from 3.5

$$\begin{pmatrix} 0.0007, & 0, & 150, & 0, & 0, & 0 \\ 0 & 0.002 & 50 & 50 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0.007 & -0.98 \\ 0 & 0 & 0 & 0 & -0.98 & 0 \end{pmatrix} \begin{pmatrix} Ds^0 \\ Dz^0 \\ Dp^0 \\ Dv^0 \\ DPG^0 \\ Dl^o \end{pmatrix} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -5.8513 \\ 5.0000 \end{pmatrix}$$

The right hand results are obtained from equations (3.6) of page 22 and (3.7) of page 22 by simplification and factorisation of Newton method/system to determine Newton steps as follows:

$DPG^o = -5.102$ by starting from row 6

In row 4, $Dz^0 - 5.102 = 0.0000$

Therefore, $Dz^0 = 5.102$

In, row (3) $Ds = - Dz$

$Ds^o = - 5.102$

In, row (1) $0.007Dz^o + 150Dp^o = 0$

$$Dp^o = \frac{0.007Dz^o}{150} = 2.3809x10^{-5}$$

In, row (2) $0.002Dz^o + 50\ Dp^o + 50Dv^o = 0$

$Dv^o = -2.2789x10^{-4}$

Finally, in row 5, $Dv^o + 0.007\ DPG^o - 0.98\ Dl^o = -5.8513$

$Dl^o = 5.9338$

$a^1_p = a^1_D = 1$, from (3.8) and from (3.9) of page 23, variables are updated as:-

$$y^1 = \begin{pmatrix} 144.8980 \\ 55.1020 \\ 7.2381x10^{-4} \\ 1.0711x10^{-3} \\ 244.8980 \\ 5.9341 \end{pmatrix}$$

$g(PG^1) = 0$

$h(PG^1) = 244.898$

$m^1 = 0.1(0.0997 + 0.0714) = (0.1711)$ from (3.10)

$= 0.1\ (0.1711) = 0.01711$

Test, from (3.12) page 24, $V_1^1 = \max\ [\max\ \{100-244.898,\ 244.898 - 300\}\ 0]$

$V_1^1 = \max[-144.898, -55.102, 0]$

$V_1^1 = 0 \leq 10^{-4}$

$V_2^1 = \frac{|0|}{245.898} = 0 < 10^{-4}$

Converged after first iteration.

Therefore, PG = 244.898

PD = 240 (Real Power Demand of the system)

P_{loss} = 4.898 (from 0.02 PG) or (PG - PD)

CHAPTER 4 RESULS AND ANALYSIS

Table 4.1: Summary of Optimisation of Load Flow Problems By Primal-Dual Interior-Point Technique over the Existing Methods (Values in P.U.).

Bus		Number of Iterations		Power Generation			Power Demand			Power Loss		
Name	**No.**	**PD-IP Tech**	**Existing**	**PD-IP Tech**	**Exist-Ing**	**% Impro vment**	**PD-IP Tech**	**Exist-ing**	**% Impro Vment**	**PD-IP Tech**	**Exist Ing**	**% Improvment**
Shiroro	1	1	≥6	0.490	0.402	22	0.480	0.374	15	0.010	0.028	64
Afam	2	1	≥6	0.087	0.060	28	0.085	0.004	2050	0.002	0.056	96
Geregu	12	1	≥6	0.179	0.120	48	0.175	0.024	625	0.004	0.096	96
Sapele	15	1	≥6	**0.174**	0.170	02	0.170	0.158	08	0.004	0.012	67
Delta	17	1	≥6	0.281	0.281	00	0.275	0.113	142	0.006	0.108	94
Kainji	24	1	≥6	0.276	0.259	07	0.270	0.088	213	0.006	0.171	96
Jebba	51	1	≥6	0.378	0.352	08	0.370	0.288	30	0.008	0.064	88

Note: The Seven Power Stations' buses are chosen for analysis as a special case of 52 bus system as they contributed to the bulk supply of power to the system.

5.1 DISCUSSION OF RESULTS

Generally, the work reveals that Primal-Dual IP load flow technique optimisation excels others as it solves one variable with linear constraints function of equality and inequality and obtains solutions at a very fast rate as it converges often at first iteration. It results in much improved larger power dispatch and consumption from system, thereby saving the system from unnecessary outages and blackouts.

Note: The Seven Power Stations' buses are chosen for analysis as a special case of 52 bus system as they contributed to the bulk supply of power to the system.

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